Robust H_∞ Filtering for 2-D Systems with Intermittent Measurements[§]

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Abstract

This paper is concerned with the problem of robust H_∞ filtering for uncertain two-dimensional (2-D) systems with intermittent measurements. The parameter uncertainty is assumed to be of polytopic type, and the measurements transmission is assumed to be imperfect, which is modeled by a stochastic variable satisfying the Bernoulli random binary distribution. Our attention is focused on the design of an H_∞ filter such that the filtering error system is stochastically stable and preserves a guaranteed H_∞ performance. This problem is solved in the parameter-dependent framework, which is much less conservative than the quadratic approach. By introducing some slack matrix variables, the coupling between the positive definite matrices and the system matrices is eliminated, which greatly facilitates the filter design procedure. The corresponding results are established in terms of linear matrix inequalities, which can be easily tested by using standard numerical software. An example is provided to show the effectiveness of the proposed approach.

Keywords: 2-D system, H_∞ filtering, intermittent measurements, linear matrix inequality, robust filtering.

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1 Introduction

The state estimation of dynamic systems with both process and measurement noise inputs has attracted considerable attention due to its application as well as theoretical importance in control and signal processing fields. In these applications, it is usually desirable to estimate the values of state variables from the system measurement data. Various schemes, such as Kalman filtering, H_∞ filtering and mixed H_2/H_∞ filtering, have been addressed in the literature. To mention a few, the filtering problem bas been addressed for uncertain systems [8, 13, 14, 16], for stochastic systems [22, 27], for time-delay systems [24, 25, 26], for Markovian jumping systems [17, 20, 21] for sample-data systems [19], and for Linear Systems over Polynomial Observations [1] and with multiple state and observation delays [2]. Among the above mentioned schemes, H_∞ filtering has been proved to be one of the most important strategies, the merit of which lies in that no statistical assumption on the noise signals is needed. In addition, H_∞ filtering has been supposed to be more robust than traditional Kalman filtering, when there exist model uncertainties in the system. Thus the H_∞ filtering is becoming more and more popular to handle the corresponding state estimation problem.

On the other hand, many practical systems can be modeled as two-dimensional (2-D) systems, such as linear image processing, multi-dimensional digital filtering and thermal processes. Therefore, over the past decades considerable attention has been devoted to the analysis and synthesis problems for 2-D systems, and many important results have been reported in the literature along the development of one-dimensional (1-D) systems. To mention a few, the problem of H_∞ filtering has been solved for 2-D systems with parameter uncertainties [6, 7], for time delay systems [3], for 2-D stochastic systems [4, 10] and the mixed H_2/H_∞ filtering has also been addressed in [23]. Earlier results on the filtering problem obtained for 1-D uncertain systems were mostly based on the notion of quadratic stability, where a positive-definite matrix was required for the entire uncertainty domain. The quadratic stability, however, has been generally regarded as being conservative, and thus recently much effort has been devoted to investigating the parameter-dependent stability. The parameter dependent approach can make the positive-definite matrices relaxed to be different for each vertex of the polytope. Similar ideas have been subsequently developed to investigate the problem of H_∞ filtering for 2-D systems. However, the improvement was achieved at the expense of setting the slack matrix variable additionally introduced to be fixed for the entire uncertainty domain. By paying careful attention to the structure of the slack matrix variable, we find the conservativeness could be further reduced.

Another feature worth mentioning is that, for the H_∞ filtering problem of 2-D systems, all the reported results are based on an implicit assumption that the communication between the physical plant and filter is perfect, that is, the signals transmitted from the plant will arrive at the filter simultaneously and perfectly. However, in many practical situations, there may be a nonzero probability that all the signals can be measured during their transmission. In other words, the systems may have intermittent measurements, which bring us new challenges. Moreover, networked systems are becoming more and more popular for the reason that they have several advantages over traditional systems, such as low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability [5, 9, 15]. If network media is introduced to filter design, the data packet dropout phenomenon, which appears in a typical network environment, will naturally induce intermittent measurements from the plant to the filter. Therefore, the problem of filter design with intermittent measurements is of significant importance and, to the best of the authors' knowledge, this problem has not been fully investigated, which motivates the present study.

In this paper, motivated by the above two aspects, we investigate the problem of robust H_∞ filter design for uncertain 2-D systems with intermittent measurements. The measurements transmitted between the plant and the filter are assumed to be imperfect. And the phenomenon of the measurements missing is assumed to satisfy the Bernoulli random binary distribution. Given a 2-D system containing polytopic parameter uncertainties, our purpose is to design an H_∞ filter such that the filtering error system is stochastically stable and preserves a guaranteed H_∞ performance. This problem is solved in the parameter-dependent framework, which is much less conservative than the quadratic approach. More specifically, we only impose part of the slack matrix variable to be fixed for the entire uncertainty domain. The corresponding results are obtained for the existence of desired filters in the form of linear matrix inequalities (LMIs), which can be solved by standard numerical software. Finally, an example is provided to illustrate the effectiveness of the proposed filter design procedures.

The remainder of this paper is organized as follows. Section 2 formulates the problem under consideration. The stability and H_∞ performance of the filtering error system is given in Section 3. In Section 4, the filter design problem is solved. An example is given in Section 5 to illustrate the effectiveness of the proposed method. Finally, conclusions are drawn in Section 6.

Notation: The notation used in the paper is standard. The superscript "T" stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all real matrices of dimension $m\times n$, and $P>0$ (≥ 0) means that P is real symmetric and positive definite (semi-definite). The notation $|\cdot|$ refers to the Euclidean vector norm and $\lambda_{\min}(\cdot)$, $\lambda_{\max}(\cdot)$ denote the minimum and the maximum eigenvalue of the corresponding matrix, respectively. In symmetric block matrices or complex matrix expressions, we use an asterisk ($*$) to represent a term that is induced by symmetry, and diag$\{\ldots\}$ stands for a block-diagonal matrix. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|\,y\}$ will, respectively, mean the expectation of x and the expectation of x conditional on y. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2 Problem Formulation

Consider the uncertain 2-D discrete systems described by the Fornasini-Marchesini (FM) model:

$$\begin{aligned}
x_{i+1,j+1} &= A_1(\alpha)\,x_{i,j+1} + A_2(\alpha)\,x_{i+1,j} + B_1(\alpha)\,\omega_{i,j+1} + B_2(\alpha)\,\omega_{i+1,j},\\
y_{i,j} &= C(\alpha)\,x_{i,j} + D(\alpha)\,\omega_{i,j},\\
z_{i,j} &= L(\alpha)\,x_{i,j}, \qquad\qquad i,j=0,1,2,\ldots,
\end{aligned} \tag{1}$$

where $x_{i,j}\in\mathbb{R}^n$ is the state vector, $y_{i,j}\in\mathbb{R}^m$ is the measured output; $z_{i,j}\in\mathbb{R}^p$ is the signal to be estimated, $\omega_{i,j}\in\mathbb{R}^q$ is the disturbance input which belongs to $l_2\{[0,\infty),[0,\infty)\}$. The system matrices $A_1(\alpha)$, $A_2(\alpha)$, $B_1(\alpha)$, $B_2(\alpha)$, $C(\alpha)$, $D(\alpha)$ and $L(\alpha)$ are appropriately dimensioned with partially unknown parameters. We assume that

$$\mathcal{R} \triangleq \begin{bmatrix} A_1(\alpha) & A_2(\alpha) & B_1(\alpha)\\ B_2(\alpha) & C(\alpha) & D(\alpha)\\ L(\alpha) & 0 & 0 \end{bmatrix} = \sum_{i=1}^{s}\alpha_i \begin{bmatrix} A_{1i} & A_{2i} & B_{1i}\\ B_{2i} & C_i & D_i\\ L_i & 0 & 0\end{bmatrix},\ \alpha\in\Gamma, \tag{2}$$

where Γ is the unit simplex:

$$\Gamma \triangleq \left\{(\alpha_1,\alpha_2,\cdots,\alpha_s): \sum_{i=1}^{s}\alpha_i = 1,\ \alpha_i\geq 0\right\}. \tag{3}$$

Remark 1 *The parameter uncertainties considered in this paper are assumed to be of polytopic type, entering into all the matrices of the system model. The polytopic uncertainty has been widely used in the problems of robust control and filtering for uncertain systems (see, for instance, [11, 12] and the references* *therein), and many practical systems possess parameter uncertainties which can be either exactly modeled or overbounded by the polytopic uncertainty $\mathcal{R}$.*

Throughout the paper, we make the following assumption on the boundary condition.

Assumption 1 *The boundary condition is assumed to satisfy the following condition:*

$$\lim_{N\to\infty} \mathbb{E}\left\{\sum_{k=0}^{N}\left(|x_{k,0}|^2+|x_{0,k}|^2\right)\right\}<\infty.$$

The objective here is to design a filter of the following form to estimate $z_{i,j}$:

$$\begin{aligned}\hat{x}_{i+1,j+1} &= A_{f1}\hat{x}_{i,j+1}+A_{f2}\hat{x}_{i+1,j}+B_{f1}\tilde{y}_{i,j+1}+B_{f2}\tilde{y}_{i+1,j},\\ \hat{z}_{i,j} &= L_f\hat{x}_{i,j},\end{aligned} \tag{4}$$

where, $\hat{x}_{i,j}\in\mathbb{R}^n$ is the filter state vector, $\tilde{y}_{i,j}\in\mathbb{R}^m$ is the input of the filter; A_{f1}, A_{f2}, B_{f1}, B_{f2}, L_f are appropriately dimensioned filter matrices to be determined.

