ΠΑΝΕΠΙΣΤΗΜΙΟ ΚΡΗΤΗΣ - ΤΜΗΜΑ ΕΦΑΡΜΟΣΜΕΝΩΝ ΜΑΘΗΜΑΤΙΚΩΝ
Archimedes Center for Modeling, Analysis & Computation
UNIVERSITY OF CRETE - DEPARTMENT OF APPLIED MATHEMATICS
Archimedes Center for Modeling, Analysis & Computation

Abruptly autofocusing waves

Nikolaos K. Efremidis and Demetrios N. Christodoulides and Ioannis Chremmos and Zhigang Chen

Original Citation:

Efremidis, Nikolaos K. and Christodoulides, Demetrios N. and Chremmos, Ioannis and Chen, Zhigang

(2011)

Abruptly autofocusing waves.

The Conference on Lasers and Electro-Optics (CLEO/QELS) (595153).

Abruptly autofocusing waves

Nikolaos K. Efremidis[1,*], Demetrios N. Christodoulides[2], Ioannis Chremmos[3], and Zhigang Chen[4]

[1] *Department of Applied Mathematics, University of Crete, 71409 Heraklion, Crete, Greece*
[2] *CREOL/College of Optics, University of Central Florida, Orlando, Florida 32816*
[3] *School of Electrical and Computer Engineering, National Technical University of Athens Zografos 15773, Athens, Greece*
[4] *Department of Physics and Astronomy, San Francisco State University, San Francisco, California 94132*
[*] *Email: nefrem@tem.uoc.gr*

Abstract: We introduce a new classes of waves that tend to autofocus in an abrupt fashion. These waves can be generated through the use of radially symmetric Airy waves.

OCIS codes: (050.1940) Diffraction; (260.2030) Dispersion; (350.5500) Propagation

The focusing characteristics of optical beams have always been an issue of great practical importance. In the case of a Gaussian wavefront the peak intensity follows a Lorentzian distribution around the focus. However, for many applications it is crucial that a beam abruptly focuses its energy right before a target while maintaining a low intensity profile until that very moment. Ideally this should be a linear property of the wave itself and not the outcome of any self-focusing effects. In medical laser treatments this feature may be highly desirable since the wave should only affect the intended area while leaving any preceding tissue intact [1]. Such behavior can also be useful in suddenly "igniting" a particular nonlinear process, such as multi-photon absorption, stimulated Raman, optical filaments in gases etc [2, 3].

To realize this fascinating prospect we introduce new families of radially symmetric (cylindrical and spherical) waves with this desired characteristic: their maximum intensity remains almost constant during propagation, while close to a particular focal point they suddenly autofocus and as a result their peak intensity can increase by orders of magnitude [4].

In the 1D case the paraxial equation is known to support the following accelerating Airy beam [5, 6]:

$$g(x,z) = \mathrm{Ai}\left(x - z^2/4 + i\alpha z\right) \exp[i\left(6\alpha^2 z - 6i\alpha\left(2x - z^2\right) + 6xz - z^3\right)/12]. \tag{1}$$

In Eq. (1), the decay parameter α ensures that the wave conveys finite energy (is thus realizable) and is typically small so as the behavior of this wave approximates in many respects [5, 6] that of an ideal ($\alpha = 0$) diffraction-free Airy wavepacket. Perhaps the most intriguing feature of this solution is its lateral parabolic acceleration.

Let us start by considering the diffraction of a radially symmetric beam propagating in a linear medium. In the paraxial approximation the normalized beam dynamics satisfy $u_z = (i/2)\,(u_r/r + u_{rr})$. The propagation of an arbitrary radially symmetric initial condition $u(r, z = 0) = u_0(r)$ can be computed in terms of the Hankel transform. In particular, we analyze the dynamics of radially symmetric Airy beams of the form $u_0(r) = \mathrm{Ai}(r_0 - r)\exp[\alpha(r_0 - r)]$ where r_0 is the initial radius of the main ring. In Fig. 1(a) the amplitude of this cylindrical Airy beam is depicted as a function of z. Our simulations show that the maximum amplitude that the beam reaches during propagation is almost constant up to the point where the beam approaches the center [Figs. 1(a)-(b)]. Close to the focal point the power of the first Airy ring is concentrated in a small area around $r = 0$. While the peak intensity remains around unity up

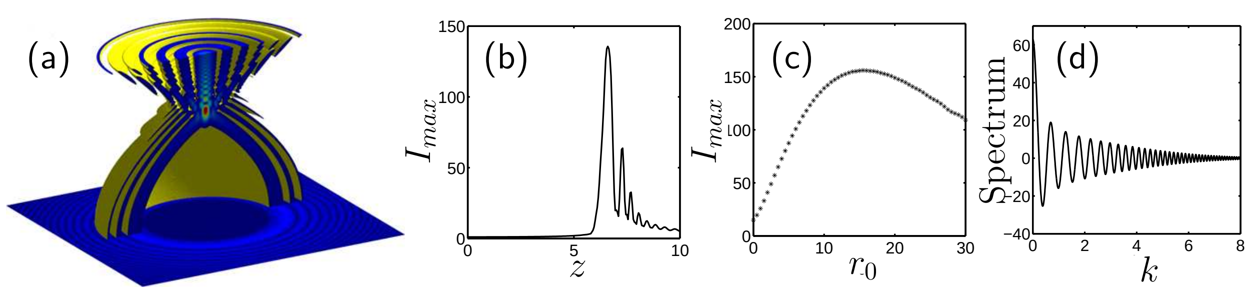

Fig. 1. Dynamics of radially symmetric Airy beams for $\alpha = 0.05$, $r_0 = 10$, and $I_{max}(z = 0) = 1$. (a) Propagation dynamics. (b) Maximum intensity as a function of z. (c) Maximum intensity that the Airy beam reaches during propagation for different values of the initial radius r_0. (d) Hankel transform of the initial condition.

