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# Primary Children's Interpretation And Use Of Illustrations In School Mathematics Textbooks and NonRoutine Problems: A SchoolBased Investigation 

## Rebecca Mary Jellis

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## Submitted for the degree of

## Doctor of Education

13 NOV 2008

## University of Durham <br> Department of Education <br> 2008 <br> 

## For Chris

## Thank you,

## Without you there would be nothing

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## Introduction

We live in a visual world. Television, movies, photographs and pictures are allpervasive, surrounding us every day with images clamouring for our attention. In order to make sense of our world, we assign a level of importance to each image we see depending on how relevant it is to our current situation and the image is then either taken into account, stored in our memories or ignored completely. As we grow older, the ability to gain meaning from these images develops, so that when, as children, we begin to read, the ability to read images becomes part of our repertoire of cues used in deciphering the patterns and meaning of words. In primary schools, children have to deal with a whole new range of images, many of which are found in the textbooks they encounter. In mathematics textbooks these images can be extensive, involving graphs and tables, representations of numbers and calculations but also in recent times illustrations have become more extensive in their role as motivators and decorators of the page. There appears to be a popular belief that when teaching mathematics, illustrations make the task easier and/or more attractive and that for children who find mathematics difficult, illustrations take on an even greater role in eliciting meaning for the pupil (Santos-Bernard, 1997). However, there is very little research investigating the pedagogical role of illustrations in school mathematics: neither the psychology of cognition and learning nor mathematics education research has paid much attention to this topic.

My interest in this topic arose not from reading through literature but rather through observing the children in my classroom when it became clear than children do not

always interpret text in the way we would expect. In this example from a child in my class 'show your working' had no mathematical meaning but rather she drew pictures of people working (a nurse and a fireman, a later picture showed an ambulance).

In 2000, schools had been sent copies of Mathematical challenges for able pupils in Key Stages 1 and 2 (2000), a book that contains puzzles and problems for children from Year 1 to Year 6. In my class of Year 3 and 4 children I had taken one of the puzzles from the Year 1 and 2 section as an introduction to this type of problem solving exercise. In this puzzle, Pete the Pirate has sixteen gold bars divided into four piles, each of a different number. The task was to make all the piles the same height in just two moves. The number of bars was not recorded in the text and I observed children pointedly counting the number of gold bars illustrated in the picture. In this case the illustration played a vital role in the understanding of the mathematical problem it posed, not only by giving the number of gold bars, but also the way they were distributed into their individual piles.

The children then tackled a puzzle from the Year 3 and 4 section entitled "Queen Esmerelda's Coins". There were aspects to this puzzle that were very similar to the Pete the Pirate question in that four piles of gold needed to be reorganised. In this case though, gold coins were used but the numbers in each pile were to be allocated according to the instructions in the text, not as in the picture. The text indicated that there was a total of twenty coins in use, yet the illustration showed far more (one pile alone had nineteen coins with a possible total of fifty-six coins across the four piles illustrated). On observing the children attempting to solve this problem, I noted a substantial number began by using the same technique they had used with the Pete the Pirate question in that they commenced by counting the coins in the illustration and used the number obtained to calculate the result. In this case the illustration, rather than containing essential information, was unhelpful and misleading and immediately led to difficulties in solving the problem whilst also de-motivating the children as they struggled to find a way in which to provide a solution.

As a result of these observations I became more aware of another difficulty children appeared to experience with mathematics questions involving problem solving. In most mathematics textbooks the illustrations appeared to play a fundamental role in the child's success or failure because in some situations, illustrations acted as an aid to the children and in others they caused unnecessary difficulties. The link between confidence and competence is an important one and if the use of confusing illustrations has a negative impact upon a child's confidence then their vision of
themselves as competent problem solvers may well be harmed. Harries and Sutherland, (2000:65) cited work conducted by Tall, (1996) which suggested that children who find mathematics questions hard are actually carrying out a more difficult kind of mathematics than those who find them easy. He found a clear tendency for low attaining pupils to rely more on primitive mathematical objects and primary processes, for example, counting on to solve addition problems, or repeated addition to solve problems which could be more effectively solved using multiplication.

Contemporary mathematics textbooks use illustrations extensively but what effect do they have upon children's mathematical success? If as Tall implies, children who have greater difficulty with mathematics find themselves using more complicated calculation methods, for instance repeated addition rather than multiplication, to solve a problem, do illustrations aid or hinder? This is especially important as these are the self same children who are likely to encounter more illustrations than their more competent peers. It seems then that it was important to research this area in order to find out just what effect text book illustrations had on children's mathematical problem solving.

In order to investigate whether this really was an issue worth researching, I conducted a small trial within my own school using children from Key Stage 2. Chapter 1 details the process of this trial from the conception and choice of methods to the collection and analysis of the data. It also highlights changes that needed to be made to the research design before the main investigation was carried out. Since the results of the trial indicated that there was indeed an enquiry worth pursuing, the rest of thesis details the research process that followed from this initial investigation. Chapter 2 reports the subsequent literature review I undertook. This forms the background to my main study, highlighting current and past research in related areas and concludes with the identification of the research questions.

Chapter 3 explains the methodology of the research project including the design of the research materials, the ethical considerations of carrying out research in schools and the recruitment of suitable school partners. The thesis then branches into two separate directions. Chapter 4 details an investigation into the most common forms of
mathematical textbooks used in primary classrooms and examines the two most popular in more detail with particular emphasis on their use of illustrations. Chapter 5 details the main investigation using examples from the Mathematical Challenges for Able Pupils in Key Stages 1 and 2 modified to allow comparison of different illustration types. As well as analysing the effects of the different illustration types, it focuses upon the effect of reading age upon mathematical performance.

Finally, Chapter 6 brings together the results of the various strands of the research project in order both to answer the original research questions and also to draw conclusions about the role of illustrations in mathematical textbooks. This section discusses the implications of the research findings in a wider context and provides ideas about how the research may be developed in the future.

## Chapter 1 -Report from Trial of Non-Routine Maths Problems

## 1.1 - Introduction

In this chapter I discuss the pilot work I carried out, noting the main points which would inform the main study. The object of the trial was to establish an answer to two points. Point 1 related to the way pupils use illustrations when solving mathematical problems. Point 2 was to establish the practicalities of conducting such a piece of research whilst working full-time. At the time I was working in a small rural primary school as a full-time class teacher of a mixed Year 3, Year 4 class. I had been told I would have support from the school to conduct such a project but it needed to fit in with the requirements of the school. Subsidiary points to be considered were whether the National Numeracy Strategy book on which the research was based was a reasonable choice of resource and the robustness of my classification system of illustrations in test items.

## 1.2 - Context of the Study

I decided to focus on the mathematics problems found in the National Numeracy Strategy book, Mathematical challenges for able pupils in Key Stages 1 and 2 (2000) as it had formed the basis of my initial observation in class (as detailed in the introduction) and was a recent publication that will have been sent to all English state schools. This book is a supplement to the National Numeracy Strategy and was expected to be used by all schools participating in the National Numeracy Strategy. Ours was one such school, so the materials found in the book could be expected to be used across state schools in England. The results then, would be potentially applicable to other schools using the same materials.

The questions in the National Numeracy Strategy book are divided into three sections, each section covering two year groups; Years 1 and 2, Years 3 and 4, and Years 5 and 6. As the title of the book indicated, "Mathematical challenges for able pupils in Key Stages 1 and $2 "$ the questions were aimed at more able pupils. The book states that pupils using this material "may need to draw on a range of skills to solve problems. These include: working systematically, sorting and classifying information, reasoning, predicting and testing hypotheses, and evaluating the solutions" (DFEE

2000:9). My view as a teacher is that these are skills that all children possess, not just able children. It is also my view that assuming that certain questions are only appropriate for particular year groups is limiting. To my mind, the issue appears to be one of using the resource to provide differentiated learning material rather than allowing the designated year groupings identified in the text to dictate access to the material.

Although a child may be familiar with the concept involved with a particular problem, it could be argued that the skills required in order to solve these problems are higher order skills.


Progression in Concept Development

Development of Mathematical Concept. For example; with the concept of addition, a child may begin by adding single digits then progressing to 2-digit numbers, 3 -digit numbers, decimals, fractions and so forth.


These higher order skills of Application and Synthesis may be applied to a range of problems from those using simple mathematical concepts to more advanced ones.

Whether the mathematical concept involved is multiplication or number bonds to ten, it should be a teacher aspiration that children are not only able to recall the numerical facts but that this knowledge is used in a range of circumstances. The problems found in the textbook require the children to use the skills of application and synthesis. The earlier sections of the book use simpler mathematical concepts which all children should be able to access, enabling them to apply their developing application and synthesis skills in a less mathematically complex environment.

Bearing this in mind, I decided to select questions taken from all three sections, Years 1 and 2, Years 3 and 4, and Years 5 and 6, with the assumption that all the children from the sample would be able to attempt at least some of the questions, the premise being that Year 3 and 4 pupils should be able to attempt those questions designed for Years 1 and 2 as well as those designed for their own age group. Similarly the Year 5 and 6 pupils should be able to access questions designed for the younger children as well as those designed for their most able peers.

## 1.3-Classification of the Illustrations

As detailed in the introduction, I had observed pupils attempting these questions in the classroom and these observations indicated that there were at least two types of illustrations to be considered. The first type was where information pertinent to the question was to be found solely in the illustration. This I called "Essential". The second occurred where the illustration merely provided decoration and did not include any information pertinent to the actual mathematical problem to be solved. This I called "Decorative". Using this rudimentary analysis of the illustrations in the book it was clear that another category of illustration was required in addition to that of "Decorative" and "Essential". This further category involved those illustrations which reflected accurately in pictures an aspect of the text which related solely to the mathematical problem. This I chose to call "Related". In conclusion, based upon my observation and interpretation I devised three categories of illustration. 1. "Essential",
2. "Related" and 3. "Decorative" and the questions were classified into these three categories dependent on the role of the illustration within the particular question. This classification focused upon the relationship between the mathematical problem found in the text and the illustration.

To summarise:

1. An Essential illustration was one that contained some or all of the information required to solve the problem and this information could only be found in the illustration and not in any of the text.
2. A Related illustration was one that reinforced or supported necessary information stated in the text. Both the text and the illustration contained the same information.
3. A Decorative illustration contained no information pertinent to the problem and merely served to make the page look more interesting.

These initial descriptive classifications were based on my professional observations in the classroom. Later I will show that an interpretive approach resulted in the requirement for an additional category. The following illustrate examples of each category.

## Essential

This illustration was classified as essential because there is no other way of knowing the position of the snakes or ladders except by the illustration. The text alone provides insufficient data.

## Snakes and ladders



Your counter is on 9.

You roll a 1 to 6 dice.
After two moves you land on 16.
Find all the different ways you can do it.
Now think of other questions you could ask.

## Related

The illustration was categorised as related because the picture reinforces information pertinent to the question. In this case the 'aliens' show either four or nine spots depending whether they are Zids or Zods.

## Decorative

This is an example of a decorative illustration. The picture shows children buying sweets in a shop. There is no clear indication in the picture of the cost of the sweet being purchased. The illustration merely sets a scene which doesn't provide any reinforcement of the information significant to the mathematical problem.

## Zids and Zods



Zids have 4 spots.
Zods have 9 spots.

Altogether some Zids and Zods have 48 spots. How many Zids are there?
How many Zods?

What if Zids have 5 spots, Zods have 7 spots, and there are 140 spots altogether? Find as many solutions as you can.


## 1.4 - Nature of the Pilot Study

The trial involved a group of Key Stage 2 pupils in my own school, none of which were on the Special Needs Register. The composition of the group is shown in Table 1.

| Year Group | Female | Male | Total |
| :--- | :--- | :--- | :--- |
| Year 3 | 3 | 7 | 10 |
| Year 4 | 5 | 5 | 10 |
| Year 5 | 6 | 6 | 12 |
| Year 6 | 7 | 5 | 12 |
| Total | $\mathbf{2 1}$ | $\mathbf{2 3}$ | $\mathbf{4 4}$ |

Table 1 Trial sample numbers by Sex and Year group
A series of booklets was created, each containing the same thirteen questions, six problems from the Year land 2 section (two essential illustrations, two related, and two decorative), three from Year 3 and 4 (one from each illustration category) and four from the Year 5 and 6 section (one related, one essential and two decorative illustrations). This distribution of questions is illustrated in Table 2:

| Year/ <br> Question type | Essential | Related | Decorative | Totals |
| :--- | :---: | :---: | :---: | :---: |
| 1 and 2 | 2 | 2 | 2 | 6 |
| 3 and 4 | 1 | 1 | 1 | 3 |
| 5 and 6 | 1 | 1 | 2 | 4 |
| Totals | 4 | 4 | 5 | 13 |

Table 2 Proportion of question types from each year group

The questions were arranged into a booklet grouped into three illustrative types, essential, related and decorative. Within each illustrative type, questions were arranged in order of difficulty starting with those designated suitable for Years 1 and 2, followed by those for Years 3 and 4 and finally those for Years 5 and 6. Three different coloured booklets were produced (green, red and blue). Each colour booklet had the illustrative types in a different order. So, for example, the Green booklet had all the Decorative questions first, starting with questions for Years 1 and 2, then 3 and 4 and finally Years 5 and 6. The Related questions came next, again ordered by year
from 1 to 6 and finally the Essential questions, once more ordered by year in the previous manner. Table 3 makes this clearer.

| Colour/Order | First | Second | Third |
| :--- | :--- | :--- | :--- |
| Green | Decorative | Related | Essential |
| Red | Related | Essential | Decorative |
| Blue | Essential | Decorative | Related |

Table 3 Order of question types within a booklet

The ordering of questions was carried out in an attempt to minimise the risk of children copying, although as all the questions were identical and the booklets could not be completed in one lesson, there was the risk that the children could have discussed the questions outside of the classroom. Each child was provided with a booklet of a particular colour and the children worked independently on the booklet over the course of several days. As I wanted the children to tackle as many of the problems as possible with as little pressure as possible I decided that this exploratory work would not be done under test conditions.

## 1.5 - Key issues arising from the pilot study

The focus of the pilot was to identify potential differences in the way children used the illustrations when solving mathematical problems. Some of the key issues that arose when I analysed the information from the pilot study were:

1. Whether questions are unambiguous or are capable of a simple and a more complex solution at the same time in light of the illustration.
2. The differences in methods used by pupils when solving problems (using mathematics alone - mathematically or by using a diagram in conjunction with mathematical understanding - visually)
3. The conflict between the mathematical requirements of the question and the real world as represented by the illustration. Some children merely try to work out what the question requires in terms of mathematics and others take the illustration more literally, using their understanding of the subject of the illustration in other contexts to help them solve the problem.
4. If the mathematics appears more challenging, the role of the illustration becomes more important in finding a solution. To some children the illustration itself may provide an answer that seems a plausible solution to the problem.
5. A combination of the text and the illustration weaves a story into which children appear to be drawn. In mathematical problem solving it may be that for some children they are unable to disassociate themselves from the story.

Rather than consider the results of every question the children encountered I shall focus upon examples from the three different categories which indicated an issue worthy of further research.

## Essential Illustration

For those questions where I had considered the illustration to be essential, the children overwhelmingly showed evidence of having used the illustration to solve the problem. Table 4 shows that in the Gold Bars question it is clear that the illustration was used to establish how many gold bars Pete had and how these were arranged. Children had realised that there were four piles at the start and that they could make either two or four piles as their solution to the problem.


The solution as given in the book is.

## 7 Gold bars <br> Move two bars from pile 1 to pile 3. Move one bar from pile 4 to pile 2.

| Year Group | 4 piles of 4 | 2 piles of 8 | Incorrect <br> solution | No answer <br> given |
| :---: | :---: | :---: | :---: | :---: |
| Year 3 | 2 | 7 | 1 | 0 |
| Year 4 | 6 | 0 | 4 | 0 |
| Year 5 | 3 | 6 | 2 | 1 |
| Year 6 | 6 | 1 | 3 | 2 |
| Total | 17 | 14 | 10 | 3 |

Table 4 Gold Bars Solutions

According to the official answer book, the only possible answer is 4 piles of 4 . In the question it asks that all the piles be made the same height, whilst simultaneously assuming that the four piles should be retained. However a problem arose that in a number of cases the pupils had not realised the inherent assumption of conserving the numbers of piles and had posited a different solution, indicating that two equal piles could also be obtained in two moves ( 2 piles of 8 ). I deemed this to be a reasonable response and recorded it as a correct solution.

Table 4 shows that the children in Years 3 and 5 overwhelmingly gave the 2 piles of 8 solution in comparison to years 4 and 6 which gave the 4 piles of 4 solution. Due to the seating arrangements in the two classrooms it would be extremely unlikely that the children copied from each other as Years 3 and 4 were mixed together in one room and Years 5 and 6 in another. Given the small numbers of pupils involved it would not be possible to find an unequivocal explanation for this result. However, one possible explanation could be that some children would count all the bars and find it easier to divide 16 by 2 rather than dividing 16 by 4 . Since this simpler approach (dividing 16 by 2) provides a solution which fits in with the requirements of the question, some children would assume that it was the correct answer, particularly as the requirement to conserve the number of piles of bars is assumed rather than made explicit in the question wording (issue 1 ).

Part of the question required the children to explain 'How did he (Pete) do it?' Those children who provided an explanation (and not all did) did so using two distinct methods, a written explanation explaining the moves (e.g. take 1 from the pile with 5 and put it on the pile with 3 then take 2 from the pile with 6 and put them on the pile with 2) or the explanation was given as a calculation (issue 2). For those who
provided a calculation, their explanation had to be inferred. For example, 6-2=4, $2+2=4,5-1=4,3+1=4$. Table 5 shows that the majority of children were able to provide an explanation in one of these two ways. As the children in one class were older that the children in the other class, it might be that the older children have a greater facility with language so were more linguistically able and inclined to give their explanation in words rather than pictures.

| Year Group | Explanation <br> indicated | Explanation via <br> calculation only | No explanation <br> indicated |
| :---: | :---: | :---: | :---: |
| Year 3 | 1 | 9 | 0 |
| Year 4 | 8 | 0 | 2 |
| Year 5 | 9 | 2 | 1 |
| Year 6 | 7 | 2 | 3 |
| Total | 25 | 13 | 6 |

Table 5 Gold Bars Explanation Given

## Mode of Picture Use

As the information within the picture is essential, it was presumed that all the children would have used the picture to find out the number of gold bars. If they had not been able to deduce this from the picture they would not have been able to attempt the question. However, children differed in their representations of the problem. This analysis relates to children indicating via labels or arrows how the picture was used to calculate or explain their solution (issue 2).

| Year | Picture used |  |  | Own drawing of piles |  |  | No <br> (rouped | Total in <br> evidence |  |  | pilot |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{4 x 4}$ | $\mathbf{2 x 8}$ | Incorrect | $\mathbf{4 x 4}$ | $\mathbf{2 \times 8}$ | Incorrect |  |  |  |  |  |
| Year 3 | 2 | 1 | 0 | 0 | 6 | 1 | 0 | 10 |  |  |  |
| Year 4 | 0 | 0 | 0 | 2 | 0 | 1 | 7 | 10 |  |  |  |
| Year 5 | 1 | 1 | 0 | 0 | 0 | 2 | 8 | 12 |  |  |  |
| Year 6 | 5 | 0 | 0 | 0 | 1 | 0 | 6 | 12 |  |  |  |
| Total | 8 | 2 | 0 | 2 | 7 | 4 | 21 | 44 |  |  |  |

Table 6 Gold Bars Picture Use

Table 6 shows that all the children in Year 3 relied heavily upon a pictorial representation (either the illustration provided or their own drawing) to calculate the answer compared with children in the other age groups.

## Related Illustration

Roly Poly was classified as Related because it reinforced the use of a six sided dice.
Pupils in school may well be familiar with a variety of dice, some of which have more or fewer sides than the conventional six sided one illustrated.

## Roly poly

The dots on opposite faces of a dice add up to 7.

1. Imagine rolling one dice.

The score is the total number of dots you can see.
You score 17.
Which number is face down?
How did you work out your answer?
2. Imagine rolling two dice.

The dice do not touch each other.


The score is the total number of dots you can see. Which numbers are face down to score 30?

The children appeared to have more difficulty with this problem as the number of children that asked 'what do you do?' rose significantly for this question. Part of the confusion arises because the dice in the illustration is not a solution to the problem. On the single dice pictured the dots you could 'see' add up to 18 . This may be a reason why, as Table 7 shows, so many used the information that the opposite faces total 7 to work out the number shown on the dice in the illustration (issue 4). Of course the picture only shows three sides of the dice but the question states "The score is the total number of dots you can see" and assumes that the pupil is able to mentally "walk around" the dice and see the sides that are not illustrated. This raises issues as to from whose viewpoint the question is being asked and requires a degree of spatial awareness that may have confused the children.

| Year Group | Answer calculated <br> using only faces <br> shown on the <br> illustrated dice | 'Don't <br> understand' | Left blank |
| :---: | :---: | :---: | :---: |
| Year 3 | 6 | 0 | 3 |
| Year 4 | 7 | 0 | 1 |
| Year 5 | 4 | 2 | 6 |
| Year 6 | 0 | 3 | 8 |
| Total | 17 | 5 | 18 |

Table 7 Roly poly Number of incorrect answers which related to the illustrated dice

Summarising, the issues that arose in the pilot study seem to show that children approach these problems in a number of different ways, and because they are still learning about handling problems such as these they may pay more attention to areas of the question that are actually merely peripheral. In discussions that took place with the children who attempted these problems it became clear that some children became more absorbed by the characters within the illustration and that this may have diverted their focus from the mathematics within the question (issue 5).

## Decorative Illustration

In the Gobstopper question the illustration is classified as decorative because there is no information in the illustration that is essential or supports information from the text. In the first part of the question the children are asked to find as many of the five different solutions as possible. In the second part no indication of the number of solutions is given.

In the second part of the question the children appeared to have worked towards the number of solutions that they gave in the preceding part. In the

pilot study, only twelve children of the forty-four attempted this part and each found five solutions. Two of the children commented that the second part was 'too hard'. For those whose answers were incorrect, their error related to a reversal of digits or they incorporated invalid coinage to their answers ( $3 \mathrm{p}, 4 \mathrm{p}$ coins). In this case, it would appear that the children were not carrying out a "reality check".

## 1.6 - Focus for further study

Those questions which in the book were aimed at Years 5 and 6 were very poorly attempted. None of the children in Years 3 and 4 attempted these questions even though they were unaware of their origin. Only two or three children from Years 5 or 6 gave correct solutions for any one question from this section and for one particular question none of the children achieved a correct solution. This indicated that only questions from the first two sections of the book (Year 1 and 2, Year 3 and 4) would be of use. Finding opportunities for discussing with pupils how they approached these problems was easy within my own class of Year 3 and 4 pupils but it proved more difficult with children in the Year 5 and 6 class. This practical aspect indicated that it would be more realistic in the main study to focus upon children in my own class. In the pilot study, children were drawn from all of Key Stage two because I wanted to sample as many pupils as possible. It soon became clear however, that having a range of year groups would introduce another variable that could make analysis of the data more complex, therefore I felt that it was best to focus upon one particular year group for the main study.

I found that both Year groups (Years 3 and 4) were affected by illustrations, at times in a positive way, at others negatively. Those in Year 3 seemed to be more affected perhaps because their comprehension skills are not as advanced, causing them greater problematical issues as they moved up the higher order skills of mathematical problem solving. I decided to focus on one particular year group (Year 3). This would give me a group in which to trial any activities (Year 4) and also provide a safeguard in that if the project went beyond the academic year the Year 3 pupils would remain in my class whilst I would lose access to the current Year 4.

The choice of problems was important to the study because I needed problems that were widely applicable within State Primary Schools. The booklet is available to all English state schools being part of the National Numeracy Strategy which has been imposed by the government upon English State schools. The illustrations themselves appeared suitable for adaptation and were readily classified.

On a descriptive level the classifications were fine, however because of the way some children interpreted the illustrations, especially those that were classified as "Related" an ambiguity arose. Some children showed that they used a "Related" illustration as if it were "Essential". For instance, in the Roly Poly question, some children totalled up the faces they could see in the illustration, rather than using the illustration as a guide. My classification of the illustrations was clearly based on their content and their relationship to the problem in the text. These classifications were based on a descriptive model because it was impossible to predict how the pupils would interpret the problems. A descriptive classification system can be rigorously applied, as opposed to an interpretive system.

Although prior to the pilot study the three classifications appeared adequate, for the main study I felt that two more categories needed to be considered. These I called "Negative Decorative" and "No Illustration".

## Negative Decorative

In some "Related" and "Decorative" problems an element in the illustration that was related to the question was drawn in excess. There were far more of the items in the picture than in the text. In this case, the illustration could not be considered as purely "Decorative" because it did show an element relevant to the mathematical task, albeit in an exaggerated manner. I therefore created an additional classification called "negative decorative" to cover this eventuality. For

example in Queen Esmerelda's Coins, the illustration reaffirms the notion of coins being organised into four separate piles as identified in the text. However, there are far more coins shown than the twenty coins stipulated in the text. Some children counted the coins in the illustration in order to obtain a solution to the problem. This had a negative effect upon their ability to calculate the answer, hence the term "Negative Decorative".

## No Illustration

Logically, in order to investigate whether an illustration has an effect on the answer to a question, the question should be also presented with no illustration at all. This final category, where there is no accompanying illustration I called "No Illustration".

## 1.7-Summary

The pilot investigation which employed my initial 3-fold classification of items raised a number of key issues associated with the way children went about finding solutions to mathematical problems that involved illustrations. It showed that there was a need to expand my classification of the types of illustrations found in mathematics textbooks. In order to investigate them further I added the two categories "Negative Decorative" and "No Illustration". It also highlighted some practical problems of carrying out the investigation within school where I had a full teaching commitment. It served to focus the study on those areas that appeared to be problematical for pupils and led to the creation of a system for classifying problems that would aid the wider investigation. The issues identified from the pilot will form the focus of the literature review in the next chapter.

## Chapter 2 - Literature Review

## 2.1 - Introduction

Problem solving in mathematics is a large area of study and within that, word problems such as those used in the trial detailed in Chapter 1 are one particular facet. The results from the trial indicated that children's interpretation of mathematics problems may be affected by the use of illustrations in different ways and that a number of key issues arose. These were identified as follows:

1. Whether questions are unambiguous or are capable of a simple and a more complex solution at the same time in light of the illustration.
2. The differences in methods used by pupils when solving problems (using maths alone - mathematically or by using a diagram in conjunction with mathematical understanding - visually)
3. The conflict between the mathematical requirements of the question and the real world as represented by the illustration. Some children merely try to work out what the question requires in terms of mathematics and others take the illustration more literally, using their understanding of the subject of the illustration in other contexts to help them solve the problem.
4. If the mathematics appears more challenging, the role of the illustration becomes more important in finding a solution. To some children the illustration itself may provide an answer that seems a plausible solution to the problem.
5. A combination of the text and the illustration weaves a story into which children appear to be drawn. In mathematical problem solving it may be that for some children they are unable to disassociate themselves from the story.

This chapter examines the available literature for evidence as to whether these issues were previously documented in other, similar situations. I was also interested in how the results of the trial related to the concept of learning and cognitive styles. Although criticised strongly in the academic press, for example by Coffield et al (2004), use of learning styles is still considered to be "best practice" among Ofsted
inspectors and LEA advisors. Similarly, as a professional teacher I was interested in whether the differences in approach I had seen were as much to do with reading ability as with mathematical ability, particularly as by definition, word problems are mediated through reading. As a result of these considerations I decided to focus on five areas. These are;

Problem Solving in Mathematics - this section reflects on some of the vast amount of work that has been conducted in the area of problem solving in general and word problems in particular. Because the topic is so vast I have chosen to focus upon pupil approaches to question interpretation and non-routine problems.

Classification of Pictorial Representations - this section explores the work of previous researchers and how the differing illustration types have been classified. This has had an impact on my decisions as to how the illustrations used in this research are classified.

Illustrations and the Reading Process - all the children in this study were at an age when learning to read was still a skill they were trying to master. Some were further along the journey than others but pictures and illustrations still formed a major part of their normal reading material. For others, illustrations will have been used as an aid to help them to decode words. Mathematical questions require the children to have a reasonable understanding of how text and pictures provide clues as to what the mathematics question is about.

Illustrations in Textbooks - over the past few decades, illustrations in textbooks have become more prolific and apparently sophisticated. This section considers the possible impact this has had upon children's learning.

Cognitive and Learning Styles - all the mathematical problems investigated in this research involved a mixture of text and visual images which required interpretation. Cognitive and learning styles concerns the way individuals respond best to these forms of information.

## 2.2 - The nature of problem solving in mathematics

In the United States of America, the National Council of Teachers of Mathematics reaffirmed the call from employers and educators and argued that problem solving should become 'the focus' of mathematics in school (Lubienski, 2000). It has been argued by many that through problem-solving experiences children learn to think strategically while learning mathematical content. As Reed (1999) points out, the basic skills for solving word problems are more complex than they first appear. Skills are needed in understanding the text, determining temporal and special relations, eliminating irrelevant information, identifying the unknown variable, selecting the correct arithmetic operations, and determining what to equate in an equation. Consequently, children need to be taught problem solving, communicating, and reasoning and this needs to be practised enough to prevent a mental overload.

The Qualifications and Curriculum Authority (QCA) give explicit guidance to teachers of Key stages 1 and 2 that;
"Children will need to be taught:
To select and apply strategies to solve problems in different contexts, checking their results;
To organise their work, using correct language, symbols and notation;
To reason logically, look for patterns, make deductions and explain them." (QCA, 2002:6)

Although there are a number of different categorisations of mathematical problems (standard, non-standard, real-world, puzzles), the common theme revolves around the degree of mathematical and non-mathematical considerations that need to be applied. This can be summarised by Cooper and Harries' model.

1: problems intended to require no extramathematical considerations.

2: Problems intended to require particular extramathematical considerations.

3: Problems intended to require general extramathematical considerations.

Increasing degree of intended realism in required answer.


Diagram 1 Cooper and Harries' model of link between realism and mathematical considerations (Cooper and Harries, 2003:453)

Teaching pupils to solve problems of this type may be carried out in a number of different ways. Boaler, (1998) conducted ethnographic three year case studies in two schools following students from Year 9 to Year 11. In one school, children followed a traditional approach to problem solving, using standard textbook questions. In another school, problem solving was taught using an open-ended and practical format. Students who followed the traditional approach developed a procedural knowledge that was of limited use to them in unfamiliar situations. Students who learned mathematics in an open, project based environment developed a conceptual understanding that provided them with advantages in a range of assessments and situations. Boaler was working with the premise that students are unable to use school-learned methods and rules because they do not fully understand them. However, the work indicated that standard textbook questions encouraged the development of procedural knowledge that is of limited use in non-school situations but if students were given open-ended, practical, and investigative work that required them to make their own decisions, plan their own routes through tasks, choose methods, and apply their mathematical knowledge, the students would benefit in a number of ways. In conclusion it was stated that the "traditional textbook approach that emphasizes computation, rules, and procedures at the expense of depth of understanding, is disadvantageous to students, primarily because it encourages learning that is inflexible, school-bound and of limited use." (Boaler, 1998:60). Because the type of questions the pupils encountered in this study were non-routine and had an open-ended format some children may not have had the ability to select an appropriate route through the task in order to find a solution. Some may have looked to illustrations to provide them with cues towards establishing an answer.

Heuvel-Panhuizen, (1999) working in the Netherlands, redesigned mathematics assessment questions with the intention of allowing students to emphasise what they could do rather than what they couldn't. When answering problems, prominence was given to the workings of the children rather than solely to the correct answer. Her team constructed the test to be easy to administer and "tried to avoid tests with extensive oral or written instructions that would result in overemphasizing listening or reading comprehension rather than an understanding of mathematics. For this reason, we have looked for tasks that require no additional information beyond the minimum of instruction needed to get the intent across. Little text is used; rather,
pictures that are self explanatory and related to meaningful situations convey the problem to be solved" (Heuvel-Panhuizen, 1999:132). The context for the problems was deemed to be important in that it allowed students to quickly grasp the purpose of the problem, little text meant the student did not have to wrestle with text and the picture provided a motivational element by being 'pleasant and inviting'. Students were considered to be inspired by the context and elicit their own strategies to solve the problem. On occasions the accompanying drawing was used directly. An example of a question type is that involving a polar bear.


Accompanying instructions are given verbally "A polar bear weighs 500 kilograms. How many children together weigh as much as one polar bear?" (Heuvel-Panhuizen, 1999:134). The problem gives no indication of the kind of procedure the children are expected to use, although long division is the intended focus and the children are expected to assume an average weight of a child.

The accompanying examples of children's work show that each child selected one particular weight within the range 25 to 35 kilograms, then used a form of repeated addition to reach a concluding answer. Heuvel-Panhuizen, notes that the "quality of this Polar bear problem lies in the fact that it makes the mathematization by the students visible and gives clues for how to guide them in the reinvention of it" (Heuvel-Panhuizen, 1999:138).

I feel that there are a number of issues with this problem that need consideration. Heuvel-Panhuizen places emphasis on the importance of context, yet a polar bear appears out of context in the Netherlands based problem, let alone the real life
situation of comparing the weight of such a creature to a certain number of children. Comparing the weight of a polar bear with the weight of a number of children is not really a "meaningful situation" (Heuvel-Panhuizen's words) but rather an example of a mathematical problem using objects that may be readily identified. The answers given by the children show that by using repeated addition, they took the mathematics out of context making it solely a mathematical problem. This is indicated by the fact that all the children used the same weight in their additions although the actual weight varied in each case. Indeed if the children had interpreted the question as reality, each child may have varied the weight of the children in their addition. Of course, if a child had used a variety of weights it would have negated the objective of assessing children's long division skills. It would appear that the only real difference between this and other mathematical problems is that children had to select the weight to use. The children have played by the classroom rules and expectations of performance, rather than genuine engagement with the possible range of relevant data. Unless they were familiar with this type of problem, it may have caused undue anxiety for some children. It has been suggested that mathematics anxiety threatens both performance and participation in mathematics and may be a fairly widespread phenomenon that begins at an early age. The use of procedures and rules which are clearer and easier to cope with than concepts in abstract situations provide students with the emotional security by providing something to help solve the problem rather than having to create one's own solution. Kyriacou noted that although much of school mathematics is embedded in real-life contexts, it is still taught and used in a way that uses this context as a means of developing and sustaining the use and understanding of mathematical operations, rather than how the mathematics was used in the real life context to solve meaningful and purposeful problems. (Kyriacou, 2005:180)

In order to investigate methods of problem solving, Kazemi (2002) interviewed ninety American fourth graders by working on multiple choice questions and juxtaposing them with similar open-ended problems. The data suggested that when answering multiple choice questions, students consider the answer choices first and do not necessarily think through the problem. Moreover, the children drew on their life experiences when the context of the problem was salient, often ignoring the parameters of the stated problem.

Cooper and Dunne, (2000) conducted a large scale project focusing on children from Year 6 and Year 9 who had taken the statutory end of key stage tests. As well as collecting quantitative data they also worked closely with individual children as they revisited the problems that had been in the key stage test. It became clear that social class had a correlation to children's success with problem solving. Children from the service class were especially likely to be successful at problem solving in realistically contextualised items because they were able to work within the parameters of the problem and be less influenced by irrelevant realistic factors. Children from the working class group were more likely to apply inappropriate real-world scenarios to mathematical problems. It became clear that the validity of items interacts with social based strategies for a solution.

Others (Kyriacou, 2005, Verschaffel et al, 2000) have also indicated that in real-world mathematical situations, adults and students do not use school-learned mathematical methods or procedures. In a school mathematics setting students were more inclined to follow standard mathematical procedures yet outside of this domain alternative mathematics was used. This indicates that the choice of mathematical procedure depended more on their environment or context than on the actual mathematics within the tasks.

Some of the problems used in this study have been taken from government issued texts. In 2002, QCA issued guidance to teachers concerning the changes to assessment in 2003. The paper included exact examples of the type used in this research.

Activities such as these can be used to help children analyse the structure of problems in order to plan a route to a solution. Methods of solving a problem include identifying the information that is relevant and the mathematics to use, breaking the problem down into smaller steps, generating and listing data, and sorting and classifying information. (QCA, 2002:7)

Activities such as these can be used to teach children how to interpret precise mathematical language, symbols, notation and diagrams and use these to communicate their mathematics. This can be modelled for them in what you draw and write and the language that you use. (QCA, 2002:9)

Activities such as these can be used to teach children to look for patterns and relationships or to explain, and later to justify, even quite simple
results. Help children to explore the mathematical structure of problems and to generalise. (QCA, 2002:11)

As of 2008, no problems of this type have appeared in the statutory tests. It may be that in a political situation where the results of statutory test are high stakes for schools and politicians this type of open ended investigative question is too hazardous for the simple reason that they would be almost impossible to mark reliably. Therefore there remains a focus on back-to-basics policies which has a direct threat to the emergence of a new form of mathematics. The current diet of problem solving with its attendant problems will remain. "The current regime of stereotyped and/or artificially realistic questions typical of many school texts and tests is not encouraging the breadth and depth of thinking that could otherwise be possible" (Cooper and Harries, 2002:20).

