NOTES

ZUGZWANGS IN CHESS STUDIES

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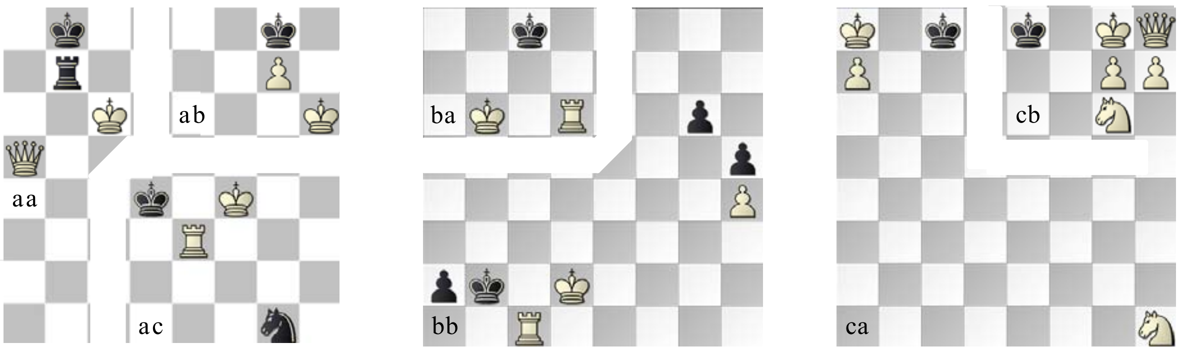
ABSTRACT

Van der Heijden's ENDGAME STUDY DATABASE IV, HHDBIV, is the definitive collection of 76,132 chess studies. The zugzwang position or *zug*, one in which the side to move would prefer not to, is a frequent theme in the literature of chess studies. In this third data-mining of HHDBIV, we report on the occurrence of sub-7-man zugs there as discovered by the use of CQL and Nalimov endgame tables (EGTs). We also mine those *Zugzwang Studies* in which a zug more significantly appears in both its White-to-move (wtm) and Black-to-move (btm) forms. We provide some illustrative and extreme examples of zugzwangs in studies.

1. INTRODUCTION

The combination of Van der Heijden's (2010) study database HHDBIV, Nalimov's sub-7-man (s7m) endgame tables (EGTs), and Bleicher's (2011) endgame analysis service has already enabled the authors (Bleicher et al, 2010) to partially check the correctness of the studies' s7m mainlines. Further, given that uniqueness of solution-move is a key property of studies, we have also identified (Haworth et al, 2011) the frequency of equi-optimal and sub-optimal moves in the studies' s7m mainlines. The impact of less than absolute uniqueness on the technical and aesthetic qualities of the studies remains a question for the future, best addressed by a combination of algorithmic analysis (Haworth, 2009) and artistic judgement.

The zugzwang position or *zug*, in which the side to move would rather not, has been a fascination for over one thousand years because of its intrinsic irony, its rarity, its effect on play and latterly because it misleads chess engines which use the null move heuristic. Zugs are more likely if there are fewer move options so they are denser in endgames with fewer men, precisely the zone where computers create EGTs. There are 25,105 sub-6-man zugs (Haworth et al, 2001) and 906,952 6-man zugs (Bleicher and Haworth, 2009), relatively fewer. The presence of Pawns and Knights, less flexible than line pieces, make zugs more prevalent.



aa) Philidor (1777); ab) KPK; ac) Ar Razi (850-)   ba) KRK, drawn with null move; bb) Polerio (1590)   ca) Nunn (1995) #1; cb) a '5-1' zug

Figure 1. Some positions illustrating the disadvantage of having the move.[2]

Fig.1 includes some modest challenges for the reader, discussed later. Nunn (1995, p6) states that "zugzwang positions often have an importance far out of proportion to their small numbers". His first endgame trilogy (Nunn, 1992, 1994, 1995) consistently features the zug theme as do reviews of EGTs (Tamplin and Haworth, 2001). Many KPK and KRK positions (Figs. 1ab and 1ba) would be drawn if the defender could *pass*. The

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[2] HHDBIV indices: aa) #956 (Nunn, 1994, #63), ac) #374, bb) #415 and ca) #55216 (Nunn, 1995, #376)

endgame study magazine *EG* mentions the words *zugzwang*, *zug* or *zz* 1,914 times over the 152 issues of its first 38 years (Costeff, 2011) and frequently published Rasmussen's endgame-specific lists of zugzwangs mined from Ken Thompson's s6m EGTs (Valois, 2011).

This note reports the identification, classification and distribution of all[3] zugs in the s7m mainlines of HHDBIV's studies. Further, all the *Zugzwang Studies* (Beasley, 2000), in which at least one s7m zug appears with both wtm and btm, have been identified and classified according to the types of those zugs.

2. THE DEFINITION OF A ZUGZWANG

Following Bleicher et al (2009), a *zugzwang* position is defined here as a position in which the side to move (stm) would prefer not to do so. This definition does not narrow the semantics of the German word by defining why the stm regrets being obliged to act. In the most familiar zugs, the stm gains a half- or a whole point by *passing*, providing three zug types – draw/win, loss/draw and most dramatically, loss/win.

However, it may be that by *passing*, i.e., playing a *null move*, the stm achieves a winning goal more quickly or loses more slowly in terms of some ply- or move-counting metric.[4] If *m* depth-metrics DT*x* have been defined,[5] this adds at least a further 2*m* types of zugzwang. Lastly, and not considered at length here, the stm, given their fallibility and lack of competence, may prefer to pass, and may objectively be thought more likely to achieve a result by passing, even though the value of the result and number of moves are not affected.