Remark 2 *Most of the previous results for filter designing are based on the implicit assumption that the communication channel between the physical plant and filter is perfect, that is, the signals transmitted from the plant will arrive at the filter completely and simultaneously. However, in many practical situations, especially in a network environment, this assumption is not always guaranteed.*

In this paper, we assume the data packet dropout (or data missing) is described by a stochastic variable, that is,

$$\tilde{y}_{i,j}=\theta_{i,j}y_{i,j}, \tag{5}$$

where the stochastic variable $\{\theta_{i,j}\}$ is a Bernoulli distributed white sequence taking the values of 0 and 1 with

$$\begin{aligned}\text{Prob}\{\theta_{i,j}=1\} &= E\{\theta_{i,j}\}=\theta,\\ \text{Prob}\{\theta_{i,j}=0\} &= 1-E\{\theta_{i,j}\}=1-\theta,\end{aligned}$$

and θ is a known positive scalar.

Remark 3 *With (5), the input $\tilde{y}_{i,j}$ of the filter is no longer equivalent to the output $y_{i,j}$ of the plant, which characterizes the real situation in many applications. The system measurement model (5) can be used to represent missing measurement or uncertain observations, which was first introduced in [18], and has been subsequently utilized in both control and signal processing problems [26].*

Based on the intermittent measurement, we have the filter in the following form

$$\begin{aligned}\hat{x}_{i+1,j+1} &= A_{f1}\hat{x}_{i,j+1}+A_{f2}\hat{x}_{i+1,j}+B_{f1}\theta_{i,j+1}y_{i,j+1}+B_{f2}\theta_{i+1,j}y_{i+1,j},\\ \hat{z}_{i,j} &= L_f\hat{x}_{i,j}.\end{aligned} \tag{6}$$

Define the augmented state vector $\tilde{x}_{i,j}=\left[x_{i,j}^T,\hat{x}_{i,j}^T\right]^T$ and the filtering error signal $\tilde{z}_{i,j}=z_{i,j}-\hat{z}_{i,j}$. Then we have the filtering error system

$$\begin{aligned}\tilde{x}_{i+1,j+1} &= \bar{A}_1(\alpha)\tilde{x}_{i,j+1}+\bar{A}_2(\alpha)\tilde{x}_{i+1,j}+\bar{\theta}_{i,j+1}\bar{A}_3(\alpha)\tilde{x}_{i,j+1}+\bar{\theta}_{i+1,j}\bar{A}_4(\alpha)\tilde{x}_{i+1,j}\\ &\quad+\bar{B}_1(\alpha)\omega_{i,j+1}+\bar{B}_2(\alpha)\omega_{i+1,j}+\bar{\theta}_{i.j+1}\bar{B}_3(\alpha)\omega_{i,j+1}+\bar{\theta}_{i+1,j}\bar{B}_4(\alpha)\omega_{i+1,j},\\ \tilde{z}_{i,j} &= \bar{L}(\alpha)\tilde{x}_{i,j},\end{aligned} \tag{7}$$

where

$$\bar{A}_l(\alpha) = \begin{bmatrix} A_l(\alpha) & 0 \\ \theta B_{fl} C(\alpha) & A_{fl} \end{bmatrix}, \quad \bar{B}_l(\alpha) = \begin{bmatrix} B_l(\alpha) \\ \theta B_{fl} D(\alpha) \end{bmatrix}, \quad l = 1, 2,$$

$$\bar{A}_3(\alpha) = \begin{bmatrix} 0 & 0 \\ B_{f1} C(\alpha) & 0 \end{bmatrix}, \quad \bar{B}_3(\alpha) = \begin{bmatrix} 0 \\ B_{f1} D(\alpha) \end{bmatrix}, \quad \bar{L}(\alpha) = \begin{bmatrix} L(\alpha) & -L_f \end{bmatrix},$$

$$\bar{A}_4(\alpha) = \begin{bmatrix} 0 & 0 \\ B_{f2} C(\alpha) & 0 \end{bmatrix}, \quad \bar{B}_4(\alpha) = \begin{bmatrix} 0 \\ B_{f2} D(\alpha) \end{bmatrix}, \quad \bar{\theta}_{i,j} = \theta_{i,j} - \theta.$$

It is clear that

$$E\left\{\bar{\theta}_{i,j}\right\} = 0, \quad E\left\{\bar{\theta}_{i,j}\bar{\theta}_{i,j}\right\} = \theta(1-\theta). \tag{8}$$

The introduction of the stochastic variable $\theta_{i,j}$ renders the filtering error system to be stochastic instead of a deterministic one. Before proceeding further, we need to introduce the following definition of stochastic stability for the filtering error system in (7), which will be essential for our derivation.

Definition 1 *(stochastic stability) The filtering error system in (7) with Assumption 1 and $\omega_{i,j} = 0$ is said to be mean-square asymptotically stable if for every initial condition $\mathbb{E}\left\{|\tilde{x}_{0,0}|^2\right\} < \infty$,*

$$\lim_{i+j\to\infty} \mathbb{E}\left\{|\tilde{x}_{i,j}|^2\right\} = 0.$$

In addition, we have the following definition.

Definition 2 *(H_∞ $performance$) Given a scalar $\gamma > 0$, the filtering error system in (7) is said to be mean-square asymptotically stable with an H_∞ disturbance attenuation level γ if it is mean-square asymptotically stable and under zero initial and boundary conditions, $||\tilde{z}||_E < \gamma||\omega||_2$ for all nonzero $\omega \triangleq \{\omega_{i,j}\} \in l_2[0,\infty)$, where*

$$||\tilde{z}||_E \triangleq \sqrt{\mathbb{E}\left\{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}|\tilde{z}_{i,j}|^2\right\}}, \quad ||\omega||_2 \triangleq \sqrt{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}|\omega_{i,j}|^2}.$$

Then, the problem to be addressed in this paper is expressed as follows

Problem HFIM2DFM (H_∞ Filtering with Intermittent Measurements for 2DFM systems**)** Consider the system in (1) with uncertainty and missing measurements described in (2) and (5) respectively. Given a real number $\gamma > 0$, design a filter in the form of (6) such that the filtering error system in (7) is mean-square asymptotically stable with an H_∞ disturbance attenuation level γ. The corresponding filter is called H_∞ filter.

3 H_∞ Filtering Analysis

In this section, the filtering analysis problem is concerned. More specifically, we assume that the filter matrices A_{f1}, A_{f2}, B_{f1}, B_{f2}, L_f in (6) are known, and we will study the condition under which the filtering error system in (7) is stochastically stable in the mean-square with a guaranteed H_∞ performance.

Theorem 1 *Consider the system in (1) and suppose the filter matrices A_{f1}, A_{f2}, B_{f1}, B_{f2}, L_f in (6) are given. Then the filtering error system in (7) for any $\alpha \in \Gamma$ is mean-square asymptotically stable with a given H_∞ performance γ, if there exist matrices $P(\alpha) > 0$ and $Q(\alpha) > 0$ satisfying*

$$\Psi \triangleq \Xi_1^T P(\alpha)\Xi_1 + \beta^2 \Xi_2^T P(\alpha)\Xi_2 + \Xi_3^T\Xi_3 + \Xi_4^T\Xi_4 + \Xi_5 < 0, \tag{9}$$

where

$$\begin{aligned}
\Xi_1 &= \begin{bmatrix} \bar{A}_1(\alpha) & \bar{A}_2(\alpha) & \bar{B}_1(\alpha) & \bar{B}_2(\alpha) \end{bmatrix},\\
\Xi_2 &= \begin{bmatrix} \bar{A}_3(\alpha) & \bar{A}_4(\alpha) & \bar{B}_3(\alpha) & \bar{B}_4(\alpha) \end{bmatrix},\\
\Xi_3 &= \begin{bmatrix} \bar{L}(\alpha) & 0 & 0 & 0 \end{bmatrix},\\
\Xi_4 &= \begin{bmatrix} 0 & \bar{L}(\alpha) & 0 & 0 \end{bmatrix},\\
\Xi_5 &= \mathrm{diag}\left\{Q(\alpha) - P(\alpha), -Q(\alpha), -\gamma^2 I, -\gamma^2 I\right\},\\
\beta &= \sqrt{\theta(1-\theta)}.
\end{aligned}$$

Proof. Consider the following index

$$\mathcal{J} \triangleq \mathcal{X}_1 - \mathcal{X}_2, \tag{10}$$

with

$$\begin{aligned}
\mathcal{X}_1 &\triangleq \mathbb{E}\left\{ \tilde{x}_{i+1,j+1}^T P(\alpha)\tilde{x}_{i+1,j+1} \middle| \tilde{x} \right\},\\
\mathcal{X}_2 &\triangleq \tilde{x}^T \,\mathrm{diag}\left\{(P(\alpha) - Q(\alpha)), Q(\alpha)\right\} \tilde{x},
\end{aligned}$$

where $\tilde{x} = \begin{bmatrix} \tilde{x}_{i,j+1}^T & \tilde{x}_{i+1,j}^T \end{bmatrix}^T$, and $P(\alpha)$, $Q(\alpha)$ are symmetric positive definite matrices to be determined.