Problem solving is an important element of the mathematics curriculum in developing a range of strategic thinking skills valued by those both within and outside the education arena. The common theme of the authors identified (Boaler, HeuvelPanhuizen, Kyriacou, Cooper and Dunne) is that within the classroom setting procedural knowledge is developed rather than investigative mathematical strategies. Those who play by the rules and ignore the real life parameters were more successful in the classroom. But as Reed identified, problem solving is a far more complex process than it first appears and this procedural knowledge developed by children may be a way in which pupils simplify such a complex problem.

## 2.3-Focus on word problems

Within the vast range of problem solving questions, one particular area is that of word problems. This type of problem encompasses the types of mathematical problems found in this study. Word problems can be viewed as mathematical exercises where a mathematical computation is hidden within the text of the problem, and the task of the pupil is to identify, and subsequently solve the computation in question. This hidden computation is often set in a real-life situation, usually in an attempt to provide an authentic context for the problem, possibly in a form that the pupil has previously experienced, for instance, shopping for sweets or distributing items fairly. Hence word problems can be seen to represent the interplay between mathematics and reality
and they give a basic experience in mathematical modelling. "In the metaphorical construction of learning, problem solving is often construed as the situation in which mathematical skill and insight are put to real use; when students solve problems, they apply their knowledge to some real world situations and do not merely perform a set of abstract exercises that can be solved in an algorithmic manner. Problem solving is seen as a decisive test of genuine skill and understanding". (Wyndhamn and Säljö, 1997:361)

In this study children were still developing their problem solving skills and their goal was to find out the mathematical task within the problem. If they had defined it correctly, the solution could be arrived at by straightforward computation. The type of word problems children in Year 3 encounter are relatively undemanding, one or two step problems. A typical one step word problem would be;

Ali picked 70 apples. He gave away 32 apples. How many apples did he have left?

The one step problem hidden within the text would be 70-32=

An example of a two step word problem would be;

Hannah invites 16 children to her party. $\frac{1}{4}$ of them arrive by car and the rest walked. How many children walked?

In this question the answer is calculated in two steps. The first being, $16 \div 4=x$, followed by the second step of $16-x=$ to give the final answer.

Even within these apparently simple word problems, many pupils seem to have difficulty learning how to choose appropriate number operations and calculation methods to solve money and real-life problems; in other words, solving word problems and applying what they know.

Appleby, (2003) who taught a class of twenty four Year 3 children found that on observation not one of her pupils was able to correctly show the calculation to given word problems. There seemed to be little or no understanding as to the purpose and task required even by the high achievers. The key problem which she identified was that the class had never been taught how to solve word problems. With the aid of a colleague she embarked on a series of sessions to develop the children's skills of extracting the key vocabulary from the problems and then applying the appropriate calculation. She reported a marked improvement in their ability to tackle word problems in that three quarters were subsequently able to show the correct calculation to a given word problem.

Appleby's work is in line with guidance on the teaching of problem solving from QCA and the National Strategy through training has, from my personal experience, concentrated upon modelling answers for children. An issue with Appleby's research is that at least a third of her class had English as an additional language which clouds the issue as to whether the children had been unsuccessful prior to this because their knowledge of the English language had not developed sufficiently or if there is a fundamental problem with children's ability to understand the language of word problems. It may be that these children have reflected Verschaffel et al's (2000) work in that when confronted with word problems in a typical school setting they solve word problems in a 'stereotyped and artificial way without relating them to any reallife experience" (Verschaffel et al, 2000:12). Through the focused sessions they have learnt school mathematics. Appleby's approach contradicts results from Wyndhamn and Säljö "which identifies that explicit training in problem solving was not particularly efficient in improving the skills of pupils. That is, poor problem solvers do not become very much better at what they are doing by copying what good problem solvers seem to be doing". (Wyndhamn and Säljö, 1997:363).

Therefore, providing approaches to problem solving to poor problem solvers is not necessarily an appropriate approach. They also need to develop their own model and ideas on how to formulate a plan when problem solving. Children are also more likely to use the arithmetic operation with which they feel most competent and/or one that has recently been discussed in the classroom (Reed, 1999). In looking at problem solving, Fairclough, (2002) considered the mathematical skills required for each
problem in parallel with the skills and procedures attributed to solving any type of problem. Her experience indicated that children develop many problem solving strategies through "training rather than teaching" (Fairclough, 2002:98).

Other authors would tend to agree. For instance, Pape suggests that:
> "Novice problem solvers begin ineffectual solution paths, based predominantly on problem exploration, and continue without monitoring their progress. In comparison, expert problem solvers read and analyze the problem but then move toward a solution using various other cognitive processes such as planning, implementing and verifying, altering their behaviour based on judgements of progress. For expert problem solvers, developing a mental representation for a problem is an ongoing process involving many transformative behaviours" (Pape, 2004:191)

This clearly links with the view of strategic thinking referring to the development of a repertoire of mental and written calculation strategies, and informed decision making about their use. It may be that the children in Appleby's study became trained to follow rules and use symbols without reflecting on, or analysing, what these rules and symbols mean in the specific context in which they are used. The general outcome may be that pupils solve certain kinds of standard problems satisfactorily, but are unable to apply their knowledge when encountering new kinds of problems. If we consider the development of mathematical problem solving as the acquiring of a series of underlying skills moving from a recall level to that of application and synthesis, some problems may require a higher level of underlying skills than the pupil currently exhibits.


Diagram 2 Progress of mathematical skills in problem solving.

When comparing the amount and kind of variability of word problems presented in Soviet and American textbooks, Stigler et al, (1986) found that solving a word problem may be quite different when the problem is unfamiliar than when the problem has become familiar. When the problem format is familiar, the child may simply use cues in the problem text to retrieve the appropriate problem scheme from memory. Soviet textbooks were seen to have greater variety and amount of word
problems than found in the American textbooks. Soviet children appeared to more successful at solving a range of word problems than those from America. The deduction was that the frequency with which children are exposed to problems of different types would relate to the ease with which problems are solved. (Stigler et al, 1986:155) The difficulty with this view is how much children are applying and synthesising and how much is recall of training. In relation to textbooks he hypothesised that "textbook makers probably are motivated to give teachers materials that will not be too difficult to teach; hence they over represent the easy problem types in their texts....children are asked to solve easier and easier problems, which in turn makes the difficult problems even more difficult and thus less likely to appear in the elementary mathematics textbooks" (Stigler et al, 1986:169). This does appear rather over-exaggerated, but it might explain the rather limited range of variety and formulaic word problems that children tend to encounter in their textbooks.

Hegarty et al, (1995) looked at two groups of college students, one group consisted of those who were described as unsuccessful problem solvers and another group of successful problem solvers. Their research focused upon eye fixation data and concluded that "unsuccessful problem solvers devote a higher percentage of their fixations to numbers and relational terms when they reread part of the problem, as compared with successful problem solvers. Overall, the unsuccessful problem solvers looked back at parts of the problem more than did successful problem solvers, thus suggesting that they are struggling to figure out how to solve the problem" (Hegarty et al, 1995:29). The unsuccessful solvers begun by selecting numbers and keywords from the problem and based their solution plan on these, they also spent their additional effort mainly in re-examining numbers. The successful problem solvers begun by trying to construct a mental model of the situation being described in the problem and plan their solution on the basis of this model and where re-examination of the problem was required they focused on informative words. The interpretation of the evidence was that problem comprehension processes play an important role in the solution of arithmetic word problems. It would appear that the successful problem solvers were able to focus on the words that inform the student what needs to done, such as "less than", "how many". These are the words that provide the cues to what needs to be done with the numbers, something unsuccessful problem solvers seem
unable to comprehend. Therefore accurate comprehension of the text is vital to chances of success.

The word problems the children encountered in this study did not follow the standard type of word problem they would normally encounter, hence I have termed them nonroutine. Rather than being straight-forward problems in simple contexts (routine problems) they require attention to both the text and usually the illustration and the addition of some common knowledge of the world in order to make sense of them. What makes them non-routine is that the children will need to decontextualise the text and illustration to find their meaning and subsequently recontextualise them in the form of an appropriate calculation (or calculations) in order to solve the problem.

## 2.4 - Issues related to the solving of word problems

The subject of this research is to look at the effect of illustrations in mathematics and a substantial part of the study looks at their role in word problems. This section explores those elements that have been identified through the pilot study that appear to have an influence upon children's use of illustrations.

### 2.4.1 - Pictorial representations and their classifications

In the pilot I classified the illustrations based upon their relationship to the text. Those illustrations that provided information necessary to the problem that was not identified in the text I classified as "Essential". Illustrations which reflected an aspect of the problem that was necessary to the calculation I classified as "Related" and those whose illustration reflected the scene only were classified as "Decorative". At the conclusion of the trial it appeared this classification based only on three types of illustration may have been too simplistic and that other classes of illustration were necessary. In this section I explore the literature to see how other researchers have classified illustrations in order to ascertain whether and how my classification might be improved.

Levis and Lentz, (1982) classified illustrations into two main types: representational and non-representational. Representational illustrations are those that show what something looks like. Non-representational illustrations are diagrammatic illustrations used to represent data, such as graphs, maps, diagrams. These two types
have also been catalogued as being realistic or schematic. Since all the illustrations used in this study would be representational this method of classification is too broad a term to be used in the study.

Duchastel and Waller (1979) classified illustrations into seven different types, namely:

> Descriptive: which shows what an object looks like
> Expressive: those that imply an emotive response in the reader
> Constructional: technical drawings that provide assembly instruction
> Functional: diagrams that illustrate a process or organisational hierarchy
> Logical-mathematical: depictions of mathematical concepts such as graphs
> Algrithmic diagrams: such as flow charts
> Data display: such as pie charts and histograms

Duchastel and Waller's classification system contains many elements that are inappropriate for the mathematical problems on which I was focussing. These problems do not use flow charts, pie charts or graphs, so a great deal of the sophistication of this approach is lost on my study. Where there are similarities to my system "descriptive" for instance, the definition is too broad for my purposes and is not precise enough to cover the illustrations I was categorising. Similarly, the "expressive" type is interesting and it is quite likely that the characters illustrated in the problems I was using would elicit an emotional response from the children. Whether this would have any effect on their ability to find the mathematical problem within the question is not nearly as clear.

When investigating the role illustrations play in school science, Stylianidou, (2002) identified those images relevant to their investigation by classifying them in a rather complicated manner.

- Images requiring interpretation of the roles of elements representing both realworld and schematic or symbolic entities (R/S)
- Images whose interpretation requires certain elements to be given importance or be highlighted, often in relation to textual/graphical features which make them salient, or do not make them salient (SEL)
- Images containing elements that require appropriate readings of symbols, and that contain examples of synonymy, homonymy and/or polysemy of symbols (SIM)
- Documents with images requiring verbal elements included within the image or used as captions (VE)
- Documents containing more than one image requiring interpretation of relationships between different images (INT)
- Images that make important use of compositional structures requiring the reading of special distributions and different representational structures (CS)

Stylianidou's model contains ideas that would be useful in the development of a system of classifying pictures, but is too complex for use in my study. It would be more appropriate for extremely content-rich illustrations and especially if there were subtle nuances to be drawn from such images. However, the dimensions within this system do not correspond with the dimensions within my own system.

Another classification system is that of Reid, (1990) who classifies illustrations into three main functions:

* Remuneration: they act as a stimulant for someone to purchase the book thereby giving the publisher a financial reward
* Perceptual: this is sub-divided into a number of categories; attracts attention to the picture, directs attention to a specific part motivates giving an incentive to look further at the book.
* Cognitive: sub-divided into: reiteration, further exposure to text representational, make concepts more concrete organisational, integrating parts of the text interpretation, make text more comprehensible transformation, presenting information in a new way pedagogic, designed to help the 'less able'

Again, this system does not fit in with the dimensions in my study, being more aligned with categorising the way that the student interacts with the question.

Santos-Bernard, (1997) focused upon two types of illustration; the cosmetic and the relevant. The cosmetic related to those illustrations that showed what something looked like but contained no information necessary to accomplish a task. This would relate to my classification of "Decorative". Her relevant illustrations showed how something looked but also contained information pertinent to the mathematical problem and were intended to be used as a direct source of information in order to accomplish a task. This category appears to cover those I have identified as "Essential" and "Related" but I believe there is a distinction between the two.

For the purpose of this study, classification of illustrations will involve those that are representational in that the illustrations represent a form of reality relevant to the text. Equally it could be said that the illustrations are descriptive because they show what
something looks like. Additionally, the cartoon nature of the illustrations encourages an emotive response from the reader giving them an expressive quality. Although others' classifications may be of value given other research foci, they do not pick out the dimensions relevant to my research focus. Principally, my representational illustrations may be classified by their related and cosmetic function (SantosBernard's work) but this classification remains rather rudimentary. There are some illustrations that are essential to the understanding of the problem and other purely cosmetic illustrations which are definitely misleading if they are taken into account when formulating a solution. I shall delay a further breakdown of these categories until the methodology chapter.

## 2.5 - Pictorial representations and the reading process

Both the literature review on word problems and the pilot study have indicated that comprehension of the problem is essential if children are to construct a schemata appropriate to the question and execute it successfully. The majority of mathematical problems that children encounter involve the reading process. The predominance of problems the children encounter in this study involve the decoding and comprehension of text. As a teacher and observer of others I know that pictures are used extensively by children when reading. For example, if a child is unable to decode a word they will typically use the illustration to help identify the word. If a noun is required the children will use the first letter of the word to see if there is an object in the illustration which begins with that letter. In discussing the text, reference will be made to the illustration. This section examines the role illustrations play in the development of children's decoding and understanding of text.

Children of ages seven and eight are still learning and refining their reading skills and most texts the children encounter contain illustrations or pictures. Reading is a complex process that involves word recognition, or the decoding of marks on the page, and comprehension, the construction of meaning from words and sentences. The skilled reader decodes in a highly automatic and efficient way and constructs a mental representation of what the text is about. (Newton, 1995).

Reading is an internal process and understanding the development of reading skills with the accompanying construction of meaning has been the central feature of reading researchers for several decades, a time that has seen children's literature undergoing tremendous changes. "Numerous contemporary picture books no longer contain simple relationships between images and written text. Some illustrations are closely aligned with the written text, providing symmetrical information, while others enhance or contradict the meaning of the written text". (Serafini, 2005:61)

This vogue for highly illustrated and colourful textbooks is the current milieu in which children are learning and developing their reading skills.

It is now acknowledged that picture books can have multiple meanings and multiple discourses and that the reading of images is one aspect of visual literacy that is required in an increasingly technological age (Walsh, 2003). Reading of pictures is a different process from reading words. Work conducted by Arizpe and Styles, (2003) identified that the reading of pictures is an equally complex act of reading as the reading of words, and that pictures can evoke different levels of responses. In contemporary society, children are increasingly presented with materials that are not linear, for example web pages, multi-media advertisements, music videos, and informational texts. In order to be successful, readers need to learn to navigate these and critically evaluate the information presented (Serafini, 2005). The implication here is that children are having to learn to read pictures as much as words and to gain a working understanding of both media sources.

The actual act of reading involves the use of sensory material, in general this will be what we see "through the eyes" or, and information we already know that is stored in our minds "behind the eyes". Smith, (2004) illustrates this process in the following diagram.


Diagram 3 Two sources of information in reading (Smith, 2004:74)

This indicates that if children are unable to comprehend meaning using information from their stored knowledge they become more reliant on the visual source. However, if within the visual text the words have been of little help, then the visual images (or illustrations) accompanying the text will become the predominant route to comprehension. As cited above, learning to read images is also a process that has to be mastered.

Most children and adults would recognise this image as a dog. It may not be a well drawn illustration of a dog but it is clearly a drawing of a dog, but you would not confuse it with an actual dog. However, if I was to say that this is a drawing of my dog Pip, as words describe both reality and depictions of reality, the generic drawing now becomes a specific drawing of a particular dog. The picture may more closely resemble another dog (say Taff) but it is still a drawing of Pip. The illustrain starts to develop substance, becoming a specific character which could be placed into any number of settings. The character develops even greater substance depending on the context. For example, if Pip is pictured in a kennel it is more believable than if Pip is pictured inside a diving suit. As they develop, children learn to read the relationship between pictures and what they depict (Serafini, 2005).

Kress, (1997) contends that the reading of images involves quite a different process to the reading of words. Images involve the arrangement and display of elements that are salient. Yet as Newton, (1995) identifies "pictures can make complex information accessible, serve to verify or shape mental models, make prior knowledge available, and motivate the reader" (Newton, 1995:128). This has direct relevance to the type of illustrations used in this study. Using as an example, in one of the mathematics problems mentioned previously, Queen Esmerelda's Coins, (page 22) we can examine this idea more closely. The illustration shows a lady with a crown on her head. The crown would be a visual symbol that the lady is either a queen or a princess. However, the text specifically names the character as Queen Esmerelda. The image then becomes one of a specific, rather than a generic, individual. To add further
substance to the image, the background infers an appropriate setting; a room in a castle with a soldier in medieval dress standing guard as she is busy dividing up her coins into different piles. The image is defining a particular reality, albeit one where the queen's cat wears a crown! Children are being lured into believing that the image portrays a particular reality. The problem in this example is that the image conflicts with the text in depicting more than the twenty stated coins. A dichotomy results in that within the image in Queen Esmerelda's Coins, some of it is true and some is false. This means that the children have to have the sophistication to accept certain elements of the image as true, but not others. This could be difficult for those who are relying on the picture for clues as to the nature of the mathematical problem.

Such a discrepancy between text and picture could cause conflict and uncertainty in a child who is relying more on the visual information, Success in solving this problem may lie in which of the two visual sources has greater influence on the child attempting it, the words or the image. Research conducted by Filippatou and Pumfrey, (1996) identified that one of the central characteristics of some children is their relatively limited ability to filter out extraneous stimuli and to focus. They also concluded that "the presence of both graphemic and pictorial information stimulates the reader who is sensitive to the semantic and syntactic constraints in language to integrate the two sources of information, both of which contribute to word identification and reading comprehension". (Filippatou and Pumfrey, 1996:269). Nevertheless, this integration may only occur when the two sources concerned are not providing conflicting information.

Due to the complexity surrounding images in picture books, Serafini (2005) argues that current picture books with their colourful illustrations that may or may not illustrate features of the text can unsettle the reader's expectations. If this is the case, are children familiar with this conflict between text and illustration and able to deal effectively with the discrepancy? From the psycholinguistic view, "comprehension will be dependent on learning to process information from the semantic, the syntactic and the orthographic dimensions of information and that the efficiency of the process relies on learning to select, within the redundancy of information available, only that which is necessary to message identification....Illustrations constitute a source of contextual information that facilitates the process of message identification through
increasing the reader's access to semantic information and reducing his reliance on orthographic information" (Donald, 1983:175/176).

In the case of Queen Esmerelda's Coins, the illustration would appear to be providing that contextual information that is missing from the text, something the child would have to imagine for themselves as they construct a mental representation of the situation. Therefore it could be argued that the illustration is an aid to the message identification process and plays an important part in the reading of the question. In order for pictures to 'work' they must activate certain information-processing skills within the learner (i.e. selecting, organizing, integrating and encoding). If these are not activated and used, negligible (or negative) effects can be expected. (Filippatou and Pumfrey, 1996) The illustrations in the mathematics context used in the study show scenes familiar to any child brought up on a diet of cartoon and 'Disney' animation, so the information-processing skills are likely to be triggered.

Pictures can become a second line of communication so it is possible for children to use them and in doing so avoid words (Newton, 1995). In work conducted by Adams, (2001) it was found that children take more time with illustrated than plain (nonillustrated) texts when reading silently. They are also found to pay more attention to the pictures when the text is relatively difficult for them. A number of researchers observing primary school children, have focused on children's use of the illustrations to show meanings beyond the scope of the literal story (Serafini (2005), Robinson (1997), Walsh (2003)). Children's reactions indicated that the pictures had a significant impact. Responses by the children were not merely literal. Many children were identifying and observing details, some were making links to their own experience, interpreting and predicting new information, making affective and evaluative comments and showing they were aware of textual features. This work indicates that illustrations are part of the reading process and have a significant impact. However, the focal attention hypothesis argues that illustrations interfere with the process of learning to read in that they are regarded as distracters in the learning process (Donald, 1983).

Filippatou and Pumfrey, (1996) identified four contrasting views of how pictures might affect pupils with different reading abilities.

## Compensatory Stance

This suggests that highly skilled readers are already skilled at extracting and remembering information presented in texts, so representational aids such as pictures are superfluous. In contrast, less skilled readers have difficulty constructing representations of textually presented information and need compensatory aids to help them adequately construct such representations

$$
\begin{aligned}
& \quad \text { General Enrichment View } \\
& \text { Here pictures may benefit skilled readers who } \\
& \text { are already able to exploit the pictures and } \\
& \text { construct even better and more detailed } \\
& \text { memory representations than they could } \\
& \text { construct from the text alone. On the other } \\
& \text { hand, less able readers struggle to form a } \\
& \text { coherent representation regardless of whether it } \\
& \text { is presented in textual or pictorial format. } \\
& \text { Therefore, pictures will enhance the recall of } \\
& \text { both relational and detail information for more } \\
& \text { highly skilled readers but have no effect for less } \\
& \text { skilled readers. }
\end{aligned}
$$

Selective Compensatory Framework
Pictures serve to compensate for skills.
Relational pictures will be superfluous for highly skilled readers, but partial pictures will not. The opposite would be the case for less skilled readers. This framework predicts that relational pictures will primarily benefit less skilled readers, and partial pictures will primarily benefit more highly skilled readers.

## Selective Enrichment

Pictures serve to enrich the memory representation of information. For less able readers who focus on more detail, then partial (non-integrative) pictures enhance recall of partial information. In contrast, if more skilled readers focus mainly on relational information, then relational (integrative) pictures would enhance recall for higher skilled readers.

Filippatou and Pumfrey, 1996:272/273

All these approaches suggest that pictures are of more importance to less able readers but that their importance to more able readers is less clear. They also relate well to the categories I created for this study. Filippatou and Pumfrey's model implies that my "Related" category might prove a stumbling block for poorer readers for the reasons given by their "Compensatory Stance" and their "Selective Compensatory Framework". In other words, whereas better readers do not have to use the illustration to aid their comprehension, poorer readers are likely to be far more reliant on the picture to fill in the gaps in their understanding. However, if the related illustration is in any way misleading the poorer readers could be compromised by their over reliance on the picture and not be equipped to realise the limitations of the illustration.

Conversely the model also concurs that less able readers would be particularly discommoded by the "Negative decorative" class which acts as a "false friend" by seeming to model the reality of the problem, but only within some elements.

Similarly, the category I call "Decorative" seems to fulfil the role of Filippatou and Pumfrey's "Selective enrichment" item.

This work suggests that the type of picture incorporated in the text is a factor but so is reading ability. Donald (1993) presented average second year readers with a continuous sixty-six word story and found that illustrations significantly facilitated word identification accuracy, the rate of self-correction and literal comprehension. A further conclusion of his research was that a crucial developmental acceleration occurs in the ability to make use of linguistic, contextual constraints between, roughly, the second and the fourth grades. His research identified that "the accuracy of those at reading age 7 drops substantially to below 90 per cent under the no illustration condition.....illustrations have a substantial effect on the meaningfulness with which the textual message is identified and that they therefore constitute a significant source of information for readers at this level. Conversely, for good readers at reading age 9, illustrations have no significant effect on either of the variables representing the accuracy of message identification, and it is clear that, for this group, illustrations do not constitute an important source of information in accessing the textual message". (Donald, 1983:179).

Donald states that at a reading age of 7, illustrations have a positive effect but for poor readers at a reading age of 9 , more reliance on the illustration to provide extra information was noted whereas good readers at reading age 9 were relatively independent of illustrations.

In work conducted by Filippatou and Pumfrey, (1996) the reader had particularly to focus on the printed text when reading, only referring to the illustration for assistance in decoding unknown words. The results suggested that the low achiever in reading (within an age group) accesses the pictures more frequently than the higher achiever, especially when the former is dealing with difficult reading material. Pictures, being the most visually salient cue were likely to be selected more often by low reading achievers for eliciting their verbal responses. Equally, less skilled readers have been found to be more susceptible to interference from illustrations than more skilled readers (Beveridge and Griffiths, 1987:31). Almost all the studies that have looked at the relationship of reading ability to the effect of pictures on text processing have
shown that less able readers are overly reliant on pictures to provide information that they are not otherwise able to elucidate from the text. The difficulty level of the text was crucial; pictures were in general helpful when associated with easier texts but had little effect on the comprehension of difficult texts. I would illustrate this in the following diagram.


Diagram 4 Reading skill and reaction to illustrative text.

For both groups focus is on details in an illustration they believe to be significant (Robinson, 1997). Donald (1983) suggests that illustrations that contain literal information, have a significant role to play in the process of early reading for good readers but that their relevance declines sharply with development. For poor readers, it is proposed that a there is a progressive dependence on illustrations which appears unlikely to be adaptive to differing situations. Therefore pictures can distract and lead the reader astray.

## 2.6 - Pictorial representations in textbooks

Before looking at the role of illustrations in textbooks it does need to be made clear that over the last few decades there has been a dramatic increase in the role and presence of illustrations in textbooks of all types, not just those used for teaching mathematics. This may be because we live in a more sophisticated multimedia world where drawings and text are expected to be stimulating, informative and of high quality. In order to illustrate this, I examined school mathematics textbooks from 1937 onwards, with a particular emphasis on the role of illustrations within the text.

2
Arithmetic

## TEST 1. MONEY: MULTIPLICATION

1. $\mathrm{f7}$ 6s. $3 \mathrm{~d} . \times 7$.
2. $£ 16 \mathrm{~s} .8 \mathrm{~d}$. $\times 3$.
3. $£ 12$ 2s. $5 \frac{1}{2} \mathrm{~d} . \times 12$.
4. 198. 11d. $\times 7$.
1. $£ 213 \mathrm{~s} .4 \mathrm{~d} . \times 3$.
2. $19 \mathrm{~s} .11 \frac{1}{2} \mathrm{~d} . \times 8$.
3. $11 \mathrm{~s}, 111 \mathrm{~d} . \times 6$
4. $7 \mathrm{~s}, 11 \mathrm{~d} . \times 10$
5. Find the cost of 5 dozen things at $\frac{1}{2} \mathrm{~d}$. each.
6. What is the difference between 6 half-crowns and 6 florins?
7. Find the postage on a 12 oz . letter, if the first 2 oz . cost $1 \frac{1}{2} \mathrm{~d}$., and every additional 2 oz ., 六d.

TEST 2. MONEY: DIVISION
$\begin{array}{lll}\text { 1. } £ 1 \div 6 . & \text { 2. } £ 2 \div 3 & \text { 3. } £ 1 \div 8\end{array}$
$\begin{array}{ll}\text { 4. } \mathrm{es} 10 \mathrm{~s} . \div 8 . & \text { 5. } £ 2417 \mathrm{~s} .6 \mathrm{c} . \div 8\end{array}$
6. $£ 642 \mathrm{~s} .31 \mathrm{~d} . \div 5 . \quad$ 7. $£ 736 \mathrm{~s} .3 \mathrm{~d} . \div 10$.
8. If 4 eggs cost 6 d ., what is the price of 1 egg ?
9. 12 articles cost $9 \mathrm{~s}, 6 \mathrm{~d}$. Find the cost of one.
10. A dozen pens cost 3s. 9d. What is the price of one ?

TEST 3. MONEY

1. Find the cost of 1 dozen articles at 3 d. each.
2. How many times is 2 s . 6 d . contained in $£ 1$ ?
3. How many farthings are equal to 10 s . ?
4. What is the value of 13 half-erowns in $£ \mathrm{~s} . \mathrm{d}$.?
5. How many shillings are equal to 5 guineas ?
6. Find the cost of twenty $1 \frac{1}{2}$ d. stamps.
7. Find the cost of 4 yds . of cloth at 1 s . 11 fd . per yd .
8. How many eggs at 2 l d . can be bought for 1 s . 6 d ., and how much is left over ?
9. 12 cups cost $7 \mathrm{~s}, 3 \mathrm{~d}$. What is the price of each ?
10. Find the cost of 12 things at 2 q d. each.

This example from 1973 shows very little change from the 1937 textbook. The page is dominated by "sums". There are only two illustrations - one being a table and the other a circle around a group of numbers that are related to a question. (Adams \& Beaumont, 1973:12)

Although the text is from a test book rather than a text used for teaching, this example from 1937 shows a complete absence of illustrations. (Fulford, 1937:2)



By the end of the decade (1979) some illustrations have begun to be used but these are marginalised with the text dominating. Using my method of classification the first illustration is "Essential" with the remaining two illustrations "Decorative". (Griffiths, 1979:36)

These two examples from 1985 and 1990 show an increase in colour and changes in the way illustrations are set out on the page. Generally though, the pages remain dominated by text and sums with the decorative illustrations a relatively minor feature of the page.

(Heinemann SPMG, 1985:8)

(Ginn, 1990:9)


By the year 2000 illustrations and colour now dominate the page. In this example each of the illustrations is essential. (Ginn, 2000:23) This example is the updated version of Ginn example from 1990 which used a decorative illustration but the dominant feature was the algorithms. The number of questions per page appears to have declined in line with the increase in illustrations and use of colour. These examples clearly indicate the rise over time of the illustrations which now are a common feature of children's mathematical experience.

Primary textbooks, especially mathematics textbooks, have received little critical attention (Harries and Sutherland, 2000, Bierhoff, 1996, Santos-Bernard, 1997). One area that has received greater attention compared to others is that of the use of illustrations in science textbooks.

Over the past 30 years there has been a huge change in the layout and presentation of science textbooks. Most noticeable has been the use of illustrations, particularly photographs, presumably in order to make the text more attractive to read and engage the interest of pupils (Kempton, 2004). Even so, there is very little research investigating the pedagogical role of photographs in school science textbooks. Neither the psychology of cognition and learning nor science education research has paid much attention to this topic (Pozzer-Ardenghi and Roth, 2004).

In 1983, Newton carried out an investigation to see if illustrations affected primary school children's comprehension skills in science. Newton cited authors who suggested that science material could be comprehended better, if accompanied by illustrations. Understanding was tested through cloze procedure and Newton concluded that the presence of illustrations does not guarantee that written material will be more easily comprehended. Complex factors, such as the relationship between the style of illustration, illustration density, and the degree of integration between the illustration and the text, also needed to be considered. The results also indicated a sex bias in that boys seemed to be benefiting more from the presence of illustrations as an aid to comprehension than girls (Newton, 1983).

Newton concluded that when the material being read was approximately matched to the reading ability of the readers, boys were able to use the illustrations to improve their comprehension ability, thus reducing or eliminating the difference in performance between themselves and girls. However, when the readability level of the material was higher than the general reading level of the readers, the girls appear to be using the illustrations more than the boys as an aid to comprehension. (Newton, 1983).

Using cloze procedure to assess comprehension may have created some extra problems as it does confine children to a limited set of possible answers and the method they choose to select answers is not always based on their comprehension of the question. From my own experience I have observed children measuring the space available for the answer in order to match a word of similar length, or selecting words in alphabetical order or the order in which they appear on the page so that comprehension is not necessarily the only strategy. Newton also reported that she assessed the readability of the selected texts (each about 200 words long) and then deleted every tenth word. The children were then asked to input an appropriate word although it is not stated if children had access to a list of suitable words. The issue I would raise concerning this technique is that by removing every tenth word, the effect of the deleted words upon the sentence could be very varied depending on the type of word removed. Small connective words may have little impact if removed but technical language of the subject may have a larger impact on sentence meaning. As such, the reliability of the varied texts could be compromised. The effect of
illustrations was found to improve the comprehension score by more than 10 per cent in one case, although the effect was not uniform. It would seem that the presence of illustrations is not always adequate for improvement of the comprehension of textual material. In addition, if children draw the wrong conclusion from an illustration this can actively prevent them from thinking or understanding a concept correctly.

Kempton, (2004) recognised that science teachers can greatly increase pupil involvement and understanding by using more visual stimuli in texts. However, the 'reading' of pictorial representations is not at all trivial or straightforward, particularly when the representations are of conceptually difficult ideas. Stylianidou, (2002) suggested that more attention needed to be paid to the construction of images, if they are to function more effectively. Pupils have to work to understand the images, notwithstanding the common view that images yield their meaning directly and simply. Pozzer-Ardenghi and Roth, (2004) investigated the role of photographs in science textbooks and concluded that it was "the reader's work of reading, the viewer's perception of the narrative and perceptual order of the photographic image and the surrounding text, and the meaning-making resources available to the reader that allows a specific interpretation of a photograph to arise" (Pozzer-Ardenghi and Roth, 2004:220).

Lemke, (2002) also recognised that every image takes an orientational stance which positions the viewer in relation to the scene, (intimate, distant; superior, subordinate) and establishes some sort of evaluative orientations of the producer/interpreter toward the scene itself (tragic, comic, normal, surprising) and does so against the background of other possible viewpoints and depictions of similar scenes. Therefore, pictures, diagrams and icons can illustrate ideas and processes but they require effort to interpret in order to aid thinking.

This implies that when meaning is not explicit not all children will grasp the intended meaning to its fullest extent. When students read textbooks, the way in which they interpret illustrations will greatly influence their understanding of the concept presented by the textbook. It is the interaction of all semiotic resources presented in the textbook, together with the illustrations, that make it possible for readers to interpret and understand what they are reading. Yet as Campbell suggested, children
generally obtain less information from pictures than adults, (Campbell, 1981). Work conducted by Stylianidou, (2002) which used illustrative diagrams as well as a picture composed of many different vehicles to show movement, identified that most of the children in the study did not appear to understand the picture primarily as a conceptual structure and that only a few children appeared to interpret the picture as carrying both narrative and conceptual meaning.

These ideas imply that when children interpret an illustration there are a number of factors which influence their understanding of it. Firstly there is the possibility that required information in the illustration will not be recognised by the child. Secondly, that the information they do gain from the illustration, if not made explicit by the illustrator, will in all likelihood, be misinterpreted. It may also be that the illustrator's view of what is explicit may be different from the child's. This is highly likely to result in a misinterpretation of the entire concept in question.

Less information read + misinterpretation of implied information $=$ misconception.

Occasional pieces of research have focused upon illustrations in mathematics textbooks. Bierhoff (1996) compared textbooks from England with those from Germany and Switzerland. The books were aimed at 8 year olds and involved the teaching of arithmetic particularly the move from working with numbers up to 20 which can be done on the fingers or with counters, to working with larger two-digit numbers. She found that textbooks play an important role in teachers' lesson planning and particularly in the approaches and methodologies teachers adopt, thus having a profound influence on teaching, and consequently on pupils' learning. Bierhoff found that the most common use of the 100 square in English textbooks was for pattern identification and colouring patterns, (i.e. groups of fives) which made it difficult to see how patterns of this kind contributed significantly to pupils' conceptualisation of number. As well as lacking a complete overview of the number system, the illustrations in the textbooks she chose did not necessarily match the accompanying exercises. English textbooks provided few ordinal orientation exercises of any sophistication which children were expected to solve without the aid of visual aids. For example the sum $4+5=$ would be illustrated as 4 objects +5 objects. In contrast, the continental textbooks at this stage provide overview illustrations of all numbers from 1 to 100 in two ways; in linear form and in squares
with rows of 10. Exercises were suggested which encouraged pupils to use these illustrations, such as finding strings of numbers, and as visual aids for calculating operations.

In her PhD thesis, which focused upon children's use of mathematical illustrations, Santos-Bernard (1997) interviewed seventy-eight 7-8 year old Mexican children whilst they were answering three arithmetic questions to see how they used and read the illustrations. Teachers were also asked to predict their pupil's success rate with the questions. She also studied two hundred and eight, 12-13 year old English students worked with talking head illustrations (cartoon type characters with a speech bubble) to see if children preferred to work with a worksheet with 'talking heads' or without.

In the Mexican study, the students who were considered to be low and medium achievers in mathematics by their teachers tended to consider the illustration as an accurate representation of reality and did not give the answer the teacher expected. Very few of these students extracted information from both sources (written and illustrative) simultaneously, even when this was necessary in order to reach the answer expected by the teachers. However, some of these children realised that the illustrations were only an approximation of reality and used them as such. Consequently, students were not giving the expected answers because of their use of illustrations but their teachers were scarcely aware of this problem. The results of Santos-Bernard's research imply that it is not obvious that low and medium achievers in mathematics acknowledge the difference between a cosmetic and a relevant illustration and as a result their mathematical success is compromised.

The conclusion from her interviews implies that;
> "the structure of each arithmetic question prompts students either to use the illustration or to disregard it ....the fact that some questions had no numbers or only one in the structure, confused some of the children. They explained that, due to the absence of two numbers which they could combine, they had to use the information in the illustration only. Other students explained that a question with no numbers or only one, could not be considered a question as such, that it was a mere statement. Sometimes they would disregard all
the information given and answer the question based on their own experience." (Santos-Bernard, 1997:215).