The presence of an *en passant* (*e.p.*) move option in the original position complicates matters. When the stm passes, the e.p. opportunity permanently disappears. The second player's null move therefore does not return to the first position and both value and depth are potentially different. An *e.p. zug* is therefore characterised by three positions rather than two. Bleicher et al (2009) identified three new types of zug, all e.p. zugs, and so classified value-critical zugs into types A1-A6 rather than just A1-A3.[6] A2 zugs are equivalent to A1 zugs unless e.p. is involved. Only 395 s7m e.p. zugs of types A4-A6 have been found (Bleicher et al, 2009)[7]. They are rare indeed, none appear in studies to date and no s7m type A5 position is known.[8]

The status of a type A zug is not entirely independent of depth metrics. In terms of a DTZ_k rule recognising a *k*-move draw-claim rule,[9] given a small enough *k*, some positions which are type A zugs become non-zugs[10] or Ai_k zugs of a different type, and some positions which are not type A zugs become Ai_k zugs.

The zugs which are value-neutral but depth-critical in terms of metric DT*x* are dubbed types B1-*x* (stm wins more quickly in DT*x* terms) and B2-*x* (stm loses more slowly). B2-*x* zugs are equivalent to B1-*x* zugs unless e.p. capture is possible. A B1-*x* zug may not be a B1 zug in terms of a different metric *y*. With three metrics, e.g., DTC, DTM and DTZ, there are $2^3=8$ possibilities for the type Bi-C/M/Z zug status of a position: table 2 shows that all eight can occur.

The *zug depth* (*zd*) of type A and B zugs can be defined in terms of the values and depths of the zug's first two positions.[11] The zug depth of a zug should not be confused with the depth of the zug's first position: they are only identical for type A2 and A6 zugs. The greater the zug depth of a type A zug, and the lesser the zug depth of a type B zug, the more subtle is the advantage of not having the move. Becker's 2005 study HHDBIV#72682 (c.f., table 3, ZP5-6) provides the two deepest A1 zugs to date.

For completeness, the value- and DT*x*-neutral zugs are hereby dubbed Types C1-*x* (stm win), C2-*x* (stm loss) and C3-(*x*) (stm draw)[12]. Type C zugs were not reviewed here because the likelihood of a result can only be assessed by using a Reference Fallible Endgame Player, e.g., that defined in (Haworth, 2003).

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[3] There may be a few false-positive or omissions but only because EGTs do not include positions with castling rights.

[4] The *null move* is a move but if it were included in *position depth* it would render that concept undefinable: the 'depth' of a position would, in most cases where two null moves cancel out, be *d*, *d*+2, *d*+4 etc.

[5] DTM ≡ Depth to Mate, DTC ≡ Depth to Conversion, DTZ ≡ Depth to Pawn-push or conversion, and going further, DTZ_k ≡ Depth to Pawn-push or conversion, moderated by a *k*-move draw rule; DTR ≡ Depth by The Rule.

[6] The most familiar zug types are labeled here A1 (draw/win), A2 (loss/draw) and A3 (loss/win).

[7] Examples: A4 (draw/win/loss, DTM =/+21/-30) 8/1p6/1k6/pP6/K7/P7/8/8 w a6; A5 (loss/win/draw, DTZ -0/+1/==) 8/8/8/2p5/1pP1p3/kP2P3/Pp1P4/1K6 b c3; A6 (loss/draw/draw, DTM -2/=/==) 8/8/8/8/pP6/p7/k1K5/1R6 b b3.

[8] Two extreme challenges for study composers: a study with an A4/5/6 zug, and an A5 zug with less than 11 men.

[9] Apart from retrograde studies, not considered here, study composition does not recognize a *k*-move draw-claim rule.

[10] HHDBIV#3292, 'Meyer' (1891), pos. 26b, 8/8/8/1K6/4N2p/1k6/8/5N2 b: an A2 but not $A2_{50}$ zug as DTZ = 57.

[11] Zug depth of a Type A1 zug ≡ depth(p2); of A2 ≡ d(p1); of A3 ≡ $d(p1)+d(p2)$; of Type B ≡ $d(p1)-d(p2)$.

[12] 8/8/3k4/1K1p4/1P6/1P6/8/8 b is arguably a C3 zug (Bleicher and Haworth, 2009): 1. … Ke5''' is the unique draw, but after the losing 1. (Ka5/Kb6)?? or 1. (Ka4/Ka6)''' Black's goals seem easier to reach.

3. SOME STATISTICS ABOUT ZUGZWANGS IN HHDBIV

Whenever possible, the stm and sntm versions of s7m positions in the mainlines of studies in HHDBIV were evaluated using the Nalimov DTM EGTs: this identified all zugs of types A1-6 and Bi-M. As there were some s7m errors in the studies (Bleicher et al, 2010) some 82 Type A zug positions were found with their first position's value incompatible with their study's designation. Similarly, 392 Type B1/2 zugs were unexpectedly found in 'Draw Study' mainlines which should not include decisive positions. These 'alien zugs' are all included in the counts of table 1 below as it may be that the designation of the study is the error. Similarly, zugs and zug studies have been included even if HHDBIV indicates[13] there is some flaw in the study.

The 9,875 type A zugs found are 1.09% of the 906,952 s7m zugs (Bleicher and Haworth, 2009). Zugs are a mere 0.000027% of s7m chess positions but 2.43% of the s7m study positions examined, i.e. 10^5 times more dense. They involve 30.45% of the studies examined. Draw study wtm A1 zugs such as ZP2-4 in table 3 are conspicuously rare, and only two zugs, ZP7 and ZP11, involve en passant.