We first prove the stochastic stability of the filtering error system in (7) with zero disturbance input $\omega_{i,j} = 0$. Then, (7) becomes

$$\begin{aligned}
\tilde{x}_{i+1,j+1} &= \bar{A}_1(\alpha)\tilde{x}_{i,j+1} + \bar{A}_2(\alpha)\tilde{x}_{i+1,j} + \bar{\theta}_{i,j+1}\bar{A}_3(\alpha)\tilde{x}_{i,j+1} + \bar{\theta}_{i+1,j}\bar{A}_4(\alpha)\tilde{x}_{i+1,j},\\
\tilde{z}_{i,j} &= \bar{L}(\alpha)\tilde{x}_{i,j}.
\end{aligned}$$

It is observed that condition (9) implies

$$\Xi_1^T P(\alpha)\Xi_1 + \beta^2 \Xi_2^T P(\alpha)\Xi_2 + \Xi_5 < 0,$$

which further implies

$$\Omega \triangleq \begin{bmatrix} \begin{bmatrix} \bar{A}_1^T(\alpha)P(\alpha)\bar{A}_1(\alpha) + Q(\alpha) \\ -P(\alpha) + \beta^2 \bar{A}_3^T(\alpha)P(\alpha)\bar{A}_3(\alpha) \end{bmatrix} & \begin{bmatrix} \bar{A}_1^T(\alpha)P(\alpha)\bar{A}_2(\alpha) \\ +\beta^2 \bar{A}_3^T(\alpha)P(\alpha)\bar{A}_4(\alpha) \end{bmatrix} \\ * & \begin{bmatrix} \bar{A}_2^T(\alpha)P(\alpha)\bar{A}_2(\alpha)+ \\ \beta^2 \bar{A}_4^T(\alpha)P(\alpha)\bar{A}_4(\alpha) - Q(\alpha) \end{bmatrix} \end{bmatrix} < 0.$$

Then along the solution of the filtering error system in (7), we have

$$\begin{aligned}
\mathcal{J} &= \mathbb{E}\left\{ \tilde{x}_{i+1,j+1}^T P(\alpha)\tilde{x}_{i+1,j+1} \middle| \tilde{x} \right\} - \tilde{x}^T \,\mathrm{diag}\left\{(P(\alpha) - Q(\alpha)), Q(\alpha)\right\} \tilde{x}\\
&= \mathbb{E}\left\{ \left(\begin{array}{c} \left[\bar{A}_1(\alpha)\tilde{x}_{i,j+1} + \bar{A}_2(\alpha)\tilde{x}_{i+1,j} + \bar{\theta}_{i,j+1}\bar{A}_3(\alpha)\tilde{x}_{i,j+1} + \bar{\theta}_{i+1,j}\bar{A}_4(\alpha)\tilde{x}_{i+1,j}\right]^T P(\alpha) \\ \times \left[\bar{A}_1(\alpha)\tilde{x}_{i,j+1} + \bar{A}_2(\alpha)\tilde{x}_{i+1,j} + \bar{\theta}_{i,j+1}\bar{A}_3(\alpha)\tilde{x}_{i,j+1} + \bar{\theta}_{i+1,j}\bar{A}_4(\alpha)\tilde{x}_{i+1,j}\right] \end{array} \right) \middle| \tilde{x} \right\}\\
&\quad -\tilde{x}^T \,\mathrm{diag}\left\{(P(\alpha) - Q(\alpha)), Q(\alpha)\right\} \tilde{x}.
\end{aligned}$$

From (8) and with $\beta = \sqrt{\theta(1-\theta)}$, it follows that

$$\mathcal{J} = \tilde{x}^T \Omega \tilde{x}.$$

This means that for all $\tilde{x} \neq 0$ we have

$$\begin{aligned} \frac{\mathcal{X}_1 - \mathcal{X}_2}{\mathcal{X}_2} &= -\frac{\tilde{x}^T(-\Omega)\tilde{x}}{\tilde{x}^T \operatorname{diag}\{(P(\alpha)-Q(\alpha)), Q(\alpha)\}\tilde{x}} \\ \leq & \quad -\frac{\lambda_{\min}(-\Omega)}{\lambda_{\max}(\operatorname{diag}\{(P(\alpha)-Q(\alpha)), Q(\alpha)\})} = \delta - 1, \end{aligned}$$

where $\delta = 1 - \frac{\lambda_{\min}(-\Omega)}{\lambda_{\max}(\operatorname{diag}\{(P(\alpha)-Q(\alpha)),Q(\alpha)\})}$ and $\mathcal{X}_1, \mathcal{X}_2$ are defined in (10). Since $\frac{\lambda_{\min}(-\Omega)}{\lambda_{\max}(\operatorname{diag}\{(P(\alpha)-Q(\alpha)),Q(\alpha)\})} > 0$, we have $\delta < 1$. Obviously,

$$\delta \geq \frac{\mathcal{X}_1}{\mathcal{X}_2} > 0.$$

That is, $\delta \in (0,1)$ and is independent of $\tilde{x}$. Thus, we obtain $\mathcal{X}_1 \leq \delta \mathcal{X}_2$, and taking expectation of both sides yields

$$\mathbb{E}\left\{\tilde{x}^T_{i+1,j+1} P(\alpha)\tilde{x}_{i+1,j+1}\right\} \leq \delta \mathbb{E}\left\{\tilde{x}^T_{i,j+1}(P(\alpha)-Q(\alpha))\tilde{x}_{i,j+1} + \tilde{x}^T_{i+1,j} Q(\alpha)\tilde{x}_{i+1,j}\right\}. \tag{11}$$

For the convenience of notations, we denote

$$\begin{aligned} \mathcal{X}_{i,j} &\triangleq \mathbb{E}\left\{\tilde{x}^T_{i,j} P(\alpha)\tilde{x}_{i,j}\right\}, \\ \mathcal{Y}_{i,j} &\triangleq \mathbb{E}\left\{\tilde{x}^T_{i,j} Q(\alpha)\tilde{x}_{i,j}\right\}. \end{aligned} \tag{12}$$

Then, (11) becomes

$$\mathcal{X}_{i+1,j+1} \leq \delta \mathcal{X}_{i,j+1} + \delta \mathcal{Y}_{i+1,j} - \delta \mathcal{Y}_{i,j+1}. \tag{13}$$

From (9), negative definite matrices $Q(\alpha) - P(\alpha)$ is obtained, that is $P(\alpha) > Q(\alpha)$, which implies

$$\mathcal{X}_{i,j} > \mathcal{Y}_{i,j}. \tag{14}$$

Upon the relationship in (13) and (14), for $i = k-1, \ldots, 0, -1$, $j = -1, 0, \ldots, k-1$, it can be established that

$$\begin{aligned} \mathcal{X}_{k,0} &= \mathcal{X}_{k,0}, \\ \mathcal{X}_{k-1,1} &\leq \delta \mathcal{X}_{k-2,1} + \delta \mathcal{Y}_{k-1,0} - \delta \mathcal{Y}_{k-2,1} \\ &\leq \delta \mathcal{X}_{k-2,1} + \delta \mathcal{X}_{k-1,0} - \delta \mathcal{Y}_{k-2,1}, \\ \mathcal{X}_{k-2,2} &\leq \delta \mathcal{X}_{k-3,2} + \delta \mathcal{Y}_{k-2,1} - \delta \mathcal{Y}_{k-3,2}, \\ &\vdots \\ \mathcal{X}_{1,k-1} &\leq \delta \mathcal{X}_{0,k-1} + \delta \mathcal{Y}_{1,k-2} - \delta \mathcal{Y}_{0,k-1} \\ &\leq \delta \mathcal{X}_{0,k-1} + \delta \mathcal{Y}_{1,k-2}, \\ \mathcal{X}_{0,k} &= \mathcal{X}_{0,k}. \end{aligned}$$

Adding both sides of the above inequalities yields

$$\sum_{l=0}^{k} \mathcal{X}_{k-l,l} \leq \delta \sum_{l=0}^{k-1} \mathcal{X}_{k-1-l,l} + \mathcal{X}_{k,0} + \mathcal{X}_{0,k}.$$

Using this relationship iteratively, we can obtain

$$\begin{aligned}\sum_{l=0}^{k} \mathcal{X}_{k-l,l} &\leq \delta^k \mathcal{X}_{0,0} + \sum_{l=0}^{k-1} \delta^l \left(\mathcal{X}_{k-l,0} + \mathcal{X}_{0,k-l}\right) \\ &\leq \sum_{l=0}^{k} \delta^l \left(\mathcal{X}_{k-l,0} + \mathcal{X}_{0,k-l}\right).\end{aligned}$$

Therefore, we have

$$\mathbb{E}\left\{\sum_{l=0}^{k} |\tilde{x}_{k-l,l}|^2\right\} \leq \mathcal{K} \sum_{l=0}^{k} \delta^l \mathbb{E}\left\{|\tilde{x}_{k-l,0}|^2 + |\tilde{x}_{0,k-l}|^2\right\}, \tag{15}$$

where

$$\mathcal{K} \triangleq \frac{\lambda_{\max}\left(P(\alpha)\right)}{\lambda_{\min}(P(\alpha))}.$$

Now, denote

$$\begin{aligned}\eta_k &= \mathbb{E}\left\{\sum_{l=0}^{k} |\tilde{x}_{k-l,l}|^2\right\}, \\ \eta_{k-l,0} &= \mathbb{E}\left\{|\tilde{x}_{k-l,0}|^2 + |\tilde{x}_{0,k-l}|^2\right\}.\end{aligned}$$