Work conducted by Harries and Sutherland (2000) compared English mathematic textbooks with international examples. Part of their consideration when looking at textbooks related to the nature of the images with which pupils engage as they read the text; this included pictures, diagrams and symbols; the ways in which pupils are introduced to links between mathematical concepts and the role of images in this respect. English pupils were found to have to cope with more distractions in the form of pictorial decoration and were introduced to a wider range of mathematical ideas at an earlier age than pupils in the other countries studied, but they are also expected to move more rapidly from the use of diagrams and pictures to the use of more abstract representation. Text from Singapore identified that word problems were used with diagrams representing a pictorial version of the word problem or linking the diagram/illustration to the symbolic notation, whereas in the English text illustrations often bore no relation to the symbolic representation. In the French textbook diagrams were used extensively throughout the texts to help pupils move between standard algorithms and representations which have been introduced for pedagogic purposes.

A concluding point from Harries and Sutherland's study was that "A strength of the Singapore, Hungarian and French texts is the use and consistency of use of appropriate representations.... The French texts also make extensive use of diagrams which are aimed at supporting pupils to make links between representations which are introduced for pedagogic purposes and more standard representations which are used by citizens outside school. These diagrams also provide a form of explanation which is almost entirely missing from the English texts." (Harries and Sutherland, 2000:63)

Santos-Bernard cited research conducted by Evans, Watson and Willows, (1987) into the reasons why publishers have begun to introduce far more illustrations into textbooks. The main findings were to:

- dress up the books
- assist the author to 'spin the magic'
- provide resting points
- support the text.

The designer and editor were found to have decided the rhythm, size, place and design of illustrations. Authors said roughly what the illustrations should contain but rarely met with illustrators. Decisions made by editors, publishers and designers were not supported by evidence from research into textbook design. The majority of people interviewed assumed that the author and editor had some knowledge of research findings on the subject, which in the vast majority of cases was not true. Trial and error, experience, personal intuition and what had sold well previously, played major roles in the decision to include illustrations. Undeniably the quality of pictures, graphs and drawings, has greatly improved in the last decade. What has not been made clear though, is the effect of these improved illustrations has had on the mathematics they are intended to support.


## Diagram 5 Eight reasons for having pictures in prose

As with many other aspects of education, what appears in a mathematics textbook does not arise by chance. It is influenced by the multifaceted aspects of an education culture as illustrated in Diagram 5 . The ways in which textbook writers view the teaching and learning of mathematics will influence how they present mathematics on a textbook page. Unlike some other countries, English schools, whilst being subject to a state controlled curriculum, are not required to use specific textbooks. This free
market has led to publishers competing for business within individual schools. In other countries mathematics textbooks are regarded as key elements of teaching and learning yet in England, books are not prioritized as a learning resource but increasingly viewed as an old-fashioned source of information. "In England, primary mathematics textbooks have received very little critical attention. The dominant view is that they provoke a routine approach to teaching and learning and are likely to be used by teachers to abdicate their responsibility to prepare and teach lessons" (Harries and Sutherland, 2000:51). When examining the reasons for textbook choice amongst teachers Bierhoff (1996) found that the criteria by which English teachers most frequently assess the suitability of a commercial mathematics scheme for their own school include, among others, the coverage of the National Curriculum, suggestions for 'stimulating activities and investigations', the number of consolidation exercises and the extent to which mathematics is presented as a 'fun subject'.

Expectations of a mathematics textbook have multiplied in recent decades; a mathematics curriculum, child appeal, their ability to cope with the individual, administrative flexibility and even the encouragement of different teaching styles. "Traditionally, they were no more than a permanent collection of suitable problems to support teaching. In their modern role, the function of primary mathematics textbooks has expanded to include attempts to:

- Help teachers respond to the mathematical requirements of society;
- Remedy and prevent weaknesses in children's levels of mathematical attainment;
- Provide a structured and sequential development of mathematics;
- Develop motivation through presentation and learning through understanding". (Gray, 1992:122)

When working with initial teacher training students, many, when invited to consider an appropriate textbook to support their teaching, focus on visual appeal. Criteria such as the need to be 'visually attractive', to have a 'child-centred approach' and to 'be accessible to children' seem to precede mathematical considerations. Words such as 'attractive', 'lively' and 'fun' permeated students' evaluations of textbooks, words repeated by those who attempt to describe the qualities of the texts they promote (Bierhoff, 1996).

Reports by the inspectorate suggest that in many primary schools mathematics is learned predominantly by pupils working independently from text books. This
suggests that textbooks play an important role in influencing the ways in which English primary teachers think about teaching and learning mathematics. In the Ginn 2005 catalogue advertising the Ginn Abacus mathematics scheme, the first point identified for the Key Stage 2 textbooks is "brightly illustrated textbooks provide independent practice of specific skills"(Ginn, 2005:31). The text also mentions minimal reading for the child. This stance on minimal reading reinforces the popular belief that when teaching children who have difficulties in learning mathematics, attractive illustrations make the task easier. The New Heinemann Maths is also promoted as "Inspiring and highly visual pupil materials that are sure to keep all your children on task" (Heinemann, 2004:7). Yet as Harries and Sutherland have pointed out "colour in textbooks can be used for a range of purposes which includes decoration, illustration, organization and mathematical analysis. In the textbooks from England and the USA colour is almost entirely decorative, and in the case of the English text it often dominates the texts and distracts from the mathematics" (Harries and Sutherland, 2000:64). As texts become more graphic the question of whose reality is being displayed may also be brought into question - the child's or the textbook writer and illustrator. Certainly in the past a middle class view dominated textbook illustrations (Gray, 1992). However as modern textbooks are expected to reflect our cultural and social requirements, representing our multi-cultural and diverse society, the consensus of what is "normal" is lost and children will have to use greater skills in reading illustrations than has otherwise been the case.

As we live in a visual world, and as existing research suggests, pictures are supposed to make significant contributions to textbooks because of their potential for improving students' retention of associated text. Children have been found to like to relate to a picture in front of them when answering questions (Kempton, 2004, Santos-Bernard, 1997). On the one hand, teachers and curriculum designers believe that images have a lot of potential as meaning-making resources, captured in the popular adage that a picture is worth ten thousand words; yet in everyday classroom life, images are often used as adjuncts that merely serve as decoration. Nevertheless, whilst students have been shown to prefer textbooks that contain illustrations when attempting to learn from textbooks, paradoxically, stronger readers appear to pay only scant attention to the pictorial information (Filippatou and Pumfrey, 1996). Decorative photographs, without captions, proved to generate greater difficulty in linking the photograph and
its associated text, as this association became subjective when explicit links are missing (Pozzer-Ardenghi and Roth, 2004). With minimal text for children to read, it may be hardly surprising that children will be encouraged to use illustrations in an attempt to gain conceptual understanding. In order to be successful, children also need the ability to interpret diagrammatic information and infer meaning. If children are to be successful and learn to accept discrepancies between pictures and reality this has to be learnt alongside other comprehension strategies.

## 2.7 - The effect of cognitive and learning styles on the way that representations are interpreted.

This aspect has been included in this research as a result of two experiences. The first is because in the pilot study it was apparent that a number of children would either draw on the illustration or draw their own illustration to help with their calculations. A secondary reason for including this area is because as a teacher I have been aware of the rise in importance that learning styles have been given within the primary school. Irrespective of the curriculum area, inset training has inevitably included an element of the importance of learning styles and that these should be accommodated within the classroom. Colleagues have also reported being asked questions by Ofsted inspectors concerning the provision for children with different learning styles.

The area of cognitive and learning styles has been a major source of research over recent years and has resulted in a great deal of misconception about the topic. In order to ameliorate this, work by Desmedt and Valcke, (2004) provided a road map for those entering the field to help recognise the main players. Concern had been expressed about the selective nature of reviews and indicated little information "about the scientific impact of the different learning and cognitive style conceptions". Using citation rate and citation links analysis it was found that a small cluster of authors have a high impact on the field with a strong relationship in their work between learning and cognitive styles.

Designers of learning materials often make the assumption that students learn in a similar manner. During the later part of the twentieth century this approach began to be questioned by psychologists. In the 1990's, Riding argued that this universal
approach ignored the important issue of individual differences in learning style and cognitive style. Sadler -Smith, (2000) defines cognitive style using Messick's definition as consistent, individual differences in preferred ways of organising and processing information and experience. Kolb (Sadler-Smith, 2000) defined learning style as a four-stage process consisting of:

- concrete experience
- observation and reflection
- formation of abstract concepts
- generalisations and the testing of the implications of these concepts in new situations

An individual's cognitive style appears to be an inbuilt core characteristic, whereas learning styles are seen as strategies which can be learned and are ways of adapting the learning materials or methods to enable an individual to deal with it as effectively as possible.

Curry's 1983 model (Diagram 6) is widely cited as describing the relationship between cognitive and learning styles using the simile of progressively deepening layers of an onion. "Cognitive personality style is the innermost layer of the onion, which relates to more fundamental and internal cognitive processes, less modifiable via instruction. The next layer of information processing style relates to how an individual prefers to process information from external stimuli. These are relatively stable, but yet still modifiable. They are influenced by the inner layer of cognitive personality style and in turn influence the outer layer of instructional preference. Instructional preference characterises the environment in which the student prefers to learn. This is influenced by the former two layers but is the least stable of the traits." (Price, 2004:681)


Diagram 6 Curry's layering effect of cognitive and learning styles.

The cognitive personality style is therefore a dominant influence in the capacity of students to organise and process information which would have a direct influence on a student's ability to tackle the different types of mathematics problems produced in this study. Riding has defined cognitive styles into two fundamental style dimensions - wholist-analytic and verbal-imagery, which affect the way in which people think about, view and respond to information and situations. This implies that if there is a mismatch between cognitive style and the material being studied, a reduction in performance can be expected. The wholist-analytic dimension concerns whether an individual organises information in wholes or in parts. The verbal-imagery dimension concerns whether an individual represents information during thinking verbally or in mental pictures.

### 2.7.1 - Wholist-analytic style

Riding, (2002) describes wholists as tending to see a situation as a whole, and are able to have an overall perspective and an appreciation of the total context. In contrast, analytics will see a situation as a collection of parts, focusing on one or two aspects of the situation at a time to the exclusion of the others. As the parts are not separated, the wholist can see a situation in the overall context. However they are at risk of being unable to separate and distinguish the individual issues that make up the whole situation.

Those individuals who tend towards an analytic style have a predisposition to focus on just one aspect of the whole at a time and this may have the effect of distorting or
exaggerating it, making it more prominent with respect to the rest. Whilst they may be good at seeing similarities and detecting differences, the negative aspect is that they have difficulty obtaining a balanced view and by focusing on one aspect they are prone to enlarging it out of its proper proportion.

In respect of organising the information in the mathematics illustrations, this might suggest that wholists, seeing the whole of the illustration, may have difficulty identifying those aspects which have particular reference to the question. In contrast, the analytic may focus on one aspect, which may or may not have a direct relevance to the mathematics, and inflate its relevance. These differing styles are important because in order to make teaching more effective there needs to be a match between the characteristics of the learner and the content or method of instruction. Equally we may be trying to shift the learner's style in which case an understanding of their preferred style is needed in order to support learners to respond to an alternative view.

### 2.7.2 - Verbal-imagery style

In this dimension people may be categorised into three types: verbalisers, bimodals or imagers. Verbalisers consider information they read, see or listen to in words or verbal associations. Imagers read, listen to or consider information they experience as mental pictures either of representations of the information itself or of associations with it. Bimodals exhibit no overall preference for either type and tend to use either mode of representation when appropriate.

A practical implication is that verbalisers may learn better by reading text and imagers by looking at pictures. Therefore verbalisers may rely on the text of a mathematics problem whereas an imager will rely on interpreting the picture.

Riding proposed a method of plotting a particular person's style by plotting the two dimensions on a grid.


## Diagram 7 Relationship model between verbal-imagery styles

Although Riding has been something of a pioneer in the field of cognitive styles others have also been active. This has resulted in a lack of any consensus on the use of terminology making comparisons difficult in a complex and confusing discipline where competing vocabularies, with overlapping categories are all vying for attention. Hayes and Allinson (in Riding and Sadler-Smith, 1997) identified twenty-two different labels including leveller-sharpener, field-dependent-independent and serialist-holist amongst many others.

One of the criticisms of Riding's work has centred upon his Cognitive Styles Analysis (CSA) test. The CSA is currently the most frequently used computerised measure of cognitive style in the UK and is also popular in other European universities (Peterson, Deary and Austin, 2004). This computer administered test relies on the participants' reaction time to identify dominant cognitive style. The theory being that the more dominant the style, the quicker a respondent will react to information presented in a particular format. Peterson, Deary and Austin, (2002), examined research conducted on the validity of the test and found that none had focused on the stability and internal consistency. Using fourteen males and thirty-six female undergraduate psychology students they created two computer programmes to present, control and record the temporal parameters of the two tasks. Task one was a replica of Riding's test except that at the end of the test no summary of the results was presented to the student whilst task two was a parallel version of Riding's test complete with a detailed summary of the results. Participants completed both tests
twice, with the second sitting approximately a week later. The conclusion drawn was that the tests in their current format were not "stable or internally consistent measures of cognitive style preference" ( Peterson et al, 2002:887). Further research conducted by Peterson et al. built upon the earlier work by putting forward suggestions for improvement of the CSA questions. This later work resulted in greater reliability of the test, especially in the area of verbal-imagery. Coffield et al. (2004) concluded that the low reliability and validity of learning style instruments strongly suggested that they should not be used in education or business.

Coffield et al (2004) have opened up the debate concerning the legitimacy of the learning style issue but acknowledges that "a thriving commercial industry has also been built to offer advice to teachers, tutors and managers on learning styles, and much of it consists of inflated claims and sweeping conclusions which go beyond the current knowledge base and the specific recommendations of particular theorists" (Coffield et al., 2004:36). Their article makes a scathing attack upon the issue of learning styles and its place in the education system. However, from a teacher perspective there seems to be an acceptance by Ofsted inspectors and Local Education Authority Advisors that such a phenomenon exists and that to disregard a child's learning style in lesson planning or presentation could be considered as professional incompetence. I was interested to discover whether learning styles could be seen to play a part in understanding mathematics problems but in order to do this I had to use a recognised assessment tool, despite Coffield's caveat as to whether any tools were really appropriate.

One of the most well known criteria for defining learning styles is the VARK system originated by Fleming and Bonwell. VARK is an acronym for the main learning preferences within Fleming and Bonwell's system, standing for Visual (seeing), Aural (hearing), Read/Write (the written word) and Kinesthetic (doing) respectively. It is administered by using a short questionnaire (either as a computer or paper version) and provides feedback in the form of scores which indicate a person's relative preference for methods of receiving and providing information within the range of the VARK system. Being short and easy to administer and score it is used extensively, however, care must be taken not to read too much into the results. Those taking the test are making subjective judgements about themselves which may affect the
reliability of the test and in doing so question the validity of the system. It is important, for instance, to realise that most people do not have a preference for just one style and a complete absence of preference for any other style. Most people are multi-modal to a greater or lesser extent. However this can also change with life experience which can be further complicated by other factors that may not be under the control of the teacher or indeed, the learner. In addition, most learning style questionnaires are aimed at graduate students or those in business, so finding an appropriate test was not easy. The most obviously relevant was the VARK questionnaire designed by Debra Jones (no date, see web site) of California, for young people, aimed at High School students.

The jungle of cognitive and learning styles is clouded further for the researcher of lower junior aged children as little of the work conducted relates to that age group. Many of the studies used in the development of cognitive and learning style analysis have used older participants such as undergraduate students. In 1999, Riding and Grimley assessed the cognitive style of 11 year olds, but their conclusion indicated a number of variables that may have affected the results, including the use of multimedia materials and gender. Despite all this research literature it is unclear if these styles are domain specific. It also seems to be unknown when the layers of the onion described by Curry (1983) develop and become fixed to form a definite style. Does age or experience help to define the style formation? It is difficult to know how fixed young children's learning styles may be at this age. If children are in the process of developing a preferred or dominant style this may be another factor influencing the performance of individual children when solving mathematics problems making it difficult to assess the influence of cognitive or learning style in the current study. Since illustrations are a visual medium then those children who react more favourably to visual images may be advantaged or disadvantaged, depending on the illustration, when solving mathematical questions. I hoped to be able to throw some light on these issues.

## 2.8 - Summary

Problem solving is seen as an important area of mathematics in which (among other things) children are expected to demonstrate their application of mathematic skills. In
order to help children to become successful problem solvers, teachers are encouraged to teach problem solving techniques but this can lead to children developing a procedural approach to problem solving rather than a skill base that is applicable to a wider range of situations. Within the whole gamut of problem solving, word problems are seen as forging the link between mathematics and reality by providing the opportunity for children to experience basic mathematical modelling. Reality within the school based context tends to be suspended, and those children who are successful can rely on their procedural knowledge where the word problems use familiar formats. In unfamiliar contexts, this procedural knowledge is not always a transferable skill which leads to success. Unsuccessful problems solvers have also been shown to focus on the unhelpful aspects of the word problem. This leads to weaknesses in their ability to decontextualise the problem and subsequently recontextualise it into a mathematical form.

Classification systems used by other researchers have proved to be too broad to be of use in the classification of illustrations to be used in this study. All the intended illustrations would fall into one or two types therefore there has been a need to develop my own classification of the illustrations further, drawing mainly on my pilot work and reflection on the items to be employed.

Illustrations are another line of communication and reading illustrations is a complex act which can evoke different levels of responses. The relationship between the illustration and what they depict generates an emotive response and some children are unable to filter out extraneous stimuli or information. Therefore, being able to read illustrations is something children need to learn as much as being able to read text. Children who are considered poorer readers are more susceptible to the influence of illustrations in their comprehension of a text than their more competent reading peers. Yet these will often be the children that are given texts with greater levels of illustrations in order to support and motivate them.

There has been an increase in the number and classification of illustrations used in textbooks over the recent years and English textbooks contain more distracters than those from other countries. Authors and illustrators rarely meet and in a competitive market the 'fun factor' of a textbook appears to be an important influence when
teachers select textbooks to use with children. Although the illustrator may intend to provide an accurate representation of reality, if the meaning of the illustration is not adequately explicit not all children will grasp the intended meaning.

Cognitive and learning styles have been seen to be the modes through which people can organise and process information. Those who work with a wholist style will see the whole of a picture or situation and in doing so may be unable to identify the finer detail. On the other hand, someone who favours the analytic style is able to focus on component parts but has difficulty understanding the whole picture. As well as these two aspects there is also that of if someone is able to organise and process information better from either reading, images, being told or by doing something kinaesthetically. A plausible assumption may be that those with an imager style would be at a greater advantage that those who tend to learn best from a kinaesthetic style given the type of word problems the children will be presented with in this study. The academic debate on learning styles is still active despite the large commercial industry, especially in education, that has developed as a result of the research. There are questions about the validity of the research, whether the testing is suitable for an educational setting, and when and if children do develop a preferred learning style as many people have been shown to be multimodal, being influenced by more than one style.

## 2.9 - Research Questions

In the light of my pilot and the literature review I decided finally to focus the research on the following questions:

1. How important are illustrations in children's understanding of mathematical problems?
2. What is the significance of a child's reading and mathematical ability upon their use of illustrations to comprehend question meaning?
3. Does a child's cognitive and learning style influence their success in decoding illustrated mathematical material?

I turn now to discuss my research design and methods.

## Chapter 3 - Methodology

## 3.1 - Introduction

This chapter aims to inform the reader how the research took place and the accompanying problems that arose, associated not only with the chosen methods but with the practicalities of carrying out the research process. The research consisted of the following components:

1. An assessment of the effect of illustrations on mathematics problem-solving strategies.
2. An investigation into the effect reading ability has on mathematical problem solving.
3. Small focus groups involving in-depth examination of mathematics problem solving.
4. Two questionnaires ascertaining children's attitudes to mathematics word problems.
5. A survey to establish the most common mathematics textbooks in use in Primary schools

Component 1 is the main area of research with the other 4 components each providing evidence to support component 1 . The relationship between all these components is shown in the following diagram.


### 3.1.1 - The Main Investigation

The aim of this investigation was to find out whether school pupils' approaches to solving mathematical word problems changed depending on the illustrations that accompanied the problems. Three schools with which I had regular contact were selected and after obtaining the head teacher's permission in each case to carry out research in their school each one was provided with a set of question booklets containing mathematical word problems. Although the questions were the same in all cases, they differed in the illustrations used. The categories of illustrations used were:

- Essential - an element of the question was represented by the illustration and nowhere else in the text.
- Decorative - there was no useful information in the illustration - it merely served to fill the page.
- Negative decorative - the illustration displayed something that was in the text, but in an exaggerated form - for instance the question mentions 4 items but the picture shows 20 items.
- Related - the illustration shows items in the same quantity as described in the text
- No illustration - text only with no illustration shown

The data arising from the different questions in differing formats were analysed in order to find out whether there was a "best" way of presenting mathematical word problems and also whether there were any disadvantages to presenting the information in specific ways.

### 3.1.2 - The Effect of Reading Ability

In this part of the research the reading ability of the children was taken into account and comparisons made between poor and good readers. The premise for this part of the investigation is that although the mathematics questions were illustrated they were largely text based, therefore better readers should be more able to discern the underlying mathematical operations and data from the question than poorer readers, all other things being equal.

### 3.1.3 - The Value of Establishing Learning Styles In Young Children

The concept of learning styles still has strong currency within the sphere of LEA advisors and Ofsted inspectors although current education research has raised a great deal of doubt over the concept and the instruments used. In theory though, if learning styles are determinable for young children and only then if the learning styles are such as to be relevant to decoding or relating to illustrations, then some children would benefit more from using illustrated mathematical problems than others. An instrument was therefore used to determine the learning styles of the children in the sample, and a comparison made between their results to determine whether the learning style had any effect on their ability to solve mathematical word problems.

### 3.1.4 - Small Focus Groups

Since young children are complex individuals, rather than just using the data from the larger scale investigation I carried out some small group discussions working through the answers children had given to some of the mathematical word problems in an attempt to find out how these children approached the question and which parts they found easy or confusing.

### 3.1.5 - Questionnaires

Two different questionnaires were used. The first was devised to augment the focus groups and to find out what children felt about the questions they had been asked to attempt - whether they were easy or hard, which ones they liked best or least and which question they felt would be best for a colleague who found mathematics difficult to attempt. The second asked children who had not taken part in the main investigation to comment on the Challenge booklet questions in their various forms and decide which form they would find most helpful in answering the questions.

### 3.1.6 - Textbook Investigation

Any research involving current classroom practice has to be related directly to what goes on in the classroom, not only in my school, but in other similar schools. Since the main focus of the research was the use of illustrations it seemed important to
examine the most widely used mathematics textbooks in order to see how they were illustrated and determine whether the forms of illustration they used either aided or hindered the pupils in their mathematical work.

## 3.2 - Research methodologies

The research used a number of instruments to ensure that there would be a range of quantitative and qualitative data that might triangulate and support evidence from the different sources. As can be seen from Table 8, the greater part of the evidence came from the Challenge question booklet which was used by pupils in all three schools. Using data from this booklet I was subsequently able to produce sub groups for the reading ability comparison and the VARK (Learning Styles) study following the VARK questionnaire. The children who took part in the textbook focus work and interviews were members of the Challenge booklet group from one school.

|  | Number of <br> children who <br> completed <br> Challenge <br> question <br> booklet and <br> questionnaire | Number of <br> children <br> involved in <br> alternative <br> question study | Number of <br> children <br> involved in <br> textbook focus <br> work and <br> interviews | Number of <br> children <br> involved in <br> Reading <br> ability <br> comparison | Number of <br> children <br> involved in <br> VARK study |
| :--- | :--- | :--- | :--- | :--- | :---: |
| School A | 79 | 0 | 0 | 45 | 32 |
| School B | 21 | 0 | 0 | 14 | 21 |
| School C | 28 | 26 | 9 | 14 | 28 |
| Total | 128 | 26 | 9 | 73 | 81 |

Table 8 Breakdown of pupil numbers by school

I had intended that I would follow up and interview children in all three schools once I had done some basic analysis of the Challenge booklet. Unfortunately, due to circumstances in my own school I was unable to do this. I was concerned that only being able to interview children I knew well and who might be influenced by the fact that I was their class teacher may have skewed the results obtained but it was not clear whether this would have a positive or a negative effect. On the one hand I might get more information from them, but on the other they might have provided the answers they thought I wanted to hear.

## 3.3 - Participant selection

The children taking part in the project were taken from three primary schools. The schools were largely self selecting via my current or previous links. One was my current school, another a school I had recently left and the final one being the school in which my sister teaches. Despite being in three different Local Education Authorities, the socio-economic catchment areas were very similar, each containing a mixture of children from professional and farming households. In two schools pupil numbers were smaller than the average whilst the third was larger than the average, being situated on the edge of a market town. Having children from very similar schools meant that a cross section of socio-economic classes was not a major feature of the study as there were no children from very poor socio-economic backgrounds. Obviously, social background still matters, even if you don't have a whole range, but if an obvious socio-economic contrast had occurred it may have added another aspect to the project, moving the focus to one where social background was of more relevance.

It was an advantage that I had direct contact with the schools involved because there were few opportunities for me to visit the other schools given my full-time teaching commitments. I was therefore reliant for data collection on the teachers in the schools I had chosen but because I knew them personally I was confident that they would follow my guidance precisely. This was an issue because it was important that all the subjects involved were treated as equally as possible and that pupils would not be given extra help or particular pupils excluded depending on the teacher's whim.

Such is the current pressure of Standardised testing in years 2 and 6, that the focus on educational development in English schools has become centred around these years with many government initiatives specifically designed to improve grades in these year groups. As a consequence, educational development in years 3 and 4 has been left somewhat in the wilderness, and seemingly of less importance. The study took place when the children were at the end of Year 3. It was correctly assumed that these children had, through three years at school, developed their basic arithmetical skills of addition, subtraction, multiplication and division. It also allowed time for the supposed dip in children's progress that occurs at the beginning of year 3 to level out
and for subsequent progress to be made. I have observed, in my own teaching, that a great deal of development takes place in these years and have always had a personal interest in this year group which is one reason why I decided to focus on Year 3 in this study. A more practical and over-riding consideration was that my own class at that time contained Year 3 children.

Children who had been identified as being on the Special Educational Needs (SEN) register for educational reasons beyond that of School Action Plus were excluded from taking part in the study. I was of the opinion that the work the children were to attempt was likely to have been too difficult for low ability children and may have distressed them.

### 3.3.1-SATs

As the study was conducted during the summer term, all the children involved had recently completed the Optional Statutory Tests for Year 3. This provided a measure of their current educational achievement. I was aware from information given at Inset training, that a third of all statutory Year 6 tests can give a false indication of the child's level, due to over or under performance on that day on those specific tasks. It therefore seemed likely that the Optional tests for Year 3 would be similarly flawed and would only give an estimated level of their current ability. In consultation with the teachers involved, I had investigated the use of other test measures which may have given a more accurate level of a child's reading and mathematics ability. However, no two schools were using the same two tests. In reading, one school was using the Salford reading test, another an NFER test and the third was using the Neale Analysis of Reading Ability. A similar picture emerged with mathematics with one school using a home made version of the one minute test, another the Young's assessment and the third was using NFER Progress in Maths. Each of these different tests for reading and mathematics can be considered to be more reliable on an individual basis than SATs, but there was no evidence to suggest that their scores and conclusions about a child's ability, whether in terms of standardised scores or reading or mathematics age were comparable. Not wishing to overload teachers or children with more tests than necessary I opted to use the information from the optional SATs along with individual teacher assessment.

One of the difficulties with using optional SATs levels is the arbitrary way in which one mark can make the difference between a child being awarded one level from another. Tennent et.al. (2006) investigated the way the differences in 11 year old children's inference ability varied for those children who had been awarded a Level 4 in the end of Key Stage 2 Reading SATs. Level 4 is the expected level of the average child at the end of Key Stage 2. It was noted how many children who were awarded a low level 4 had poor inference skills, similar to those children one would expect from a child being awarded a high Level 3. It is this area between the top end of one level and the lower end of the next level where inconsistencies can occur when a generalised national Curriculum Level such as Level 3 or 4 is used. The advantage of the $A, B$, and $C$ sub level system, where $A$ is the highest level and $C$ the lowest, is the wide range of skills and understanding that may be achieved within one particular National Curriculum level, can be subdivided to take into account this wide range of attainment that can be seen within one level. There may well be some inconsistencies between those children whose marks place them on the border between one sublevel and the next, but this will be much reduced in comparison with the boundary between one whole level and the next. Hence the sub levelling of the optional Year 3 SATs gave a reasonably accurate measure of a child's previous attainment in reading and mathematics. The optional SATs of this age group do not provide sublevels for those children deemed to be assessed as attaining Level 4. This is a potential weakness but as Tennent et al (2006) have suggested, and considering their age, these children are likely to be working at the lower end of level 4.

I had asked the teachers involved to indicate if the test results were significantly different from their Teacher Assessment and for those few children where this occurred they were excluded from the study. Therefore, an assumption has been made that the SATs results were a reasonably valid measure of the child's ability. Any child who was working below level 2 was excluded from the study as the work was likely to be too challenging and inappropriate for their educational needs. This resulted in a sample size of 128 children across the three schools. The investigation required that the children be assigned different coloured booklets, which is explained in greater detail later in this chapter but the group could also be sub-classified by using their Mathematics and Reading SATS results as shown in Tables 9 and 10.

| Booklet <br> Colour | 2 C | 2 B | 2 A | 3 C | 3 B | 3 A | 4 | Total |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Red | 0 | 0 | 1 | 5 | 3 | 6 | 10 | 25 |
| Blue | 0 | 2 | 4 | 3 | 3 | 7 | 9 | 28 |
| Green | 1 | 2 | 0 | 4 | 5 | 5 | 8 | 25 |
| Yellow | 0 | 2 | 0 | 3 | 4 | 6 | 10 | 25 |
| Purple | 3 | 0 | 2 | 3 | 1 | 7 | 9 | 25 |
| Total | 4 | 6 | 7 | 18 | 16 | 31 | 46 | 128 |

Table 9 Optional Year 3 SATs Reading Results for sample group (A represents the top end of the level)

| Booklet <br> Colour | 2 C | 2 B | 2 A | 3 C | 3 B | 3 A | 4 | Total |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Red | 0 | 2 | 4 | 5 | 8 | 2 | 4 | 25 |
| Blue | 1 | 1 | 2 | 6 | 10 | 5 | 3 | 28 |
| Green | 0 | 1 | 3 | 7 | 6 | 1 | 7 | 25 |
| Yellow | 0 | 0 | 4 | 9 | 4 | 4 | 4 | 25 |
| Purple | 0 | 1 | 4 | 7 | 3 | 5 | 5 | 25 |
| Total | 1 | 5 | 17 | 34 | 31 | 17 | 23 | 128 |

Table 10 Optional Year 3 SATs Mathematics Results for sample group (A represents the top end of the level)

## 3.4 - Researcher role

The problems of conducting research in one's own classroom using the children you teach are manifold. Due to the teacher pupil relationship there may have been a greater tendency for the children to behave in a manner they assumed would please me and equally my interpretation of their actions may have been clouded by my knowledge of them. It was for these reasons that when formulating the research plan I thought it important to look beyond my classroom and to use evidence from other schools. This would not only provide me with more quantitative data but would also minimalise the risk of data being biased by the pupil/teacher relationship.

Permission to carry out the research had been obtained from the headteachers of all three schools. This was a verbal agreement but when I spoke to the individual class teachers about the project they each signed a consent form. Informed consent can be something of a misnomer in education research in that permission is given by someone on behalf of the children rather than by the children themselves. Consent is therefore assumed rather than informed. The children who completed the Challenge Booklet were informed that the work they were being asked to do was "to find out if
we can make maths easier to understand and that illustrations were a feature of the investigation".

As the research progressed, it became increasingly obvious that any qualitative data would come from my own class, bringing into greater play some particular ethical issues of education research. The question of how much to divulge to the children about the research was difficult because their knowing the prime objective may have influenced them to act in a manner that was not part of their normal behaviour. Conversely, it could be argued that the research was not about the children but about an aspect of learning and by informing the children completely, the children may have behaved in a way that gave a greater insight to the problem. On the subject of the researcher being the teacher, Homan (2002) states that "it may be that an attitude of trust or loyalty or a climate of reciprocal favour is made to apply in the giving of consent. Pupils may overcome apprehension or misgivings only because in other situations the teacher has secured their trust. Students or parents of pupils may be disinclined to refuse to participate for fear of losing favour or gaining a reputation for being uncooperative" (Homan, 2002:37). As I was using standard classroom materials I was able to incorporate these into the overall teaching plan for the class. I do not believe that the education of the children was affected in a negative way. If anything, the research was a positive experience for the children and myself as I gained a greater understanding of their thought processes. The ethical issue of using human subjects whose education may be compromised by the research is one that needs consideration and one I had to balance between my teaching and research responsibilities.

Having contact with the children throughout their primary years could have enabled me to access other children as they moved through the school so that another group of Year 3 children could have been included the following year. I decided though, that I would gain enough data from the sample I had and therefore did not need to carry out the investigation with more pupils.

### 3.4.1-My Context

When I began this project I was working as the Deputy Headteacher in a small rural primary school. It had been agreed that I would be allowed two days per half-term dedicated to my research project and any other time I could elicit as part of my management time which equated to half a day per week. It was on this basis that I planned the project as, apart from the distance, there would be no problem visiting the other schools involved. In the time between the September and the following May I planned out the project and began to collect data for the background and textbook analyses. After May, the children had completed their optional SATs and following this the selected children completed the Challenge Booklet. Since school holidays in the three local authorities coincided, the two days I was allocated were essential in order to allow me to visit the other schools to interview the children involved. Unfortunately it was at this time that budget cuts in my own school resulted in my management time and the two research days being cut. This meant that I had no time within the term to follow up the findings from the Challenge Booklet anywhere except in my own school.

In the following academic year I commenced study for the National Professional Qualification for Headship (NPQH). I had been asked to do this qualification previously but because of my research commitments I felt it would conflict with the limited professional development time I had. At the introduction meeting when I mentioned the research project I was told to "stop doing that because it won't do you any good and it's not valued". Since it was of value to me I continued and insisted that it be included as a target as part of my Performance Management in school. However, even then, during my review meetings I was asked if "I was still bothering with that thing". I was finding it very difficult to fit the research alongside my teaching commitments which meant that not only was I unable to visit the other schools but it was also taking longer than expected to analyse the data. That year also saw all teaching staff, including myself, served with redundancy notices. A change in Key Stage 1 numbers as the children moved to Key Stage 2 had resulted in a substantial loss of funding and the school was forced to reduce the teaching staff by one member. This was duly done but meant that my class size increased to thirty-six
children with a concomitant increase in the time I needed to spend on my classroom duties.

The following Easter saw the retirement of the serving Headteacher and as Deputy Head I was given the post of acting Head. I still had a classroom responsibility and was teaching 0.6. In the following summer I was appointed as Headteacher and although I had no classroom responsibility I was still teaching 0.5 across the school. The Governing Body agreed that I could have one day per month for my research but only if this did not conflict with my Headteacher duties. As I have no Deputy or Senior Management Team this, in effect, was a hollow promise.

What these events have all resulted in, is that once the research project was in motion I was unable to follow my original plan, which resulted in some aspects only being able to be conducted in my own school and then often at the same time as I was teaching resulting in the project taking far longer than anticipated. I also feel that it is a sad reflection of the lack of importance research as professional development has in schools.

The classroom teacher can have the greatest impact upon a child's education and is invariably in contact with the child on a daily basis as they learn and develop, a good teacher will observe and question what a child is doing. To do this effectively the teacher must question the practice that occurs in their own classroom and as such should be an integral part of education research and development. It is my belief that the influences that have an impact upon classroom practice and ultimately a child's education, come from government, research from academic institutions and classroom practitioners and this partnership should be equal. However, at the moment I would suggest that classroom practitioner research is held in low regard and a change needs to take place within the school and system itself but also from the other members of the partnership (governments and academics) to give teacher research the status it deserves.

## 3.5 - Instruments used to gather data

I decided to use authentic mathematical materials that the children were likely to encounter in any primary school classroom. The reason for this was that the inspiration for the study originated from my observation of children using such materials in the classroom indicating that children were already using illustrations as a source of mathematical information to aid their calculations. As a result of findings in the trial (Chapter 1), it highlighted inappropriate use of illustrations and I had wondered just how widespread the problem actually was. Using anything other than authentic learning materials would have compromised the value of the study.

At an LEA numeracy conference I had the opportunity to ask the delegates if they used the mathematics challenges document and if so, whether they were used with all the class or just the most able. This was an impromptu data collection exercise, as I took advantage of being in the right place at the right time, being able to address the delegates briefly having asked permission from the organisers on the day. There were approximately one hundred delegates and a show of hands indicated that all used the challenges across the whole range of ability groups, selecting challenges they deemed appropriate to the children's ability and experience. Although unplanned, this did confirm my notion that the use of the challenges document was something that was not confined to the most able but used with a full range of ability levels. As such, I concluded the challenges to be an authentic teaching and learning material and a worthy topic for research. It was at this point that the materials for research divided into two strands - textbook use and the challenge problems.