Item	in Win Studies			No. of	in Draw Studies			No. of	Total Positions			Total
	wtm	btm	all	Studies	wtm	btm	all	Studies	wtm	btm	all	Studies
Positions evaluated	145,874	117,262	263,136	11,993	75,506	67,527	143,033	5,171	221,380	184,789	406,169	17,164
'e.p.' positions evaluated	53	19	72	72	6	31	37	37	59	50	109	109
A1	38	2	40	31	24	3,648	3,672	1,735	62	3,650	3,712	1,766
A2	0	5,829	5,829	3,258	40	1	41	32	40	5,830	5,870	3,290
A3	1	292	293	270	0	0	0	0	1	292	293	270
All Ai	39	6,123	6,162	3,515	64	3,649	3,713	1,759	103	9,772	9,875	5,274
All legitimate Ai	0	6,121	6,121	3,491	24	3,648	3,672	1,735	24	9,769	9,793	5,226
B1-M	5,800	1	5,801	3,071	1	255	256	134	5,801	256	6,057	3,205
B2-M	0	9,178	9,178	4,968	135	1	136	60	135	9,179	9,314	5,028
All Bi-M	5,800	9,179	14,979	6,045	136	256	392	178	5,936	9,435	15,371	6,223
All legitimate Bi-M	5,800	9,178	14,978	6,044	0	0	0	0	5,800	9,178	14,978	6,044
A1 Zugzwang Studies	---	---	---	0	---	---	---	505	---	---	---	505
A2 Zugzwang Studies	---	---	---	706	---	---	---	0	---	---	---	706
A3 Zugzwang Studies	---	---	---	16	---	---	---	0	---	---	---	16
B1-M Zugzwang Studies	---	---	---	329	---	---	---	0	---	---	---	329
B2-M Zugzwang Studies	---	---	---	0	---	---	---	0	---	---	---	0

Table 1. A statistical profile of Type A1-A6 and B1-B2 zugs.

Id	HHdbIV	First mainline zug	Force	DTC		DTM		DTZ		Zug depth			Notes
	Study #			stm	sntm	stm	sntm	stm	sntm	DTC	DTM	DTZ	
BB0	4647	8/8/8/8/4nk1K/5b2/8/5Q2 w - 4	KQKBN	24	-26	35	-37	24	-26	-2	-2	-2	Karstedt (1905); not B1-C, -M or -Z
BB1	17741	8/pK6/8/8/8/2Q5/r7/1k6 w - 1	KQKRP	13	-16	30	-32	**7**	**-6**	-3	-2	**1**	Dedrle (1937); B1-Z
BB2	72292	3k4/8/1p1P4/1P2K3/8/8/8/8 w - 22	KPPKP	2	-3	**16**	**-13**	2	-3	-1	**3**	-1	Fontein (2005); B1-M
BB3	11518	8/3k1p2/5P2/3KP3/8/8/8/8 w - - 0 1	KPPKP	3	-4	**16**	**-13**	**2**	**-1**	-1	**3**	**1**	Rabinovich (1927); B1-M/Z
BB4	957	2Q5/8/3p4/3kr3/5K2/8/8/8 w - 1	KQKRP	**16**	**-15**	37	-37	10	-10	**1**	0	0	Philidor (1777); B1-C
BB5	2341	8/8/8/8/5K2/1Q1p4/3kr3/8 w - 6	KQKRP	**9**	**-6**	26	-28	**8**	**-6**	**3**	-2	**2**	Von Guretzky Cornitz (1864); B1-C/Z
BB6	1009	8/8/5K1k/4N3/7p/7N/8/8 w - 36	KNNKP	**13**	**-10**	**14**	**-11**	3	-6	**3**	**3**	-3	Chapais (1780); B1-C/M
BB7	223	5k2/8/5P1p/4K2P/8/8/8/8 w - 1	KPPKP	**7**	**-4**	**16**	**-13**	**6**	**-3**	**3**	**3**	**3**	Lasker; B1-C/M/Z

Table 2. B1-*x* zugs showing all eight values of metric-dependant type B status.

Id	HHdbIV	Mainline zug	Force	Zug type	Zug depth	Notes
ZP1	13,062	5k1K/7P/5P2/8/8/8/p7/8 b - 6	KPPKP	A1	x 1	Aizenshtat (1929); minDTx-zd s7m A1 zug
ZP2	923	8/8/6p1/8/7P/2p5/p1K5/k7 w - 6	KPKPPP	A1	m 5	Ponziani (1769); minDTM-zd s7m wtm A1 zug
ZP3	1,600	8/8/8/8/8/p4K2/1p5k/1R6 w - 4	KRKPP	A1	m 5	Kling and Horwitz (1848); minDTM-zd s7m wtm A1 zug
ZP4	53181	8/8/8/8/6p1/5bQ1/6pK/5k2 w - 6	KQKBPP	A1	m 28	Kovalenko (1985); maxDTM-zd s7m wtm A1 zug
ZP5	72682	k4b2/3K4/1r1N4/5N2/8/8/8/8 b - 6	KNNKRP	A1	m 225	Becker (2005); maxDTM-zd s7m A1 zug; also pos. 10b
ZP6	72682	k2r1b2/8/3NK3/5N2/8/8/8/8 b - 13	KNNKRP	A1	m 225	Becker (2005); maxDTM-zd s7m A1 zug
ZP7	30080	8/8/8/3k4/2pP4/4K3/1P6/8 b - d3 5	KPPKP	A2	m 25, z 2	Richter (1958); only type A *e.p. zug* ; zd = m18 without e.p.
ZP8	45949	8/8/8/8/p7/kpK5/p7/N7 b - 5	KNKPPP	A3	m 11	Travasoni (1978); minDTM-zd s7m A3 zug
ZP9	20548	8/8/2K5/k7/5R2/2P3p1/5p2/8 b - 5	KRPKPP	A3	m 57	Kuvatov (1941); maxDTM-zd s7m A3 zug
ZP10	76065	k7/2K2n2/8/3N4/8/3pB3/8/8 w - 15	KBNKNP	B1-C/M	m 56	Vandecasteele (2010); maxDTM-zd s7m B1 zug
ZP11	8954	8/8/8/8/3k1Pp1/8/4K1P1/8 b f3 2	KPPKP	B2-C/M/Z	m 3, z 1	Dedrle (1923); only type B *e.p. zug;* e.p. not critical
ZP12	29830	8/8/8/4B3/8/3NK3/4n3/5k2 b - 6	KBNKN	B2-C/M/Z	m 70, c/z 62	Laznicka (1958); maxDTM-zd s7m B2 zug
ZP13	60443	k1N5/B7/K7/4p3/n7/8/8/8 b - 3	KBNKNP	B2-C/M	m 70	Vandecasteele & Missiaen (1992); maxDTM-zd s7m B2 zug

Table 3. Selected s7m zugzwang positions from studies in HHDBIV.[14]

[13] In HHDBIV, an '*@n*' in the study title indicates a flaw, the code being defined in (Van der Heijden, 2010).