Therefore, the inequality in (15) becomes

$$\eta_k \leq \mathcal{K} \sum_{l=0}^{k} \delta^l \eta_{k-l,0}.$$

Then for $\mathcal{K} = 0, 1, \ldots, N$, we have

$$\begin{aligned}\eta_0 &\leq \mathcal{K}\eta_{0,0}, \\ \eta_1 &\leq \mathcal{K}\left[\delta\eta_{0,0} + \eta_{1,0}\right], \\ \eta_2 &\leq \mathcal{K}\left[\delta^2\eta_{0,0} + \delta\eta_{1,0} + \eta_{2,0}\right], \\ &\vdots \\ \eta_N &\leq \mathcal{K}\left[\delta^N\eta_{0,0} + \delta^{N-1}\eta_{1,0} + \cdots + \eta_{N,0}\right].\end{aligned}$$

Summing up both sides of the above inequalities we have

$$\begin{aligned}\sum_{k=0}^{N} \eta_k &\leq \mathcal{K}\left(1 + \delta + \cdots + \delta^N\right)\eta_{0,0} \\ &\quad + \mathcal{K}\left(1 + \delta + \cdots + \delta^{N-1}\right)\eta_{1,0} + \cdots + \delta\eta_{N,0} \\ &\leq \mathcal{K}\left(1 + \delta + \cdots + \delta^N\right)\eta_{0,0} + \mathcal{K}\left(1 + \delta + \cdots + \delta^N\right)\eta_{1,0} \\ &\quad + \mathcal{K}\left(1 + \delta + \cdots + \delta^N\right)\eta_{N,0} \\ &= \mathcal{K}\frac{1 - \delta^{N+1}}{1 - \delta} \sum_{k=0}^{N} \eta_{k,0}.\end{aligned}$$

Then, under Assumption 1, for every initial condition $\eta_0 < \infty$, the right side of this inequality is bounded, which means $\lim_{k\to\infty} \eta_k = 0$, that is, $\mathbb{E}\left\{|\tilde{x}_{i,j}|^2\right\} \to 0$ as $i + j \to \infty$. According to Definition 1, the filtering error system in (7) with $\omega_{i,j} = 0$ is stochastically stable in the mean square.

Now, the H_∞ performance for the filtering error system in (7) will be established. To this end, assume zero initial and boundary conditions, that is, $\tilde{x}_{i,j} = 0$ for $i = 0$ or $j = 0$. An index is introduced as

$$\mathcal{I} \triangleq \tilde{z}^T\tilde{z} - \gamma^2\tilde{\omega}^T\tilde{\omega} + \mathcal{J},$$

where $\tilde{z} \triangleq \begin{bmatrix} \tilde{z}_{i,j+1}^T & \tilde{z}_{i+1,j}^T \end{bmatrix}^T$, $\tilde{\omega} \triangleq \begin{bmatrix} \omega_{i,j+1}^T & \omega_{i+1,j}^T \end{bmatrix}^T$ and $\mathcal{J}$ is defined in (10). Then along the solution of the filtering error system in (7), we have

$$\begin{aligned}\mathcal{I} &= \tilde{x}_{i,j+1}^T \bar{L}^T(\alpha)\bar{L}(\alpha)\tilde{x}_{i,j+1} + \tilde{x}_{i+1,j}^T \bar{L}^T(\alpha)\bar{L}(\alpha)\tilde{x}_{i+1,j} - \gamma^2\omega_{i,j+1}^T\omega_{i,j+1} - \gamma^2\omega_{i+1,j}^T\omega_{i+1,j} + \mathcal{J} \\ &= \xi^T\Psi\xi,\end{aligned}$$

where $\xi \triangleq \begin{bmatrix} \tilde{x}_{i,j+1}^T & \tilde{x}_{i+1,j}^T & \omega_{i,j+1}^T & \omega_{i+1,j}^T \end{bmatrix}^T$ and Ψ is defined in (9). Then for any $\xi \neq 0$, we have $\mathcal{I} < 0$, that is,

$$\mathbb{E}\left\{ \tilde{x}_{i+1,j+1}^T P(\alpha)\tilde{x}_{i+1,j+1} \middle| \tilde{x} \right\} < \tilde{x}^T \operatorname{diag}\left\{(P(\alpha) - Q(\alpha)), Q(\alpha)\right\}\tilde{x} - \tilde{z}^T\tilde{z} + \gamma^2\tilde{\omega}^T\tilde{\omega}.$$

Taking expectation of both sides yields

$$\begin{aligned}\mathbb{E}\left\{\tilde{x}_{i+1,j+1}^T P(\alpha)\tilde{x}_{i+1,j+1}\right\} \;\leq\; & \mathbb{E}\left\{\tilde{x}_{i,j+1}^T(P(\alpha) - Q(\alpha))\tilde{x}_{i,j+1} + \tilde{x}_{i+1,j}^T Q(\alpha)\tilde{x}_{i+1,j}\right. \\ & \left.-\tilde{z}_{i,j+1}^T\tilde{z}_{i,j+1} - \tilde{z}_{i+1,j}^T\tilde{z}_{i+1,j}\right\} + \gamma^2\omega_{i,j+1}^T\omega_{i,j+1} + \gamma^2\omega_{i+1,j}^T\omega_{i+1,j}. \qquad (16)\end{aligned}$$

From the relationship in (16) and the notations in (12), for $i = k-1, \dots, 0, -1$, $j = -1, 0, \dots, k-1$, it can be established that

$$\begin{aligned}\mathcal{X}_{k,0} &= \mathcal{X}_{k,0}, \\ \mathcal{X}_{k-1,1} &< \mathcal{X}_{k-2,1} + \mathcal{Y}_{k-1,0} - \mathcal{Y}_{k-2,1} - \mathbb{E}\left\{\tilde{z}_{k-2,1}^T\tilde{z}_{k-2,1} + \tilde{z}_{k-1,0}^T\tilde{z}_{k-1,0}\right\} \\ &\quad +\gamma^2\omega_{k-2,1}^T\omega_{k-2,1} + \gamma^2\omega_{k-1,0}^T\omega_{k-1,0}, \\ \mathcal{X}_{k-2,2} &< \mathcal{X}_{k-3,2} + \mathcal{Y}_{k-2,1} - \mathcal{Y}_{k-3,2} - \mathbb{E}\left\{\tilde{z}_{k-3,2}^T\tilde{z}_{k-3,2} + \tilde{z}_{k-2,1}^T\tilde{z}_{k-2,1}\right\} \\ &\quad +\gamma^2\omega_{k-3,2}^T\omega_{k-3,2} + \gamma^2\omega_{k-2,1}^T\omega_{k-2,1}, \\ &\;\;\vdots \\ \mathcal{X}_{1,k-1} &< \mathcal{X}_{0,k-1} + \mathcal{Y}_{1,k-2} - \mathcal{Y}_{0,k-1} - \mathbb{E}\left\{\tilde{z}_{0,k-1}^T\tilde{z}_{0,k-1} + \tilde{z}_{1,k-2}^T\tilde{z}_{1,k-2}\right\} \\ &\quad +\gamma^2\omega_{0,k-1}^T\omega_{0,k-1} + \gamma^2\omega_{1,k-2}^T\omega_{1,k-2}, \\ \mathcal{X}_{0,k} &= \mathcal{X}_{0,k}.\end{aligned}$$

Adding both sides of the above inequalities and considering the zero initial and boundary conditions, we have

$$\sum_{l=0}^{k}\mathcal{X}_{k-l,l} < \sum_{l=0}^{k-1}\mathcal{X}_{k-1-l,l} - 2\mathbb{E}\left\{\sum_{l=0}^{k-1}\tilde{z}_{k-1-l,l}^T\tilde{z}_{k-1-l,l}\right\} + 2\gamma^2\sum_{l=0}^{k-1}\omega_{k-1-l,l}^T\omega_{k-1-l,l}.$$

Summing up both sides of this inequality from $k = 0$ to $k = N$, we have

$$\mathbb{E}\left\{\sum_{k=0}^{N}\sum_{l=0}^{k-1}\tilde{z}_{k-1-l,l}^T\tilde{z}_{k-1-l,l}\right\} < \gamma^2\sum_{k=0}^{N}\sum_{l=0}^{k-1}\omega_{k-1-l,l}^T\omega_{k-1-l,l} - \frac{1}{2}\sum_{l=0}^{N}\mathcal{X}_{N-l,l}.$$

Therefore, we have

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}\sum_{l=0}^{k-1}\tilde{z}_{k-1-l,l}^T\tilde{z}_{k-1-l,l}\right\} < \gamma^2\sum_{k=0}^{\infty}\sum_{l=0}^{k-1}\omega_{k-1-l,l}^T\omega_{k-1-l,l},$$

that is, $||\tilde{z}||_E < \gamma||\omega||_2$ for all nonzero $\omega_{i,j}$, and the proof is completed. □

When the communication links existing between the plant and the filter is perfect, that is, there is no data dropout during their transmission, the condition in Theorem 1 reduces to the following corollary.

Corollary 1 *Consider the system in (1) and suppose the filter matrices A_{f1}, A_{f2}, B_{f1}, B_{f2}, L_f in (6) are given. When $\theta = 1$, the filtering error system in (7) for any $\alpha \in \Gamma$ is asymptotically stable with a given H_∞ performance γ, if there exist matrices $P(\alpha) > 0$ and $Q(\alpha) > 0$ satisfying*

$$\Xi_1^T P(\alpha)\Xi_1 + \Xi_3^T\Xi_3 + \Xi_4^T\Xi_4 + \Xi_5 < 0,$$

where Ξ_1, Ξ_3, Ξ_4 and Ξ_5 are given in (9).