My own experience indicated that two main sources of written mathematics were in use; standard textbooks and a more recent government document of the time, Mathematical challenges for able pupils in Key Stages 1 and 2(2000). The introduction of the Numeracy Strategy and greater use of ICT, meant that teachers were being discouraged from using textbooks, yet they still appeared to be a feature of classroom practice. However there remains a wealth of publishers of mathematical textbooks so one of the first questions I needed to answer was how widespread textbook use was and which textbooks were in current use. The use of the Mathematical challenges material was slightly more problematical because as the title
suggests, the problems were aimed at able pupils. In the book the challenges are divided into three sections; those suggested as suitable for Years 1 and 2, Years 3 and 4 and finally Years 5 and 6. In my own classroom practice I used problems from all three sections, although principally the first section aimed at Years 1 and 2. Indications were such that this type of problem solving would become a more frequent feature of testing materials, so it appeared unfair to deprive a substantial number of children who were not considered to be from the more able group the opportunity to encounter these types of non-routine problems.

### 3.5.1 - Mathematical challenges

From the literature review it became clear that illustrations can be classified in a number of ways and not all typologies seemed appropriate or particularly easy to use. I therefore decided upon an expansion of the work conducted by Santos-Bernard, (1997) who worked with the principle of relevant and cosmetic illustrations and the more influential work conducted by Shuard and Rothery, (1984) who categorised illustrations into three types; decorative, related and essential. Ultimately I elected to divide the illustrations into four categories because I felt that Shuard and Rothery had missed an important type of illustration. The four categories were;

- Essential - In this category some information essential to the question is found in the illustration which is not found in the text. The child therefore needs to 'read' the visual material with close attention along with the text although it may not always be clear which part of the illustration is essential, especially if it also contains decorative aspects.
- Related - In this category the illustration repeats or emphasises information stated in the text. It may increase the ways in which the reader can extract meaning from the text as the visual clue embodies the information and reinforces aspects of these abstract problems. Like the essential group, the reader has to be able to focus on the related information and not be distracted by decorative elements.
> - Decorative - These illustrations serve no instructional purpose and their function is solely to make the work more attractive to read by lightening the text or breaking up the information.
- Negative Decorative - This is a category I used in addition to those recorded in the literature. Like the decorative illustration they serve no instructional purpose but the illustrator tends to show an excessive number of those elements that may be essential or related to the problem solving. For example, when the text states how many coins are used as part of the problem the illustration shows a number of coins in excess of the number stated in the text. These illustrations can have a negative effect if the reader gives greater credence to the illustration rather than to the text.

Having decided upon a categorisation format I then applied this to all the mathematics challenges in sections 1 and 2 (those suitable for years 1,2,3 and 4). Alongside the illustration I also determined which mathematical concept was being tested. I decided not to use examples from the Year 5 and 6 section because from the initial trial, few children had been able to complete the questions and as I was giving the questions to Year 3 children with a range of abilities (not just the most able) I wanted everyone to have some chance of accessing the mathematics. Despite what appeared to be very clear criteria, I did find a few examples where it was difficult to decide which category the illustration related to. This highlighted a possible weakness in the method, in that it was my interpretation of the question, rather than using an impartial formula for categorisation.

As the focus for the study was number operations, those dealing with data handling or shape were discounted. Of the remaining questions, I selected three from Years 1 and 2 section and six from the Year 3 and 4 section. Part of the decision was based upon how easy it would be to alter the presentation of the question to ensure that a variety of illustration types could be used for any particular question. Having categorised the illustrations into four groups I wanted to be able to present each chosen question in each of the four ways to different children. I was also making the assumption that the illustration had an effect on the child's ability to tackle the problem so I included another question type where no illustration was included. As it was the effect of the
illustration I was hoping to focus upon, the basic question format and text was not altered except to ensure the illustration category matched the text information. To have changed the text as well as the illustration would have meant that another variable was incorporated which would have made analysis of the answers confusing as to whether the text or illustration was having the greatest effect upon a child's comprehension and subsequent calculation.

Of the total of nine questions, five questions were selected for alteration. For each of these five questions there would be five different versions (essential, related, decorative, negative decorative and no illustration). The other four questions were not altered and acted as control questions to be given to all children. The four control questions were an example of an essential, related, decorative and negative decorative illustration. No control question was included that had no illustration. I felt that nine of the challenges were sufficient for the children to deal with in the allotted time. Having selected the questions and altered those as necessary I composed five different booklets. The sequence of the questions was the same in each booklet and began with those from section 1 as I hoped these easier questions would give the children confidence before they tackled the more complex challenges from section 2. Each booklet type was given a different colour front page so that the teacher was able to organise the booklets so that children who had the same question format would not sit next to each other. The booklet colours were blue, red, yellow, green and purple (Appendix 1).

|  | Snakes <br> and <br> Ladders <br> Control <br> Question | Gold Bars | Fireworks | Roly <br> Poly <br> Control <br> Question | Spaceship | Ski Lift <br> Control <br> Question | Queen <br> Esmerelda's <br> Coins | Duck <br> Ponds | Three <br> Monkeys <br> Control <br> Question |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Blue | Essential | Negative <br> Decorative | Decorative | Related | Related | Decorative | Essential | No Picture | Negative <br> Decorative |
| Red | Essential | Decorative | Related | Related | Essential | Decorative | No Picture | Negative <br> Decorative | Negative <br> Decorative |
| Yellow | Essential | Essential | No Picture | Related | Negative <br> Decorative | Decorative | Decorative | Related | Negative <br> Decorative |
| Green | Essential | Related | Essential | Related | No Picture | Decorative | Negative <br> Decorative | Decorative | Negative <br> Decorative |
| Purple | Essential | No <br> Picture | Negative <br> Decorative | Related | Decorative | Decorative | Related | Essential | Negative <br> Decorative |

Table 11 Question type and booklet colour

Having devised and printed the booklets I met with each of the teachers who would be administering the test. I asked them to treat it as a formal test as I wanted to minimise the likelihood of the children copying or working collaboratively. Children were given two forty minute sessions to complete the booklets. Asking the children to complete the booklet in one session was too much, but I was aware that there was the possibility that children may have discussed the questions in the intervening time. I suggested that the sessions occurred either on the following day or two days later. The problem with timed tests is that some participants may display anxiety solely due to the timed element. In work conducted with adults on mathematical tasks, Walen and Williams found that the "anxiety produced by not being able to meet the demands of an external time frame focused their attention away from the mathematics even more; time became the 'hot' issue...It was not the mathematics, but rather imposition of a foreign temporal structure over their mathematical knowing, that led to their discomfort and eventual paralysis in mathematical testing situations. "(Walen and Williams 2002:375). Despite this, I felt it was better to treat the booklet as a test, for issues mentioned previously and the difficulty in classroom management it could create if some children were working on the booklet whilst others were working on alternative tasks. By giving the teacher's specific instructions as regards the administration of the booklets as a test, I felt this would ensure greater consistency of teacher input between the three schools. I also believe that children understand that in a test situation it is 'against the rules' to talk or collaborate in any way. If children, irrespective of the booklet colour, hence illustration type had collaborated, the results would have had to be considered to be invalid.

I had decided on forty minutes duration for the test following two trial sessions with a group of Year 4 pupils where some were given thirty minutes and others an hour. Thirty minutes appeared too short because it made the children anxious about completing the booklet in time whilst those having the longer time started to become bored and appeared to lose concentration. Given a set amount of time would also help with the reliability of the data. If different children in different schools had been given different times in order to complete the booklet it would add another variable of time to the analysis where some children may have been given limited time and others unlimited. There appeared to be little problem with the layout of the booklet or the questions. Since one question was later to prove extremely difficult for the Year 3
children it may have been better to have trialled the booklets with Year 3 children, but time and access to another group was problematical.

Once they had completed the booklet, the children were asked to complete a questionnaire which focused on three questions (Appendix 2). This was a simple questionnaire which had been based upon a questionnaire used by Santos-Bernard (1997) in her research. Children were asked to explain which question they preferred and why. The intention of this question was to establish whether there were certain questions the children responded to better as a group than others and if this was related to the type of illustration. The second question asked them to consider which question would they think would be suited to someone who found mathematics difficult. By putting the question through a third person I hoped to establish the type of question and illustration most suited to mathematic problem solving. The final question related directly to the pictures, asking the children to explain what they liked or found useful about the pictures. This was intended to focus the children's attention directly on the illustrations.

I had considered using a questionnaire based upon a Likert scale which would have given me further quantitive data, but I felt this would not provide me with the detailed qualitative information I felt was necessary. Therefore I decided to ask children to explain their reasoning in detail as it would form the basis for follow up interviews. Due to the difficulties I later encountered in being able to interview children, this qualitative data acquired from the questionnaire was extremely valuable being something I don't believe would have occurred from a questionnaire which produced quantitive data. For small children, explaining things in detail is difficult, hence the limit of three questions that addressed the most important aspects of the study. The wording of the questions was based upon that used by Santos-Bernard (1997).

Following this, I conducted a questionnaire with a different group of children from those who had completed the Challenge Booklet. These children came from my own school and were from Years 2 and 4. In this the children were presented with each of the five questions (Appendix 3) that had been altered with each type being seen at the same time. They were told that the questions were basically the same, having the same problem and answer, but each had a different picture to go with the text. They were asked to look carefully at each of the questions and decide which format they
thought would enable them to complete the question. Although I did not seek this information, all the children also commented on which illustration they felt was 'the worst'. I would have liked to again use Year 3 children but circumstances prevented this and I did not want to use the same children who had completed the booklet in case they were influenced by their experience. However, it could also be argued that their experience of answering the questions could have made them better informed and therefore a good source of information about question design.

When all the booklets were returned I had intended to select particular children and interview them in greater detail about the questions and their answers, going through their reasoning with them. Unfortunately cutbacks in the school budget meant that supply cover was not available and I no longer had any time out of my classroom. This prevented me from visiting the other schools as planned and meant that due to my now full timetable teaching commitments it took far longer than expected to carry out a basic analysis of the booklets. This meant that by the time I had selected those answers of interest, the children involved would have forgotten their thinking process at the time. Also I was now limited to using only children in my own class whilst also teaching, which considerably reduced the sample size. Only being able to work with children from my own class meant that my consideration of the children's responses may have been influenced by the teacher/pupil relationship. This was a disappointing moment in the research. However as an alternative I followed the example by Cooper and Harries (2005) and asked children to comment on another child's work. Within these discussion groups there was the child whose work was being discussed even though they were unaware of this. In Cooper and Harries' research children were presented with another child's answers and asked to explain how they might have been found. I followed a similar procedure. I was able to do this with a small group of children in my class.

### 3.5.2 - Textbook study

In order to answer the question of what textbooks were in use, how they were used and how frequently they are used, I sent out a questionnaire (Appendix 4) to fifty schools in my own Local Education Authority (LEA). The LEA is divided into four regions and I randomly chose either twelve or thirteen schools from each region using
the LEA directory. For each region I tried to ensure that a mixture of school types was present - rural versus urban; varying role numbers of large and small schools; faith and secular schools. In my accompanying letter I explained the nature of the survey and that being a full time teacher myself I had constructed the questionnaire to take a minimal amount of time. Accompanying this plea for professional sympathy I included a stamped addressed envelope.

Of the fifty requests sent out, thirty five were returned. Following this high return rate I wrote a letter to all the schools thanking them for their response. Since the schools were totally anonymous I had no idea which schools had in fact returned the forms, so sent my thank you letter to all fifty schools. This resulted in a few consciences being pricked and a further four were duly returned. In total I received thirty nine replies (78\%).

The questionnaire was divided into three parts and focused on the number operations of addition, subtraction, multiplication and division for Year 3. Respondents were asked to indicate which textbook(s) they most readily used for number work, how frequently it was used in a week and what prompted them to use that textbook (Appendix 4). The introduction of the Numeracy Strategy dictated the format of the numeracy lesson in that the lesson began with a whole class activity followed by individual or small group work before returning to a whole class plenary. It is during the twenty minute individual or group work that textbooks would be used. Therefore I was fairly confident that there would be a consistency of textbook use in the classroom.

The results indicated that two main textbooks were in use - Ginn Abacus and the New Heinemann Mathematics schemes. Having confirmed the overwhelming use of these books I decided to analyse the use of illustrations within the books. Like the Challenges book I focused on those questions which dealt with basic number operations of addition, subtraction, multiplication and division. Any question which dealt with the additional concept of money or measurement was ignored. The analysis used the following criteria.

| Page (for ease of referral) |
| :--- |
| Concept (addition, subtraction, multiplication or division) |
| Number of parts (A question was often divided into parts e.g. a, b, c etc. although each was a <br> separate sum |
| Number of words in the general introductory instruction |
| Number of words per part |
| Illustration (if the illustration was essential, related, decorative, negative decorative or no <br> illustration present) |
| Is illustration related to concept |
| RJ view of ease of understanding (my own view on how easy it was for the mathematics to be <br> identified) |

Table 12 Textbook Analysis Criteria

The last three points were open to interpretation, but they did allow me greater understanding when deciding which questions to use from the textbook. I particularly wanted illustrations to be the focus rather than just the mathematics. As Campbell, (1981) has highlighted, adults have less difficulty reading illustrations than children, so it appears justifiable that as an adult, if I found an illustration difficult to interpret a child would be likely to find it even more difficult. The textbooks differed slightly from the Challenge Booklet in that many of their illustrations acted as a canvas on which the maths was written rather than a separate illustration allied to the question. This required the creation of another category (Decorative Background) where the overall illustration of the page provided essential information to the solution of each individual problem.

Initially I had planned to select a number of questions from each scheme and work with children from each of the schools, taping interviews with children as they completed the questions. However, due to my teaching commitments this had to be abandoned and I was only able to work with children in my own class. I decided not to adapt the questions as I had done with the challenges questions because my artistic skills are not that good and I was aware that there may be copyright implications. This experience of questioning the children as they worked through the questions made their thought processes clearer to me, something I had not been able to do with the Challenge Booklet questions. The two methods were complimentary in that the Challenge Booklet gave me much more quantitative data whereas this textbook process gave greater qualitative data.

## 3.6 - VARK

As stated in the literature review, VARK is a method of establishing an individual's preferred way of accessing information. The questions and analysis procedure are well established and therefore were not altered (Appendix 5). This activity was conducted later in the research process and not all children completed the questionnaire as I had lost contact with the staff in one of the schools and a number of children had moved away. Not only did time constraints prevent the completion of the questionnaire when the main research was taking place but I felt the nature of the questions were not particularly relevant to young children despite the title indicating its suitability ("The VARK questionnaire - for younger people"). Although taking place only a few months later (in the following autumn term when the children had moved to Year 4 and possibly a new teacher) it must be recognised that the children are likely to have developed in the intervening time so the results may have been compromised by this.

Of the 128 children in the total sample, I was only able to get 81 VARK returns. Depending upon the results a person may show a strong indication for one style or be multimodal - in other words there is no significant difference between a preference for two, three, or four styles. Surprisingly, all the children's results indicated a multimodal result, with three children exhibiting three main styles and the remaining having all four styles. My interpretation of this was that either the children have not yet developed sufficiently to have an identifiable style, that the questionnaire was invalid for children of this age group, or that the concept of learning styles is a misnomer. These results gave me no clear cut way of defining their learning style for comparative analysis.

Because the children's results had indicated a multimodal style I decided that in order to distinguish between them I would focus on the area where they scored highest even though the difference between their highest and next highest preferences was only a matter of one or two points. This resulted in 24 children showing a visual preference, 26 an aural preference, 14 a reading preference and 17 a kinaesthetic preference. The hypothesis would be that those children who indicated a visual preference would focus more on the illustrations, so the questions which had a
decorative or negative decorative illustration should have misled them greater than the other children all things being equal. Because the sample size in this group was small (81), I only focused on those questions where a substantial number (at least $50 \%$ ) of children achieved a correct answer. However upon analysis a picture emerged that indicates that there was no significant difference in performance between each of the VARK groups.

It has to be acknowledged that the sample size is small but there are concerns with the data and the way in which the information was collected as well as the premise of cognitive and learning styles. The VARK questionnaire may not have been valid for this age group of children, and as they indicated that they were all multimodal I elected to focus on their primary learning style, even though the results suggested that this was not a strongly preferred style These are weaknesses of the research which cannot be improved upon in this particular project. However if we were to consider that the data does have some validity, it would appear to indicate that children at this age have not established a dominant cognitive learning style.

As a classroom practitioner I am aware of the high emphasis cognitive, and in particular learning styles, is being given by those who influence classroom practice, namely government bodies and LEA advisors. I had to conclude from the results of this small study and the literature review that I found no evidence that cognitive and learning styles, if they in fact exist, are an important factor for an individual's performance when solving mathematical problems of this nature. I therefore decided that the results of the VARK investigation would not be used further within the research project.

## 3.7 - Validity of the data collection

Any research must be shown to be valid. This means that it should be shown to measure what it claims to do. Internal validity refers to the results within the study and usually concerns causality. As the focus of the study was to elicit whether children's mathematical performance was affected by different types of illustrations, changing the illustrations for the challenges questions was an appropriate way in which to tests this thesis. As much as was possible the same language and illustration
style was used for each of the different question versions. This meant that the presentation of the versions varied only by the detail in the various illustrations. The essential illustration always contained some information that was crucial to the question and could only be found in the illustration. Usually it was the only way the number of items required to answer the question could be ascertained. The related illustrations reaffirmed essential information that could also be found in the text. The decorative illustration showed an example of an item similar to that in the question whilst the negative decorative showed a number in excess of the actual number of items identified in the text. The question with no illustration used the same wording as the decorative and negative decorative questions because text was the only clue to the mathematical requirements of the question. The Duck Pond question caused difficulty in matching these criteria and is discussed further in the analysis of the results. The illustration criteria, although reliant solely on my interpretation, were sufficiently clear to make the necessary judgements for the characterisation of the illustrations. As the sample size was relatively small, I used four control questions so that for each type of illustration there would be a larger number for analysis. The difference in mathematical difficulty of the individual questions may have reduced the validity of the research because the difficulty of the question itself may have had a greater impact on performance than the effect of the illustration.

> I believe that using different children to comment when selecting which type of question from all five types (Appendix 3), was justified, because first impressions were important in understanding how influential an illustration is on the initial understanding of a question. I considered that if the same children who had completed the questions had done this exercise they may have been unduly influenced by their experience when answering the questions. Although as stated earlier, their experience might have made them better judges of appropriate question design.

The selection of which textbook questions to use was more difficult because I deliberately selected questions that appeared problematic. Because of this, I always presented the children with less complex questions prior to the focus question and always in the context of our numeracy curriculum. As the children who took part in the textbook investigation also took part in the challenges booklet and 'post test' interviews, this formed a triangulation of evidence.

Gathering data by audio-tape can be difficult due to acoustic problems and the difficulty in interpreting the evidence reliably. The effects of the teacher/pupil relationship could have an effect on the outcome but, as Hammersley, (1993:229) indicates "the teacher is more able to interpret a tape than a stranger is, given an adequate degree of self-critical awareness" As there was no alternative adult available to interview the children it was inevitable that the teacher/pupil relationship influenced the interviews. Being aware of this, when interviewing the children, I aimed to pose questions that would allow the child to expand their explanation but not lead them down a particular path.

The teacher questionnaire on textbook use was helpful because it gave me information on how widespread the use of textbooks was in schools. In the absence of this, as a practising teacher I may have made unfair assumptions about the use of textbooks based on my own classroom practice. The children's questionnaire gave the children an opportunity to express their opinions beyond the evidence of their answers to the items alone. Their answers may have been compromised due to poor writing skills or an inability to expand and express their ideas adequately. However I do consider this was more informative than a Likert questionnaire which was unlikely to have given the children opportunities to express fully their ideas. The VARK questionnaire, by its reputation should have been a valid instrument for determining preferred style for accessing information. Its use may have been compromised by being delivered later to the group with not all the sample completing it as well as an unknown number of children who tried to manipulate their answers.

In consideration of external validity there are two factors to bear in mind. One being the range of abilities of the children involved and the other being the socio-economic make-up of the sample. In consideration of the ability range of the children in the study, the SATs results for mathematics and reading indicate that a wide range of abilities was present in the sample as would be expected for this age of children. However children with identified Special Education Needs were not included and any further study would have to take this into account.

The schools that took part in the study were partly self-selecting and although each class included children from a range of socio-economic backgrounds, they were not representative of society as a whole. None of the schools were in urban environments or essentially poor economic households. The work by Cooper and Dunne, (2000) indicates that socio-economic status can have an effect upon children's understanding and success in mathematical word problems, especially those that contain an element of realism, which is an important element of the questions in this study. Because of this importance of socio-economic effect across the social spectrum, the external validity of this study is inevitably limited by the nature of my samples.

## 3.8 - Reliability of the data collection

Reliability is where different researchers can repeat a piece of research and obtain precisely the same results. Although more likely to occur in the world of scientific research, such reliability is unlikely to be achieved in educational research because the mix of causal variables will be so different each time - the children and their experiences prior to and at the time of the research is just one aspect that could affect this. Using authentic, publicly available learning materials that have been judged suitable for the children at the end of Year 3 would imply that this research could be conducted by another researcher and whilst their results may be different I am confident a strong element of reliability would be achieved.

## 3.9 - Analysis of the data collected

The data collected from the Challenge Booklet and Textbook research was a mixture of quantitative and qualitative data which I feel complemented each other. The quantitative data gave descriptive validity whilst the in-depth qualitative data provided deeper insights to the patterns identified in the quantitative data.

### 3.9.1 - Quantitive analysis

All the quantitative data was entered into Microsoft Excel. Using this program different analyses were undertaken. Where the results from small group have been analysed the sample size does become very small which must be taken account of in
the interpretation of the results. To aid comparison, percentages have been used when presenting data from different sized groups. All percentages have been rounded to the nearest whole number which in some cases means that the totals do not always add up to 100 per cent.

One of my research questions was to examine the significance of a child's reading and mathematical ability upon their use of illustrations to comprehend question meaning. The quantitative data that the booklet work produced would be used to answer this question. The sample group was heavily weighted in favour of those children who could be considered to be competent readers. Of the 128 children in the sample, 77 children had a reading level of 3 A or 4 , which may be considered as above the average for this age group, whilst only 23 had a reading level of 2A or below (of which only five were 2 B and one 2 C ). In order to answer the research question I needed to find two groups whose results I could compare. An expected level for this age would be 3B.

The children did not necessarily achieve the same curriculum level in their reading and mathematics SATs results. In order to try to split the sample group I gave each level a nominal score ( 1 for $2 \mathrm{C}, 3$ for $2 \mathrm{~B}, 5$ for 2 A and so on, until level 4 equalled 13 points). This resulted in pupils scoring between 4 and 26 points, with the bulk of the pupils being in the higher points bracket. A line had to be chosen to divide these sets but in reality this dividing line was not clear cut. One answer was to divide them so that the children who scored 14 or less points would be considered as one group and anything over this as another group. Fourteen points could be achieved by scoring 3C in mathematics and reading. A level of 3 C can be considered to be in the lower average range for this age group therefore fourteen points would be a distinguishing mark between those children clearly achieving as expected from those who are working at the lower end of expectations. As can be seen on Table 13, there is a significant difference in the number of children who gained 16 points as opposed to 14 points which is also the median point.

| Points scored by combining SATS Maths and Reading Scores |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | Total |
| Blue Booklet |  | 2 |  | 1 | 2 | 3 | 4 | 2 | 4 | 2 | 6 | 2 | 28 |
| Green Booklet |  | 2 | 1 |  |  | 2 | 6 | 3 | 2 | 2 | 1 | 6 | 25 |
| Yellow Booklet |  |  |  |  | 2 | 4 | 4 | 2 | 3 | 2 | 5 | 3 | 25 |
| Red Booklet |  |  | 1 |  | 1 | 2 | 6 | 5 |  | 5 | 2 | 3 | 25 |
| Purple Booklet | 1 | 1 | 1 |  | 4 | 1 | 1 | 3 | 1 | 5 | 3 | 4 | 25 |
| Total | 1 | 5 | 3 | 1 | 9 | 12 | 21 | 15 | 10 | 16 | 17 | 18 | 128 |

Table 13 SATs level points related to booklet colour

The expected curriculum level for a child at the end of Year 3 would be 3B or above. Therefore it would make sense if the dividing line between the two proposed groups was at the 3 C point. However, since pupils did not always get the same level for each subject, the question arose of would it be their result in their mathematics or their reading from which the level would be taken? I found no satisfactory answer to this.

Following this I decided to look at the results of those children who gained a 2A, 2B or 2 C in reading and compared their mathematics result to see if it was above, equal or below their reading level. I then did the same comparison between the child's reading and mathematics results for all the other levels. A very definite pattern emerged as can be seen in Figure 1. Children who gained 3C or below in reading were more likely to have a mathematics level equal or higher to their reading level. Children of 3B and above in their reading were likely to have a lower level in mathematics.


Figure 1 Percentage of Pupil's Mathematics level compared with their reading level

The difference between the levels also became more pronounced the higher the reading level. Those who had been awarded a level 3 C or below in reading tended to have a mathematics level adjacent to their own. For example a child who had a 3C in reading may have a 2 A or 3 B in mathematics. The reading levels higher than 3 C had a much wider range. For example the child who achieved a 3 A in reading was more likely to have been awarded a 3 C or even a 2 A in mathematics. A child who achieved level 4 in reading may have got a 3 B in mathematics. It has to be acknowledged that level 4 was the ceiling in the Year 3 optional SATs, so no child could be recorded as having a higher level in mathematics if they were awarded a level 4 in reading. As discussed above, the marking scheme that awards these levels is partly arbitrary, in that one mark could make the difference between one level and another. Therefore the wider range in between the mathematics and reading levels at the higher end is even more significant.

Following this analysis and given that I had to split the groups using some type of criteria, it appeared reasonable to split the sample based upon the children's reading level at 3C. Those who were awarded 3C or below formed one group ( $\mathrm{n}=35$ ) whilst those who achieved 3B and above ( $\mathrm{n}=93$ ) formed another group. This still resulted in
uneven numbers in the two groups but reaffirmed the split point in the point score exercise above. For the 3C and below group there were six children from the red booklet group, nine from the blue, seven from the green, five from the yellow and eight from the purple booklet group.

### 3.9.2 - Qualitative analysis

The qualitative data investigation, interviews and observations, took place with individual or pairs of children. Except for my own name all other names were anonymised to minimise the possibility of individuals being identified. The remarks made by the children on the two questionnaires, gave an in-depth view to the children's understanding of how important illustrations are in children's understanding of mathematical problems.

In the analysis of the questionnaire I had considered using a software program such as Nvivo or Atlas.ti in an attempt to investigate trends arising from the children's answers, but the range of codes produced from the questionnaire was so limited that similarities and trends were self evident making the use of such software redundant.

### 3.10 - Summary

This chapter summarised the various elements of the research I carried out. I believed that it was important to examine the subject from a number of different viewpoints. This resulted in a number of qualitative and quantitative approaches involving both the main target group of pupils and others to provide corroboration. Generally the methodology worked well and apart from some minor changes from the pilot project to the main investigation it was substantially unaltered. The main problem that arose was a difficulty with making time for the investigation as a number of the elements were reliant on being examined within the same academic term. Changes to the staffing of the school where I work and consequent reduction in the time I had available for research purposes whilst I was in school seriously constrained some of the follow-up work I had hoped to do. I was able however, to work laterally and widen the scope to involve other pupils in the research which, with hindsight has proved to be a strength rather than a weakness. As will be seen in the results chapter
that follows, the investigation certainly raised some interesting points concerning the child's perspective in approaching mathematical word problems. Similarly, the differences in approach to such questions depending on the child's reading ability produced a number of points that required further analysis. Given the various constraints that arose during the data collection phase of the research it was encouraging that so much information arose over what was a fairly short time scale.

## Chapter 4 - Textbook Investigation

## 4.1 - Introduction

As detailed previously in the literature review, mathematics textbooks for use in Primary schools have changed radically over the last few decades and one of the principal changes has been the incorporation of illustrations into the page layout. For primary age children, old fashioned textbooks containing bland pages of sums have been replaced by textbooks incorporating colourful and attractive illustrations. In this chapter I present an analysis of the results of an investigation into textbook use amongst teachers, an exploration of the distribution of textbooks used in our current mathematics curriculum for one LEA, the extent to which the different illustration types are distributed throughout the textbooks and finally explore the ways that different children work with the material in order to illustrate the role these images have in children's mathematical calculations.

## 4.2 - Textbook Survey

Inspectorate reports have suggested that during the independent working time of the Numeracy Strategy lesson, children were predominantly seen working independently from textbooks. This view concurred with what I had seen in my own extensive observations of teacher's practices within the Numeracy lesson. Bierhoff (1996) found that textbooks play an important role in teachers' lesson planning, and particularly in the approaches and methodologies teachers adopt. Yet as Harries and Sutherland identified, textbooks are increasingly viewed as an old-fashioned source of information that provoke a routine approach to teaching and learning (Harries and Sutherland 2000). It is but a short step from using a routine approach to solve simple mathematics problems in textbooks to providing pupils with a formulaic approach to solving word problems. Over reliance on textbook examples to provide suitable problems may be unwise if the problems provided within the textbook are not clear or do not provide a wide range of problem types. Using illustrations may make questions appear to be very different when they are not and may also serve only to
confuse pupils who are still developing problem solving skills by making the problems appear much harder than they are.

As described in the methodology chapter (Chapter 3), I used a questionnaire to obtain information from practising teachers regarding the type of mathematics textbooks used in schools, how frequently they were used in a week and what motivated teachers in their choice and use of a particular book (Appendix 4). Of the fifty requests for information sent out to a variety of primary schools within my Local Education Authority, thirty-nine questionnaires were returned. The questionnaire was not intended to provide an in-depth knowledge of teacher's approaches and methodologies but simply to find out what (if any) textbooks were used, why they were chosen, how they were used in the classroom and the extent to which children would be exposed to working with them during any particular week. I deliberately made the questionnaire very straightforward, in order to obtain as many replies as possible, considering the likelihood of the teacher having to complete questionnaires in an already busy working day.

### 4.2.1 - Most commonly used textbooks for the teaching of number work.



Figure 2 Mathematics textbooks used in the teaching of number

As indicated by the results shown in Figure 2, Abacus was by far the most commonly used textbook. This was not surprising as when the National Numeracy Strategy was introduced, Abacus was quick to publish teaching material in line with the new strategy demands and was highly recommended by LEA teams. New Heinemann Mathematics, which came onto the market shortly after Abacus, was also widely used and Figure 2 shows that this was the second most common book used in classrooms.

A significant number of respondents indicated that no textbook was in regular use. Some of these respondents indicated that they 'aren't encouraged to use them now'. This may be as a result of short term plans produced by the National Numeracy Strategy which incorporated activities that were not textbook, but worksheet, based. One respondent indicated that they make regular use of photocopiable worksheets from the material incorporated in the LCP file.

### 4.2.2 - Reasons why teachers use mathematics textbooks

In the survey, teachers were presented with a range of possible options to indicate why they use the textbook. They could tick as many statements as they felt were appropriate and add others if their view had not been identified. All those teachers who had indicated their use of a textbook in question one answered this question. The results were as follows:

Reasons Teachers Gave For Using A Textbook


Figure 3 Reasons teachers gave for using a textbook

In the teacher's opinion matching the teaching objective to the textbook was a major reason for selecting a textbook. The ability to differentiate the work so that pupils can work independently is a significant point, reaffirming my view that pupils will tend to be working independently when using the textbooks but also, more significantly, it means children are working from these books with potentially little guidance or support from the teacher at the time, feedback tending to be given during the marking stage. If the illustrations are a contributing factor to causing children confusion in their calculations, the teacher may well assign a child's incorrect answer entirely to a misunderstanding of the objective alone rather than to their misunderstanding of the question due to the way the problem is presented. In the survey, the illustrations and the attractiveness of the page were also identified by a number of teachers as being important. If added together, the results from these two separate items would make a total of sixteen responses which would place it as second in order of reasons for textbook choice. This links with the work conducted by Bierhoff (1996) who found that the visual appeal of a textbook was a major factor in student teacher's selection of textbooks. It also reaffirms the marketing statement by Ginn that the brightly illustrated textbooks allow pupils the opportunity to practice skills independently. In light of the focus of this thesis and in order to address these issues, an analysis of the two most commonly used textbooks was undertaken. Coincidentally, my school uses both of these, New Heinemann Maths and Abacus although before I carried out the survey I was unaware that they were also the most commonly used within the LEA.

In the Heinemann Catalogue the New Heinemann Maths is promoted as being;
"Motivating - Inspiring and highly visual pupil materials that are sure to keep all your children on task. Textbooks. NHM Textbooks combine solid mathematical content with clear layouts and a child-friendly way to encourage further practice of skills. They provide opportunities to use and apply mathematical understanding in new situations. The Textbooks: reinforce fluency of number facts and mental calculations strategies, provide problem solving activities" (Heinemann 2005:12).

The Abacus scheme, produced by Ginn, promote their textbooks as being;
"Brightly illustrated textbooks providing independent practice of specific
skills. Clear and appropriate reading levels, with an easy-to-use layout.

Include investigations or open-ended skills focus to encourage creativity.
Problem solving included throughout" (Ginn 2005:14).

The people who write this promotional blurb have obviously done their research! In a short space they manage to use a great number of words and phrases that will immediately resonate with teachers.

From the Heinemann blurb:

- "inspiring"
- "highly visual"
- "keep all your children on task"
- "child-friendly"
- "problem solving"

And from the Abacus blurb:

- "brightly illustrated"
- "independent practice"
- "easy-to-use layout"
- "creativity"
- "problem solving"

However, it is important to note that for both these firms, the attractiveness of the page is a major selling point and is usually mentioned in the sales literature before references to the mathematical objectives within the textbook. It is also interesting to note how the attractiveness of the material is linked to a 'child-friendly' approach, indicating the intention that the materials are designed in such a way to make the mathematics easily accessible to children. The Abacus blurb also mentions "creativity" although there is little scope within the textbook questions to demonstrate this given that they have a strict marking scheme with a limited number of correct answers.

### 4.2.3 - Frequency of textbook use in the teaching of number work

From the textbook survey, twenty-nine (74\%) of the respondents used textbooks in the teaching of number work. The number of times the textbook was used per week was as follows.

Frequency of Textbook Use in The Teaching of Number


Figure 4 Frequency of textbook use in the teaching of number

Figure 4 shows that overwhelmingly, children are engaged in using textbooks on a regular basis. Out of a total sample size of thirty-nine respondents, $67 \%$ of them indicated that children would engage with a mathematics textbook at least twice a week. From my own professional knowledge it is likely that these would be used during the twenty minute independent working part of the numeracy hour lesson although this was not specifically identified in the questionnaire. It is clear from the results of the questionnaire that textbooks are still very much part of the children's mathematical experience.

## 4.3 - Textbook Analysis

From the results of the survey, two main textbooks in common use were the Abacus and New Heinemann Maths. Few of the respondents were using any other textbooks so I decided to concentrate on these two main schemes.

The Abacus scheme has a number of components for each Year group. By Year 3 these include three textbooks, Number 1, Number 2 and Shape, Data and Measures. Unlike Abacus, Heinemann have the one main textbook which incorporates activities covering all the Mathematics Attainment Targets. For both schemes there are also a number of supplementary materials available.

My analysis of these textbooks focused on the illustrations but only in the context of the four number operations, addition, subtraction, multiplication and division. Questions involving shape, space and measure were disregarded as were questions that dealt with number sequencing, fractions or similar. I also elected not to include questions that involved another mathematical element such as measurement or money because it would mean it would be more difficult to compare the results when an additional concept is involved.

The analysis of the illustrations followed the same pattern as those of the main study. I decided whether the illustration was essential, related, decorative or negatively decorative. Using Microsoft Excel I decided to record the data using the following fields.

| Concept |
| :--- |
| Number of parts for each question |
| Number of words (general instruction) |
| Number of words per part of question |
| Illustration |
| Is illustration related to the concept |

Table 14 Textbook Analysis Criteria

The number of words in the instructions could give some indication of the level of reading competence expected of the pupils as well as the amount of interpretation of the illustrations and instructions children were expected to do. In her work, HeuvelPanhuizen (1999) avoided questions with extensive written instructions on the basis that it resulted in an overemphasis on reading comprehension rather than an understanding of mathematics and the assertion that the pictures were self explanatory.

### 4.3.1 - Heinemann Textbook

I began my analysis with the Heinemann textbook. I immediately encountered a problem when defining the illustrations. The following illustration, "Javelin Throws", is an example of the problem I encountered. At the top of the page are a number of sums that have been written as part of the illustration. The illustration is decorative, but uses the whole page to provide information about the context upon which the sums are based. If I was unsure or needed reminding what a javelin was, the illustration would flesh out the concept for me. For the analysis of the textbooks I therefore incorporated a new illustrative category - "Decorative Background". This is used when the illustration is part of the decorative background to the page. This is different from the previous category of decorative which would usually just incorporate a small image somewhere on the page.