[14] $c \equiv$ DTC, $m \equiv$ DTM, $x \equiv$ DTx, $z \equiv$ DTZ

4. THE ZUGZWANG STUDY

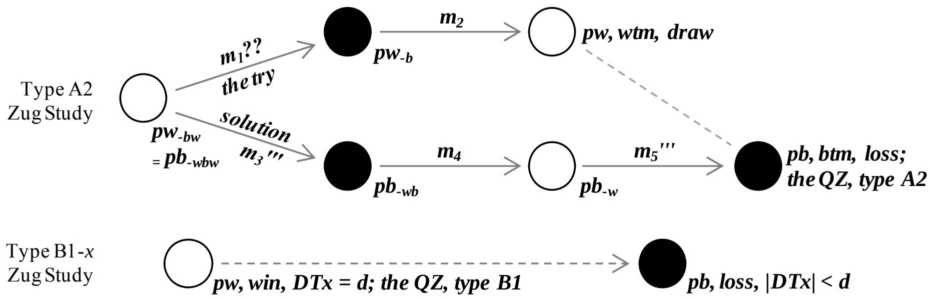

Figure 2. Type A2 and Type B1-*x* Zugzwang Study scenarios.

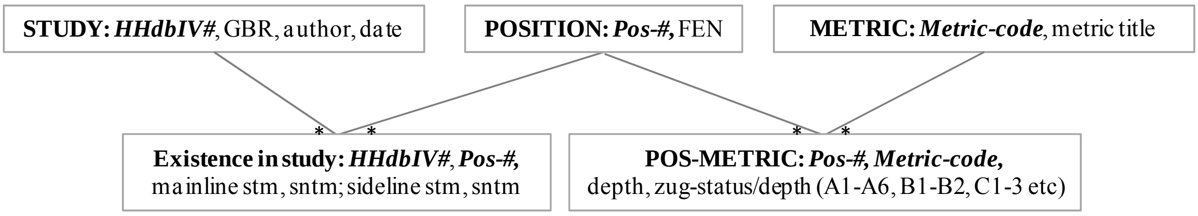

Figure 3. Data model showing how positions' and studies' zug status and zug-depth are related and defined.

Id	HHdbIV Index #	Mainline zug QZ	Force	Zug type	Study Zug depth	Notes
ZS1	17284	5k1K/7P/5P2/8/8/8/p7/8 b - 13	KPPKP	A1	c/m/z 1	Herbstman (1936); minDTC/M/Z-zd A1 zug study
ZS2	67333	k6b/3N3r/2K5/1N6/8/8/8/8 b - 3	KNNKRB	A1	m 225	Becker (2000); maxDTM-zd s7m A1 zug study
ZS3	374	8/8/8/3k1K2/4R3/8/6n1 b - 4	KRKN	A2	m 17	Ar Razi (850-); oldest zug st. (Beasley, 2000) q.v. Fig. 1ac
ZS4	65529	8/5p2/6k1/2p1P3/2P1K3/8/8/8 b - 2	KPPKPP	A2	m 19	Beasley (1998) q.v. (Beasley, 2000, Fig. 4)
ZS5	62486	k7/n1K5/8/8/b7/8/R7/8 b - 5	KRKBN	A2	m 28	Nunn (1994) q.v. (Beasley, 2000, Fig. 5 and 5a)
ZS6	66356	1r1k4/8/2Q5/8/8/8/1p6/1K6 b - 3	KQKRP	A2	m 27	Beasley (1999) q.v. (Beasley, 2000, Fig. 6)
ZS7	65538	8/8/8/5KPk/8/8/8/5N1n b - 3	KNPKN	A2	m 16	Beasley (1998) q.v. (Beasley, 2000, Fig. 7a)
ZS8	66360	8/8/8/1p6/1K6/N2N4/k7/8 b - 3	KNNKP	A2	m 4	Beasley (1999) q.v. (Beasley, 2000, Fig. 9)
ZS9	14336	6bk/4Np2/5K2/6N1/8/8/8/8 b - 9	KNNKBP	A2	c/m/z 1	Korolkov (1931); minDTC/M/Z-zd A2 zug study
ZS10	59216	N7/N7/8/2k4p/7K/8/8/8 b - 7	KNNKP	A2	m 102	Randviir (1991); nearly the maxDTM-zd s7m A2 zug study
ZS11	34545	8/8/4k3/7p/1pN4K/4N3/8/8 b - 4	KNNKPP	A2	m 103	Soukup Bardon (1965); maxDTM-zd s7m A2 zug study
ZS12	65419	8/p7/8/5pK1/4kP2/8/P7/8 b - 5	KPPKPP	A3	m 34	Dashkovsky (1998); Trébuchet representative
ZS13	40874	8/8/8/1pKp4/1P1P4/2k5/8 b - 9	KPPKPP	A3	m 31	Kralin (1973)
ZS14	44148	5kb1/5p2/5P1K/8/8/8/8/8 b - 4	KPKBP	A3	m 38	Pogosyants (1976)
ZS15	44536	8/2nKPp2/5k2/8/8/8/8/8 b - 8	KPKNP	A3	m 37	Yakimchik (1977)
ZS16	59964	8/8/8/2p5/K1P5/pB6/1k6 b - 7	KBPKPP	A3	m 31	Pervakov (1991)
ZS17	75166	8/8/3pPKn1/3k4/8/8/8/8 b - 8	KPKNP	A3	m 40	Katsnelson (2008)
ZS18	65276	5k2/8/4P1P1/8/8/1p1p4/8/2K5 b - 8	KPPKPP	A3	m 26	Khatyamov (1998); minDTM-zd s7m A3 zug study
ZS19	45748	8/8/8/3kpK2/1p6/1P2P3/8 b - 11	KPPKPP	A3	m 44	Zinar (1978); maxDTM-zd s7m A3 zug study
ZS20	16062	8/8/8/8/2N1K1kp/7N/8 w - 1	KNNKP	B1-C/M/Z	c/m/z 1	Troitzky (1934); minDTC/M/Z-zd B1 zug study; (~pos. 8b)
ZS21	19081	8/8/7K/5k2/6bb/4Q3/8/8 w - 1	KQKBB	B1/C/M/Z	c/z 14	Dedrle (1939); maxDTC/Z-zd s6m B1 zug study; (~pos. 6b)
ZS22	41474	7k/r3bQ2/8/8/8/4K3/8 w - 11	KQKRB	B1-C/M/Z	m 14	Dobrescu (1974); maxDTM-zd s6m B1 zug study; (~pos. 13b)
ZS23	30075	8/4p3/3bK3/8/8/5N2/8/5B1k w - 11	KBNKBP	B1-M	m 20	Joitsa (1958); maxDTM-zd s7m B1 zug study; (~pos. 15b)