4 H_∞ Filter Design

Theorem 1 addresses the H_∞ filtering problem for the system in (7) where the filter matrixes A_{f1}, A_{f2}, B_{f1}, B_{f2}, L_f are all known. However, our eventual purpose is to determine the filter matrices. In this section, we will give a parameter-dependent approach to solve the robust filter design problem for the uncertain systems. To reduce the design conservatism, in what follows, firstly we give the following proposition, which eliminates the products between the positive-definite matrix $P(\alpha)$ and system matrices.

Proposition 1 *Consider the system in (1) and suppose the filter matrices A_{f1}, A_{f2}, B_{f1}, B_{f2}, L_f in (6) are given. Then the filtering error system in (7) for any $\alpha \in \Gamma$ is asymptotically stable with a given H_∞ performance γ, if there exist matrices $P(\alpha) > 0$, $Q(\alpha) > 0$ and $J(\alpha)$ satisfying*

$$\begin{bmatrix} -I & 0 & 0 & 0 & \bar{L}(\alpha) & 0 & 0 & 0 \\ * & -I & 0 & 0 & 0 & \bar{L}(\alpha) & 0 & 0 \\ * & * & \Pi & 0 & J^T(\alpha)\bar{A}_1(\alpha) & J^T(\alpha)\bar{A}_2(\alpha) & J^T(\alpha)\bar{B}_1(\alpha) & J^T(\alpha)\bar{B}_2(\alpha) \\ * & * & * & \Pi & \beta J^T(\alpha)\bar{A}_3(\alpha) & \beta J^T(\alpha)\bar{A}_4(\alpha) & \beta J^T(\alpha)\bar{B}_3(\alpha) & \beta J^T(\alpha)\bar{B}_4(\alpha) \\ * & * & * & * & Q(\alpha) - P(\alpha) & 0 & 0 & 0 \\ * & * & * & * & * & -Q(\alpha) & 0 & 0 \\ * & * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{17}$$

where $\Pi = P(\alpha) - J^T(\alpha) - J(\alpha)$.

Proof. If we can prove (17) is equivalent to (9), the proposition can be easily established. By virtue of the Schur complement, (9) is equivalent to

$$\begin{bmatrix} -I & 0 & 0 & 0 & \bar{L}(\alpha) & 0 & 0 & 0 \\ * & -I & 0 & 0 & 0 & \bar{L}(\alpha) & 0 & 0 \\ * & * & -P(\alpha) & 0 & P(\alpha)\bar{A}_1(\alpha) & P(\alpha)\bar{A}_2(\alpha) & P(\alpha)\bar{B}_1(\alpha) & P(\alpha)\bar{B}_2(\alpha) \\ * & * & * & -P(\alpha) & \beta P(\alpha)\bar{A}_3(\alpha) & \beta P(\alpha)\bar{A}_4(\alpha) & \beta P(\alpha)\bar{B}_3(\alpha) & \beta P(\alpha)\bar{B}_4(\alpha) \\ * & * & * & * & Q(\alpha) - P(\alpha) & 0 & 0 & 0 \\ * & * & * & * & * & -Q(\alpha) & 0 & 0 \\ * & * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{18}$$

On the one hand, suppose there exist matrices $P(\alpha) > 0$, $Q(\alpha) > 0$ and $J(\alpha)$ satisfying (17). From the negative definite matrix (17), we know the fact that $J + J^T - P(\alpha) > 0$ and $P(\alpha) > 0$, so that $J^{-1}(\alpha)$

exists. In addition, by noticing $\left(P(\alpha)-J^T(\alpha)\right)P^{-1}(\alpha)\left(P(\alpha)-J(\alpha)\right)\geq 0$, we have $-J^T(\alpha)P^{-1}(\alpha)J(\alpha)\leq P(\alpha)-J^T(\alpha)-J(\alpha)$, which together with (17) yields

$$\left[\begin{array}{cccccccc} -I & 0 & 0 & 0 & \bar{L}(\alpha) & 0 & 0 & 0 \\ * & -I & 0 & 0 & 0 & \bar{L}(\alpha) & 0 & 0 \\ * & * & -\bar{\Pi} & 0 & J^T(\alpha)\bar{A}_1(\alpha) & J^T(\alpha)\bar{A}_2(\alpha) & J^T(\alpha)\bar{B}_1(\alpha) & J^T(\alpha)\bar{B}_2(\alpha) \\ * & * & * & -\bar{\Pi} & \beta J^T(\alpha)\bar{A}_3(\alpha) & \beta J^T(\alpha)\bar{A}_4(\alpha) & \beta J^T(\alpha)\bar{B}_3(\alpha) & \beta J^T(\alpha)\bar{B}_4(\alpha) \\ * & * & * & * & Q(\alpha)-P(\alpha) & 0 & 0 & 0 \\ * & * & * & * & * & -Q(\alpha) & 0 & 0 \\ * & * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{array}\right]<0, \tag{19}$$

where $\bar{\Pi}=J^T(\alpha)P^{-1}(\alpha)J(\alpha)$. Performing a congruence transformations to (19) by $\operatorname{diag}\left\{I,\,I,\,J^{-1}(\alpha)P(\alpha)\,,\,J^{-1}(\alpha)P(\alpha),\,I,\,I,\,I,\,I\right\}$, we obtain (18). On the other hand, if (18) holds, by selecting $J(\alpha)=J^T(\alpha)=P(\alpha)$, the negative definite matrix (17) is established. The proof is completed. □

Remark 4 *The introduction of slack variable $J(\alpha)$ enables us to realize the parameter dependence, that is to use a different positive definite matrix P_i for each vertex of the polytope. Moreover, $J(\alpha)$ is also allowed to be α-dependent, that is, there is no need setting the introduced slack matrix $J(\alpha)$ to be constant for the entire uncertainty domain. As $J(\alpha)$ is a general matrix, that is, no structural restriction is imposed, which is potential to yield to less conservative results for the robust filter design.*

Based on the above idea, in what follows, we present a new filtering result. It is observed that we only need to set part of $J(\alpha)$ to be constant for the entire uncertainty domain. More specifically, for the uncertain case, we structure $J(\alpha)$ as the following structure

$$J(\alpha)=\left[\begin{array}{cc} J_1(\alpha) & J_2(\alpha) \\ J_4 & J_3 \end{array}\right]. \tag{20}$$

The $(3,3)$ block of (17) implies $P(\alpha)-J(\alpha)-J^T(\alpha)<0$ and $P(\alpha)>0$, therefore, $J(\alpha)$ is nonsingular. With $J(\alpha)$ is given in (20), there is no loss of generality in assuming that J_4 and J_3 are invertible. Let matrices $P(\alpha)$, $Q(\alpha)$ be partitioned as:

$$P(\alpha)=\left[\begin{array}{cc} P_1(\alpha) & P_2(\alpha) \\ * & P_3(\alpha) \end{array}\right],\qquad Q(\alpha)=\left[\begin{array}{cc} Q_1(\alpha) & Q_2(\alpha) \\ * & Q_3(\alpha) \end{array}\right],$$

and introduce matrix

$$T\triangleq\left[\begin{array}{cc} I & 0 \\ 0 & J_3^{-1}J_4 \end{array}\right], \tag{21}$$

and define

$$\begin{aligned} \bar{P}(\alpha) &= \left[\begin{array}{cc} \bar{P}_1(\alpha) & \bar{P}_2(\alpha) \\ * & \bar{P}_3(\alpha) \end{array}\right]\triangleq T^T P(\alpha) T, \\ \bar{Q}(\alpha) &= \left[\begin{array}{cc} \bar{Q}_1(\alpha) & \bar{Q}_2(\alpha) \\ * & \bar{Q}_3(\alpha) \end{array}\right]\triangleq T^T Q(\alpha) T. \end{aligned}$$

Performing congruence transformations to (17) by diag $\{I, I, T, T, T, T, I, I\}$ and taking into account (7), we obtain

$$\begin{bmatrix} -I & 0 & 0 & 0 & \tilde{L}(\alpha) & 0 & 0 \\ * & -I & 0 & 0 & 0 & \tilde{L}(\alpha) & 0 \\ * & * & \Gamma_1 & 0 & \Gamma_2 & \Gamma_3 & \Gamma_4 \\ * & * & * & \Gamma_1 & \beta\Gamma_5 & \beta\Gamma_6 & \beta\Gamma_7 \\ * & * & * & * & \bar{Q}(\alpha) - \bar{P}(\alpha) & 0 & 0 \\ * & * & * & * & * & -\bar{Q}(\alpha) & 0 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{22}$$