The first section of the page also fitted my criterion of only including data from the four number operations. However the section towards the bottom of the page required the children to apply their subtraction skills to the measurement of the javelin throws. The questions in this part of the page were not included in the analysis because it involved an additional mathematical concept - that of measurement.


The total results from the analysis of all the pages I analysed within the Heinemann textbook have been subdivided into the four number operations and are shown in Table 15.

|  | 0 words | $1-5$ words | $6-10$ words | $11+$ words | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Addition | 39 | 40 | 13 | 6 | 98 |
| Subtraction | 66 | 19 | 19 | 17 | 121 |
| Multiplication | 15 | 6 | 5 | 11 | 37 |
| Division | 24 | 22 | 6 | 7 | 69 |
| Total | 144 | 87 | 43 | 41 | 315 |

Table 15 Number of words used in general question instructions

These results indicate that very few, if any, words are included in the general instructions. With few written instructions, children will have to interpret the visual information to decipher what they were expected to do. However as Campbell (1981) suggested, children generally obtain less information from pictures than adults so it is possible that with no written instruction, they are more likely to misinterpret the requirements of the question.

Although I am by no means advocating long extended instructions, it does raise the question as to how much useful information can be provided in fewer than five words. Even 'copy and complete' can be open to misinterpretation. Teachers indicated that they used the textbooks because they provided children with independent work but for this I would assume that some additional information must be necessary. However, if they are used for truly independent working that support may not always be available.

If the written word is not present, the role the illustrations play in transmitting information must be greater. The illustration type has been subdivided by the four operations (Table 16).

|  | Essential | Decorative <br> Background | Related | Decorative | Negative <br> Decorative | No <br> Picture | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Addition | 32 | 38 | 5 | 3 | 1 | 17 | 96 |
| Subtraction | 45 | 28 | 1 | 6 | 0 | 30 | 110 |
| Multiplication | 18 | 7 | 1 | 2 | 0 | 10 | 38 |
| Division | 18 | 10 | 5 | 7 | 0 | 19 | 59 |
| Total | 113 | 83 | 12 | 18 | 1 | 76 | 303 |

Table 16 Illustration type associated with different number operation

These results, with $65 \%$ falling into the 2 "essential" categories, indicate how important the illustration is to the understanding of the question. With minimal written instruction, the illustration becomes the main source of information. Potentially uncomplicated sums (e.g. 55-34) have a tendency to be presented in the Decorative Background category. This provides evidence of how children may be reliant on the illustrations to provide them with instructional information but equally the illustration may act as the main distracter causing children to misinterpret or make mistakes due to poor concentration.

If children are frequently required to use the illustrations to understand meaning, how frequently are the illustrations linked to the mathematical concept? The results for the four operations in my investigation are illustrated in Table 17.

|  | Clear Link to <br> Maths Concept | No Clear Link to <br> Maths Concept | Total |
| :--- | :--- | :--- | :--- |
| Addition | 5 | 74 | 79 |
| Subtraction | 3 | 76 | 79 |
| Multiplication | 1 | 26 | 27 |
| Division | 0 | 40 | 40 |
| Total | 9 | 216 | 225 |

Table $\mathbf{1 7}$ Link between number operation, illustration and mathematical concept

These results show that in most cases there is no clear link between the illustration and the mathematical concept involved, so children are being required to further interpret a non-related illustration within a mathematical setting.

### 4.3.2 - Abacus Textbooks

Information from the Abacus scheme was taken from the two Year 3 textbooks, Number 1, and Number 2. The presentation of the illustrations in the Abacus scheme involved the use of more muted and pastel colours when compared to the Heinemann scheme.

One noticeable difference between the two schemes is the number of words used in the general instructions. The Abacus scheme uses far more words and for some activities (Table 18), especially the explore activities, there was substantial text, often involving thirty words and occasionally as high as sixty-seven words.

|  | 0 words | $1-5$ words | $6-10$ words | $11+$ words | Total |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Addition | 1 | 14 | 25 | 44 | 84 |
| Subtraction | 0 | 10 | 14 | 18 | 42 |
| Multiplication | 0 | 9 | 21 | 13 | 43 |
| Division | 0 | 5 | 11 | 12 | 28 |
| Total | 1 | 38 | 71 | 87 | 197 |

Table 18 Number of words used in general instructions

After these general instructions very few words were used for subsequent parts of the questions. Presumably the authors considered the general instructions adequate as little further text instruction was given for the remaining parts of the questions.

Despite the often lengthy general instructions, the illustrations in the Abacus scheme (Table 19) are often found to be essential to the calculation (56\%) compared with the Heinemann scheme which contained fewer instructions with only $37 \%$ of essential illustrations. In the Abacus scheme there are noticeably fewer Decorative Background illustrations compared with the Heinemann scheme (2\% compared with 27\%).

|  | Essential | Decorative <br> Background | Related | Decorative | Negative <br> Decorative | No <br> Picture | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Addition | 46 | 2 | 17 | 2 | 5 | 12 | 84 |
| Subtraction | 24 | 1 | 9 | 3 | 3 | 2 | 42 |
| Multiplication | 26 | 1 | 5 | 2 | 1 | 8 | 43 |
| Division | 14 | 0 | 9 | 0 | 2 | 3 | 28 |
| Total | 110 | 4 | 40 | 7 | 11 | 25 | 197 |

Table 19 Abacus - Illustration type compared with number operation

Like the Heinemann scheme, few of the illustrations had a direct relationship to the mathematical concept in question (Table 20).

|  | Clear Link to <br> Maths Concept | No Clear Link to <br> Maths Concept | Total |
| :--- | :--- | :--- | :--- |
| Addition | 2 | 70 | 72 |
| Subtraction | 0 | 40 | 40 |
| Multiplication | 2 | 33 | 35 |
| Division | 0 | 25 | 25 |
| Total | 4 | 168 | 172 |

Table 20 Abacus - Link between number operation, illustration and mathematical concept

Since I conducted this analysis, Abacus has produced another version of the scheme. In this version the noticeable difference is the change in the illustrations replacing the muted colours with very vibrant colours.

## 4.4 - Summary of the Textbook Analysis

The preceding analysis shows that a number of different textbooks are routinely used in school classrooms. Of those in current use, the Heinemann and the Abacus schemes appear to be the most popular. Teachers use them because they feel they meet the objectives, allow for independent working, are easy to manage and are pleasing to the eye. In English schools, textbooks have traditionally been used to provide individual drill and practice exercises to help children learn mathematics. What is surprising is the variation between the two textbooks I examined in detail, (Abacus and Heinemann). The Heinemann textbook had higher numbers of questions where there were no or just a few (below 5) words in the question, whereas the Abacus textbook had much wordier questions. It would follow then, that pupils using the Abacus textbook would need to read more in order to find out what they had to do to answer the mathematical question and consequently, poorer readers could be disadvantaged to a greater extent by the Abacus textbook than the by Heinemann one. If the Heinemann textbook had been designed with a particular mathematical philosophy in mind it would link well with the work by Heuval-Panhuizen as detailed above in that this work is characterised by a conscious decision to keep the text within the question to an absolute minimum.

In terms of illustrations, the Heinemann and Abacus textbooks both contained a large number of illustrations that would be characterised as "Essential", but the Abacus textbook had far less illustrations that would be characterised as "Decorative Background" than the Heinemann one. Presumably, this is because the Abacus books, being more wordy tended to rely more on words to "set the scene" and Heinemann tended to use illustrations for the same purpose.

As has already been stated, there has been a move over time for textbooks to be more highly decorative and colourful than was previously the case, but this may not necessarily be a positive move. As detailed previously, Harries and Sutherland (2000)
noted that English textbooks contained far more potentially distracting decorative features than their continental counterparts. French textbooks for instance, made more extensive use of diagrams that provided more explanation, and linked directly to the mathematical concepts being tested. It could be argued that whereas diagrams in French textbooks are included for sound pedagogical reasons, pictures in English textbooks are provided more for entertainment and ornamental reasons. Both Heinemann and Abacus set great store by this aspect of their textbooks, Heinemann marketing them as "Inspiring and highly visual" and Abacus as "brightly illustrated". However, having observed mathematics lessons in French schools I have noticed that there is a difference in the way that textbooks are used. In the French schools that I observed, the textbooks were used as a whole class resource, with the class working together under the direction of the teacher, whereas in England they tend to be used for independent, individual or small group work.

In both the textbooks analysed there was a lack of supporting information about the mathematical concept in question. In the majority of cases, the sums were presented as a series of unrelated tasks and there was no extra information available as to why the task was required or how the tasks on the page could be related to each other. This lack of background information meant that key links between the tasks set might not be made by the pupil working alone. Even though the entire page might cover sums where a certain number is added, the child might not notice this and just treat each sum as an unrelated calculation to be carried out as quickly as possible. This is made worse by the distractions caused by the extra elements of the illustration present only to provide decoration.

The analysis of the textbooks shows that there is an overwhelming need for the children to access the illustrations in the books in order for the mathematical problem to be retrieved, but for these illustrations to be less distracting.

The next question to explore is how the children themselves use these illustrations, if they in fact do, to interpret the mathematical task facing them.

## 4.5 - Children's Perceptions of Textbooks.

For this part of the research I studied pairs of children as they worked through a page from the textbooks that I had chosen. There were three boys and five girls. I have also included information from another boy $I$ had observed during a normal mathematics lesson but as I had not planned to work specifically with him I did not record the conversation. As I was having to conduct these investigations at the same time as running a busy classroom with thirty six children and no additional adult help, this limited my ability to widen the sample group further. Children were selected by the fact that they;

- were in Year 3
- they took part in the other maths question study
- had proved themselves to be capable of achieving the mathematical concept in question, having done this on a previous occasion
- had loud enough voices so their discussions could be recorded above the general buzz of the rest of the class.

Two pages were chosen from the Heinemann Textbook and two from the Abacus scheme. They were chosen because they;

- had a high reliance on the illustration
- they required concepts that the children had previously achieved
- my previous experience of using these in the classroom had noted problems of interpretation although I had been unable to investigate why.

The children sat with me and I explained that there were some questions I would like them to do but I wanted to know about their thinking and working out which was why they were not working with the rest of their group. The conversations were recorded on a small handheld digital recorder although the children were unaware of this. This deliberate decision to record the children without their direct permission was potentially unethical. I took this decision because if the children were aware of their conversations being recorded there was a risk that the responses they gave would not be truly representative of their thinking. They may well have been inclined to talk more about what they thought I would want to hear than what they were actually thinking, which would have given false information. Prior to this investigation I had formally approached the Headteacher and Governors explaining what I would like to do and reassuring them that it was for the purpose of educational research and gained their permission to conduct this research. Since it is normal for these bodies to act as
gatekeepers for the children's welfare I felt that I had gained sufficient permission to record the children, albeit covertly. Of course, for ethical reasons, these recordings will be destroyed following the conclusion of the research.

### 4.5.1 - Types of Illustrations in the Examples Explored

Unlike the examples in the "Mathematical challenges for able pupils in Key Stages 1 and 2 " booklet, the illustrations in the Heinemann and Abacus text books tended to fall into two main categories "Essential" and "Decorative Background". In this analysis I used the following illustration types:

From the Heinemann textbook:

- Rockets (Decorative Background)
- Paint Mixer (Essential)
- Teams of Four question (Related) *
- Cards and Dice questions (Decorative Background) *
* on the same page of the book

From the Abacus textbook:

- Chopping Logs (Decorative Background)
- Hands (Essential)


### 4.5.2 - Misconceptions which arise from interpreting the illustrations

In order to investigate the children's use of illustrations in solving these textbook problems I presented children with specific problems and questioned them as they were solving the problems. In each case the pupil's page is presented, including any calculations they wrote down as they were solving the problems, followed by a transcript of the concurrent discussion we had. The transcript includes my comments explaining what I felt was happening at the time. The first problems relate to questions taken from the Heinemann textbook and being typical of the analysis findings include very little text. The illustrations act as Decorative Background.


This is the work produced by Robert, whose Year 3 optional SATs gave him a national curriculum level of $3 B$.

The following is the work of Susan, a Year 3 girl whose SATs gave her a curriculum level of 3 A .


Following initial settling down, the tape transcript ran as follows;

| Transcript |
| :--- |
| RJ With this first one what do you think you are |

R Add 11 onto these numbers.
RJ OK. What do you think S?
S I think you hmm, have to add 11 onto these numbers.
RJ OK. So how are you going to go about adding that 11 on?
S You could start with the littlest number and add the big number.
R That's 39
S Add 10 onto the 28 and then added 1 on.
RJ Excellent. Have a go at the next one then.
S That's 56 I added 10 onto 40 that made 50 and added 1 onto the 5 made 56.
R Equals 68. Added 10 onto 57 equals 67 then add 1.
RJ Excellent. Well done! You're whizzing on, doing really well.
S 84. That's the first rocket done.
R On this next rocket you have to add 9 onto it.
RJ OK. So what strategy are you going to use this time?
R Add 10 take away 1.
S $\quad 28$ add 10 equals 38 . Then yeah, I have to take 1 away because 9 is 1 less than 10 . OK next one then.
S $\quad 64.62$ add 9 equals er, equals 71.
R I've done it. What's the next rocket about.
S You are adding 21 on.
R er I'll add 10 add 11
S Add 20 and then add 1. 95 that one (referring to question 2d).

The children work quietly at questions 3 a and 3 b
S $\quad 66$ add 20 equals 86 , er 87 , yeah, 87
R $\quad 72$ add 20, 92 , add 1, 93 . I've finished!
S Next one. Add 10 then add 9. I could add 18 then add 1 . No. I'll add 20 then take one away. 22 add 20.
R It's 42. 41!
S Next ones 35, add 10 equals, that would be 55. Yeah take away 154 . Number 3,59 add 19, hmm these are hard
R Add 10 would be 69

Here the children are taking note of the illustrations because they comment on the way the sums have been laid out on a rocket rather than indicating the first few sums have been completed.

Throughout this part the children have indicated and reinforced a strategy for successfully adding on near multiples of ten to any number.

| S | 79 |  |
| :---: | :---: | :---: |
| R | 78 |  |
| RJ | You have been doing well. For number 1 you added 11, then we added 9, number 3 we added 21 and 4 we added 19. Now onto number 5 . |  |
| R | We're adding |  |
| S | We're adding nothing | The children had developed a strategy and procedure for the |
| R | It tells you the answer adding 30. |  |
| S | It tells you the answer. That's silly. | addition of near multiples of |
| RJ | Yes it does, so what do you think you have to do? | ten. The situation with this sum was different and their |
| R | It's adding 30 , it's adding 30 , adding 30 | procedural knowledge now |
| S | It tells you the answer Yea. 25 add 29 makes 55 . No. 25 add 20 equals 40 , not 69 . | begun to show potential limitations. |
| R | That's 69. er. |  |
| S | hmm. I don't know what to do. |  |
| RJ | What is making that one a little bit trickier? | As the children become unsuccessful they are consistently looking back at parts of the problem and struggling in trying to figure out how to solve the problem spending a great deal of time re-examining numbers. |
| S | Adding 29 and it's odd. |  |
| RJ | It's odd, what do you mean it's odd? |  |
| S | 29 is an odd number |  |
| RJ | OK and what was 19? |  |
| S | That was odd, so that's not right. |  |
| RJ | You were pointing out the middle one can you use that at all? |  |
| S | hmm, er |  |
| R | er |  |
| RJ | Does this one in the middle help you at all? |  |
| S | hmm sort of. |  |
| R | No. Its got a different number to the others. |  |
| RJ | S, you said sort of. |  |
| S | It has 25. You could add 30 then take away |  |
|  | 1. I think it's 30 you just need to take away |  |
|  | 1 . |  |
| R | I know 74. |  |
| S | Do we need to do that one (points to the rocket containing 5 a and 5 b ). |  |
| RJ | What do you think? What do you think it's there for? |  |
| S | Cause er, cause, er, it's always using 25. |  |
| RJ | Right so each of those has used a 25 but the other numbers are different aren't they. |  |
| R | 29, 30, 31 |  |
| S | They count one |  |
| RJ | Do you think that helps you then? |  |
| S | No. I'll just ignore that. I'll add 25 and 31. 25 er 35 , equals 20 add 30 is 55 and then it's |  |
|  | 56. |  |
| R | We've got the tricky one out the way. |  |
| S | Yeah! |  |



What this transcript demonstrates is the way that the drill and practice of adding multiples of 9 or 11 reaffirmed their rules about mathematics. The children worked confidently and speedily through questions 1 to 4 . However, question 5 caused them great difficulty (the point when the children reach question five is identified by the red text in the transcript). The illustration of a rocket shows an example solution in between two questions. The middle solution adds 30 to 25 , the other two questions add 29 and 31 respectively. The inference is that they should see the pattern of adding 30 to be similar to adding 29 or 31 . However, the children found the mathematics confusing and the illustration caused greater confusion as they tried to read the rocket from the nose backwards, perhaps because they assumed that it was a rocket travelling forwards to be a clue to the calculations required. Having no text to guide their thinking only reinforced their need to use the picture. Their previous experience of questions 1 to 4 had established what they needed to do and they had been successful. Having the central solution in a different colour was probably meant to highlight this as different from the two questions either side, but it actually caused some of the confusion so that they were unable to recognise how they could use the information to make a quick calculation of the sums on either side. After a long debate they reverted back to their successful method that had worked for the previous questions. I believe that their decision to revert back to their previous strategy was a success. The confusion of the picture in question 5 had been detrimental because it made them doubt the rules they were learning and there was not sufficient information available for them to modify their understanding of the rules.

In the next example taken from the Heinemann Textbook the picture is essential to the calculation. Again two children were consulted as they attempted to answer the
question. David and Claire both have been awarded a national curriculum level 3A in their optional SAT.


David's work (his answers have been written faintly in pencil on the colour mixing pots related to the question).


The transcript of this session went as follows.

| Transcript | Comment |  |
| :--- | :--- | :--- |
| RU $\quad$We have this problem here that I'll like <br> you to look at and see how you would <br> solve it. (just over two minutes passed <br> with the children making no comment). | Because the meaning of the <br> question has not been made <br> explicit, the children are unable <br> to grasp the intended meaning to |  |
|  | You've been looking at it for a while now. <br> What do you think? | its fullest extent. |
| C I don't know |  |  |



This pair highlighted a number of very interesting notions. At first it was very unclear to them what the question required. The question obviously had two parts ( $4 a \operatorname{and} 4 b$ ) but the illustration indicated one single question. They were unable interpret the illustration as a 'factory line production'. With no text to introduce the concept, they
had to rely solely on their interpretation of the illustration which they were failing to do.

Claire did seem able eventually to distinguished the two parts of the diagram but unfortunately assumed the two numbers had to be added together rather than the algebraic equation of $52+x=83$. David however saw the diagram as a whole and quickly made the connection between the two yellow containers when added together make the 83. In my opinion it seems unlikely that this was the intention of the designers. I fully expect the question was to be of two separate parts, $52+x=83$ and part $b, x+31=60$. For David, this logic of using the two yellow containers had to be compromised in order to find the amount needed in the blue container. This he was successfully able to do.

The illustration appears to have been designed with children's experience of colour mixing in mind. However the two children were not able to transfer their experience of colour mixing to that shown. Not only was there the problem of measuring the paint but they demonstrated clearly how two separate curriculum subjects are seen in defined roles. Their knowledge of art was very domain specific. This domain specific idea was reinforced in that 'maths is maths' and under no circumstances should art be allowed or expected to intrude into the mathematics arena.

The next two problems are taken from the Abacus scheme. The first of which
 involved the workings of Hilary, a Year 3 girl who obtained level 3B in the optional SATs and Louise, who was awarded level 3B as well. The illustration is related in that a $\log$ is seen being split in two, reinforcing the written instruction.

Hilary produced the following written work.


This is the work produced by Louise

The transcript referring to this exercise was as follows;

| Transcript | Comment |
| :---: | :---: |
| H You have to split the log into two parts so it's not the $\log$ but the number. |  |
| L 2 and 3 make 5 |  |
| H But on this first one it is 6 and half of 6 is 3 so you must be splitting the $\log$ in half. Half of 5 is 2 and a half. | The objective of the exercise was to identify whole number bonds yet one child |
| L You can't split 5 cause it's odd. | is being convinced by the |
| H You can if you split the odd number left, it makes half, so half of 5 is 2 and a half. | illustration that dividing the number into two equal parts |
| L What about 7 then? | is required. In this the |
| H That will be 3 and a half. | illustration is conflicting |
| L Well 4 and 3 make 7 so it could be 4 and 3 . | with the objective. |
| H Maybe there are lots of answers. You write down yours and I'll write down mine. The pictures shows the log cut in half so it must be equal parts. |  |
| (L appears to be persuaded by H's reasoning and for |  |
| question 4 , divides 5 exactly by 2 . However, H |  |
| appears to be persuaded by L and uses whole numbers. |  |

For the remainder of the exercise whole number pairs are used by both children).

The discussion the children had at the beginning of the exercise illustrates how children will make a literal interpretation of the illustration. It could also indicate that Hilary, by only working with halves of numbers, was working within the realms of an arithmetic operation with which she felt "safest".

Due to my classroom obligations the final pair of children had to work independently. I had begun the session with them but quickly found another commitment which meant they were left working as a pair. Their conversation continued to be taped. In fact I believe it was a fortuitous opportunity as it gave an indication of their true workings without the pressure of an adult being present.

The pair consisted of Zoe who achieved 3C in the optional SATs and Bobby who achieved level 3B. The mathematical concept involved the children multiplying groups of five. As with the other pairs it was a concept the children had achieved in other work prior to the activity.


Work by Zoe.


Work produced by Bobby.

I explained that I wanted them to think aloud as they worked on the task, even if I had to leave them. At this point an unexpected visitor entered the classroom so the children were left for the next few minutes. The following transcript is taken from the point when I had left them but before they had begun the task.


| Z | What |  |
| :---: | :---: | :---: |
| B | 4 the other one is a thumb. |  |
| Z | This says it 3 times 5 , it even gives you the answer so it must be 5 fingers on a hand. |  |
| B | That's silly because you have 4 fingers and 1 thumb so the book's wrong. It should be 3 times 4 which is, 4 add 4 add 4,4 add 4 is 8 add another 4 is $10,11,12.12$. |  |
| (the conversation ceases for a while as the children work independently.) |  |  |
| B | This is all wrong. The hands don't even match. One is missing. (I believe $B$ is referring to question 5 which has an odd number of hands shown) | A conflict occurred here because the illustration did not match the reality of people having two hands. However |
| Z | Maybe it's like ..... (At this point $Z$ uses the name of a child in the class who following a lower arm amputation only has one hand) | because the children through personal experience were familiar with the notion that |
| B | lower arm amputation only has one hand They won't know about ..... It's wrong again. | familiar with the notion that not everyone has two hands, they were trying to justify the 'mistake' in the book. |

The work from Bobby indicates his belief that as the hands were not in pairs for questions 8 and 9 he continued to assume one hand was missing so the sum was not attempted. It was interesting to note how he continued to work within the belief of four fingers per hand whereas Zoe followed the conventional expectation of multiples of five. She saw this as a mathematics problem whereas Bobby was being influenced by his real life experiences.

### 4.5.3 - Critique of a page from a mathematics textbook

The final observation concerned a boy working on page 60 of the Heinemann textbook (shown below). He had problems determining the requirements of the problems illustrated on the page which is not really surprising in that, even to an adult, the question requirements are quite vague and open to interpretation. Since this page is typical of current textbooks I feel that at this point an in-depth critique of the whole page would be appropriate here. The questions on the page are designed (I believe) to reinforce children's multiplication and division skills.


1. I found that I could interpret the first question in three different ways. It is divided into two parts and uses a group of 27 children. The first part asks "How many teams of 4 can be made?". Presumably the requirement here is a simple division, $27 \div$ $4=6$ with 3 children left over. But some very able children could understand the question in a different way in terms of the number of combinations or permutations possible. In this case the question would be much harder, but by no means impossible. If the first part of the question is interpreted in this way then the second part of the first question seems laughably easy. Alternatively, if the question is interpreted in its simpler form, then the second part of the question is asking for almost exactly the same calculation as the first part of the question. The only difference between the two parts of the question involves the considerations of the remaining 3 children.

However, a third, even simpler solution is possible. The illustration actually shows the 27 children in groups of 4 except for one larger group of 7 . A child reliant on the illustration could just count the various groups in the picture and come up with a correct answer. Using this method they would not need to carry out any mathematical operations apart from counting.
2. The second question on the page shows a picture of what look like playing cards with an ethereal number 52 above them. It seems that in this case the illustration is just to show that we are dealing with playing cards and the number shows how many there are. Many children would be confused by this seamless movement between illustration and information and tried to count the cards in the picture. In this case the ethereal number just got in the way. Indeed, with such a small amount of written information on the page it is unclear even to an adult that this is exactly the conclusion we must draw about the number of cards involved. Only the fact that there are 52 cards in a standard playing card pack would tend to sway one's mind. However, crucially, this information is much less likely to be a known by a child and even if it is, they may be equally aware of other card games, Happy Families or Top Trumps for example, that do not use 52 cards.
3. The third question involves a picture of a pile of coloured dice again with an ethereal number 17 hovering over it. The question asks "how many pairs of dice?" but gives no indication of how they should be paired. Again a number of solutions could exist. The simplest answer would be $17 \div 2=8$ with 1 left over but the dice could also be paired by colour, either the same colour as a pair or a different colour as a pair. Similarly the dice could be paired by position or by which number was uppermost.
4. Question 4 shows a pile of dominoes again with an ethereal number 38 floating above it. The question asks how many dominoes each player would have if they were shared equally between 5 players. Again the simple answer is $38 \div 5=7$ with 3 left over. Crucially though, the dominoes come in two different colours, a most unusual way to play a game where you are trying to conceal what you have in your hand! Again an alternative answer would be to try to count the number of dominoes in the
illustration and use that number to formulate an answer. Another would be to attempt to divide the dominoes based on the numbers they show.
5. Finally question 5 is a simple division. 25 jigsaws need to be stored on shelves and each shelf can hold 3 jigsaws. How many shelves are needed? Of course the answer is meant to be $25 \div 3=9$ shelves, 8 with 3 on and 1 with 1 on it. The illustration in this case however is extremely misleading. If one looks closely at the illustration some things strike immediately. The child has placed only one jigsaw on a shelf before starting another shelf. The size of the jigsaws in the picture mean that you couldn't fit three on a shelf and the boy is having trouble reaching the fourth shelf - he couldn't possibly reach to place a jigsaw on a ninth shelf!

It is important here to realise that although the questions detailed above are severely compromised by their inability to clearly detail what the child is required to do is by no means unusual. This is typical of a page from any textbook in common use in school classrooms today. In addition these textbooks are meant to be self explanatory and that the child should be able to work through the questions without much reference to the teacher. In this sense they are directly analogous to the "drill and practice" books that have been used routinely in mathematics teaching for much of the last century. As has been shown though, the desire to make the pages interesting and colourful has had a devastating effect on the actual content - in some cases making it too dense to be understood by the client group - a child learning mathematics.

If the misunderstandings given above seem far fetched I include here an example of work from a child that had tried to solve the questions on this page. My observations were recorded immediately following the lesson. The boy, Alan, was awarded level 3C in the Year 3 optional SATs. The page (page 60, above) is taken from the Heinemann textbook and contains a mixture of essential and related pictures.

In question 1 (a related illustration) he did not count the total number of children in the illustration but did count the groups to check there were four children in each group. He then focused on the group of seven and split that into a group of four and a
group of three. He then counted up all the groups of four illustrated on the picture, including that from the group of seven.

For question two he counted up the number of cards he could see (20) and divided that by the ten. He made no reference to the 52 indicated. It may have been that he was unaware that there are 52 cards in a pack so the number had no significant meaning and just prevented him from being able to count the number of cards clearly.

On question three he made up pairs of dice only using those in the picture and pairing them only by the same colour. He therefore gave the answer of two pairs of red with one left over, two black pairs and three blue pairs with one left over. This gave a total of 16 dice.

On question four he tried to count the number of dominoes but the 38 was 'in the way' so he guessed it to be about 20 based on his counting of the playing cards. Question five (a decorative illustration) he did not attempt because there "wasn't enough jigsaws you can see". This question was then done as a conventional division sum which he worked out correctly using his three times table. From his SATs results Alan would be considered as being a low achiever in reading and it may be because of this that he was accessing the pictures more frequently and became over reliant on them for information. Hegarty (1995) identified that unsuccessful problem solvers looked back at parts of the problem which indicated that they were struggling to figure out how to solve the problem. Alan appears to have followed this trend being unable to focus on the words that would inform him what to do but tried to work from the illustration alone.

At no time during his working did he make any reference to the numbers indicated on the centre of the illustration apart from when they were 'in the way' which prevented him from counting the number of cards, dominoes or dice. What is of interest is that where he was unable to use the picture at all (question five) he relied totally on his mathematical knowledge and skills. It was pure coincidence that I happened to be sitting at his table when he was completing this work and as such I was able to observe why his answers were incorrect and to an extent understand the logic of his working. If he had been working independently, a reason why teachers chose these
teaching materials, and I had only seen his answers, my understanding of his mistakes would have been based upon the division concept rather than his misinterpretation of the sum presentation. My subsequent teaching would therefore have been unlikely to address his specific need.

### 4.5.4 - Illustrations as a stimulus for focused discussion

A number of key points arose from the observations of children answering textbook questions. The first point involved the degree to which the children are distracted by their own experiences, maybe giving more weight to their underlying knowledge of the world than the question demands. For example the child who saw the log being cut into equal parts came away with the impression that the number in the question also had to be divided into equal parts although this was not really the case.

In each case, if the question differs a little from reality, it is unclear how much credence the child will place on the picture alone and how much will be provided by his or her understanding of the world. In many cases, the child places more weight on the things they believe that they know about the world than on those they do not yet know. However, some children are able to suspend reality and focus purely upon the mathematical element of the problem. This may be clearly an issue of relative mental development. Some children are more able to "think outside the box" than others. Providing an illustration that provides an opportunity for ambiguity may allow some children to apply their creativity in a situation where this is not actually required. An example to illustrate this would be the child who saw a link with what they had seen on the television programme "Red Dwarf" and had a different interpretation of the number of fingers one has. The impression here was not that they misunderstood the illustration but that there was more than one way that the illustration could have been interpreted. Since children are often asked to be creative it cannot be surprising when they are creative in situations where this is not required and is not the intention of the task.

This is clearly illustrated by the child stating that paint mixing is nothing to do with mathematics, but art. In this situation mathematical modelling would seem to be an entirely appropriate way in which to present the problem, based as it is on the ratios
of the primary colours required to create the colour required, but in this case the relationship was not one that was easily grasped by the pupil. Although the intention of the designer may have been to provide a question that seems to be child-friendly and appropriate, because it is not explicit and children are not as adept at reading illustrations as adults, a degree of confusion arises as to the requirements of the problem and the incorrect solution is provided due to the ambiguity of the question arising from the way the problem is presented. The case of Alan is a good example. As he wasn't able to access the written information he therefore became over-reliant on the illustration, resulting in a complete misunderstanding of the nature of the problem and the solution required.

The second point involved the application of mathematical knowledge. Children tend to look for a procedural way of solving the problem and once they discover a procedure that seems so work, assume that they have discovered the appropriate way to answer the question. For example, some textbooks present the information in a simple formulaic way (adding eleven to a series of numbers for instance). Once they understand the rules of the question, children find them easy to answer because they are able to follow a procedural strategy. Each new answer confirms that their strategy is indeed the correct one and they feel increasingly competent to solve such problems. If other similar questions are posed in a totally different way however, children often miss the link with what they have practiced before and find such questions difficult or completely incomprehensible. In many cases these problems are as incomprehensible to adults as they are to children.


For example, what is this question asking the child to do? In this case the middle sum is an example to be used to solve the two questions (a) and (b) but this is not very clear and goes against our natural inclination to read both text and mathematical sums from left to right. To add to the confusion, the children paid more attention to the illustrated movement of the rocket and attempted to read the problem from the nose of the rocket backwards towards the left.

## 4.6 - Summary

During this chapter I have shown both that textbooks are widely used by teachers and that the presentation and illustrations on the page do have an effect upon children's problem-solving work. The analysis of the two textbook schemes indicates how widespread essential pictures are in the books, sometimes appearing to be at the expense of detailed written instructions. The work conducted with the children illustrates how reliant the children are on using the pictures and that for some it prevented the children from understanding the intended task. Children, especially those who are poor readers appear to read the pictures just as much as they do the text. This leads to a myriad of interpretations of the requirements of the question and consequently having a direct result upon their mathematical success, often impacting negatively on their work.

## Chapter 5 - Challenge Book Questions

### 5.1 Introduction

In this section I examine the results obtained from the challenge book questions. Each task within the challenge booklets is introduced and the differences between each illustration type explained. On examination of the answers the children provided, a number of key themes emerged. These are described and illustrated with examples from the various question types. In order to investigate the effect of reading ability on a child's ability to solve mathematical problems a comparison is made between the results from a group of poorer readers and those from a group of better readers. A discussion and summary then follows, relating the findings to the literature review.

## 5.2-Overview of the tasks

All the questions in the challenge booklet originated from the Mathematical Challenges for able pupils in Key Stages 1 and 2 (2000) document although the accompanying illustrations were adapted (by changing the illustration in some way or removing it altogether) so that they matched the group of illustration types developed in this study. For the control questions, i.e. those questions that appeared in their original form in all test booklets, the original illustration was used. There were four control questions, Snakes and Ladders, Roly Poly, Ski Lift and Three Monkeys. These are described and illustrated below. All the other question types appear in the appendix in all their different formats. The different coloured question booklets were created as described in the methodology chapter, with the questions presented in the order shown in Table 21.

|  | Snakes <br> and <br> Ladders <br> Control <br> Question | Gold Bars | Fireworks | Roly <br> Poly <br> Control <br> Question | Spaceship | Ski Lift <br> Control <br> Question | Queen <br> Esmerelda's <br> Coins | Duck <br> Ponds | Three <br> Monkeys <br> Control <br> Question |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Blue | Essential | Negative <br> Decorative | Decorative | Related | Related | Decorative | Essential | No Picture | Negative <br> Decorative |


| Red | Essential | Decorative | Related | Related | Essential | Decorative | No Picture | Negative <br> Decorative | Negative <br> Decorative |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Yellow | Essential | Essential | No Picture | Related | Negative <br> Decorative | Decorative | Decorative | Related | Negative <br> Decorative |
| Green | Essential | Related | Essential | Related | No Picture | Decorative | Negative <br> Decorative | Decorative | Negative <br> Decorative |
| Purple | Essential | No <br> Picture | Negative <br> Decorative | Related | Decorative | Decorative | Related | Essential | Negative <br> Decorative |

Table 21 Question type and booklet colour

Snakes and Ladders was the first task in the booklet and was used as a control
 question, appearing in all booklets in the same format. Only the original version was used. The picture in this question was essential because it was the only way children would be able to identify where the snakes' heads were, and therefore recognise that landing a counter on those squares meant travelling down a snake. I made the assumption that all the children would be familiar with the rules of a snakes and ladders game. The counter needed to be moved 7 squares avoiding the second and the sixth squares (it begins on square 9 and finishes on square 16). Therefore, avoiding the snakes, there are four possible ways to get to 16 in two throws: 1 then $6 ; 3$ then $4 ; 4$ then $3 ; 5$ then 2.