Table 4. Selected zugzwang studies from HHDBIV.

Beasley (2000) defines a Zugzwang Study as one in which at least one zug position *p*, playing a larger part, appears both in the mainline and, later or in a sideline, with the other side to move. Here, the type and zug-depth of any such *qualifying zug(s)* QZ are associated with that zug study. In the A2 zug study of Figure 2, White wins by forcing Black into an A2 zug in the mainline, having avoided a published sideline in which it has to play from what is the drawing, 'second position' side of the zug. Table 4 gives more examples.

Figure 2 also illustrates a type B1-*x* zug study in which zug position *p* appears in the mainline, first won with wtm and then lost (more quickly) with btm. Studies may be found with *p* appearing, both wtm and btm, but fail to be type B1 zug studies because (a) *btm* appears before *wtm*, (b) *pb* is deeper than *pw* or (c) one or both of *pw* and *pb* are not wins for White. Some manual inspection of potential zug studies was necessary as none of these failure modes are automatically detectable by CQL.[15] A B2-*x* zug study, one whose mainline has losing position *pb* followed by a DT*x*-deeper winning position *pw* would suggest at least a minor dual in the study as White has regressed in DT*x* terms. No B2-M zug studies have been identified in HHDBIV.

In studies, White is challenged to win or draw, and the concept of *depth* is currently only associated with wins and losses. However, it is possible to define DTC/Z depths for some draws,[16] so type B3-*x* drawing zug studies are possible in principle.[17]

Figure 3 shows how the zug-depths of QZs in a study may be used to define the zug-depth of a study for a given metric. The minimum and maximum DT*x* zug depths of a zugzwang study derive from the DT*x* zug depth(s) of the qualifying zug(s) QZ: if there is only one QZ, these define a DT*x* zug depth for the study.

Table 4 lists some selected zug studies with their QZs. There are only 16 type A3 zug studies: two are in fact infeasible[18] and seven are Trébuchet-like and represented by ZS12. Using sub-6m DTC and DTZ EGTs, it was possible to identify type B1-C/Z zug studies where a QZ was s6m. Of the 180 such type B1-*x* zug studies, 163 are type B1-C, 156 are type B1-M and 151 are type B1-Z.

5. OBSERVATIONS ON SOME ZUGZWANGS

While chess has its roots in the scene of battle, it takes its *alternate move rule* from the world of board games. Figures 1ab (KPK) and 1ba (KRK) demonstrate that if the defender had the advantage of a null move, they could often avoid or delay mate. It would be interesting to compute EGTs for such a variant of chess.

The following notation is used to add compact comment on the moves:

° ≡ only physical move available, z(*d*, *type*) ≡ zugzwang position (DTM zug depth *d*, type),
', **"** and **"'** ≡ DTM-metric-optimal, only DTM-metric-optimal and unique, value-preserving move.

Study compositions should certainly entertain but often educate as well: the surprise and irony of the zugzwang does both. Ar Razi's composition demonstrates significant interest, remarkable over a thousand years ago, in the subtleties and abstract concepts of chess: the seeds of algebra were planted about the same time. It culminates in Fig.1ac (ZS3) and, as published, ends **4. ... Kc4' 5. Kg3' Kd4' 6. Kf2**, in fact another B2-C/Z zug. The artistic content of Polerio's 1590 study (Fig. 1bb) lies in the immediate sacrifice of the strongest piece on the board even if this key move is not difficult to find: **1. Ra1"' Kxa1 2. Kc2"'** z5(A2) **g5° 3. hxg5"' h4°** enforced pawn race **4. g6"' h3° 5. g7"' h2° 6. g8=Q"' h1=(Q/R/B/N)°** failing to prevent **7. Qg7#"'.**

Nunn (1994) notes that in a 'near terminal' KQKR position, White can usually force the Philidor position (Fig. 1aa), a Bi-C/M/Z zug with a zug depth of 3. Black to move concedes quickly: **1. ... Rb1" 2. Qd8+' Ka7° 3. Qd4+" (Ka8/Kb8)" 4. Qh8+" Ka7" 5. Qh7+".** The wtm win is slower as White has to transfer the move by a familiar five ply triangulation manoeuvre: **1. Qe5+' Ka8' 2. Qa1+' Kb8" 3. Qa5"** returning to the original position but with btm in a B2-C/M/Z zug.

Fig.1cb features a most frustrating if unlikely 5-1 zug, one of an increasingly bizarre sequence of similar *n*-1 zugs. The Knight is on the wrong foot, releasing its control of e7 at exactly the wrong moment. As can be explained by a parity argument,[19] the Knight is unable to transfer the move, even if it can go on a 58-zug tour of the available board as seems possible. Nunn (1995, §1.1) makes the same point about Fig.1ca's KNP(a7)K position before analysing KNPKN and contributing over forty studies in that endgame.