where

$$\begin{aligned} \Gamma_1 &= \begin{bmatrix} \bar{P}_1(\alpha) - J_1^T(\alpha) - J_1(\alpha) & \bar{P}_2(\alpha) - J_2(\alpha)J_3^{-1}J_4 - J_4^T J_3^{-1} J_4 \\ * & \bar{P}_3(\alpha) - J_4^T J_3^{-T} J_4 - J_4^T J_3^{-1} J_4 \end{bmatrix}, \\ \Gamma_2 &= \begin{bmatrix} J_1^T(\alpha)A_1(\alpha) + \theta J_4^T B_{f1} C(\alpha) & J_4^T A_{f1} J_3^{-1} J_4 \\ J_4^T J_3^{-T} J_2^T(\alpha) A_1(\alpha) + \theta J_4^T B_{f1} C(\alpha) & J_4^T A_{f1} J_3^{-1} J_4 \end{bmatrix}, \quad \tilde{L}(\alpha) = \begin{bmatrix} L(\alpha) & -L_f J_3^{-1} J_4 \end{bmatrix}, \\ \Gamma_3 &= \begin{bmatrix} J_1^T(\alpha)A_2(\alpha) + \theta J_4^T B_{f2} C(\alpha) & J_4^T A_{f2} J_3^{-1} J_4 \\ J_4^T J_3^{-T} J_2^T(\alpha) A_2(\alpha) + \theta J_4^T B_{f2} \bar{C}(\alpha) & J_4^T A_{f2} J_3^{-1} J_4 \end{bmatrix}, \\ \Gamma_4 &= \begin{bmatrix} J_1^T(\alpha)B_1(\alpha) + \theta J_4^T B_{f1} D(\alpha) & J_1^T(\alpha) B_2(\alpha) + \theta J_4^T B_{f2} D(\alpha) \\ J_4^T J_3^{-T} J_2^T(\alpha) B_1(\alpha) + \theta J_4^T B_{f1} D(\alpha) & J_4^T J_3^{-T} J_2^T(\alpha) B_2(\alpha) + \theta J_4^T B_{f2} D(\alpha) \end{bmatrix}, \\ \Gamma_5 &= \begin{bmatrix} J_4^T B_{f1} C(\alpha) & 0 \\ J_4^T B_{f1} C(\alpha) & 0 \end{bmatrix}, \quad \Gamma_6 = \begin{bmatrix} J_4^T B_{f2} C(\alpha) & 0 \\ J_4^T B_{f2} C(\alpha) & 0 \end{bmatrix}, \quad \Gamma_7 = \begin{bmatrix} J_4^T B_{f1} D(\alpha) & J_4^T B_{f2} D(\alpha) \\ J_4^T B_{f1} D(\alpha) & J_4^T B_{f2} D(\alpha) \end{bmatrix}. \end{aligned}$$

Define $F(\alpha) \triangleq J_1(\alpha)$, $V(\alpha) \triangleq J_2(\alpha)J_3^{-1}J_4$, $W \triangleq J_4^T J_3^{-1} J_4$ and

$$\begin{bmatrix} \bar{A}_{f1} & \bar{B}_{f2} \\ \bar{A}_{f2} & \bar{B}_{f2} \\ \bar{L}_f & 0 \end{bmatrix} \triangleq \begin{bmatrix} J_4^T & 0 & 0 \\ 0 & J_4^T & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} A_{f1} & B_{f1} \\ A_{f2} & B_{f2} \\ L_f & 0 \end{bmatrix} \begin{bmatrix} J_3^{-1} J_4 & 0 \\ 0 & I \end{bmatrix},$$

substituting the above matrices into (22) we obtain

$$\Theta(\alpha) \triangleq \begin{bmatrix} -I & 0 & 0 & 0 & \hat{L}(\alpha) & 0 & 0 \\ * & -I & 0 & 0 & 0 & \hat{L}(\alpha) & 0 \\ * & * & \Upsilon_1 & 0 & \Upsilon_2 & \Upsilon_3 & \Upsilon_4 \\ * & * & * & \Upsilon_1 & \beta\Upsilon_5 & \beta\Upsilon_6 & \beta\Upsilon_7 \\ * & * & * & * & \bar{Q}(\alpha) - \bar{P}(\alpha) & 0 & 0 \\ * & * & * & * & * & -\bar{Q}(\alpha) & 0 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{23}$$

where

$$\begin{aligned}
\Upsilon_1 &= \bar{P}(\alpha) - \begin{bmatrix} F(\alpha)+F^T(\alpha) & V(\alpha)+W \\ * & W^T+W \end{bmatrix}, \\
\Upsilon_2 &= \begin{bmatrix} F^T(\alpha)A_1(\alpha)+\theta\bar{B}_{f1}C(\alpha) & \bar{A}_{f1} \\ V^T(\alpha)A_1(\alpha)+\theta\bar{B}_{f1}C(\alpha) & \bar{A}_{f1} \end{bmatrix}, \quad \hat{L}(\alpha) = \begin{bmatrix} L(\alpha) & -\bar{L}_f \end{bmatrix}, \\
\Upsilon_3 &= \begin{bmatrix} F^T(\alpha)A_2(\alpha)+\theta\bar{B}_{f2}C(\alpha) & \bar{A}_{f2} \\ V^T(\alpha)A_2(\alpha)+\theta\bar{B}_{f2}C(\alpha) & \bar{A}_{f2} \end{bmatrix}, \\
\Upsilon_4 &= \begin{bmatrix} F^T(\alpha)B_1(\alpha)+\theta\bar{B}_{f1}D(\alpha) & F^T(\alpha)B_2(\alpha)+\theta\bar{B}_{f2}D(\alpha) \\ V^T(\alpha)B_1(\alpha)+\theta\bar{B}_{f1}D(\alpha) & V^T(\alpha)B_2(\alpha)+\theta\bar{B}_{f2}D(\alpha) \end{bmatrix}, \\
\Upsilon_5 &= \begin{bmatrix} \bar{B}_{f1}C(\alpha) & 0 \\ \bar{B}_{f1}C(\alpha) & 0 \end{bmatrix}, \Upsilon_6 = \begin{bmatrix} \bar{B}_{f2}C(\alpha) & 0 \\ \bar{B}_{f2}C(\alpha) & 0 \end{bmatrix}, \Upsilon_7 = \begin{bmatrix} \bar{B}_{f1}D(\alpha) & \bar{B}_{f2}D(\alpha) \\ \bar{B}_{f1}D(\alpha) & \bar{B}_{f2}D(\alpha) \end{bmatrix}.
\end{aligned}$$

Based on this, we obtain the following theorem.

Theorem 2 *Consider the 2DFM system in (1). For a given positive constant γ, an admissible robust H_∞ filter in the form of (4) exists if there exist matrices $\bar{P}(\alpha) = \begin{bmatrix} \bar{P}_1(\alpha) & \bar{P}_2(\alpha) \\ * & \bar{P}_3(\alpha) \end{bmatrix} > 0$, $\bar{Q}(\alpha) = \begin{bmatrix} \bar{Q}_1(\alpha) & \bar{Q}_2(\alpha) \\ * & \bar{Q}_3(\alpha) \end{bmatrix} > 0$, $F(\alpha)$, $V(\alpha)$, W and matrices $\bar{A}_{f1}$, $\bar{A}_{f2}$, $\bar{B}_{f1}$, $\bar{B}_{f2}$, $\bar{L}_f$ satisfying (23). Moreover, under the above conditions, the matrices for the filter in (4) are given by*

$$\begin{bmatrix} A_{f1} & B_{f1} \\ A_{f2} & B_{f2} \\ L_f & 0 \end{bmatrix} = \begin{bmatrix} W^{-1} & 0 & 0 \\ 0 & W^{-1} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \bar{A}_{f1} & \bar{B}_{f2} \\ \bar{A}_{f2} & \bar{B}_{f2} \\ \bar{L}_f & 0 \end{bmatrix}. \tag{24}$$

Proof. Suppose there exist matrices $\bar{P}(\alpha)$, $\bar{Q}(\alpha)$, $F(\alpha)$, $V(\alpha)$, W, $\bar{A}_{f1}$, $\bar{A}_{f2}$, $\bar{B}_{f1}$, $\bar{B}_{f2}$, and $\bar{L}_f$ satisfying (23). Noting the $(4,4)$ block in the negative definite matrix (23) together with $\bar{P}_3(\alpha) > 0$, which implies W is nonsingular. Thus, we can always find square and nonsingular matrices J_3 and J_4 satisfying $W = J_4^T J_3^{-1} J_4$. Now, let T as in (21) and matrices

$$P(\alpha) \triangleq T^{-T}\bar{P}(\alpha)T^{-1}, \quad Q(\alpha) \triangleq T^{-T}\bar{Q}(\alpha)T^{-1}, \quad J(\alpha) \triangleq \begin{bmatrix} F(\alpha) & V(\alpha)J_4^{-1}J_3 \\ J_4 & J_3 \end{bmatrix},$$

$$\begin{bmatrix} A_{f1} & B_{f1} \\ A_{f2} & B_{f2} \\ L_f & 0 \end{bmatrix} \triangleq \begin{bmatrix} J_4^{-T} & 0 & 0 \\ 0 & J_4^{-T} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \bar{A}_{f1} & \bar{B}_{f2} \\ \bar{A}_{f2} & \bar{B}_{f2} \\ \bar{L}_f & 0 \end{bmatrix} \begin{bmatrix} J_4^{-1}J_3 & 0 \\ 0 & I \end{bmatrix}. \tag{25}$$

Now, by some algebraic matrix manipulations and taking into account the above definition, (22) is equivalent to

$$\begin{bmatrix}
-I & 0 & 0 & 0 & \bar{L}(\alpha)T & 0 & 0 & 0 \\
* & -I & 0 & 0 & 0 & \bar{L}(\alpha)T & 0 & 0 \\
* & * & T^T\Pi T & 0 & V^T(\alpha)\bar{A}_1(\alpha)T & V^T(\alpha)\bar{A}_2(\alpha)T & V^T(\alpha)\bar{B}_1(\alpha) & V^T(\alpha)\bar{B}_2(\alpha) \\
* & * & * & T^T\Pi T & \beta V^T(\alpha)\bar{A}_3(\alpha)T & \beta V^T(\alpha)\bar{A}_4(\alpha)T & \beta V^T(\alpha)\bar{B}_3(\alpha) & \beta V^T(\alpha)\bar{B}_4(\alpha) \\
* & * & * & * & T^T(Q(\alpha)-P(\alpha))T & 0 & 0 & 0 \\
* & * & * & * & * & -T^TQ(\alpha)T & 0 & 0 \\
* & * & * & * & * & * & -\gamma^2 I & 0 \\
* & * & * & * & * & * & * & -\gamma^2 I
\end{bmatrix} < 0, \tag{26}$$

where $\Pi = P(\alpha) - J^T(\alpha) - J(\alpha)$, $V(\alpha) = J(\alpha)T$ .Performing congruence transformations to (26) by diag $\{I, I, T^{-1}, T^{-1}, T^{-1}, T^{-1}, I, I\}$ yields (17).