The second task in the booklet was Gold Bars. The item was revised in order to provide the required five illustration types and distributed amongst the booklets as shown in Table 20. In this task, Pete the Pirate has to redistribute the piles of gold bars so that, within two moves, each pile contains the same number of gold bars. The official answer to this question is "Move two bars from pile 1 to pile 3. Move one bar from pile 4 to pile 2" (DFEE 2000:102) This official answer implies that Pete has
to end up with the same number of piles as he started with, a fact not emphasised in the question. Due to this ambiguity an alternative solution is possible - move pile 2 to pile 4 (or vice versa) and move pile 3 to pile 1 (or vice versa) - to make two piles of eight. Since the task is ambiguous I decided to accept both answers as correct, the official correct answer (4 piles of 4 ) and the alternative correct answer (2 piles of 8).

| Booklet Colour | Illustration Type | Question | Characteristic |
| :---: | :---: | :---: | :---: |
| Blue | Negative Decorative |  | I altered this illustration by adding extra gold bars to each pile in order to ensure the illustration contained far more gold bars than indicated in the question. The large numbers could cause confusion. |
| Green | Related | Gold Bars <br> Pete is a pirate <br> His gold bars are in four piles. <br> In the first pile he has 6 bars In the second pile he has 3 bars. In the thind pile he has 2 bars. <br> He can move one or more bars at a time | The illustration used is the original but I added to the text so that the text reflected accurately the number shown in the illustration. This is likely to make miscounting less likely. |


| Yellow | Essential |  | Original Question, therefore no changes were made. <br> Children have to count the bars in order to find out how many Pete has. |
| :---: | :---: | :---: | :---: |
| Red | Decorative |  | For this category I incorporated into the text, sentences which give the correct number of bars required to answer the question. I incorporated only two gold bars in the illustration to illustrate a gold bar but not enough are seen to reflect the text information. It could encourage children to draw gold bars onto the picture thereby concentrating on their calculation. |
| Purple | No Picture | Gold Bars <br> Pete is a pirate. <br> His gold bars are in four piles. <br> In the first pile he has 6 bars. <br> In the second pile he has 3 bars. <br> In the third pile he has 2 bars. <br> In the fourth pile he has 5 bars. <br> He can move one or more bars at a time. <br> He made all the piles the same height <br> He made just two moves. <br> How did he do it? | For this category I removed all the illustration but added to the text the required information concerning the gold bar arrangement. Having just the words may oblige the children to consider this as a mathematical calculation. |

The next question, Fireworks, consisted of two parts, each part requiring two answers. The item was revised in order to provide the required five illustration types and distributed amongst the booklets as shown in Table 20. The premise was that Emma was setting off fireworks and particular fireworks produced either 3 stars or 4 stars when they were lit. In the first part, Emma's fireworks made 19 stars in total, so
there are two possible ways in which this can be achieved - 5 fireworks which made 3 stars and 1 which made 4 stars; or 1 which made 3 stars and 4 which made 4 stars. Although the child was obviously required to use calculation in order to find an appropriate solution, an additional complication arose because, in the original version, the illustration itself provides a valid solution. By simply counting the number of stars in the original illustration a correct solution could be achieved, even though the child had not carried out any calculations to solve the problem.

| Booklet Type | Illustration Type | Illustration | Characteristic |
| :---: | :---: | :---: | :---: |
| Blue | Decorative | Fireworks <br> Emma had some fireworks. Some made 3 stars. Some made 4 stars. <br> Altogether Emma's fireworks made 19 stars. How many of them made 3 stars? <br> Find two different answers. <br> What if Emma's fireworks made 25 stars? Find two different answers. | To make this illustration decorative I included an example of the fireworks but the number of stars seen coming from the fireworks does not match the text. They show either two, three or five which could mislead the children from the three or four stars stated in the question. |
| Green | Essential | Fireworks <br> Emma had some fireworks. <br> Each of the two types made different numbers of stars. <br> Altogether Emma's fireworks made 19 stars. How many of them made 3 stars? <br> Find two different answers. <br> What if Emma's fireworks made 25 stars? Find two different answers. | To make this illustration essential I removed the text which gave the numbers of stars which each type of firework could make. Therefore the children have to interpret the illustration to find out how many stars the two types could make. |


| Yellow | No Picture | Fireworks <br> Emma had some fireworks. <br> Some made 3 stars. <br> Some made 4 stars. <br> Altogether Emma's fireworks made 19 stars. How many of them made 3 stars? <br> Find two different answers. <br> What if Emma's fireworks made 25 stars? <br> Find two different answers. | No illustration, therefore the illustration has been removed although no changes to the text was required. The children's comprehension of the questions has to come from the text alone. |
| :---: | :---: | :---: | :---: |
| Red | Related | Fireworks <br> Emma had some fireworks. Some made 3 stars. <br> Some made 4 stars. <br> Altogether Emma's fireworks made 19 stars. <br> How many of them made 3 stars? <br> Find two different answers. <br> What if Emma's fireworks made 25 stars? Find two different answers. | This is the original question therefore no changes were made. The illustration reinforces the numbers of stars each type makes as explained in the text. |
| Purple | Negative Decorative | Fireworks <br> Emma had some fireworks Some made 3 stars. Some made 4 stars. <br> Altogether Emma's fireworks made 19 stars. How many of them made 3 stars? <br> Find two different answers. <br> What if Emma's fireworks made 25 stars? Find two different answers. | To make this illustration fit the category I included an excessive number of fireworks. Therefore far more stars than the 19 or 25 described in the text are shown. The fireworks however do only display the three or four stars. |

The next task, Roly Poly, a control question, required the pupil to use their understanding of a dice to find out which number is face down. This task utilises six sided dice and the picture is categorised as related because it reaffirmed the number of six sided dice required (dice of many differing sides are in common use in school classrooms). The task states that "the score is the total number of dots you can see", but the question does assume that in order to 'see' the

## Roly poly

The dots on opposite faces of a dice add up to 7 .

1. Imagine rolling one dice.

The score is the total number of dots you can see.
You score 17.
Which number is face down?
How did you work out your answer?
2. Imagine rolling two dice.

The dice do not touch each other.


The score is the total number of dots you can see. Which numbers are face down to score 30 ? child would have to move so that they were able to see all four sides and the top of the dice. If you could only see the dice from a stationary position then only three faces would be seen. If these faces were the three highest numbers 6,5 and 4 the maximum number that could be "seen" would be 15 , therefore 17 would be impossible. Depending on how the word 'see' is interpreted, this might be seen by the pupil as a trick question.

In the next task, Spaceship, the question was again presented using the five different illustration types. In this task there are two types of aliens, with either 3 legs (Tripods) or 2 legs (Bipods). Given that there must be at least two of each type of alien, the question gives the total number of legs present and the task requires the pupil to calculate how many of each type of alien could be present.

| Booklet Colour | Illustration Type | Illustration | Characteristics |
| :---: | :---: | :---: | :---: |
| Blue | Related | Spaceship <br> (69) 400 Siven $\qquad$ $N=2=0$ <br> Some Tripods and Bipods flew from planet Zeno. There were at least two of each of them. <br> Tripods have 3 legs. Bipods have 2 legs. There were 23 legs altogether. <br> How many Tripods were there? How many Bipods? <br> Find two different answers. | The original version therefore no changes were made. <br> The illustration reaffirms the information in the text in that the aliens are shown with either two or three legs. |
| Green | No Picture | Spaceship <br> Some Tripods and Bipods flew from planet Zeno. There were at least two of each of them. <br> Tripods have 3 legs. Bipods have 2 legs. There were 23 legs altogether. <br> How many Tripods were there? How many Bipods? <br> Find two different answers. | The illustration has been removed but the text is unchanged from the original. The relevant characteristics of the aliens are contained within the text. |
| Yellow | Negative Decorative | Spaceship <br> 26 as <br> d 450 <br>  <br>  <br> Some Tripods and Bipods flew from planet Zeno. There were at least two of each of them. <br> Tripods have 3 legs. <br> Bipods have 2 legs. There were 23 legs altogether. <br> How many Tripods were there? How many Bipods? <br> Find two different answers. | To fit this category I included an excessive number of aliens. This was to be the only point of potential confusion therefore the aliens have the requisite number of legs although the total seen is in excess of the total identified in the text. |


| Red | Essential | Spaceship <br> Some Tripods and Bipods flew from planet Zeno. There were at least two of each of them. <br> There were 23 legs altogether. <br> How many Tripods were there? How many Bipods? <br> Find two different answers. | The text has been altered to remove the sentences which tell the numbers of the legs each alien type has. The prefixes tri and bi, could give a clue to the children but the number of legs for each alien type can only be obtained from the illustration. It is not explicit in the text. |
| :---: | :---: | :---: | :---: |
| Purple | Decorative | Spaceship <br> Some Tripods and Bipods flew from planet Zeno. <br> There were at least two of each of them. <br> Tripods have 3 legs. <br> Bipods have 2 legs. <br> There were 23 legs altogether. <br> How many Tripods were there? <br> How many Bipods? | Rather than the aliens, in this version their spaceship is shown in the illustration. I elected to do this because the role of a decorative illustration is to set the seen and a spaceship links with the title of the question and the notion of aliens. |

Ski Lift (another control question) was the next task the children encountered in the

## Ski lift

On a ski lift the chairs are equally spaced.
They are numbered in order from 1.
Kelly went skiing.
She got in chair 10 to $\mathbf{g o}$ to the top of the slopes.
Exactly half way to the top, she passed chair 100 On its way down.

How many chairs are there On the skd lift?


Make up more problems like this.
booklet. This was a control decorative question. The illustration shows a girl with skiing equipment on an individual ski lift. It cannot be considered as a related illustration since there is no further information supporting the text, for example the number of the chair lift is not shown or a wider view of a ski lift
scene. Most pupils thought this question was rather easy but in fact it was far harder than expected principally because a good understanding of how a ski lift works is essential to finding a correct solution.

Queen Esmerelda's Coins was the next task. This had been divided into the five illustration types. This question followed a similar format to Gold Bars in that four piles of coins were included. Rather than making equal height piles, the twenty coins had to be distributed between four piles according to different ratios.

| Booklet Colour | Illustration Type | Illustration | Characteristics |
| :---: | :---: | :---: | :---: |
| Blue | Essential | Queen Esmerelda's coins <br> Queen Esmerelda had some gold coins. <br> She put them in four piles. <br> - The first pile had four more coins than the second. <br> - The second had one less coin than the third. <br> - The fourth pile had twice as many coins as the second. <br> How many gold coins did Esmerelda put in each pile? | I altered the text and illustration so that the only way in which the children can identify how many coins Queen Esmerelda has is to count the coins in the picture. |
| Green | Negative Decorative | Queen Esmerelda's coins <br> Queen Esmerelda had 20 gold coins. <br> She put them in four piles. <br> - The first pile had four more coins than the second. <br> - The second had one less coin than the third. <br> - The fourth pile had twice as many coins as the second. <br> How many gold coins did Esmerelda put in each pile? | This is the original version therefore was not altered. <br> The illustration shows far more than the twenty coins Queen Esmerelda is supposed to have. |


| Yellow | Decorative | Queen Esmerelda's coins <br> Queen Esmerelda had 20 gold coins. <br> She put them in four piles. <br> The first pile had four more coins than the second. <br> The second had one less coin than the third. <br> - The fourth pile had twice as many coins as the second. <br> How many gold coins did Esmerelda put in each pile? | In this version the piles of gold coins have been made more equal in height, although there are still a few more than the stated twenty. I did this so that the four piles are illustrated but is only decorative to the scene. |
| :---: | :---: | :---: | :---: |
| Red | No Picture | Queen Esmerelda's coins <br> Queen Esmerelda had 20 gold coins. <br> She put them in four piles. <br> The first pile had four more coins than the second. <br> The second had one less coin than the third. <br> The fourth pile had twice as many coins as the second. <br> How many gold coins did Esmerelda put in each pile? | Having no illustration, all the information about the number of coins has to be retrieved from the text. The text was not altered from the original as it contained all the relevant information. |
| Purple | Related | Queen Esmerelda's coins <br> Queen Esmerelda had 20 gold coins. <br> She put them in four piles. <br> - The first pile had four more coins than the second. <br> - The second had one less coin than the third. <br> - The fourth pile had twice as many coins as the second. <br> How many gold coins did Esmerelda put in each pile? | To reinforce the information in the text I altered the text so that only twenty coins in four piles can be seen although the numbers in each pile does not reflect the requirements of the question because from experience of the fireworks question where I had witnessed children using the related illustration |


|  |  | which gave the <br> correct answer, 1 <br> wanted to ensure <br> the children would <br> have to consider <br> performing a <br> mathematical <br> calculation. |
| :--- | :--- | :--- | :--- |

Duck Ponds was the next task, again illustrated in five different ways. Using fourteen ducks each time, the ducks had to be distributed between three or four ponds (depending on the part of the question) with the number of ducks in each pond having an algebraic relationship to the other ponds in the sequence. For instance, in the first part, there were four ponds and fourteen ducks. Each pond could hold either five ducks or two ducks, so the solution would be two ponds each containing two ducks .

| Booklet Colour | Illustration Type | Illustration | Characteristics |
| :---: | :---: | :---: | :---: |
| Blue | No Picture | Duck Ponds <br> Use 14 ducks eech time. <br> 1. There are four ponds. Make each pond hold two ducks or five ducks. <br> 2. There are three ponds. Make each pond hold twice as many ducks as the one before. <br> 3. There are four ponds. Make each pond hold one less duck than the one before. | The number of ducks and the number of ponds is only available via the text. |
| Green | Decorative | Duck Ponds <br> Use 14 ducks each time. <br> 1. There are four ponds. Make each pond hold two ducks or five ducks. <br> 2. There are three ponds. Make each pond hold twice as many ducks as the one before. <br> 3. There are four ponds. Make each pond hold one less duck than the one before. | I elected to use the illustration from the original version because it only illustrates ducks on the water and gives no indication of the total number of ducks or ponds. |

Yellow Related

| Purple | Essential | To fit with the <br> requirements of <br> the question I |
| :--- | :--- | :--- |
| altered the |  |  |
| illustration so that |  |  |
| the number of |  |  |
| ducks and ponds |  |  |
| can only be |  |  |
| inferred through |  |  |
| the illustrations. |  |  |
| The text did not |  |  |
| require alteration |  |  |
| from the original. |  |  |

Because of the two illustrative features of this question, the ducks and the ponds, this question was difficult to classify and alter which resulted in a compromise between the ducks and ponds depending upon the format. I had wanted to keep to the original format as much as possible. The version with no illustration resulted in minor textual changes so that the number of ponds could be included in the text. In the decorative format I removed the pond illustration but kept the original ducks. I had considered showing some ducks on a pond as the only illustration but this would have totally removed all illustrative elements of the original. I also had misgivings about the related version even though this was the original. In the trial I had considered this to be an "essential" item because of the pond illustration is the only way in which the number of ponds is specified. However I was unsure if the pond can be counted as a true illustration within the context of this study. Therefore, I focused upon the ducks in which case the illustration was related because it reinforced ducks on a pond. However, because it does not reaffirm the fact that fourteen are required it could be considered decorative. On reflection, because of these inconsistencies, and my inability to clearly define this question using my illustration types I feel that with hindsight, this question should not have been used.


Each of them ate a different odd number of nuts.
How many nuts did each of the monkeys eat? Find as many different ways to do it as you can.

The final task in the booklet was a control question with a negative decorative picture - Three Monkeys. In this question, three monkeys eat a total of 25 nuts, with each one eating a different odd number of nuts. The children were asked to find as many different ways as possible the monkeys could have eaten the nuts. There
were ten possible answers.

## 5.3-Overall results from illustration type

An analysis was carried out in order to determine whether pupil success was associated with particular illustration types. The number of correct answers for each illustration type were calculated and the results are discussed here. For ease of comparison, the results are presented as percentages rounded to the nearest whole number.


Figure 5 Percentage of Correct Answers for Different Illustration Types

This chart (Figure 5) shows the percentage of successful answers for each question that was presented to the children using the five different illustration types (control questions are not included). Where a high percentage of children were able to give a correct solution, this was closely linked to the illustration being of a related or essential type. This is more clearly demonstrated when the percentages for all five questions are calculated based upon the illustration type (Figure 6) .

Total Percentage of Correct Answers for Each Question Type


Figure 6 Total Percentage of Correct Answers for Each Question Type

The illustration type that produced the lowest percentage of correct answers was the negative decorative illustration.

When the control questions are considered alone (Figure 7) they show that the greatest number of correct answers once again were associated with the essential and related question types. Since the various questions could have a number of different parts (Snakes and Ladders could have up to 4 correct answers, Roly Poly only one for each of the two parts, Ski lift one answer and Monkeys up to ten) I decided that the mean number of correct answers would form the basis for this graph.

Control Questions - Correct Answers (Mean) by Type


Figure 7 Control Questions - Correct Answers (Mean) by Type

Using this method of calculation reaffirms that the related and essential illustration types were associated with more correct answers than the other illustrative types. The graph does not include data from the no picture category as there was no control question that had no picture.

If the results of the control questions are incorporated into the results of the other questions, the results confirm that the essential and related illustration types were associated with a greater number of correct answers than the decorative and negative decorative questions (Figure 8).

Total Number of Correct Answers for Different Picture Types


Figure 8 Total Number of Correct Answers for Different Picture Types

To summarise, in all the questions asked the children were most successful when presented with a question of the related or essential illustration types. The illustration type associated with the least number of correct answers was the negative decorative type. In most cases, an illustration that was negatively decorative proved to be worse than having no illustration at all. This means that if we were to illustrate a mathematics question, providing an illustration that adds nothing but looks like it might provide information is likely to be extremely misleading. This begs the question as to how children tend to use the illustrations provided in mathematics questions. On further analysis it became clear that irrespective of the illustration type, there were a number of common themes that characterised the way the vast majority of children used the illustration in order to help them solve the problem. A discussion of these themes may also provide clues as to why illustrations of the essential and related types were associated far more with pupil success than the negative decorative type.

## 5.4 - Themes Arising from Pupil Answers to the Challenge Book Question

When the results from the Challenge Book questions were examined, a number of key themes emerged. These themes illustrate the particular way in which the children in
the study approached the questions and highlights areas where the illustration had an effect on the child's approach to solving the problems. There were four main themes identified, namely:

- An over reliance on the illustration
- A reality based perspective to the solution
- Once the information has been retrieved from the illustration it becomes superfluous as they focus purely on their calculations
- Children use their own drawings to interpret and solve the problem

I shall introduce each one and illustrate it with examples of children's work.

### 5.4.1. An over reliance on the illustration.

The solutions that the children gave provided clear evidence that they were relying heavily on the illustration for information and as a model to help them solve the problem. Of all the questions which showed an illustration, $27 \%$ of the children's answers displayed evidence (in the form of pencil marks or extra items drawn into the picture) that the children had used the illustration directly to help them understand or solve the question. Since it is reasonable to assume that some children will have used this approach but not left any marks on the illustration the real figure is likely to be greater than this. Use of the illustration in this way was irrespective of the type of illustration presented to the children, negative decorative illustrations showed as much marking as the essential or related illustrations. It appears as if the illustration is the first "port of call" when a child attempts to find a solution to a mathematical problem, especially if the question appears to be unclear or difficult. If this is indeed the case, it follows then, that those questions that were illustrated with a negative decorative type of picture substantially reduced the chances of the child achieving a correct solution compared to a child whose illustration was essential or related.

In this example of Gold Bars, the child has relied totally upon the picture, disregarding the discrepancy between the text, which explained the correct number of bars in each pile, and the picture which does not reflect this. (Blue Booklet, Girl, (Maths 3B, Reading 4)


This inappropriate use of the illustration was repeated with the Spaceship question where children had a tendency (sixteen of the twenty-five children whose illustration was negative decorative) to count the aliens in the picture itself. (Yellow Booklet, Girl, Maths 3C, Reading 3C)

In this example of a related version of Queen Esmerelda's Coins the twenty coins were distributed between four piles but not with the appropriate connection between each pile. Those that were successful showed evidence of having begun by counting the coins in the illustration and then used trial and error to redistribute the coins in the correct ratios. This indicates that the reliance on the illustration gave an initial concrete base that the child could

then use to develop their next stage of abstract thinking.

The Roly Poly question again highlighted an issue that reflected the reliance the children placed on the illustration. Few children were able to give a positive response to this question. Of the 97 incorrect responses for part 1, 59 (61\%) related directly to the dice shown in the picture. In part 2, 43 responses related to the dice pictured. The implication of this is that when the problem appears too difficult children are very likely to rely on the picture for assistance irrespective of whether the illustration actually provides any meaningful information.

1. Imagine roiling one dica.

The score is the total number of dots you can see.
You score 17.
Which number is face down? 3
How did you work out your answer?
2. Imagine roiling two dicz. 4 no lo make,

The dice do not touch each other.

(Green Booklet, Boy, Maths 4, Reading 4)
For those children whose illustration for the Fireworks question was essential or related an interesting discrepancy between parts 1 and 2 was noted. The apparent high success rate for part one may well be a false result. This is because the illustration actually gives one of the two answers. All those answers that are counted as correct relate to the solution shown on the illustration, only one child gave both answers. When faced with part 2, those children who may have relied on the illustration may have had no mathematical basis upon which to attempt part 2 and so gave up.

The evidence strongly indicates that irrespective of the illustration type, the children assume that the illustration is essential to the mathematical process of answering the
questions. The illustration appears to provide concrete support to their understanding and calculations notwithstanding the discrepancies between the illustration and the text. It would indicate that generally the illustration is of more value than the text in children's understanding and calculation processes in mathematical problem solving.

### 5.4.2. A reality based perspective to the solution

In this instance the child may visualise the problem in order to solve it by putting themselves or a third party into the situation described in the text. This gives a personal reality based perspective to the solution which in some instances overrode the mathematical considerations. At times this familiarity with the subject matter interfered with their understanding of the problem.


An example of this imaginative perspective was seen in the Roly Poly question where children needed to be able to imagine the dots on five sides of the dice, even though they could not be seen directly, demanding not just an imaginative perspective, but visualising the dice from a changing point of view. Of course the question could have
been solved solely by using a calculation method. The total number of dots on a dice is 21,17 from 21 equals 4 . Yet because it used an object with which all the children were familiar they appear to have relied on their visualisation skills and where this was too difficult, the dice illustrated in the question was the visual source of choice. In order for the children to have matched their answer to that of the dice shown in the illustration they would have had to imagined the spots on the dice from various perspectives.

The above example shows how the child has tried to visualise the numbers on the various sides of the dice. He appears to have imagined the dots in number sequence and adjusted his conclusion when four is shown at the top. The right hand dice on part two indicates this clearly where the sides have been labelled with the numbers one to four, the top five and the bottom six. This may well be his strategy because each dice shown has a six on the bottom. Obviously he has disregarded the text information that the opposite sides of a dice always add up to seven. (Green Booklet, Boy, Maths 4, Reading 4).

All the children who were successful with this question had achieved level 3A or 4 in their mathematics SATs paper. However, not all children working at level 3A or 4 were successful, with some misled into using the pictured dice directly. It may be that those who were successful were confident enough to move out of a stationary perspective.

1. Imagine rolling one dice. The score is the total number of dots you can sez. You score 17.
Which number is face down? 6 How did you work ourt your answer? pecause 1 looked one arourrd the dice arrd the bottom
2. Imagine rollina two dicz.
(Green Booklet, Girl, Maths 3C, Reading 3B)

Although this child made an incorrect calculation, it would appear that she understood the need for a changing point of view.

For some children the request to imagine themselves rolling the dice acts as a trigger for them to make a literal image of themselves rolling a dice. In this situation the mathematics element appears to fade into the background.

1. Imagine rolling one dice.

The score is the total number of dots you can see. You score 17.
Which number is face down? How did you work out your answer?
2. Imagine rolling two dice.

The dice do not touch each other.
(Red Booklet, Girl, Maths 3B, Reading 4)

In the Gold bars question, some children also provided answers that related to the mechanics of performing the actual task, rather than as a mathematical abstraction. In these cases the child tended to focus on the whole task as a reality based problem rather than as a mathematical problem. Since the question text asks "How did he do it?" this answer may not be as inappropriate as it first seems.

He Lifted one at a time
and putit downerve he wanted
(Red Booklet, Boy, Maths 2A, Reading 3A)
Another example from a child who focused upon the physical aspect of Pete moving the bars, and in this case sand, to make the piles of equal heights.
(Red Booklet, Girl, Maths 2A, Reading 4).


In the previous examples children's extensive knowledge of the reality prevented them from tackling the mathematics correctly. However, a measure of that reality is required. If that is missing then children are equally disadvantaged. Too little knowledge is as bad as too much. The Ski Lift question which was the decorative control question presented a major difficulty. Of the 122 children who attempted this question, only one child was successful in answering it correctly. Six children did not respond. The evidence clearly shows that this question was too difficult because they did not have the knowledge of how a ski lift operated.

The processes apparently generating incorrect answers fell into three main categories. Children who answered this question either subtracted ten from the hundred, assumed one hundred was the maximum number of chairs or added the ten to the one hundred (Figure 9).

Ski Lift - Incorrect Answers


Figure 9 Ski Lift incorrect answers

The child who was successful drew a detailed picture of the Ski Lift.

(Purple Booklet, Girl, maths 4, Reading 4)
This was the only drawing that gave any indication that the child knew how a ski lift operates. Of the one hundred and twenty-eight children, eighty children tried a arithmetic calculation and eighteen used words.

Of the children in my own class, only one child had ever been skiing in a place where ski lifts were in operation. A degree of skiing knowledge was necessary in order for the children to even attempt this question. I would suggest that not many British children of this age would go skiing although many of the children are of the socioeconomic class who may go skiing in the future. Therefore the decorative illustration is inadequate. If a question of this type was to be used, a picture that illustrates the mechanism of a ski lift would be of greater use, possibly making a very difficult question more accessible.

This Ski Lift question demonstrates that in some cases, a clear understanding of the context is essential in order to solve a problem. In this study this reality perspective was essential if one was to attempt a correct answer for the ski lift question as a working knowledge of how a ski lift operates was required. Equally with the Fireworks question the fact that fireworks can only be set off once may have been a factor in preventing children from giving the required two solutions to the question.

It would appear that children are not always clear when their knowledge of the world and an imaginative perspective is an asset to mathematical problem solving and when reality has to be limited in order achieve the correct mathematical solution. The problem may well be compounded when mathematical problems are set in a context with which the children are familiar. Some children appear to be unable to distinguish between a mathematical context where reality can be restricted in favour of the mathematics and a reality based context where more legitimate practical considerations are taken into account.

### 5.4.3. Once the information has been retrieved from the illustration it becomes superfluous as they focus purely on their calculations

Unlike the previous theme where an imaginative perspective influenced the children's understanding and calculations often with a negative result, in this instance there was a clear indication that the mathematics overrode the constraints of reality that were an essential part of the problem.

The control question Snakes and Ladders demonstrated how once the basic information had been retrieved from the illustration it could then be discarded in lieu of focused attention upon the calculations. Some children gave incorrect responses because they focused upon finding as many different ways of possible to make seven irrespective of the snake's position. The assumption is that they used the picture to calculate how many squares were needed from the starting position to the end, but then they took no further account of the information in the picture. For them, they had extracted a mathematical number bond question from the information within the illustration.

The following example is typical of the children's calculation. This one has been separated into two calculations for each roll of the dice, so that the first line refers to a roll of 5 then 2 ; the second line a roll of 4 and 3 etc. His own question follows a similar pattern to that indicated by the initial posed question.

$$
\begin{aligned}
& q+5=1414+2=16 \\
& 9+4=1313+3=16 \\
& 9+3=1212+4=16 \\
& 9+2=1111+5=16 \\
& 9+1=1010+6=16
\end{aligned}
$$



(Purple Booklet, Boy, Maths 4, Reading 3A)
For another child, the number bond focus was making 16, the final square on the snakes and ladders grid and failed to relate the task to the realistic constraints imposed by the question.
8 and 8
12 and 4
10 and 6
1 and 15
2 and 14
3 and 13
4 and 12
5 and 11
6 and 10
11 and 5
13 and 3
14 and 2
15 +1
(Red Booklet, Girl, Maths 4, Reading 4)
Of the forty incorrect responses, thirty six had included a combination that involved them travelling down a snake. It seems that these children paid no attention to the realistic constraint embedded in the picture.

The requirements of the question were also discarded in favour of a purely mathematical solution in this example from the Fireworks question. The child has counted the number of fireworks shown in the picture (11) and divided it by two, presumably because the question explains that there are two types, those that produce three stars and those that produce four stars. Recognising that a fraction of a firework is not a possibility she has decided to divide the two types as near as possible into whole numbers that will result in a total of eleven.

(Purple Booklet, Boy, Maths 3A, Reading 3A) Part 1

An issue which arose when children had focused upon using calculations was that often their solutions tended to be rather involved and long winded (shown in the following two examples) and as the children attempted to work through the problem using their limited calculation knowledge they lost track of the goal they were aiming for. This meant that as the calculation became more difficult the children showed a tendency to lose track in the calculation process often backtracking and crossing out in order to make sense of the problem they were trying to solve.

(Yellow Booklet, Girl, Maths 3A, Reading 4)


### 5.4.4. Children use their own drawings to help interpret and solve the problem

Fifty-one percent of the questions answered by the children included their own drawings within their calculations and seventy-nine percent of the children drew a picture for at least one of the questions they answered. Often these drawings resembled the objects involved in the question. This would indicate that not only
were drawings giving the children a measure of confidence about using digits in their calculations, but by copying the objects shown in the illustration they acted as a form of concrete apparatus.

This example from the Firework question, indicates a misunderstanding of the question in that the child has based their calculation upon 25 fireworks rather than the 25 stars produced by the fireworks. The excessive number of fireworks shown in the illustration may have been a contributing factor to this child's misunderstanding but clearly shows how children's own drawings that replicate those shown in the questions is a part of their calculations.

(Purple Booklet, Girl, maths 3B, Reading 3B)

With the monkeys question, all the children that were successful in finding at least one combination had drawn a picture to illustrate their working and results. It would appear therefore that for some children the drawing of their own interpretation of the mathematics is helpful to their comprehension and calculation.

(Purple Booklet, Girl, Maths 4, Reading 4)

The following example of a child's correct working is of interest as it appears he has made piles of twenty-five nuts which he has then proceeded to divide up.

(Green Booklet, Boy
(Christopher), Maths 3A,
Reading 4)

This work, from the Gold Bars question, shows that alongside their drawing of the initial piles (being an exact copy of the gold bars shown in the essential illustration) the child has provided an explanation in words and pictures as to how Pete could move the gold bars to get four piles of four. (Yellow Booklet, Boy, Maths 4, Reading 4)


The Duck Ponds question also showed a proliferation of drawn ducks in the children's calculations and answers. In this example the boy has drawn the correct number of ducks for part 1 but in part 2 he has continued to use two and five ducks for the first two ponds and then drawn seven ducks in the final pond. For part three the first pond holds four ducks and subsequent ponds hold one duck less. This implies that for the

third part he has focused on the 'one less' instruction rather than the total number of fourteen ducks stated in the text.
(Blue Booklet, Boy, Maths 3B, Reading 3A)
The following is an example where the question has been misread or misunderstood and that a child's own illustrations can confuse things even more. The question involves the distribution of a number of ducks into a number of ponds according to a fixed set of rules. The instructions state that fourteen ducks should be used to answer the question each time. However, the child did not take notice of this instruction and just read the instruction "make each pond hold two ducks or five ducks". He chose to make each pond hold two ducks. Again, ignoring the fourteen duck requirement he

1. Make each pond hold two dueks or five ducks.

2. Make each pond hold twice as many ducks as the one before.

3. Make each pond hold one less duck than the one before.

then misreads the next two instructions and assumes that in each case it refers to the previous question part. This results in his solution to the final question involving negative numbers of ducks. In the first part he has drawn the ducks but in the second and third part he has reverted to using digits.

This is possibly due to his
misunderstanding of the question - it would have been impossible to draw zero or minus one ducks. (Purple Booklet, Boy, Maths 3C, Reading 2C)

Where no illustration was provided, children would still draw their own versions of the problem rather than relying on digits or marks to help them in their calculations. In the Spaceship question, eighteen of the twenty five children who had no illustration drew their own pictures. This again indicates that a visual representation can aid understanding and calculation.

(Green Booklet, Boy, Maths 3C, Reading 3B)

The fact that so many children use their own drawings in the calculation process may indicate that at this level of mathematical competence actually drawing the items provides a concrete base for solving potentially difficult problems when children are not completely confident in using abstract numbers and calculations. The fact that they elected to draw similar representations rather than marks to replace numbers would also indicate that developmentally they are still very much reliant on visual, concrete apparatus. The level the children had achieved in their mathematics and reading SATs appears to have made little difference as to whether children chose to use their own their drawings or not which would indicate that ability is not necessarily a factor in whether or not children rely on drawing pictures to help themselves solve mathematical problems.

### 5.5 Results from the interviews

In addition to the examples from the Challenge Booklets, the interviews with small groups of children reinforced two of the themes mentioned above, specifically theme 2 (a reality based perspective to the solution) and theme 3, (once the information has been retrieved from the illustration it becomes superfluous as they focus purely on their calculations). The interviews did not provide evidence for any of the other themes.

Theme 2 (a reality based perspective to the solution) may be illustrated by an interview with Bobby and Claire. In this example, Bobby is so taken with his knowledge of pirates and his opinion of the physique of the pirate in the picture that he finds it difficult to solve the mathematics within the question.


Bobby provided a correct answer to Gold Bars of 2 piles of 8 but after a wrong calculation. Claire, who was asked to take part in the discussion, gave an incorrect answer to the original question. Both the children had a version of the question where the illustration was decorative. Although Bobby's working showed an incorrect and a correct answer, I only showed the children the incorrect one as I wanted to see if they could explain how it could be done correctly.

| Transcript | Comment |
| :--- | :--- |
| RJ: As you can see, this person made a wrong | Introduction to task. |
| calculation. I want us to try to work out what he did |  |
| first with the wrong answer and then how it could |  |
| have been done right. |  |
| C: Pete has four piles of gold. |  |
| B: No he has one. Look and it's only got two bars of |  |
| gold. | Bobby thinking is being <br> C: But this says four piles so this is right. |
| B: Maybe we can't see the rest because it's buried in a |  |
| treasure chest but then he must have three chests and | of pirates rather than the |
| pathematical element. |  |
| pirates always have coins and jewels they don't have |  |
| bars. |  |
| C: Shut up and stop being silly. He has four piles | Claire quickly focused |

because it says and in one pile he has 6, in another 3, another 2 and another 5 . So how many has he got altogether?
B: Five and three make eight add two makes ten and then six makes sixteen. So why does it only show two bars. But he's drawn lots.
C: He's drawn the bars in the piles. The first one has six. One, two, three, four, five, six (counted the 6 bars drawn in pile one). The second has three. You check. The next two and the last five. Is that right?
B: So he's drawn the bars. Now what do we do?
C: They have to be the same height but he can only do it in two moves.
B: If he moved one bar to here making four then two bars to here making four. That makes six, four, four and two. That's wrong. Does this two bars mean there is two bars or that he moves these two bars?
RJ: I don't know. That's something I was hoping you might be able to work out.
B: If it was two bars then he's made six, four, two and three so it's still wrong because they're not the same.
C: It's the middle bars that's the problem we should move those cause the others are taller and you begin with the biggest.
RJ: How do you mean?
C: When you add numbers together, you start with the biggest number even if it doesn't come first and add on the other numbers. The six and five piles don't move.
B: They'd be heavy to move so he just moves the three and two. If he puts the three on the six that makes nine and the two cause it's next to five on there that makes nine and seven and then if he moves one from the nine to the seven that means they both have eight.
C: But that's three moves and you can't do three moves.
B: Stop getting cross. It's not my work we're trying to think like someone else what they would do not what I'd do. Move the two onto the six and the three on the five that makes eight each in two goes. Like this. (Bobby then drew an almost replica illustration of that shown on his working).
RJ: How would you have done it then Bobby?
B: He had sixteen bars divided by four makes
err....four. So each pile has four.
RJ: Can you make each pile have four then in just two moves.
B: Move two from the six on the three next to it, that makes five then move one from there to the two that

Bobby's attention to the mathematics but Bobby is still influenced by the information in the illustration.

Recognition that the text has been illustrated to show the piles of bars that Pete originally started with.

The children recognise that adding one to the second pile is a correct move as will adding two to the third pile. Unfortunately they focus on moving the bars from the fourth pile, leaving the first intact their reasoning being that the tallest pile is complete.

Bobby is once again influenced by the reality of moving heavy gold bars recognising that you wouldn't want to lift them far just adding from the centre piles to the two outside piles although he does find a solution of two piles of eight it is not done in the required two moves.

The mathematical solution Bobby has calculated would give four piles of four but once again the reality of lifting gold bars and the appearance of Pete leads
makes three. No. Cause you need to move one from the five onto the three and that makes three moves.
RJ : Does he have to move them onto the pile next to each other?
B: Gold Bars are heavy and he looks a bit weak and skinny. His arms and legs are tiny so he doesn't have many muscles.
C: Maybe the parrot picks them up in it's beak!
$\mathrm{B}: \mathrm{Ha}, \mathrm{Ha}$ and I suppose the crab helps as well.
RJ: But the question says he, that doesn't mean he had help. So lets focus back on the numbers and just imagine he's really strong.
B: If you move one from the five and put it on the three.
C: and if you move two from the six, that makes four and put it on the two that makes four.
RJ: Each pile then has four bars and you did it in two moves. OK you've been thinking hard this morning so one last question. You've come up with two different answers. Two piles of eight and four piles of four and each could be done in two moves. Is one answer more correct than the other?
$B$ and C: No
B: They both do it in two moves and are the same height.

Bobby to dismiss his mathematical solution and focus on Pete as a true life character.

The absurdity of the image refocuses the children to the mathematics and they quickly recognise a correct solution.

Recognition that two solutions are possible.

An example of Theme 3 (once the information has been retrieved from the illustration it becomes superfluous as they focus purely on their calculations) is provided by the interview with David and Louise. David's answers show that once he has discovered that it is about snakes and ladders - a game that is familiar to him - he ignores any other clues from the illustration. His solutions are presented as a series of number bonds but he makes no link with the fact that in snakes and ladders some combinations cause the player to slide down a snake and lose position.