In Fig.4a, White wins with the move or not: the key to winning is to have the right parity by playing **1. ... a4"'** or **1. a3"'**, the latter an example of the chess study theme *festina lente*. Thus, **1. a4?? Nf3"'** z27(A1) **2. a5 Nd4"'** z26(A1) **3. a6 Nb5"'** z25(A1) **4. Kb7 Nd6+"'** =, while **1. a3"'** z28(A2) **Nf3' 2. a4"'** z27(A2) **Nd4' 3. a5"'** z26(A2) **Nb5' 4. a6"** z25(A2) **Kd6'** and White wins in nine moves. In Fig.4b, btm is a draw but wtm wins via a sequence of zugs: **4. Kh6"'** and then **4. ... Ne7' 5. Nd6"'** z27(A2) **Ng6" 6. Nf5"** z26(A2) **Kf7" 7. Nd4"** z25(A2) **Ne7" 8. Kh7"'** or **4. ... Kf7' 5. Nd6+"' Kg8" 6. Nf5"** z26(A2) **Kf7" 7. Nd4"** z25(A2) **Nf8.**

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[15] Comparison of *pw's* and *pb's* DT*x* revealed that White had not played DT*x*-optimally in some 100 B1-*x* zug studies: Some 70 of these have previously unnoticed *duals* whose artistic significance could now be assessed. Example: HHDBIV#16062, Troitzky (1934), 8/8/8/8/4K2p/4N1kN/8 w: 2.Nd4" rather than 2.Ke4

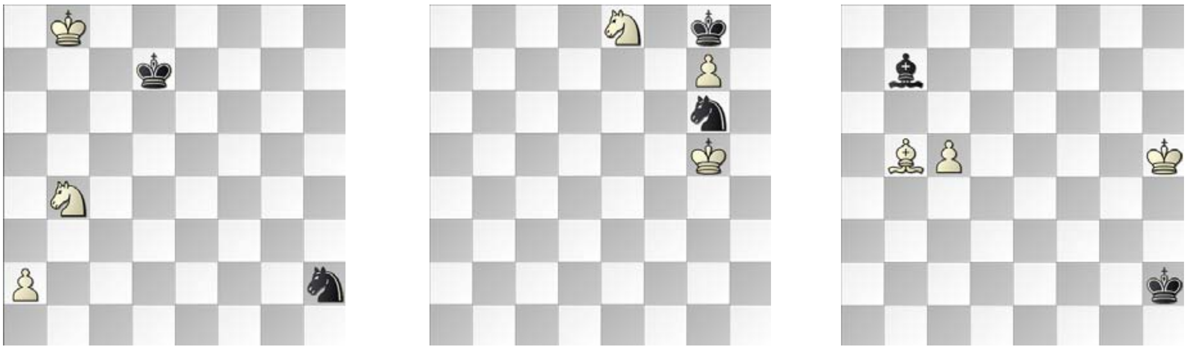
[16] Assuming players' motivations, e.g., defender seeks the DTC/Z draw-goal, attacker seeks to avoid attaining it.

[17] e.g., HHDBIV#18, 8/8/8/8/3k3p/7K/5p2/8 w (DTC/Z=5 plies); HHDBIV#73, 8/8/2n5/8/1pK5/p7/8/k1B5 w (DTC/Z=1 ply) **1. Bxa3"'**; HHDBIV#8064, 7K/8/k1P5/7p/8/8/8/8 w (DTC=11 plies, DTZ=7 plies).

[18] HHDBIV#38856 (Marysko, 1970) and #68444 (Blundell, 2001).

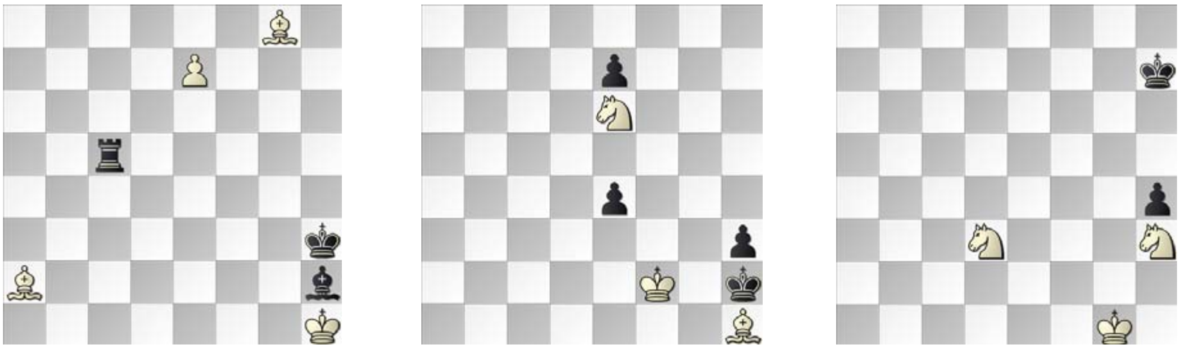
[19] Elkies (1996) also exploits the concept of parity in a sequence of ingenious, didactic Pawn endgames.