From the above proof, we know that if condition (23) has a set of feasible solutions, then the filter with a state-space realization A_{f1}, A_{f2}, B_{f1}, B_{f2}, L_f guarantees the filtering error system in (7) to be stochastically stable and with a prescribed H_∞ performance. Let us denote the transfer function from $\tilde{y}_{i,j}$ to $\hat{z}_{i,j}$ by $T_{\hat{z}\tilde{y}}(z_1, z_2) = L_f (z_1 z_2 I - z_1 A_{f1} - z_2 A_{f2})^{-1} (z_1 B_{f1} + z_2 B_{f2})$. Substituting the filter matrices with (25) and considering the relationship $W = J_4^T J_3^{-1} J_4$, we have

$$\begin{aligned} T_{\hat{z}\tilde{y}}(z_1, z_2) &= \bar{L}_f J_4^{-1} J_3 \left(z_1 z_2 I - z_1 J_4^{-T} \bar{A}_{f1} J_4^{-1} J_3 - z_2 J_4^{-T} \bar{A}_{f2} J_4^{-1} J_3 \right)^{-1} \left(z_1 J_4^{-T} \bar{B}_{f1} + z_2 J_4^{-T} \bar{B}_{f2} \right) \\ &= \bar{L}_f \left(z_1 z_2 I - z_1 W^{-1} \bar{A}_{f1} - z_2 W^{-1} \bar{A}_{f2} \right)^{-1} \left(z_1 W^{-1} \bar{B}_{f1} + z_2 W^{-1} \bar{B}_{f2} \right). \end{aligned}$$

which means (24) is established and the proof is completed. □

Remark 5 *Theorem 2 tells us that not only the positive definite matrices $P(\alpha)$ and $Q(\alpha)$ are allowed to be dependent on the uncertain parameter α, but the general slack matrices $F(\alpha)$ and $V(\alpha)$ are also allowed to be α-dependent. This is different from the existing results in this field, which require the slack matrices to be fixed for the entire uncertainty domain. It is worth noting that, as they are dependent on the parameter α, the condition in (23) cannot be directly employed for filter design. One way to facilitate Theorem 2 for the construction of a filter is to convert (23) into a finite set of LMI constraints. The following theorem gives a possible way to achieve this.*

Theorem 3 *Consider the 2DFM system in (1). For a given positive constant γ, an admissible robust H_∞ filter in the form of (4) exists, if there exist matrices $\bar{P}_i = \begin{bmatrix} \bar{P}_{1i} & \bar{P}_{2i} \\ * & \bar{P}_{3i} \end{bmatrix} > 0$, $\bar{Q}_i = \begin{bmatrix} \bar{Q}_{1i} & \bar{Q}_{2i} \\ * & \bar{Q}_{3i} \end{bmatrix} > 0$, F_i, V_i, W and matrices $\bar{A}_{f1}$, $\bar{A}_{f2}$, $\bar{B}_{f1}$, $\bar{B}_{f2}$, $\bar{L}_f$ satisfying*

$$\Theta_{ij} + \Theta_{ji} < 0, \qquad 1 \le i \le j \le s, \tag{27}$$

where

$$\Theta_{ij} \triangleq \begin{bmatrix} -I & 0 & 0 & 0 & \tilde{L}_i & 0 & 0 \\ * & -I & 0 & 0 & 0 & \tilde{L}_i & 0 \\ * & * & \Psi_1 & 0 & \Psi_2 & \Psi_3 & \Psi_4 \\ * & * & * & \Psi_1 & \beta\Psi_5 & \beta\Psi_6 & \beta\Psi_7 \\ * & * & * & * & \bar{Q}_i - \bar{P}_i & 0 & 0 \\ * & * & * & * & * & -\bar{Q}_i & 0 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{28}$$

$$\begin{aligned} \Psi_1 &= \bar{P}_i - \begin{bmatrix} F_i + F_i^T & V_i + W \\ * & W^T + W \end{bmatrix}, \quad \tilde{L}_i = \begin{bmatrix} L_i & -\bar{L}_{fi} \end{bmatrix}, \\ \Psi_2 &= \begin{bmatrix} F_i^T A_{1j} + \theta \bar{B}_{f1} C_j & \bar{A}_{f1} \\ V_i^T A_{1j} + \theta \bar{B}_{f1} C_j & \bar{A}_{f1} \end{bmatrix}, \quad \Psi_3 = \begin{bmatrix} F_i^T A_{2j} + \theta \bar{B}_{f2} C_j & \bar{A}_{f2} \\ V_i^T A_{2j} + \theta \bar{B}_{f2} C_j & \bar{A}_{f2} \end{bmatrix}, \\ \Psi_4 &= \begin{bmatrix} F_i^T B_{1j} + \theta \bar{B}_{f1} D_j & F_i^T B_{2j} + \theta \bar{B}_{f2} D_j \\ V_i^T B_{1j} + \theta \bar{B}_{f1} D_j & V_i^T B_{2j} + \theta \bar{B}_{f2} D_j \end{bmatrix}, \\ \Psi_5 &= \begin{bmatrix} \bar{B}_{f1} C_j & 0 \\ \bar{B}_{f1} C_j & 0 \end{bmatrix}, \quad \Psi_6 = \begin{bmatrix} \bar{B}_{f2} C_j & 0 \\ \bar{B}_{f2} C_j & 0 \end{bmatrix}, \quad \Psi_7 = \begin{bmatrix} \bar{B}_{f1} D_j & \bar{B}_{f2} D_j \\ \bar{B}_{f1} D_j & \bar{B}_{f2} D_j \end{bmatrix}, \end{aligned}$$

moreover, if the above condition is satisfied, the matrices for the filter in (4) are given by (24).

Proof. Suppose there exist matrices $\bar{P}(\alpha) > 0$, $\bar{Q}(\alpha) > 0$, $F(\alpha)$, $V(\alpha)$, W, $\bar{A}_{f1}$, $\bar{A}_{f2}$, $\bar{B}_{f1}$, $\bar{B}_{f2}$ and L_f satisfying (23), then the filter in the form of (4) exist. Now, we use these matrices and α in the unit simplex Γ to assume the following form:

$$\begin{aligned}\bar{P}(\alpha) &= \sum_{i=1}^{s} \alpha_i \bar{P}_i, \quad \bar{Q}(\alpha) = \sum_{i=1}^{s} \alpha_i \bar{Q}_i, \\ V(\alpha) &= \sum_{i=1}^{s} \alpha_i V_i, \quad F(\alpha) = \sum_{i=1}^{s} \alpha_i F_i.\end{aligned} \tag{29}$$

By virtue of (29), it is easy to rewrite $\Theta(\alpha)$ in (23) as

$$\Theta(\alpha) = \sum_{j=1}^{s}\sum_{i=1}^{s} \alpha_i \alpha_i \Theta_{ij} = \sum_{i=1}^{s} \alpha_i^2 \Theta_{ii} + \sum_{i=1}^{s}\sum_{j=i+1}^{s} \alpha_i \alpha_j \left(\Theta_{ij} + \Theta_{ji}\right), \tag{30}$$

where Θ_{ij} takes the form of (28). On the other hand, from (27) we have

$$\Theta_{ii} < 0, \qquad i = 1, \cdots, s, \tag{31}$$

$$\Theta_{ij} + \Theta_{ji} < 0, \qquad 1 \le i < j \le s. \tag{32}$$

Considering $\sum_{i=1}^{s} \alpha_i = 1$, $\alpha_i \ge 0$, then from (30)-(32) we have $\Theta(\alpha) < 0$. Based on Theorem 2, there exists a filter in the form of (4) such that the filtering error system in (7) is stochastically stable with a given H_∞ performance. □

5 An Illustrative Example

In this section, we use an example to illustrate the effectiveness of the theoretical results developed above. **Example:** Consider the model of the static field [6], which is described by differential equation:

$$\eta_{i+1,j+1} = \alpha_1 \eta_{i,j+1} + \alpha_1 \eta_{i+1,j} - \alpha_1 \alpha_2 \eta_{i,j} + \omega_{1(i,j)},$$

where $\eta_{i,j}$ is the state of the field at spacial coordinates (i, j), and α_1, α_2 are, respectively, the vertical and horizontal correlative coefficients of the random field, satisfying

$$\alpha_1^2 < 1, \qquad \alpha_2^2 < 1.$$

Defining the augmented state vector $x_{i,j} = \begin{bmatrix} \eta_{i,j+1}^T - \alpha_2 \eta_{i,j}^T & \eta_{i,j}^T \end{bmatrix}^T$, and assume that the measured equation and the signal to be estimated are

$$\begin{aligned} y_{i,j} &= \alpha_1 \eta_{i,j+1} + (1 - \alpha_1 \alpha_2) \eta_{i,j} + \omega_{2(i,j)}, \\ z_{i,j} &= \eta_{i,j}. \end{aligned}$$

It is easy to transform the above equation into a 2-DFM model in the form of (1), with the corresponding system matrices given by

$$\begin{aligned} A_1(\alpha) &= \begin{bmatrix} \alpha_1 & 0 \\ 0 & 0 \end{bmatrix}, \; A_2(\alpha) = \begin{bmatrix} 0 & 0 \\ 1 & \alpha_2 \end{bmatrix}, \quad B_1(\alpha) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \\ B_2(\alpha) &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \; C(\alpha) = \begin{bmatrix} \alpha_1 & 1 \end{bmatrix}, \; D(\alpha) = \begin{bmatrix} 0 & 1 \end{bmatrix}, \; L(\alpha) = \begin{bmatrix} 0 & 1 \end{bmatrix}. \end{aligned}$$

It is assumed that measurements transmitted between the plant and the filter are imperfect, that is, the data may be lost during their transmission. Based on the above assumption, our purpose is to design a filter in the form of (4) such that the resulting filtering error system in (7) is mean-square asymptotically stable with a guaranteed H_∞ noise attenuation performance.