The interview is formed around a discussion about a particular child's solution to the snakes and ladders problem. The work was David's originally, though he didn't realise this in the interview, and Louise was chosen because she had been successful at solving this problem herself. David had written a list of calculations which did not take account of the position of the snakes.

```
6+1 (David's original answers)
5+2
```

$4+3$
$3+4$
$2+5$
$1+6$

| Transcript |
| :--- |
| RJ: Can you remember when we did these? It was a little |
| while ago. We are going to have a look at this person's |
| work and I'm not going to tell you whose it is but I want |
| us to try to work out what they did because the answer |
| isn't quite right. OK? |
| D: The counter is on 9 and you have to get to 16. |
| RJ: Have you both played Snakes and Ladders? Do you |
| know the rules? |

D: Yeah. You throw two dice and then move your counter. If you land at a ladder you go up to the top and if you land on a snake you go down it and have to start all over again.
L: But you have to land on the snake's head and you have to throw a six to start. The first one who gets to 100 is the winner.
RJ: Why 100 ?
D: Cause that's where home is, not like this one which only goes to 16 .
L: It could be a game for really young children, like reception who are just learning to play. But not for us. RJ: OK So we're fine with the game so let's look at this answer.
L: It's an easy question this because you only have to move from 9 to 16 that's ... 16 take away 9 . (uses fingers to count on from nine to sixteen) Seven.
RJ: But seven is more than six and the dice goes to six. D: But you only have one dice. Which means it must be for reception cause they wouldn't play with two dice because they can't add. They're only learning to count.
L: So we need to make seven using the digits one to six.
D: One and six, two and five. (As the children say these I write them down on a whiteboard)
L: Three and four, Four and three.
D: Five and two, Six and one.
L: Seven and nought. Oh no you can't get a seven only six.
RJ: You got carried away there. Apart from your seven and nought how do your answers match these on the paper?
D: They're the same but you said it was wrong.
RJ : I could be lying, just to trick you.
L: True. But you said it in that voice which means you weren't.

| (I wait for a few them a counter) <br> RJ: Use the counter and move it as you would the throws based on your list we've written down on the whiteboard. D: Start at nine, move one. (moves counter) Then six. One, two, three, four, five, six. Home. Your go. <br> L: Two and five won't work. You go down a snake. Look I'll show you. (places counter on the 9 and moves two places, going down the snake. That's the mistake. He's forgotten about the snakes. <br> RJ: Ok you're right. That's the mistake, he didn't think about the snakes. Are there any others on the list which would be wrong? <br> D: Six and one. All the others are OK. <br> RJ: OK. So why do you think he got it wrong by writing down the answers which go down the snake? <br> D: He's worked out you just have to make seven and then wrote down all the ways you make seven. If you look his numbers go down from six, five, four, three, two, one. Just like we did. He didn't think about the snakes. RJ: It was an easy mistake to make. Do you think you would have made this mistake? <br> D: No because I know how to play snakes and ladders and I'm not that stupid (even though he did!). L: No. It was an easy sum and they can sometimes trick | Using the counter on the illustrated board refocuses their attention to the demands of the illustration making the children realise the mistake that had been made. <br> David considers it to be a stupid mistake to disregard the snakes, something he would never do whilst Louise interprets it as a trick and that the illustration, whilst providing essential information, is also intentionally misleading the reader. |
| :---: | :---: |

It seems clear from this extract that children are likely to make the same mistake over and over again unless their attention is drawn back to the illustration and they are asked to consider their answers in the light of the extra information provided by the illustration. Even Louise, who got the original question right is drawn into the number bonds solution and only changes her stance when she actually makes the moves on the snakes and ladders board.

Since the themes described above were very clearly displayed throughout the solutions analysed it is clear that the illustration provided with a mathematical problem plays an important role in the child's understanding of the problem. Children appear to expect that if an illustration accompanies a problem that it is there to provide extra information or to augment the information already provided. Illustrations that do not provide information or even worse, provide information that is at odds with the question serve to confuse and mislead children. In many cases they are not discerning enough to see that the illustration is beguiling them but will
attempt to build the information it contains into their solution regardless of whether this makes the solution harder or even impossible to achieve. Similarly, children are likely to rely solely on the illustration for evidence on how to solve the problem or, if the medium is very familiar will disregard the question entirely and build their own model to obtain a solution. In all cases this illustrates that children at this stage of their learning are not sophisticated enough to distinguish whether or not the illustration is there to help them. In consequence it is vital that illustrators are required to restrict their illustrations to items within the question and not to elaborate or exaggerate the figures involved.

## 5.6 - Pupil Perceptions of the challenges

In the final part of the booklet the children were asked to complete the questionnaire (Appendix 2). They were asked answer the following three questions.

1. Which question would you prefer to work with and why?
2. Imagine someone who found maths really hard, which question do you think would be the easiest for them and why?
3. What did you like or find useful about the pictures?

The first question was aimed at finding out which of the tasks the children preferred and whether the illustration type was a factor in their selection. The purpose of the second question was to ascertain which of the tasks they thought was the easiest in order to investigate whether the illustration had an influence on their selection. I specifically phrased the question with a third person in mind because I felt that it would help to remove some of the child's personal baggage of their personal experience and provide a more objective view of the questions. The third question was specifically aimed at collecting their view of how useful the illustrations were to them in answering the questions, particularly as I had noticed previously that children tend to try to make use of the illustration in solving a problem regardless of whether the illustration actually provides them with useful or appropriate information.

### 5.6.1-Question Preference

In answer to the first item asking which question they preferred and why, the results were as follows (Figure 10).

The Question They Preferred


Figure 10 Question Preference

Except for the Ski Lift question, all the other questions were considered to be a favourite by at least one child. However I chose the most popular of these by using those questions that had been chosen by at least ten children. Four questions fell into this category, being Snakes and Ladders, Gold Bars, Three Monkeys and Duck Ponds. Their reasons for choosing these tasks as their favourite were broken down as follows.

|  | It is a <br> challenge | It is easy <br> to <br> understand | It is fun | I enjoy <br> number <br> facts | The <br> picture <br> helps | Other | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Snakes and <br> Ladders | 3 | 32 | 7 | 1 | 22 | 0 | 65 |
| Gold Bars | 4 | 11 | 3 | 0 | 6 | I like pirates <br> $(1)$ | 25 |
| Three <br> Monkeys | 0 | 9 | 2 | 1 | 0 | 0 | 12 |


| Duck <br> Ponds | 2 | 4 | 1 | 0 | 2 | Lots of <br> helpful <br> information <br> $(1)$ | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Total | $9(8 \%)$ | $56(50 \%)$ | 13 <br> $(12 \%)$ | $2(2 \%)$ | $30(28 \%)$ | $2(2 \%)$ | 112 |

Table 22 Reasons for question preference
Table 22 clearly indicates that "It is easy to understand" is the most common reason for selecting a particular question and that Snakes and Ladders was the most popular question. Since Snakes and ladders was a control question and only appeared in one illustration format - "Essential" it would appear that the picture is an important factor in this understanding especially as $28 \%$ felt that the picture helped. This implies that the children are using the illustration to aid their comprehension irrespective of whether this was the intention of the illustrator.

The following comments made by the children in their answers to the questionnaire offer some further support for this interpretation of the data.

(Purple Booklet, Girl, Maths 3A, Reading 3A)

This girl enjoys counting things in pictures so a decorative or negative decorative illustration would be likely to cause confusion. She evidently uses a picture to find information or have information confirmed. If the picture does not match the text, her strategy for tackling problems could be undermined. She may also feel tricked or betrayed if the text and illustration don't match because she is looking for the pictures
to give the clue. For her, as well as being motivating by being informative, the illustration does have an entertaining role in that it is fun.

Which question would you prefer to work with and why? Cold bars because the picture helped you get to the easy answere. I also fund it case because you can look at the picture and by the 4 piles gold bar's you hove to divide
(Yellow Booklet, Boy, Maths 4, Reading 4)

This child appears to have gained a great deal from the information provided in the illustration. In the case of Gold Bars this was essential for this child in order that he could easily solve the task.


(Blue Booklet, Girl, Maths 3A, Reading 4)

This child appears to enjoy the challenge of finding different ways of finding number patterns (Snakes and Ladders) but unfortunately she was not successful with this question herself because she took no notice of the position of the snakes in the illustration and wrote a series of number bonds that make seven.

Question one asked which of the questions found in the booklet the child preferred but at this stage did not specifically ask about the illustrations. However the answers that the children gave predominantly mentioned the illustration as being an important
aspect of the question. It seemed that an appropriate illustration made the question seem easier to understand and more fun.

### 5.6.2 - Question difficulty

In answer to question two on the questionnaire, 'which maths question they thought someone who has difficulty with maths would prefer to work with' the results were as follows (Figure 11).

Most Likely Question a Poor Mathematician Would Prefer


Figure 11 Most Likely Question a Poor Mathematician Would Prefer

Five children misunderstood the question and instead of identifying a particular question they identified an individual in the school who they considered to have difficulty with mathematics, so their results are not included.

Of the remaining children who did select a question, the reasons given for those questions where ten or more children chose the same question, are as follows.

|  | You can <br> count <br> things | It is easy to <br> understand / <br> work out | It is fun | The picture <br> helps | Other | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Snakes and <br> Ladders | 1 | 44 | 1 | 21 | 0 | 67 |
| Gold Bars | 1 | 8 | 1 | 5 | 0 | 15 |
| Space Ship | 2 | 3 | 0 | 3 | Helps you <br> know your <br> tables (2) | 10 |
| Total | $4(4 \%)$ | $55(60 \%)$ | $2(2 \%)$ | $29(31 \%)$ | $2(2 \%)$ | 92 |

Table 23 Reasons given for the most likely questions a poor mathematician would prefer

Unsurprisingly the fact that a question appears easy is the primary reason for children selecting a question for someone who finds difficulty with mathematics. Interesting again is the view that the illustration will help and we can assume that the idea you can count things is also linked to the illustration. This again emphasises that the illustration is considered by the children to be important in comprehending and calculating a mathematical problem.

Imagine someone who found maths really hard which question do you think would be the easiest for them and why?nates
and loaders

$$
\begin{aligned}
& \text { and Fireworks because the pictur } \\
& \text { give you a cue so you can } \\
& \text { count to find out the answer. }
\end{aligned}
$$

pictures
(Purple Booklet, Girl, Maths 3A, Reading 3A)

(Yellow Booklet, Boy, Maths 4, Reading 3A)

Imagine someone who found maths really hard which question do you think would be the easiest for them and why? Well if they only knew their $3 x+4 x$ table space ship would be best because you are asking your hnolage of the ore $3 x+t x$ tale le.

This child appears to recognise that mathematical knowledge is extremely important since she argues that before you could attempt to answer the Spaceship question you would have to know your three and four times tables.
(Yellow Booklet, Boy, Maths 4, Reading 4)

Over all, the children's responses suggest that they feel that easy questions involve low numbers and good illustrations to back up the requirements of the task. The Snakes and Ladders question was chosen by many children, presumably because it would be familiar to most children of their age and involved small numbers so the calculations wouldn't be too difficult.

### 5.6.3 - Perceptions of illustrations

The final question on the questionnaire asked the children 'what did you like or find useful about the pictures?' Since children tended to give the same sort of answers these are tabulated below. Surprisingly, although the question was a positive one, some children provided negative or neutral answers to the question. For each comment type I interpreted the statement as being either positive, negative or neutral in order to group the statements for later analysis.

| Explanation of the pictures | Number of <br> children | Comment |
| :--- | :--- | :--- |
| They help you to understand | 29 | Positive |
| Count things | 13 | Positive |
| You can 'see it' | 9 | Positive |
| Tell you the answer | 9 | Positive |
| Showed you what to do | 8 | Positive |
| They give clues/easy hint | 7 | Positive |
| They are useful | 5 | Positive |
| Fun | 5 | Positive |
| Help you to work it out | 4 | Positive |
| They gave detail | 2 | Positive |
| Look interesting | 2 | Positive |
| Give more information | 2 | Positive |
| Made maths easy/explained things | 2 | Positive |
| They told you what or how many you need | 1 | Positive |
| Helped you if stuck | 1 | Positive |
| Help think of problems | 1 | Positive |
| Nothing | 4 | Neutral |
| Some you need to use | 1 | Neutral |
| Sometimes wrong | 1 | Negative |


| Had little to do with the sum | 1 | Negative |
| :--- | :--- | :--- |
| Some made it harder | 1 | Negative |

Table 24 Explanation of the children's ideas about the pictures

When these statements were grouped by comment type into positive, neutral and negative, the following results were obtained.

Children's Comments About The Illustrations


Figure 12 Children's Comments About The Illustrations

Predominantly the children's comments were positive. Within this large positive group the children overwhelmingly see the illustrations as a source of information rather than merely as something that decorates the page.

Here are some typical comments made by the children.

(Purple Booklet, Girl, Maths 3A, Reading 3A)

This girl seems to believe that illustrations can be helpful when problem solving but at the same time becomes emotionally involved with the characters finding them 'funny' which could act as a motivational element but may also act as a distracter.

What did you like or find useful about the pictures?
Some of them they didn't help at all
but some only how many' re if they helpful for really
(Yellow Booklet, Boy, Maths 4, Reading 4)
This child recognises that illustrations can be a source of essential information but that their usefulness should not always be relied upon.

(Yellow Booklet, Boy, Maths 3B, Reading 3A)
This boy's answer reflects the sentiments of many other children in that they feel that by being able to count items in a picture aids their computation skills.

The children indicate that they are reading the illustrations and using them to provide clues to solving the mathematical problem. Because of this reliance on the picture, illustrations that are merely decorative (and especially negative decorative) are likely to mislead children. This may be why some of the children commented that the picture can 'trick you' or is 'unhelpful'. Unfortunately the children who wrote these comments were not asked to identify the particular questions where this was the case.

Interviewing those children to elicit how they felt 'tricked' by these particular questions would have been useful, but time constraints meant that it was not possible.

Throughout the analysis of the children's answers to all the questions, their perception that illustrations are an important feature in helping them to understand and solve the questions is very clear. For the four problems identified for analysis in question 1, $28 \%$ of the children felt the illustration was the reason for their choice of question preference. For question 2 , of the three problems analysed $31 \%$ identified that the illustration would help a person to solve the question and 100 of the 128 children held positive views about the illustration in answer to question 3. The perceived usefulness of the illustration is also a central theme in their comments. This indicates that to the children, the illustration is extremely important, irrespective of whether in truth it leads them to a correct solution.

## 5.7-Small Scale Survey - Comparison of illustrations

In relation to the five questions where alternative illustration versions had been provided, a group of twenty-six children studied all the versions and were asked which version they felt would be the best to help them work out the answer (Appendix 3). This group of children had not taken part in the original study, being a mixture of children from Year 2 and Year 4. This was because I felt that using children who had completed the booklet may have compromised the validity of the data as a result of their prior experience of thinking about the items. The results are shown in Table 25;

| Question | Illustration Type |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Essential | Related | Decorative | Negative <br> Decorative | No <br> Picture | Total |
| Gold Bars | 2 | 22 | 0 | 2 | 0 | 26 |
| Fireworks | 6 | 18 | 0 | 1 | 1 | 26 |
| Spaceship | 2 | 16 | 1 | 5 | 2 | 26 |
| Queen <br> Esmerelda's <br> Coins | 6 | 10 | 6 | 2 | 2 | 26 |
| Duck Ponds | 16 | 5 | 0 | 3 | 2 | 26 |
| Total | $32(25 \%)$ | $71(55 \%)$ | $7(5 \%)$ | $13(10 \%)$ | $7(5 \%)$ | 130 |

Table 25 Results for Alternative Illustration Types

It can be seen that the related illustration was seen by this group as being the illustration that would be the best to help them work out the answer. The next most popular was the essential illustration type. This finding is in line with the results obtained from the challenge booklet in that these two illustration types are linked with the greatest number of correct answers.

Examples of their comments, using the children's spelling:

- it has near anuff legs.(Spaceship - related)
- 'it tells you how many stars the fireworks make like 4 and 3 and it matches the picture' (Fireworks - related)
- 'the best because you'v got all the writing and you'v got the pitcher to match the words' (Gold Bars - related)
- 'it has the right amount of coins like the words' (Queen Esmerelda's Coins related)
- 'it tells the truth' (Gold Bars - related)
- 'the picture matches the instructions' (Gold Bars - related)
- 'it tells you what to do but it does not tell you the numbers on the blockes so you would have to work it out' (Gold Bars - essential)
- 'I like this one because it has lots and lots and lots of them (aliens) on it (Spaceships - negative decorative)
- 'it shows the right number of ducks and the right number of ponds' (Duck Ponds - related)
- 'it shows you the picture and tells you how many stars each rocket has and it gives you the anser in the picture' (Fireworks - related)

It can be seen that the children value an illustration that reflects the numbers found in the text. This provides a reinforcement that the numbers they are concentrating upon in order to find a solution are the correct ones.

Although I did not seek this information, all the children also commented on which illustration was 'the worst'. This being the question presentation that they felt would hinder their understanding and calculation of the question the most. Their results are shown in Table 26.

| Question | Illustration Type |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Essential | Related | Decorative | Negative <br> Decorative | No <br> Picture | Total |
| Gold Bars | 5 | 0 | 4 | 3 | 14 | 26 |
| Fireworks | 0 | 0 | 1 | 14 | 11 | 26 |
| Spaceship | 5 | 2 | 2 | 10 | 7 | 26 |
| Queen <br> Esmerelda's | 1 | 2 | 0 | 1 | 22 | 26 |


| Coins |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Duck Ponds | 1 | 0 | 2 | 1 | 22 | 26 |
| Total | $12(9 \%)$ | $4(3 \%)$ | $9(7 \%)$ | $29(22 \%)$ | $76(58 \%)$ | 130 |

Table 26 Alternative Illustration Types Seen as 'the worst'
Examples of their comments, using the children's spelling:

- 'it would be the worst because it has far to many legs' (Spaceship - negative decorative)
- 'it has no picture to work from' (Fireworks - no picture)
- 'it tells you but it does not show you' (Queen Esmerelda's Coins - no picture)
- 'it's rong' (Gold Bars - negative decorative)
- 'it has far far far too many coins' (Queen Esmerelda's Coins - negative decorative)
- 'you can hadly sey the duclings' (Duck Ponds - negative decorative)
- 'it looks realy hard and coplicated' (Fireworks - negative decorative)
- 'it doesn't tell you how many legs they have'(Spaceship - decorative)
- 'its not telling the truth' (Gold Bars - negative decorative)

It can be seen that pictures that show numbers that do not match the text, or indeed no picture at all are very confusing for children faced with this type of problem.

It is of interest here that a task containing no picture at all is seen by the children as worse than a negative picture. In fact though, the children who had actually tried to solve the problems had far less success when the picture was of the negative decorative type than those with no picture.

## 5.8 - Analysis of children's responses by reading ability

One of the arguments in the Literature Review proposed that many of the studies that have looked at the relationship of reading ability to the effect of pictures on text processing have shown that less able readers are more adversely affected by the presence of pictures than more able readers. (Beveridge and Griffiths, 1987:31).

Filippatou and Pumfrey described two approaches to the use of illustrations in problem solving depending on the reading ability of the child being tested.

## 1. Compensatory Stance

This suggests that highly skilled readers are already skilled at extracting and remembering information presented in texts, so representational aids such as pictures are superfluous

Pictures serve to compensate for skills. ..relational pictures will primarily benefit less skilled readers, and partial pictures will primarily benefit more highly skilled readers. (Filippatou and Pumfrey, 1996:272/273)

If, Filippatou and Pumfrey are correct, poorer readers are likely to be far more reliant on the picture to fill in the gaps in their understanding since they would be relying on the Selective Compensatory Framework. However, if the illustration was in any way misleading, then poorer readers' attempted solutions could be compromised by their over reliance on the picture because these children might not be equipped to realise the limitations of the illustration, unlike more able readers for whom the picture is only partly the source of information within an illustrated problem. The more able readers' use of the Compensatory Stance means that often, pictures are superfluous or only used to reinforce rather than as the primary source of information.

This argument formed the basis for one of the research questions as to whether there is a difference between able and less able readers.

## What is the significance of a child's reading and mathematical ability upon their use of illustrations to comprehend question meaning?

To investigate this question I have compared the results of those children who may be considered less able readers with those of their more able peers. As shown in the methodology chapter, children who had a reading level (based on SAT scores) of 3C or lower were more likely to have a higher mathematics level than their reading level. Children whose reading level was 3B and above were more likely to have a mathematics level lower than their reading level, and the higher the reading level the greater the disparity.

In order to investigate this, I selected a number of pupils from the study and divided them into two groups depending on their SATs reading score. Pupils were selected so there was an equal spread of mathematics scores within each group. This resulted in a sample of pupils distributed according to the table below:

| Reading Level | Mathematics Level |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | 2C/2B | 2A | 3C | 3B |
| 3C and below (n=35) | $2(6 \%)$ | $8(23 \%)$ | $18(51 \%)$ | $7(20 \%)$ |


| "Poorer Readers" |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| 3B and above (n=38) <br> "Better Readers" | $1(3 \%)$ | $9(24 \%)$ | $17(45 \%)$ | $11(29 \%)$ |

Table 27 Reading and Mathematics Level for the final focus group

Although the two groups were similar, there remained some difference between the groups. In order to compare groups more easily, percentages rounded to the nearest whole number are used rather than the actual numbers. Those in the group whose reading level was 3C and below were called "Poorer Readers", those with a reading level of 3B and above were called "Better Readers".

Some of the questions in the challenge booklet resulted in a very high proportion of children being unable to answer the questions correctly so not all the questions or parts of questions have been used in this comparison. For instance, it would be inappropriate to compare results from the Ski Lift question when none of the children in this study group made a correct response. Therefore only the following questions are used in the comparison;

Snakes and Ladders
Gold Bars
Fireworks (part 1 only)
Duck Ponds (part 1 only)
Snakes and Ladders was a control question but the results from the other three questions will also be broken down by illustration type.

In the following table, the results for each question are shown in order to compare the Poorer Readers with the Better Readers.

|  | Correct <br> answer | Incorrect <br> answer | No Response |
| :---: | :---: | :---: | :---: |
| Snakes and Ladders |  |  |  |
| Poorer Readers | $48 \%$ | $46 \%$ | $6 \%$ |
| Better Readers | $56 \%$ | $34 \%$ | $10 \%$ |
| Gold Bars |  |  |  |
| Poorer Readers | $37 \%$ | $60 \%$ | $3 \%$ |
| Better Readers | $46 \%$ | $49 \%$ | $5 \%$ |
| Duck Ponds - Part 1 |  |  |  |
| Poorer Readers | $34 \%$ | $54 \%$ | $11 \%$ |
| Better Readers | $56 \%$ | $33 \%$ | $10 \%$ |
| Fireworks Part 1 |  |  |  |


| Poorer Readers | $20 \%$ | $77 \%$ | $3 \%$ |
| ---: | :--- | :--- | :--- |
| Better Readers | $36 \%$ | $59 \%$ | $5 \%$ |

Table 28 Reading Focus Group Results

The preceding table (Table 28) shows that those in the Better Readers group got more answers correct than those in the Poorer Readers group. As both groups had an equal spread of mathematical ability, it would appear that their reading level affected their performance. However, although this seems to be the case, it alone does not answer the question concerning the role illustrations play in task success. In order to investigate this further I examined the results based upon their booklet colour, which determined the type of illustration the children worked with. However, because of the small sample size, once this was divided between the five booklet colours the sample size for each type became very small.

Snakes and Ladders was a control question where the illustration was essential and therefore was not included in this analysis.

| Gold Bars |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Group/ Illustration Type | Correct <br> answer | Incorrect <br> answer | No <br> Response | Total |
| Blue Poorer Readers (Neg. Dec) | 0 | 9 | 0 | 9 |
| Blue Better Readers (Neg. Dec) | 2 | 5 | 0 | 7 |
| Green Poorer Readers (Related) | 5 | 3 | 0 | 7 |
| Green Better Readers (Related) | 5 | 2 | 0 | 7 |
| Yellow Poorer Readers (Essential) | 3 | 2 | 0 | 5 |
| Yellow Better Readers (Essential) | 3 | 5 | 0 | 8 |
| Red Poorer Readers (Decorative) | 3 | 3 | 0 | 6 |
| Red Better Readers (Decorative) | 3 | 5 | 2 | 10 |
| Purple Poorer Readers (No Picture) | 2 | 5 | 1 | 8 |
| Purple Better Readers (No Picture) | 5 | 1 | 0 | 6 |
| Total | 31 | 39 | 3 | 73 |

Table 29 Gold Bars Reading Focus Group and Illustration Type Results

As can be seen in Table 29, for the Gold Bars question, although the group size is small, those who had the negative decorative illustration (marked in blue) did poorly, especially the poorer readers, compared to the success of those whose illustration was related to the question (marked in red).

| Duck Ponds - Part 1 Only |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Group/ Illustration Type | Correct <br> answer | Incorrect <br> answer | No <br> Response | Total |  |
| Blue Poorer Readers (No Picture) | 3 | 6 | 0 | 9 |  |
| Blue Better Readers (No Picture) | 3 | 3 | 1 | 7 |  |
| Green Poorer Readers (Decorative) | 1 | 3 | 3 | 7 |  |
| Green Better Readers (Decorative) | 4 | 2 | 1 | 7 |  |
| Yellow Poorer Readers (Related) | 1 | 5 | 0 | 6 |  |
| Yellow Better Readers (Related) | 6 | 1 | 1 | 8 |  |
| Red Poorer Readers (Negative Dec.) | 1 | 5 | 0 | 6 |  |
| Red Better Readers (Negative Dec.) | 3 | 5 | 2 | 10 |  |
| Purple Poorer Readers (Essential) | 5 | 2 | 1 | 8 |  |
| Purple Better Readers (Essential) | 4 | 2 | 0 | 6 |  |
| Total | 31 | 32 | 10 | 73 |  |

Table 30 Duck Ponds (Part 1) Reading Focus Group and Illustration Type Results

Table 30 shows Better readers were very successful when the illustration was related (shown in red) although for the Poorer Readers this was not the case. As in Gold Bars above, both groups did poorly when the illustration was negative decorative (shown in blue).

| Fireworks - Part 1 Only | Correct <br> answer | Incorrect <br> answer | No <br> Response | Total |
| :--- | :---: | :---: | :---: | :---: |
| Blue Poorer Readers (Decorative) | 1 | 7 | 1 | 9 |
| Blue Better Readers (Decorative) | 2 | 4 | 1 | 7 |
| Green Poorer Readers (Essential) | 4 | 3 | 0 | 7 |
| Green Better Readers (Essential) | 4 | 3 | 0 | 7 |
| Yellow Poorer Readers(No Picture) | 0 | 5 | 0 | 5 |
| Yellow Better Readers (No Picture) | 1 | 7 | 0 | 8 |
| Red Poorer Readers (Related) | 1 | 5 | 0 | 6 |
| Red Better Readers (Related) | 5 | 5 | 0 | 10 |
| Purple Poorer Readers (Negative <br> Dec.) | 1 | 7 | 0 | 8 |
| Purple Better Readers (Negative <br> Dec.) | 2 | 4 | 0 | 6 |
| Total | 21 | 50 | 2 | 73 |

Table 31 Fireworks Reading Focus Group and Illustration Type Results

For the Fireworks question there were significantly more incorrect answers than for the Gold Bars and Duck ponds question. However, it can still be seen that as in the Duck Ponds question Better readers were very successful when the illustration was related (shown in red) although for the Poorer Readers this was not the case.

These individual results seem to show a relationship where the related illustration type is associated with more correct answers and the negative decorative with more incorrect answers. This may be seen more clearly when the results of the different illustration types are totalled. Figure 13 clearly shows that the related and essential illustrations were associated with a greater level of success than other illustration types, especially over the negative decorative type of illustration. Figure 13 also shows that in all cases the Better Readers group had greater success than the Poorer Readers group.

Question Success Rate by Group and Illustration Type


Figure 13 Question Success Rate by Group and Illustration Type

The difference between these two groups was their reading ability, their mathematics levels being similar across the two groups. This suggests then, that the difference in success between the two groups may be due to their reading ability and may well lie in the children's ability to comprehend the question. There are several studies in the literature that would lend support to this view (Newton, 1995, Filippatou and Pumfrey, 1996, Beveridge and Griffiths, 1987, Arizpe and Styles, 2003).

The results of this study would appear to confirm this relationship between reading ability and ability to comprehend mathematical questions. Although the words in the questions were all within the reading ability of all the children, those children in the

Poorer Readers group would seem to have had greater difficulty comprehending the meaning of the text. If this were the case it would mean that they were more likely to look to the picture to provide a context for the words they had read because, as Smith (2004) states, if within the visual text the words have been of little help, then the visual images (or illustrations) accompanying the text will become the predominant route to comprehension. If these children referred to the illustrations more frequently than their Better Reader peers, the decorative and negative decorative illustrations may well have hindered, or even misled, their comprehension of the question. Even when the illustration was related to the text, the children in the Poorer Readers group still had less success. This may have been because they were susceptible to illustrative interference.

When I worked with the children who were shown all the question types and asked them to select the one they felt would be the best to help them solve the problem, I worked individually with one particular boy. He had a reading level of 2 A and a mathematics level of 3B (this would put him in the Poorer Readers group) but was very poor at writing, (level 1). I therefore wrote down his spoken comments. In all the cases he selected the negative decorative illustration as being the most helpful. Each time he was shown a different question he was completely enthralled by a vast number of aliens or fireworks or whatever was shown. If he was typical of those in the Poorer Readers group, this could explain how children in this group are much more susceptible to interference than those in the Better Readers group.

Previous research into the role of illustrations and comprehension had focused upon either children's understanding of narrative text or on scientific concepts. This research has focused upon mathematics, and in particular, non-routine problem solving. It would appear reasonable to conclude that some illustrations can aid comprehension but because poorer readers either rely on illustrations for comprehension or become distracted by inappropriate information in the illustration, inappropriate illustrations can be extremely misleading. This may be because children can develop a dependence upon the illustration particularly if they are poor readers, possibly distracting them from utilising other problem solving skills they may possess. Alternatively, reading ability and the ability to find meaning in illustrations may both partly depend on visual processing capabilities.

## 5.9 - Discussion and summary

The analyses of data from the challenge booklets indicated that the children were most successful with those illustrations that were either essential or related. However, this measure of success is based only upon the children achieving a correct answer. In addition, the relation of the content of the illustration to the mathematical task illustration had a marked affect upon children's understanding and interpretation of each problem. At the beginning of this chapter I identified a number of themes that resulted from an analysis of the children's answers to the Challenge Book questions. These were:

1. An over reliance on the illustration
2. A reality based perspective to the solution
3. Once the information has been retrieved from the illustration it becomes superfluous as they focus purely on their calculations
4. Children use their own drawings to help interpret and solve the problem

These themes were borne out both by the answers the children gave and the previous research identified in the Literature Review. They also provide an insight into how the children tackled the problems and highlighted typical methods they use in order to formulate a solution.

In his work, Serafini (2005) argues that current picture books with their colourful illustrations that may or may not illustrate features of the text can unsettle the reader's expectations. In my own work, because the children often interpreted the illustration as essential regardless of whether it was or was not essential, this resulted in a number of incorrect answers being formulated.

Children indicated that the pictures allowed them to 'imagine' or to help construct a mental model of the situation which appears to endorse Newton's view that "pictures can make complex information accessible, serve to verify or shape mental models, make prior knowledge available, and motivate the reader" (Newton, 1995:128). In order for pictures to 'work' they must activate appropriate information-processing skills within the learner (i.e. selecting, organizing, integrating and encoding). If these skills are not activated and used, or are used carelessly or inappropriately, negative effects can be expected. (Filippatou and Pumfrey, 1996). This was observed in my study where the negative decorative illustration type was generally associated with
incorrect answers and confusion about the requirements of the question. This corresponds with theme 1 , an over reliance on the illustration, whereby children would use the illustration as a basis for their calculation despite it being at odds with the text.

Amongst others, Lubienski, (2000) has argued that through problem-solving experiences children learn to think strategically while learning mathematical content. In my study it was clear that children were thinking strategically but not that they were learning new mathematical skills. For instance, in the Roly Poly question they had to thing strategically in order to imagine the dice from various viewpoints but it was not clear that they were developing new mathematical strategies to help them solve the problem. Actually, if anything they were reinforcing the skills they had (or felt most comfortable with). Wyndhamn and Säljö (1997) argued that problem solving allows for mathematical skill and insight is put to real use. The mathematical problems the children encountered in this part of my study required the children to think strategically. This can be seen in the case of those children who, when solving a problem, began with an estimated answer which they then adapted. Once the children had recognised that identifying two numbers which make seven was required to solve the Snakes and Ladders questions they then focused on this aspect even though it might result in the counter landing on a snake. These children clearly knew about number bonds and were happy to find two numbers that added up to seven. What they had difficulty with was interpreting this number bond sequence in the context of the question where some of the number bonds (those that would cause their counter to land on a snake) were not part of the solution. This corresponds well with Theme 3, once the information has been retrieved from the illustration it becomes superfluous as they focus purely on their calculations, and links with Kazemi's (2002) study, where students were noted as not necessarily thinking through the complete problem.

However, as Reed (1999) points out the basic skills for solving word problems are more complex than they first appear. Skills are needed in understanding the text, determining temporal and special relations, eliminating irrelevant information, identifying the unknown variable, selecting the correct arithmetic operations, and determining what to equate in an equation. It was the need to use this wide range of skills which caused the children a number of problems, particularly as children are
likely to pay more attention to the pictures when the text is relatively difficult for them to comprehend in a mathematical context. For some, eliminating the irrelevant information found in the illustration proved a stumbling block, regardless as to whether this was the excessive items found in the negative decorative group, such as the fireworks or the dice illustrated in the related Roly Poly question.

In such a complex process it is hardly surprising that children attempt to ground their work within a context they understand. Unfortunately for some, who are unable to recognise the limitations of their own personal experience, the result tends to be based upon their own reality rather than the mathematical requirements of the question. This is the basis for theme 2 - a reality based perspective to the solution.

In some circumstances children tended to use other strategies to help them understand the requirements of the problem solving process. Drawing their own illustrations provided a concrete basis for their mathematics which enable the children to 'get a handle' on the mathematical process, rather than using mathematical symbols or language. This then is the observation that resulted in theme 4 , children use their own drawings to help interpret and solve the problem.

These results from the question booklets study indicate that the children often act on the assumption that an illustration is helpful to their understanding of a mathematics question, even though a decorative or negative decorative illustration type is recognised by some children as misleading. In order for children to be able to make appropriate decisions they will have needed to learn, as Serafini (2005) indicates, to navigate and critically evaluate the information that is presented in the illustrations. However my findings would appear to indicate that this is a process children of this age are developing rather than have mastered.

One very clear finding was the large number of children who were unable to answer the question correctly when the illustration was of a negative decorative type. I would hypothesise that a cause of this may be children's inadequate skills at discarding information that is inappropriate. As an adult, and assuming I am better at reading illustrations than children, I would look at the negative decorative illustration of Queen Esmeralda's coins and recognise immediately that there are more than twenty
coins shown and in consequence immediately discard the illustration as a source of mathematical information, although it may still have some value in illustrating the context of the question. Indeed, Filippatou and Pumfrey, (1996) identified that one of the central characteristics of some children is their relatively limited ability to filter out extraneous stimuli and to focus.

In the Gold Bars question a significant number of the children whose illustration was negatively decorative used the illustration to work out their answer. It could be that when there is a low number of items shown, as in the Gold Bars decorative illustration, the illustration will either be discarded immediately or will act as a base for them to work up from, but when the illustration shows an excessive number of items there is a greater tendency to work from the illustration itself. It may be that this behaviour has been learnt from their previous early textbook experience where there is often a one-to-one relationship between a mathematics question and the number of items illustrated. It may too be true that this is a particular problem in English schools. Bierhoff, and Harries and Sutherland showed that English pupils have to cope with more in the form of pictorial decoration distractions in their textbooks than their European counterparts.

These questions could be considered as multi-layered in that there are several layers to the question and the mathematics that have to be processed before a final answer is reached. In this context, the presentation of the mathematics is as a story that has to be peeled away before the pure mathematics can be accessed but not all children can successfully separate the mathematics from the story. Children who are less adept at keeping track and dealing with this multi-layered thinking may become more reliant on the actual illustration to help them with this visualisation than their more competent peers. This may be a reason why so many children used the dice shown in the picture in the Roly Poly question because they were more confident in the visualisation of the dice shown than in imagining another dice.

What all of the children in the extracts have demonstrated is that problem solving can be a very complex process. The following model seems to capture, via its steps, the processes required for a successful solution. I have described in this chapter.