In connection with Fig.4c, it is worth noting that the btm positions with Bb5Pc5/bb7 and any of Kh4/kh2, Kg4/kg2, Kg3/kg1, Kf4/kf2 or Kh3/kh1 are type A2 zugs. After **1. Kh4"'** z44(A2), Nunn's study continues **1. ... Kg2 2. Kg4"'** z37(A2) **Kf2' 3. Kf4"'** z36(A2) **Bg2' 4. Ke5' Ke3" 5. Bd7"' Kd3' 6. Be6"' Kc3" 7. Bd5"'** winning after **7. ... Kb4' 8. c6' Bf1" 9. Kd6"' Ba6" 10. Bg2' Ka5" 11. Kc5"' Bc8' 12. Bf1"' Bg4' 13. c7' Bf5" 14. Bc4".** Alternative sidelines are **1. ... Bc8" 2. Bc6"'** z43(A2), **1. ... Bf3 2. Ba4' Kg2" 3. Kg5"' Kg3" 4. Kf5"' Kf2" 5. Kf4"' Bb7" 6. Bb5"'** and Black is in the Kf4/kf2 zug trap again, or **1. ...Kh1 2. Kg5 Kg1' 3. Kf5"' Kf2" 4. Kf4"'** z36(A2). White walks the unique path to goal as befits a chess study, in this case setting and avoiding myriad zug traps along the way.



a) HHDBIV#66134: Nunn (1995) #37 | b) HHDBIV#31369: Olmutsky (1960) position 4w; (Nunn, 1995) #42 | c) HHDBIV#63231: Nunn (1995) #272

Figure 4. Further studies featuring zugzwang positions.



a) HHDBIV#62681: 'draw', Kasparyan (1995), 23 A1 zugs | b) HHDBIV#28659: 'win', O'Donovan (1956), 11 A2 zugs | c) HHDBIV#3292: 'win', Meyer (1891), 52 zugs

Figure 5. Some studies featuring record numbers of zugs of a type.

Finally, we identify those studies in HHDBIV whose mainlines feature the most s7m zugzwangs of each type. Combinations of Knights and Pawns dominate the records as expected but do not actually monopolise. HHDBIV#62681 is a Draw Study by Kasparyan (1995) with 23 Type A1 zugs: Black is repeatedly one null move away from winning. Draw studies should not feature zugs of types A2, A3, B1 and B2 and Win studies should not feature zugs of type A1 so we ignore those that do appear. Four win studies feature a maximum of 11 type A2 zugs: HHDBIV#17618 by Troitzky (1937), #28659 by O'Donovan (1956), #34913 by Missiaen (1965) and #63931 also by Missiaen (1996). HHDBIV#65419 by Dashkovsky (1998), involving a Trébuchet position and facing pawns, has three type A3 zugs. HHDBIV#3292 (position published by Meyer (1891) as 'mate in 79', correct EGT-derived solution published in HHDBII#2822) features 52 zugs in all.

HHDBIV#62681, Fig. 5a, EG#10355, Kasparyan (1995), KBBPKRB, 6B1/4P3/8/2r5/8/7k/B6b/7K w:
1. Bae6+ Kg3"' 2. Bc4"' Ra5 3. Ba2 Rf5 4. Bac4"' Re5 5. e8=Q Rxe8"' 6. Bce6"' z8 **Rf8 7. Bef7"'** z8 **Ra8 8. Ba2"'** z8 **Ra3 9. Bgb3"'** z7 **Ra6 10. Be6"'** z7 **Ra4 11. Bec4"'** z6 **Ra5 12. Bd5"'** z6 **Ra7 13. Bf7"'** z9 **Rb7 14. Bab3"'** z9 **Rb5 15. Bfd5"'** z8 **Rb4 16. Bdc4"'** z7 **Rb6 17. Be6"'** z8 **Rb8 18. Bg8"'** z9 **Rc8 19. Bbc4"'** z9 **Rd8 20. Bcd5"'** z8 **Rd7 21. Bgf7"'** z9 **Re7 22. Bde6"'** z9 **Rc7 23. Bc4"'** z10 **Rc6 24. Bfe6"'** z8 **Rc5 25. Bed5"'** z10 **Rc7 26. Bf7"'** z10 **Rd7 27. Bcd5"'** z9 **Rd6 28. Bfe6"'** z7 **=.**

HHDBIV#17618, Troitzky (1937), KNNKP, 1k6/8/5N2/3K4/2p5/8/2N5/8 w:
1. Kd6"' z33 **c3 2. Nd5"'** z25 **Kc8' 3. Ke7"'** z1(B2-M) **Kb8" 4. Kd8"** z23 **Kb7" 5. Kd7"'** z22 **Ka7" 6. Kc7"' Ka6" 7. Kc6"'** z20 **Ka7" 8. Ne7"** z19 **Ka6" 9. Nc8"** z18 **Ka5° 10. Nb6"'** z17 **Ka6° 11. Nc4"** z16 **Ka7° 12. Nd6" Ka6" 13. Nb7"**

z14 **Ka7° 14. Nc5" Kb8"** z1(B1-M) **15. Kd7"'** z1(B2-M) **Ka7" 16. Kc7" Ka8°** z1(B1-M) **17. Kb6" Kb8° 18. Nb7" Kc8" 19. Kc6"'** z8 **Kb8° 20. Nd6" Ka7" 21. Kb5"'** z1(B2-M) **Kb8" 22. Kb6" Ka8°** z1(B1-M) **23. Kc7"** z1(B2-M) **Ka7° 24. Nb4" c2' 25. Nc8+"' Ka8° 26. Nc6" c1=Q' 27. Nb6#"'.**

HHDBIV#28659, Fig. 5b, O'Donovan (1956), KBNKPPP, 8/4p3/4N3/8/4p3/7p/5K1k/7B w:
1. Bf3"' exf3' 2. Ng5' z16 **Kh1" 3. Ne4"** z15 **Kh2" 4. Ng3'** z14 **e5"** z4(B1-M/Z) **5. Ne4' Kh1°** z4(B1-M) **6. Nd2' Kh2"** z2(B1-M/Z) **7. Nxf3+" Kh1° 8. Nd2"'** z10 **Kh2" 9. Ne4"** z9 **Kh1° 10. Nf6" z8 Kh2" 11. Ng4+" Kh1° 12. Kf1"** z6 **e4" 13. Nf2+"' Kh2° 14. Nxe4"'** z4 **Kh1° 15. Kf2"** z3 **Kh2" 16. Nd2"** z2 **Kh1° 17. Nf1"** z1 **h2° 18. Ng3#"'.**