First, assume the uncertain parameters α_1 and α_2 are given by $0.15 \le \alpha_1 \le 0.45$, $0.35 \le \alpha_2 \le 0.85$. Thus the above parameter uncertainty can be represented by a four-vertex polytope. The stochastic variable is assumed to be $\theta_{i,j} = 1$ ($\theta = 1$), which means that the measurements always reach the input of the filter successfully. Using the method proposed in (27), the minimum H_∞ performance $\gamma^* = 2.4989$ is obtained and the associated filter matrices are given by

$$\begin{aligned} A_{f1} &= \begin{bmatrix} 0.4445 & -0.1658 \\ 0.0096 & -0.0062 \end{bmatrix}, \quad A_{f2} = \begin{bmatrix} -0.0847 & 0.0109 \\ 0.2526 & 0.2379 \end{bmatrix}, \\ B_{f1} &= \begin{bmatrix} -0.1696 \\ -0.0131 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} 0.0340 \\ -0.5975 \end{bmatrix}, \quad L_f = \begin{bmatrix} -0.0036 & -1.0010 \end{bmatrix}. \end{aligned}$$

Now, assume the data may be lost during their transmission. Suppose $\theta = 0.8$, that is, in the communication link, the probability of the data packet missing is 0.2. With the above assumption, we apply the filter design method in Theorem 3, and the achieved H_∞ disturbance attenuation level is $\gamma^* = 3.7487$ with the corresponding filter matrices

$$\begin{aligned} A_{f1} &= \begin{bmatrix} 0.4950 & -0.0795 \\ -0.2211 & 0.0325 \end{bmatrix}, \quad A_{f2} = \begin{bmatrix} 0.0366 & -0.0241 \\ 0.3558 & 0.4773 \end{bmatrix}, \\ B_{f1} &= \begin{bmatrix} -0.0835 \\ 0.0234 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} -0.0876 \\ -0.5332 \end{bmatrix}, \quad L_f = \begin{bmatrix} -0.6449 & -0.9351 \end{bmatrix}. \end{aligned}$$

In the following, suppose $\theta = 0.8$, we shall show the effectiveness of the designed H_∞ filter by presenting simulation results. The data packet dropouts is shown in Figure 1, which is generated randomly according to $\theta = 0.8$. To show the asymptotic stability of the filtering error system, we assume $\omega_{i,j} = 0$ and let the initial boundary conditions generated randomly. The obtained filtering error signal $\tilde{z}_{i,j}$ is shown in Figure 2, from which we can see the estimation error response converges to zero under the preceding conditions.

To illustrate the performance of the designed filter, we assume the zero boundary conditions, and let the external disturbance $\omega_{i,j}$ be

$$\omega_{i,j} = \begin{cases} \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T & 3 \le i,j \le 19 \\ 0 & \text{otherwise} \end{cases}.$$

Figure 3 shows the response of the filtering error signal $\tilde{z}_{i,j}$. By calculation, we have $||\tilde{z}||_2 = 1.5729$, $||\omega||_2 = 1.700$, which yields $\gamma^* = 0.9252$, which are below the corresponding prescribed value 3.7487, showing the effectiveness of the filter design method.

Finally, Table 1 shows the minimum guaranteed performances γ^* in terms of the feasibility of (27) for different of θ, from which we can see that the smaller the value of θ, the larger the value of γ^*. This is reasonable, as θ smaller implies higher chance of measurements missing, and thus worse disturbance attenuation performance γ^*.

θ	1	0.95	0.9	0.85	0.8	0.75
γ^*	2.4989	2.5844	2.9804	3.3882	3.7487	4.0809

Table 1. γ^* for different values of θ.

6 Conclusions

In this paper, the problem of robust H_∞ filter design for uncertain 2-D systems with parameter uncertainties and missing measurements has been investigated. A stochastic variable satisfying the Bernoulli random binary distribution is utilized to characterize the data missing phenomenon. A parameter-dependent technique has been used to design an H_∞ filter such that the filtering error system is stochastically stable and preserves a guaranteed H_∞ performance, which is much less conservative than the quadratic approach. Some slack matrices have been introduced to facilitate the H_∞ filter design, and only part of the slack matrix variable has been imposed to be fixed for the entire uncertainty domain. The corresponding results are in the form of linear matrix inequalities, which can be solved by standard numerical software. An example has shown the effectiveness of the filter design approaches presented in this paper.

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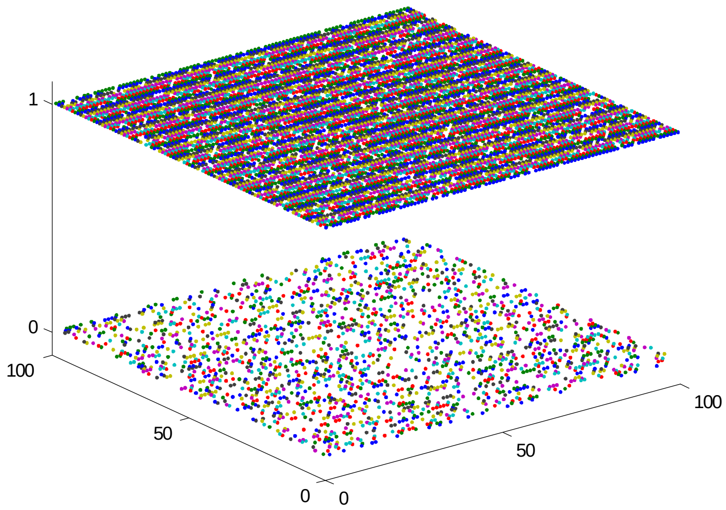

Figure 1. Data packet dropout

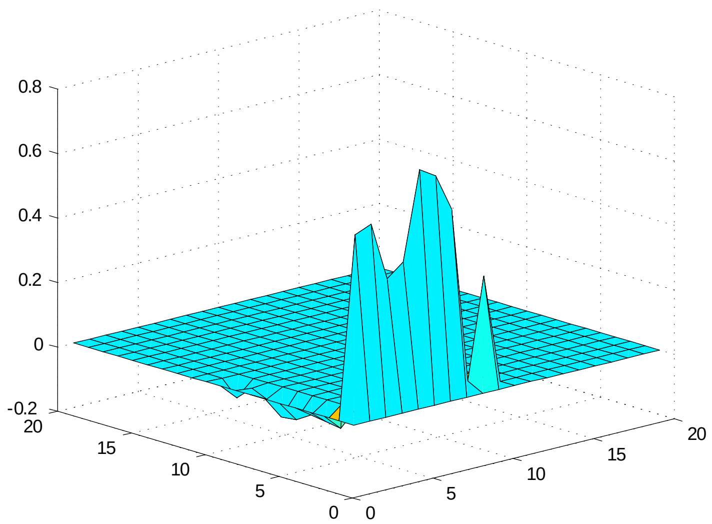

Figure 2. Filter error with $\omega_{i,j} = 0$

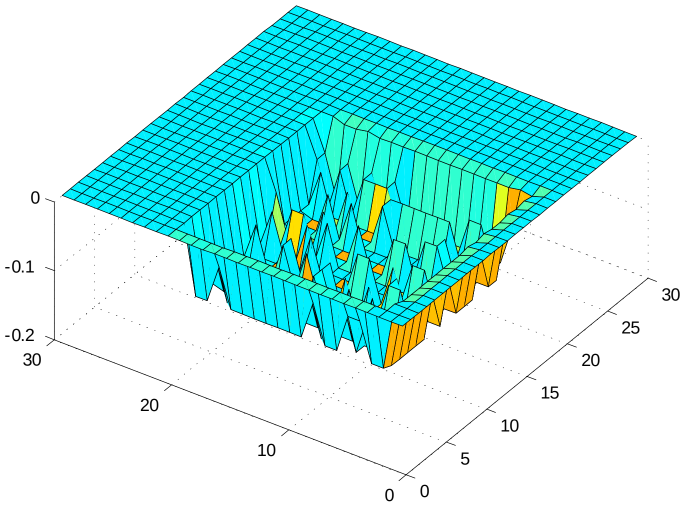

Figure 3. Estimation error with $\omega_{i,j} \neq 0$