1. Reading text and illustration

This stepwise model of problem solving suggests that in order to formulate a successful solution, each step needs to be considered before moving onto the next one. If, as in the case of many children in the sample, one or more steps is missed out, then the final result may not be the one required or vital information may not have been gleaned from the question making it more likely that the question will be answered incorrectly.

| Step | Action |
| :--- | :--- |
| 1 | In this step the student reads the question and the illustration in order to <br> deduce what is required by the task |
| 2 | The student discards any information that is not relevant. This is a key <br> point in the understanding of questions containing illustrations particularly <br> those where the illustration is at odds with the question |
| 3 | The student identifies any other information they require that may not be in <br> the question or the illustration, but may be general knowledge or other <br> knowledge of the world. This is another key point - small children do not <br> have the same knowledge of the world as adults and may attach importance <br> to different factors than adults would. In addition, children generally obtain <br> less information from pictures than adults (Campbell, 1981). |


| 4. | In this step the student is identifying the numbers that they need to carry <br> out the calculation. If they use an exaggerated illustration to provide these <br> numbers they are unlikely to be successful |
| :--- | :--- |
| 5. | The student needs to decide what calculation is required to solve the <br> problem. |
| 6. | The student calculates the solution. Some calculation types can overlap, <br> multiplication can be carried out by direct multiplication (by knowing one's <br> tables) or by repeated addition. Both can be sources of error. |
| 7. | The answer is checked to ensure that it fits all the parameters of the <br> question. |

However, children do not always move straightforwardly up the model but will refer back to different stages and it may be possible that the more confident a child is with mathematics the less likely they will refer to previous stages in the model. The less confident child may continually refer back to the text or the illustration because they are not able to confidently eliminate irrelevant information. At each step a child could make a wrong judgement which will have an effect upon their success in the other steps. To summarise this:

Less information read + misinterpretation of implied information $=$ misconception.

In the Gold Bars discussion on page 164, Bobby was unable to eliminate irrelevant information so repeatedly referred back to the illustration. Initially because it did not match the text, later because the pirate did not appear strong enough to move the gold bars and because aspects of real life parameters affected his judgement he was unable to select the correct arithmetic operation because he was floundering on the steps below.

The investigation using the different Challenge Booklets highlighted the significance of illustrations in children's perception of mathematical word problems. Irrespective of the type of illustration, the data shows that children will tend to interpret and use the illustration as if it were essential. It also shows that some children have not yet developed the discrimination skills to tell when an illustration is merely providing a
context and when it actually portrays an essential part of the mathematical problem. In addition, the comparison between the Poorer and Better Readers groups' answers to the Challenge Book questions highlights the importance of a child's reading ability in relation to solving mathematical word problems. Just being able to read the words is not enough, the child must be a confident enough reader to understand the context of the text in order to draw out the salient points of the mathematical problem. Indeed, it could be argued that with this age of children, reading ability is more important than mathematical ability in solving these types of problems.

Children are influenced by, and do use illustrations in order to solve mathematical problems, but not necessarily in the way that adults would expect them to. The corollary of this is that illustrators and teachers need to be very much more aware of the way that children use illustrations in problem solving than is currently the case.

## Chapter 6 - Conclusion

This project started because of an observation I had made whilst teaching the children in my class. The trial that followed this observation indicated that children were using illustrations to aid their understanding of questions and in consequence this influenced their calculations, sometimes with negative results. Information from current literature indicated that the role of illustrations in mathematics had only received minimal attention. The relationship between the written text and illustrations is a complex one and little researched, especially in specific subject areas.

The results of the subsequent literature review resulted in three research questions. The first question was a natural progression from the initial classroom observation in asking how important are illustrations in children's understanding of mathematical problems. In order for children to answer the problem solving questions they have to read the questions. As many children are still refining their reading skills, illustrations are often used as a clue to text comprehension. The second question arose from the literature review and was linked to the reading process in that it asked what is the significance of a child's reading and mathematical ability upon their use of illustrations to comprehend question meaning? Illustrations are a visual image, so a natural progression of the investigation was to explore the learning styles of children and their influence which became the focus of the third question in that does a child's cognitive and learning style influence their success in decoding illustrated mathematical material?

During the research project not everything went as planned. The enforced restrictions that occurred because of our school budget meant that I had no opportunity to work with children outside my own classroom. I had planned to interview children from other schools but this had to be abandoned and interviewing my own children in the classroom was difficult whilst also teaching. This meant that the amount of qualitative data I was hoping to collect was greatly reduced, forcing me to place greater reliance on the quantitative data that I had collected.

## 6.1 - Answers to Research Questions

## How important are illustrations in children's understanding of mathematical problems?

The results of my research showed that illustrations are very important in children's interpretation of a mathematical problem. The evidence shows that they find it difficult to discriminate between illustrations that provide meaningful information in the context of the question and those which mislead, meaning that they tend to use the information in the illustration irrespective of whether it is actually appropriate or relevant. The children in this study demonstrated that they actually engage in a complex process of interpretation when reading texts containing illustrations.

The results from the study indicate that questions that contained an essential or related illustration produced far greater levels of success (in terms of correct answers) than any other category. It would appear that related pictures increased recall of related information. This means that illustrations serve as more than convenient motivational vehicles. Those questions that were not accompanied by an illustration did not produce any better results than those where the illustration as purely decorative. It would appear that illustrations can enhance comprehension of mathematical questions and that the children tended to rely more on the illustration in order to comprehend problem solving questions which are more difficult to understand than plain arithmetic questions. The more abstract the mathematical problem, the greater chance there is that children will resort to illustrations to infer the meaning of the question. Since children tend to interpret illustrations as essential, those that were negatively decorative had the greatest effect of misleading the child, distracting their attention away from the textual part of the problem where the relevant information required to solve the problem would be found.

The work conducted with pairs of children indicated that children did use the illustrations to check their understanding and to compare with the information taken from the text. However, if the text was not clear they were more likely to interpret the illustration rather than the text. Far more use was made by the children of the pictures
than of the print. As such, it would be safe to conclude that the younger and less competent the reader, the greater the influence of illustrations will be.

An important finding from this research is that it would appear that children seem to think illustrations are really important and that their perspective of importance often contradicts the intended purpose of many illustrations they encounter in their work. This is likely to be because they appear unable to identify whether an illustration is decorative or essential.

## What is the significance of a child's reading and mathematical ability upon their use of illustrations to comprehend question meaning?

My research showed that when solving mathematical problems, a child's reading ability is more significant than their mathematical ability. In this research the results of two groups were compared. Both groups had similar levels of ability in mathematics but one group had a higher level of reading ability than the other. Despite a similar mathematics ability there was a difference in performance between the two groups, with the more able readers being more successful in solving mathematical problems. It also showed that poorer readers have more difficulty detecting specific causal mathematical relationships in text, even if they are also presented pictorially. In addition, the evidence showed that those children who were better readers would appear to have a higher level of tolerance of ambiguity and uncertainty and were more capable of correctly interpreting the mathematical requirement in the texts. It would seem that reading this kind of material is less a practice of reading skills and more one of information gathering, and the more able readers appear better at sifting through text and illustrations to locate relevant information from a plethora of extraneous information.

## Does a child's cognitive and learning style influence their success in decoding illustrated mathematical material?

There is controversy concerning the issue of cognitive and learning styles and their importance in education. The results from the research have done little to resolve the debate. The results from the VARK questionnaire were inconclusive as all the
children came out as multimodal across all the VARK categories. This may have been because the test was not suitable for children of this age, that for these young children a dominant style has not yet developed or that the notion of learning styles is a misnomer.

From the work done in my research the children do appear to respond to the illustrations and design their own mental images when solving problems but this may reflect a developmental phase rather than a cognitive or learning style. Therefore it is inconclusive whether cognitive and learning style influence these children's success in interpreting illustrated mathematical material. It may be that the more we see pictures, the more we will think in terms of pictures, so that illustrations may provide a framework for mental images. This framework may then act as a foundation that will enable the student to create helpful images.

### 6.2 Research Implications

The results from this research indicate that in any given situation a proportion of children will be compromised in their mathematics calculation by the presence of illustrations. My research highlighted a number of themes that characterised the way that children interact with illustrations when solving mathematical problems. These were:

1. An over reliance on the illustration
2. A reality based perspective to the solution
3. Once the information has been retrieved from the illustration it becomes superfluous as they focus purely on their calculations
4. Children use their own drawings to help interpret and solve the problem

Since these themes figured so strongly in the research, any illustrator commissioned to illustrate mathematical textbooks would be wise to bear them in mind. They summarise the ways that children use pictures to solve problems and if the intention is to provide illustrations that motivate and encourage learning (as many textbooks claim their illustrations do) then they should be considered at an early stage.

Harries and Sutherland concluded that "Unless we understand the ways in which pupils learn from engaging with texts we will not be able to produce electronic texts
which facilitate and enhance the learning of mathematics". (Harries and Sutherland, 2000:65). Illustrations and visual imagery are part of this process of finding out how children engage with texts. With the introduction of interactive and multi-media technology into schools, teachers and education software, designers must take into account the role of illustrations beyond that of the engager and motivator.

Illustrations can be of benefit to children in supporting them as they try to solve mathematical problems. However, issues have arisen because of the indiscriminate use of illustrations. A dialogue between the writer and the illustrator of a mathematics textbook must be established in order to make clear to the illustrator the potential pitfalls associated with children's interpretation of illustrations. Adults are more adept at reading illustrations than children so the latter must be consulted in future illustrative design. The results from this research project indicate that it is not sufficient just to include illustrations but the type of illustration is also extremely important. Illustrations that mislead through providing extraneous information or are merely decorative can be detrimental to a child's understanding and concomitantly to their immediate success. Attainment is always normative - some pupils must inevitably do less well than others, and it is this sense of doing less well than one's peers that can all too easily lead to disaffection and anxiety in mathematics, and create a vicious circle of low self-confidence and underachievement.

Other research highlighted in the literature review has indicated that the use of illustrations with written textual material can facilitate children's learning to read, motivate them, aid retention and improve the comprehension abilities of competent readers to improve their mathematical problem solving skills. The results of this research reinforce this finding that reading ability does affect pupil performance. It also shows that the indiscriminate use of illustrations can undermine the development of a more reflective and strategic approach to thinking about mathematics particularly for lower-attaining pupils.

All those involved in education need to be aware of the pitfalls that can occur because children have misread or relied upon an inappropriate illustration. It may not be the mathematical concept that the child needs support with, but rather the reading of mathematical texts, especially those using copious illustrations. There is a valuable
lesson here for test and text designers and writers. This small and intensive study has been able to tell a great deal and there is a very clear message about the positive and negative consequences different types of illustrations can have.

### 6.3 Further Research

There is a large amount of scope for either large-scale or small-scale investigations into the role of illustrations in the teaching and learning of mathematics. Expanding the sample size and socio-economic spectrum would certainly help to provide a firmer statistical basis to either confirm or refute the findings from this study. Equally, detailed case studies would provide greater insights into the thinking processes involved in this area. Longitudinal and replica investigations of children at different age groups and differential experience would help to see when children become more able to differentiate between relevant and irrelevant illustrations for information gathering. In a world of increasing globalisation, international comparisons would also be beneficial in raising publishers' awareness of the effect illustrations can have on mathematical understanding as there is no reason to believe that this is purely a English phenomenon. Certainly more detailed work on the effect reading ability has upon mathematical performance would be of benefit. This would naturally develop into the exploration of possible differences between boys and girls as the work of other researchers has already shown that girls do appear to have an advantage where reading comprehension is concerned.

In respect of cognitive and learning styles, further work could be done to indicate at what age or time of life, these become fixed (if indeed they do) and if these styles are generic or domain specific. Once this becomes better established then their role in the learning of mathematics and illustrative problem solving would be more relevant.

Lastly, my own professional practice has been and will be affected by this work both through its findings and through the act of carrying it out. The findings from this study can be disseminated amongst my colleagues at my own school and in the wider educational community through for example, inset training. My own knowledge of the research process has been enhanced, particularly that of the role of teacher-led research in school. This is a point about which I feel very strongly. Teachers have an
ideal opportunity to observe, question and research practice. My experience is that this is not valued within the world of practice, and it should be.

## Bibliography

Adams M J (2001) Beginning to read: thinking and learning about print. Bradford Book MIT Press Cambridge pp 333-374

Appleby J (2003) Solving word problems in the primary classroom Primary Mathematics Spring

Arizpe E and Styles M (2003) Children reading pictures Interpreting Visual Texts. London: Routledge Falmer pp 19-52

Asoko H and de Bóo M (2000) Using models, analogies and illustrations to help children think about science ideas. Primary Science Review No 65 pp 25-27

Asoko H and de Bóo M (2001) Analogies and Illustrations; representing ideas in primary science Association for Science Education Black Bear Press Cambridge pp 1-5

Bennett J (1991) Learning to Read with Picture Books Thimble Press Stroud pp 3-9
Beveridge M and Griffiths V (1987) The effect of pictures on the reading processes of less able readers: a miscue analysis approach Journal of Research in Reading Vol. 10 No 1 Blackwell, Oxford pp 29-42

Bierhoff H (1996) Laying the Foundations of Numeracy: a comparison of primary school textbooks in Britain, Germany and Switzerland National Institute of Economic and Social Research London pp 3-36

Blair et. al (2000) New Heinemann Maths Textbook 3 Heinemann Educational London

Bloom P (2000) How Children Learn the Meanings of Words Bradford London pp 171-190

Boaler J (1998) Open and Closed Mathematics: student experiences and understanding Journal for Research in Mathematics Education Vol. 29 No 1 pp 41-62

Campbell P (1981) What do children see in mathematics textbook pictures? Arithmetic Teacher Vol. 59, 2 pp 12-16

Coffield F, Moseley D, Hall E and Ecclestone K (2004) Should we be using learning styles - what research has to say to practice. Learning \& Skills Research Centre pp 36-48

Cooper B (2004) Dilemmas in designing problems in 'realistic' school mathematics: a sociological overview and some research findings. In Olssen M (Ed) Culture and Learning: Access and Opportunity in the Classroom Greenwood Press Connecticut USA pp 183-202

Cooper B and Dunne M (2000) Assessing Children's mathematical Knowledge; social class, sex and problem solving Open University Press Buckingham

Cooper B and Harries T (2002) Children's Responses to Constructing 'Realistic' mathematics problems; just how realistic are children ready to be? Educational Studies in Mathematics No 49 Kluwer Academic Netherlands pp 1-23

Cooper B and Harries T (2003) Children's Use of Realistic Considerations In Problem Solving: some English Evidence Journal of Mathematical Behaviour No 22 pp 451-465

Cooper B and Harries T (2005) Making Sense of realistic Word Problems: Portraying Working Class "Failure" on a Division With Remainder Problem International Journal of Research and Method in Education Vol. 28, No 2 Routledge pp 147-169

Desmedt $E$ and Valcke $M$ (2004) Mapping the learning styles "jungle": an overview of the literature based on citation analysis Educational Psychology Vol. 24 Carfax Publishing pp 445-464

DfEE (2000) Mathematical challenges for able pupils in Key Stages 1 and 2 DfEE London

Donald D R (1983) The use and value of illustrations as contextual information for readers at different progress and development levels. British Journal of Educational Psychology Vol. 53 pp 175-185

Dowling P (1992) A touch of class: ability, social class and intertext in SMP 11-16 in Pimm D and Love E Hodder (Ed) Teaching and Learning School Mathematics Stoughton pp 137-152

Duchastel P and Waller R (1979) Pictorial illustrations in instructional texts. Educational Technology Vol. 19 No 11 pp 20-25

Evans M, Watson C and Willows D (1987) A naturalistic inquiry into illustrations in instructional textbooks. In Houghton H and Willow D (Eds) The Psychology of Illustrations Vol. 2 New York Springer-Verlag pp 87-115

Fairclough R (2002) Developing problem-solving skills in mathematics in Koshy V and Murray (Ed) Unlocking Numeracy David Fulton London pp 79-105

Filippatou D and Pumfrey P (1996) Pictures, Titles, Reading Accuracy and Reading Comprehension: a research review (1973-95) Educational Research Vol. 38 Number 3 pp 259-287

Fleming and Bonwell (2005) VARK a guide to learning styles. www.vark-learn.com
Gibbs S (2003) Do Pictures Make a Difference? A Test of the Hypothesis that Performance in Tests of Phonological Awareness is Eased by the Presence of Pictures Educational Psychology in Practice, Vol. 19 No 3 Carfax Publishing pp 219-228

Gray E (1992) The primary mathematics textbook: intermediary in the cycle of change in Pimm D and Love E Hodder (Ed) Teaching and Learning School Mathematics Stoughton pp 123-152

Hammersley M (1993) Controversies in Classroom Research Open University Press Buckingham pp 222-234

Harries T and Sutherland R (2000) Primary School Mathematics Textbooks: an international comparison. Ed Thompson I Issues in teaching numeracy in primary schools Open University Press Buckingham pp 51-66

Heinemann Numeracy, Science and ICT catalogue 2004 Heinemann
Heuvel-Panhuizen (1999) Context problems and assessment: ideas from the Netherlands in Thompson I (Ed) Issues in teaching numeracy in primary schools Open University Press Buckingham pp 130-142

Homan R (2002) The principle of assumed Consent: the Ethics of Gatekeeping in McNamee M and Bridges D (Ed) The Ethics of Educational Research Blackwell Publishing Oxford pp 30-47

## Jones D Vark Questionnaire For Young People www.vark-learn.com

Kemmis S (1993) Action Research in Hammersley M (Ed) Educational Research The Open University Press Buckingham pp 177-190

Kempton T (2004) Using paintings and cartoons to teach ethics in science School Science Review Vol. 86 No 315 pp 75-82

Kress G (1997) Visual and verbal modes of representation in electronically mediated communication: The potentials of new forms of text. In Snyder I (Ed) Page to screen. Syndey: Allen and Unwin pp 53-79

Kyriacou C (2005) The Impact of Daily Mathematics lessons in England on Pupil Confidence and Competence in Early Mathematics: a systematic review British Journal of Educational Studies Vol 53 No 2 Blackwell pp 168-186

Lemke J L (2002) Multiplying meaning: visual and verbal semiotics in scientific text for JR Martin and R Veel (Eds.) Reading Science Routledge

Levie W and Lentz R (1982) Effects of text illustrations: a review of research. Educational Communication and Technology Journal Vol. 30 No 4 pp 195-232

Lubienski S (2000) Problem Solving as a means towards mathematics for all: an exploratory look through a class lens Journal for Research in Mathematics Education Vol. 31 No 4 pp 454-482

Merttens R and Kirkby D (1997) Ginn Abacus 3 Number Textbook 1 Reed Educational Aylesbury

Merttens R and Kirkby D (1997) Ginn Abacus 3 Number Textbook 2 Reed Educational Aylesbury

Newton D (1995) The Role of pictures in Learning to Read Educational Studies Vol. 21 No. 1 Carfax pp 119-130

Newton L (1983) The effect of illustrations on the readability of some junior school textbooks Reading Vol. 17 No 1 UK Reading Association pp 43-54

Pape S (2004) Middle School Children's Problem Solving Behavior: a cognitive analysis from a reading comprehension perspective. Journal for Research in Mathematics Education Vol. 35 No 3 pp 187-219

Peterson E, Deary I and Aurtin E (2002) The reliability of Riding's Cognitive Style Analysis test Personality and Individual Differences Vol. 34 Pergamon pp 881-891

Peterson E, Deary I and Aurtin E (2005) A new measure of Verbal-Imagery Cognitive Style; VICS Personality and Individual Differences Vol. 38 Elsevier pp 1269-1281

Peterson E, Deary I and Aurtin E (2005) Are intelligence and personality related to verbal-imagery and wholistic-analytic cognitive styles? Personality and Individual Differences Vol. 39 Elsevier pp 201-213

Pithers R T (2002) Cognitive Learning Style: a review of the field dependent-field independent approach. Journal of Vocational Education and Training Vol. 4 No 1 pp 117-132

Pozzer-Ardenghi L and Roth W-M.(2004) Making Sense of Photographs Science Education Vol. 89 No 2 pp 219-238

Price L (2004) Individual Differences in Learning: cognitive control, cognitive style and learning style. Educational Psychology Vol. 24 Carfax Publishing pp 681-698

QCA (2002) Changes to assessment 2003: guidance for teachers of key stages 1 and 2 Qualifications and Curriculum Authority London

Reed S (1999) Word Problems Research and Curriculum Reform Lawrence Erlbaum Associates New Jersey

Reid D (1990) The role of pictures in learning biology: Part 2, picture - text processing. Journal of Biological Education Vol. 24 No 4 pp 251-258

Rezaei A and Katz L (2003) Evaluation of the reliability and validity of the cognitive styles analysis Personality and Individual Differences Vol. 36 Pergamon pp 13171327

Riding R (2002) School Learning and Cognitive Style David Fulton London

Riding R and Grimley G (1999) Cognitive style, gender and learning from multimedia materials in 11-year-old children British Journal of Educational Technology Vol. 30 Blackwell Oxford pp 43-56

Riding R and Sadler-Smith E (1997) Cognitive style and learning strategies: some implications for training design. International Journal of Training and Development. Blackwell Oxford. pp 199-208

Robinson M (1997) Children Reading Print and Television Falmer Press London pp 22-32

Sadler-Smith E (2001) The relationship between learning style and cognitive style Personality and Individual Differences No 30 Pergamon pp 609-616

Santos-Bernard (1997) The Use of Illustrations in School Mathematics Textbooks Presentation of Information Ph.D Thesis Nottingham University

Serafini F (2005) Voices in the Park, Voices in the Classroom: Readers Responding to post-modern Picture Books Reading Research and Instruction Vol. 44 No 3 pp 4563

Shuard H and Rothery A (1984) Children Reading Mathematics John Murray London pp 45-66

Smith F (2004) Understanding Reading Lawrence Erlbaum Associates London Pp 6-86

Stigler J, Fuson K, Ham M and Sook Kim M (1986) An analysis of addition and subtraction word problems in American and Soviet elementary mathematics textbooks Cognition and Instruction Vol 3 No 3 Lawrence Erlbaum Associates pp 153-171

Stylianidou F (2002) Analysis of science textbook pictures about energy and pupils' readings of them International Journal of Science Education Vol. 24 No 3 Taylor and Francis. pp 257-283

Tennent W, Stainthorp R and Stuart M (2006) Assessing reading at Key Stage 2: SATs as measures of children's inferential abilities British Educational Research Journal Vol 32 No 1 Routledge, London pp 1-16

Verschaffel L, Greer B, and De Corte E (2000) Making Sense of Word Problems Swets and Zeitlinger Lisse pp 3-14

Walen S and Williams S (2002) A Matter of Time: emotional responses to timed mathematics tests. Educational Studies in Mathematics No 49 Kluwer Academic Netherlands pp 361-378

Walsh M (2003) 'Reading pictures': what do they reveal? Young children's reading of visual texts. READING literacy and language November 2003 pp 123-130

Wyndhamn J and Säljö R (1997) Word problems and mathematical reasoning - a study of children's mastery of reference and meaning in textual realities Learning and Instruction Vol 7 No 4 pp 361-382 Pergamon

## APPENDICES

Appendix 1 Challenge Booklets. Blue, Green, Yellow, Red, Purple

Appendix 2 Challenge Booklet Questionnaire

Appendix 3 Alternative Illustration Types and Questionnaire

Appendix 4 Textbook Survey<br>Questionnaire

Appendix 5 VARK Questionnaire

## Best copy available

## Poor print quality

## Maths Question Project Booklet <br> 

The questions in this booklet are intended to help us understand how we can improve the design of maths questions so that we can make questions easier to understand. You will have two, 40 minute sessions to answer all the questions.
Later you will be asked to comment on the questions.
You must treat these questions as if you were in a maths test.

- There are nine questions to answer, with only one question per page.

There is plenty of space on each page for you to do your working out and write your answer.

* If there is a question you are stuck on, move onto another question and come back to that one later.

今 You must work on your own. If there is a word you have trouble reading you may ask your teacher for help.

* Some of the questions you may have seen before in your maths work. They may have been altered so do read the questions carefully.


## Snakes and ladders



Your counter is on 9.

You roll a 1 to 6 dice.
After two moves you land on 16.

Find all the different ways you can do it.

Now think of other questions you could ask.

## Gold Bars

Pete is a pirate.
His gold bars are in four piles.
In the first pile he has 6 bars.
In the second pile he has 3 bars.
In the third pile he has 2 bars.
In the fourth pile he has 5 bars.
He can move one or more bars at a time.


He made all the piles the same height.
He made just two moves.
How did he do it?

## Fireworks

Emma had some fireworks.
Some made 3 stars.
Some made 4 stars.


Altogether Emma's fireworks made 19 stars.
How many of them made 3 stars?
Find two different answers.

What if Emma's fireworks made 25 stars?
Find two different answers.


Roly poly

The dots on opposite faces of a dice add up to 7.

1. Imagine rolling one dice.

The score is the total number
of dots you can see.
You score 17.


Which number is face down?
How did you work out your answer?
2. Imagine rolling two dice.

The dice do not touch each other.


The score is the total number of dots you can see. Which numbers are face down to score 30?


Some Tripods and Bipods flew from planet Zeno.
There were at least two of each of them.
Tripods have 3 legs.
Bipods have 2 legs.
There were 23 legs altogether.
How many Tripods were there? How many Bipods?

Find two different answers.


## Ski lift

On a ski lift the chairs are equally spaced.
They are numbered in order from 1.

Kelly went skiing.
She got in chair 10 to go to the top of the slopes.
Exactly half way to the top, she passed chair 100 On its way down.

How many chairs are there On the ski lift?


Make up more problems like this.

## Queen Esmerelda's coins

Queen Esmerelda had some gold coins.
She put them in four piles.


The first pile had four more coins than the second. The second had one less coin than the third.
The fourth pile had twice as many coins as the second.

How many gold coins did Esmerelda put in each pile?

## Duck Ponds

Use 14 ducks each time.

1. There are four ponds. Make each pond hold two ducks or five ducks.
2. There are three ponds. Make each pond hold twice as many ducks as the one before.
3. There are four ponds. Make each pond hold one less duck than the one before.

## Three monkeys



Three monkeys ate a total of 25 nuts.
Each of them ate a different odd number of nuts.

How many nuts did each of the monkeys eat?
Find as many different ways to do it as you can.

## Maths Question Project Booklet

## Child

## Class

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## Snakes and ladders



Your counter is on 9.

You roll a 1 to 6 dice.
After two moves you land on 16.
Find all the different ways you can do it.
Now think of other questions you could ask.

## Gold Bars

Pete is a pirate.
His gold bars are in four piles.


In the first pile he has 6 bars.
In the second pile he has 3 bars.
In the third pile he has 2 bars.
In the fourth pile he has 5 bars.
He can move one or more bars at a time.


He made all the piles the same height. He made just two moves. How did he do it?

## Fireworks

Emma had some fireworks.
Each of the two types made different numbers of stars.


Altogether Emma's fireworks made 19 stars. How many of them made 3 stars?
Find two different answers.

What if Emma's fireworks made 25 stars? Find two different answers.


## Roly poly

The dots on opposite faces of a dice add up to 7.

1. Imagine rolling one dice.

The score is the total number of dots you can see.
You score 17.


Which number is face down?
How did you work out your answer?
2. Imagine rolling two dice.

The dice do not touch each other.


The score is the total number of dots you can see. Which numbers are face down to score 30?

## Spaceship

Some Tripods and Bipods flew from planet Zeno. There were at least two of each of them.

Tripods have 3 legs.
Bipods have 2 legs.
There were 23 legs altogether.
How many Tripods were there?
How many Bipods?
Find two different answers.

## Ski lift

On a ski lift the chairs are equally spaced.
They are numbered in order from 1.

Kelly went skiing.
She got in chair 10 to go to the top of the slopes.

Exactly half way to the top, she passed chair 100 On its way down.

How many chairs are there On the ski lift?


Make up more problems like this.

Queen Esmerelda's coins

Queen Esmerelda had 20 gold coins.
She put them in four piles.


## Duck Ponds

Use 14 ducks each time.


1. There are four ponds. Make each pond hold two ducks or five ducks.
2. There are three ponds. Make each pond hold twice as many ducks as the one before.
3. There are four ponds. Make each pond hold one less duck than the one before.

## Three monkeys



Three monkeys ate a total of 25 nuts.
Each of them ate a different odd number of nuts.

How many nuts did each of the monkeys eat?
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Gold Bars
Pete is a pirate.
His gold bars are in piles.


He made all the piles the same height.
He made just two moves.
How did he do it?


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He made all the piles the same height.

He made just two moves.

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He can move one or more bars at a time.
He made all the piles the same height.
He made just two moves.
How did he do it?

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There were 23 legs altogether.
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## Ski lift

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Make up more problems like this.

## Queen Esmerelda's coiris

Queen Esmerelda had 20 gold coins.

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## Duck Ponds

Use this number of ducks each time.


1. Make each pond hold two ducks or five ducks.

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3. Make each pond hold one less duck than the one before.


## Three monkeys



Three monkeys ate a total of 25 nuts.
Each of them ate a different odd number of nuts.

How many nuts did each of the monkeys eat?
Find as many different ways to do it as you can.

This is the final and important part of your task. Now you have answered the maths questions in your booklet, I need you to think carefully about each one when you answer these questions. Do not worry about your spelling but do try to write neatly.

Which questions would you prefer to work with and why?

Imagine someone who found maths really hard, which question do you think would be the easiest for them and why?

What did you like or find useful about the pictures?

## Thank you and well done ©

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B

## d Bars

## is a pirate.

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Appendix 3 Gold Bars

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In the first pile he has 6 bars.
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In the fourth pile he has 5 bars.
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He made just two moves.
How did he do it?

E


## 4

## ireworks

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## A

Spaceship


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Bipods have 2 legs.
There were 23 legs altogether.
How many Tripods were there?
How many Bipods?

Find two different answers.


## D

Spaceship


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There were at least two of each of them.
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There were 23 legs alfagethem.
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How many Bipods?

## 3

Spaceship
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Bipods have 2 legs.
There were 23 legs altogether.
How many Tripods were there?
How many Bipeds?
Find two different answers.

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- The fourth pile had twice as many coins as the second.

How many gold coins did Esmerelda put in each pile?

1
C.
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The third.
The fourth pile had twice as many
coins as the second.
How many gold coins did Esmerelda
post in each pile?

## leen Esmerelda's coins

 the second had one less coin than the third.
The fourth pile had turice as many coins as the second.
many gold coins did Esmerelda put in each pile?

## A.

## Duck Ponds



1. Alake each pond hold two ducks or five cucks.

2. Make each pond hold twice as many ducks as the one before.

3. Make each pand hold one less duck than the ome before.


## D

## Suck Ponds

se 14 ducks each time.


1. Abake each pond frold two ducks or five ducks.

2. Make each pond hold twice as many ducks as the one before.

3. Make each pond hold one less duck than the one before.


Duck Ponds
Use 14 ducks each time.



1. There are four ponds. Ahake each pond hold two ducks or five ducks.
2. There are three ponds. Make each pond hold twice as many ducks as the one before.
3. There are four ponds. Make each pond hold one less duck than the one before.

## $C$

## Duck Ponds

Use 14 ducks each time.

1. There are four ponds. Make each pond hold two ducks or five ducks.
2. There are three ponds. Make each pond hold twice as many ducks as the one before.
3. There are four ponds. Aake each pond hold one less duck than the one before.

## Euck Ponds

Use 14 ducks each time.


1. There are four ponds. Make each pond bold two ducks or five ducks.
2. There are three ponds. Make each pond hold twice as many ducks as the one before.
3. There are fouse ponds. Wiake each pond moid one less duck tham the one before.

You will be shown five different versions of the same question. You will need to select which yersion you think would be the easiest. You will also need to write down your reasons why you chose that version.

Gold Bars Version $\qquad$

Five Worts Version $\qquad$

Spaceship Version $\qquad$

Queen Esmereida's Coins Version

Dacs Pancls Version $\qquad$

Please tick which textbook you would consider to be the main textbook you would use for number work (addition, subtraction, division, multiplication) in your classroom.

| Abacus | Cambridge Maths |
| :--- | :--- |
| Ginn | Letts |
| New Heinemann | SPMG |
| Other (please specify) |  |

During a week whose focus is number, how frequently would you say the textbook is used by someone in your class?

| Daily | Three or four times in the week |
| :--- | :--- |
| Twice during the week | Once during the week |
| Not at all |  |

What is it about the text book that encourages you to use it? (Tick as many as you like)

| Well illustrated | Matches teaching objectives |
| :--- | :--- |
| Attractive looking | Children can work independently |
| Differentiated work | Teacher time saving |
| Other (please specify) |  |

Thank you for taking time to complete and return this questionnaire. It is greatly appreciated.

## 

## The YaRK Questionnare - for Younger People

Debra Jones, Coordinator of the University of California Gateways Project, wanted to develop a VARK questionnaire for high school students which has now been done. Sponsored by the University of California, Office of the President. The Gateways Project is an outreach project designed to reach "under-served students and assure their successful articulation to higher learning". This version of VARK is therefore modified for High School students. Debra anticipates using an online version of VARK with 30,000 students and the version is likely to be modified after these trials. Acknowledgment of this draft version should be made to Debra Jones, Cabrillo College, Aptos, California 95003 (831-479-5071), Charles C. Bonwell and Neil Fleming.

## How Do I Learn Best?

This questionnaire aims to find out something about your preferences for the way you work with information. You will have a preferred learning style and one part of that learning style is your preference for the intake and output of ideas and information.

- Choose the answer which best explains your preference and circle the letter next to the answer.
- Please select more than one response if a single answer does not match your perception.
- Leave blank any question that does not apply, but try to answer at least 10 of the 13 questions.

1. When you have a few minutes with nothing better to do would you be more likely to:
a. stare into space or doodle.
b. talk to yourself or to others.
c. pick something up to read.
d. do something practical, like fix something or straighten up your room.
2. You are not sure whether a word should be spelled 'dependent' or 'dependant'. Do you:?
a. look it up in the dictionary.
b. see the word in your mind and choose by the way it looks
c. sound it out in your mind.
d. write both versions down on paper and choose one.
3. You want to plan a surprise party for your best friend's birthday. Do you:?
a. talk about it on the phone with your other friends.
b. make lists of what to do and what to buy.
c. picture the party activities in your mind.
d. invite friends and let it develop.
4. You are going to make or build something special for your family. Do you:?
a. make something without the need for instructions.
b. thumb through some books and magazincs looking for ideas.
c. refer to a specific handbook where there are good instructions.
d. talk it over with some friends
5. You are really pleased with your acceptance for a summer program. This is also of interest to two friends. Do you:?
a. take them to see the program in action.
b. show them the brochure and information you've found about it
c. start practising the activities you'll be doing in the program.
d. describe to your friends the activities you'll be doing each day of the program.
6. You are about to buy a new CD player. Other than price, what would most influence your decision?
a. the salesperson telling you about it.
b. reading the details about it.
c. playing with the controls and listening to it.
d. it looks really nice and it is something you could picture in your room.
7. Recall a time in your life when you learned how to play a new board game or computer game. How did you leam best? By:
a. watching others do it first
b. reading instructions.
c. listening to somebody explaining it.
d. doing it or trying it for yourself.
8. After reading a play you need to do a project on it for your English class. Would you prefer to:?
a. read a speech from the play in front of the class.
:b. draw a poster showing something that happened in the play.
c. act out a scene from the play.
d. write your own review on the play
9. You are about to try to hook up your parent's new computer. Would you first:?
a. unpack the box and start trying to put the pieces together.
b. read the manual that comes with the computer.
c. telephonc a friend and ask questions about it.
d. look at the pictures in the manual and on the box
10. You need to give directions to two friends to go to a house nearby. Do you:
a. draw a map on a piece of paper.
b. tell them the directions.
c. write down the directions on a piece of paper.
d. walk them over there yourself.
11. You have a problem with your knee and it hurts when you play your favourite sport. Would you prefer that the doctor:
a. describe to you what is wrong.
b. give you an article or brochure that explains the common problems with knees.
c. show you a diagram of what is wrong.
d. demonstrate with a model what is wrong.
12. A new movie has arrived in town. What would most influence your decision to go (or not go)?
a. you hear friends talking about it
b. you read what others say about it in a magazine.
c. you see a preview of it.
d. it is similar to others you have liked.
13.. Do you prefer a teacher who likes to use:?
a. a textbook and handouts.
b. diagrams, charts, pictures and.slides.
c. field trips, labs and hands-on sessions.
d. Class discussions and guest speakers.

## Name :

| Question | a category | b category | c category | d category |
| :---: | :---: | :---: | :---: | :---: |
| 3 | A | ® | © | K |

Scoring Chart

| Question | a category | b category | c category | d category |
| :---: | :---: | :---: | :---: | :---: |
| 1 | V | A | R | K |
| 2 | R | V | A | K |
| 3 | A | R | V | K |
| 4 | K | V | R | A |
| 5 | V | R | A | K |
| 6 | A | R | K | V |
| 7 | V | R | A | K |
| 8 | A | V | K | R |
| 9 | K | R | A | V |
| 10 | V | A | R | K |
| 11 | A | R | V | K |
| 12 | A | R | V | K |
| 13 | R | V | K | A |


| Total number of V's circled |  |
| :--- | :--- |
| Total number of A's circled |  |
| Total number of R's circled |  |
| Total number of K's circled |  |

Total:


Letter in circle
Number below circle