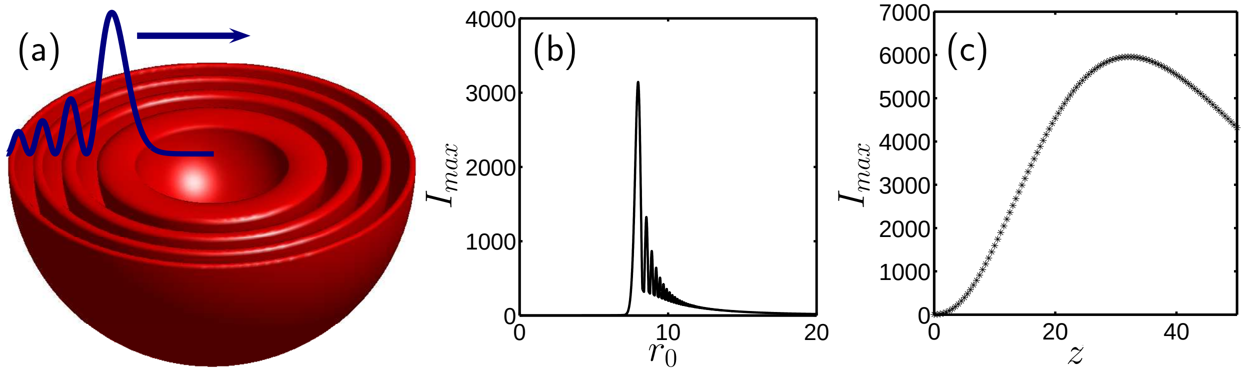

Fig. 2. (Color online) (a) Isointensity hemisphere of the Airy wave. (b), (c) Same as in Fig. 1 for the three-dimensional Airy solution given by Eq. (2) for $\alpha = 0.05$, $r_0 = 15$, and $I_{max}(z = 0) = 1$.

to $z \approx 6$ it then very rapidly increases by more than 135 times at the focal point [Fig. 1(b)]. For longer propagation distances the maximum intensity starts to decrease. In Fig. 1(c) the maximum intensity that the beam reaches during propagation is shown as a function of r_0. Note that large intensity contrasts are possible for a wide range of values of r_0. Much higher values of I_{max} are possible by further suppressing diffraction (decreasing α). Fig. 1(d) depicts the Hankel transform of the input field profile of this beam as a function of the radial spectral component k.

Particularly engineered superpositions of Airy functions can exhibit enhanced properties as compared to radially symmetric Airy beams. Here we introduce the transformations $x'(r, r_0, \theta_0) = r\cos(\theta_0) + r_0\sqrt{2}/2$, $y'(r, r_0, \theta_0) = -r\sin(\theta_0) + r_0\sqrt{2}/2$, which are used to define the following radially symmetric superposition of two-dimensional Airy beams $u(r, r_0, z) = \int_0^{2\pi} g(x'(r, r_0, \theta_0), z)g(y'(r, r_0, \theta_0), z)d\theta_0$, where $g(x, z)$ is given by Eq. (1). In comparison with Fig. 1 we notice two main advantages of this latter configuration: (a) The intensity contrasts achieved are larger. (b) The maximum intensities of the subsequent oscillations are smaller.

Three-dimensional spherically symmetric abruptly autofocusing waves are also possible in the region of anomalous dispersion provided that dispersion and diffraction effects are equalized. The corresponding normalized spatio-temporal equation reads $u_z = (i/2)(2u_r/r + u_{rr})$ where $r = \sqrt{x^2 + y^2 + t^2}$ and supports the following exact Airy-like solution

$$u(r, z) = [g(r_0 - r, z) - g(r_0 + r, z)]/r, \tag{2}$$

where $g(r, z)$ is given by Eq. (1) and close to the origin $u(r, z) \approx -2g_{r_0}(r_0, z) - g_{r_0 r_0 r_0}(r_0, z)(r^2/3)$. Since the energy is initially spread out on a 3D ring even higher intensity contrasts are attained [see Fig. 2].

Nonlinear processes and, in particular, multiphonon absorption and Kerr are the fundamental phenomena that accompany the aforementioned potential application of abrupt autofocusing. For this reason we study the autofocusing dynamics in the presence of these phenomena. The interplay between Kerr nonlinearity can lead to instabilities, depending on the strength of the nonlinearity. In addition, strong Kerr nonlinearities might lead to self-focusing that can play a synergetic role to abrupt autofocusing.

References

1. T. Juhasz, F. Loesel, R. Kurtz, C. Horvath, J. Bille, and G. Mourou, "Corneal refractive surgery with femtosecond lasers," IEEE J. Sel. Top. Quantum Electron. **5**, 902–910 (1999).
2. K. M. Davis, K. Miura, N. Sugimoto, and K. Hirao, "Writing waveguides in glass with a femtosecond laser," Opt. Lett. **21**, 1729–1731 (1996).
3. M. Mlejnek, E. M. Wright, and J. V. Moloney, "Dynamic spatial replenishment of femtosecond pulses propagating in air," Opt. Lett. **23**, 382–384 (1998).
4. N. K. Efremidis and D. N. Chirstodoulides, "Abruptly autofocusing waves," Opt. Lett. **35**, 4045–4047 (2010).
5. G. A. Siviloglou and D. N. Christodoulides, "Accelerating finite energy Airy beams," Opt. Lett. **32**, 979–981 (2007).
6. G. A. Siviloglou, J. Broky, A. Dogariu, and D. N. Christodoulides, "Observation of Accelerating Airy Beams," Phys. Rev. Lett. **99**, 213901 (2007).