HHDBIV#65419, Table 4 ZS12, Dashkovsky (1998), KPPKPP, 8/pk6/8/5p2/5P2/8/P4K2/8 w:
1. Kg3"' Kc6' 2. Kh4"' Kd5' 3. Kh5"' Kd4 4. Kg6"' Ke4' 5. Kg5"' z34(A3) **a6' 6. a3"'** z32(A3) **a5' 7. a4"'** z30(A3)**.**

HHDBIV#3292, Fig. 5c, Meyer (1891), KNNKP, 8/7k/8/8/7p/3N3N/8/6K1 w, see also (Berger, 1922),
18 wtm B1, 31 btm B2 and 3 btm A2 zugs, the latter being positions 26b, 38b and 81b:
1. Kf2" Kg6' 2. Ke3" Kf5' 3. Kd4" Kg4' 4. Ndf2+" Kf5" 5. Kd5" Kf6" z1 **6. Ne4+" Ke7"** z38 **7. Kc6" Ke6" 8. Neg5+" Ke5' 9. Kc5"** z7 **Kf6" 10. Kd6'** z1 **Kf5" 11. Kd5"** z1 **Kf6" 12. Ke4"** z1 **Ke7" 13. Ke5"** z9 **Kd7" 14. Kd5"** z9 **Kc7" 15. Nf7'** z11 **Kb6" 16. Nd6"** z9 **Ka5" 17. Kc5" Ka4" 18. Ne4" Kb3" 19. Kb5"** z2 **Kb2" 20. Kb4"** z7 **Kc2' 21. Kc4"** z7 **Kb2' 22. Nd2"** z5 **Ka3' 23. Nf1"** z7 **Kb2" 24. Nf2" Ka3" 25. Kb5"** z9 **Kb3" 26. Ne4"** z63(A2, DTZ=57) **Kc2' 27. Kc4" Kb2"** z3 **28. Nc5' Kc2" 29. Nh2" Kd2" 30. Kd4"'** z2 **Ke2' 31. Ne4"** z5 **h3"** z9 **32. Ng5" Kd2" 33. Ngf3+" Kc2" 34. Kc4" Kb2" 35. Nd4" Ka2"** z5 **36. Kc3" Ka3" 37. Nb3" Ka4' 38. Kc4"'** z51(A2) **Ka3° 39. Nc5" Kb2' 40. Kd3" Kb1" 41. Kc3" Kc1"** z1 **42. Ne4' Kd1' 43. Nf2+' Ke2' 44. Nfg4"' Kd1"** z2 **45. Ne3+" Kc1"** z5 **46. Kc4" Kb2" 47. Kb4"** z21 **Ka1" 48. Ka3'** z5 **Kb1° 49. Kb3"** z5 **Kc1" 50. Kc3'** z5 **Kb1° 51. Nc4"** z2 **Ka2" 52. Kc2"** z4 **Ka1° 53. Kb3"** z5 **Kb1° 54. Nd2+" Kc1" 55. Kc3"** z12 **Kd1° 56. Nb3" Ke1" 57. Kd4" Ke2"** z1 **58. Ke4"** z1 **Ke1" 59. Ke3"** z3 **Kd1° 60. Kd3"** z5 **Ke1° 61. Nd4' Kd1" 62. Ne2"** z16 **Ke1° 63. Nc3"** z1 **Kf2° 64. Kd2" Kg2" 65. Ke3' Kg3"** z1 **66. Ne2+"' Kg2"** z16 **67. Nd4" Kg3"** z1 **68. Ndf3" Kg2°** z1 **69. Nd2" Kg3"** z1 **70. Ndf1+" Kh4" 71. Kf4"' Kh5° 72. Kf5"' Kh6" 73. Kf6"' Kh5' 74. Ne3" Kh6"** z1 **75. Neg4+" Kh7" 76. Kf7" Kh8°** z9 **77. Ne3' Kh7° 78. Nf5" Kh8°** z7 **79. Kg6" Kg8° 80. Ng7" Kf8" 81. Kf6"'** z8(A2) **Kg8° 82. Ne6" Kh7" 83. Kg5"** z1 **Kg8" 84. Kg6" Kh8°** z1 **85. Kf7"** z1 **Kh7° 86. Ng4" h2' 87. Nf8+' Kh8° 88. Nf6' h1=Q' 89. Ng6#"'.**

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References

Beasley, J. (2000). Creating reciprocal zugzwang studies. EBUR Vol. 12, No. 2, pp. 8-12.

Berger, J. (19222). *Theorie und Praxis der Endspeil*. 2nd edition, esp. positions #398a and #401b.

Bleicher, E. (2011). http://www.k4it.de/ Sub-7-man chess EGT-query service.

Bleicher, E. and Haworth, G.McC. (2009). 6-man Chess and Zugzwangs. *Advances in Computer Games* (ed. H. Jaap van den Herik and P. Spronck), pp. 123-135. Proceedings of ACG12, Pamplona. Lecture Notes in Computer Science, 6048. ISSN 0302-9743. Springer-Verlag.

Bleicher, E., Haworth, G.McC. and van der Heijden, H.M.J.F. (2010). Data-mining chess databases. ICGA Journal, Vol. 33, No. 4, pp. 212-214.

Costeff, G. (2011). *EG*, Issues 1-152, pdf format. http://gadycosteff.com/eg/eg.html.

Elkies, N.D., (1996). On Numbers and Endgames: Combinatorial Game Theory in Chess Endgames. *Games of No Chance* (ed. R.J. Nowakowski), pp. 135-150. Proceedings of the MSRI conference on Combinatorial Games. CUP, ISBN 0-521-64652-9. See also http://library.msri.org/books/Book29/files/elkies.pdf.

Haworth, G.McC. (2003). Reference Fallible Endgame Play. *ICGA Journal*, Vol. 26, No. 2, pp. 81-91.

Haworth, G. McC. (2009). The Scorched Earth Algorithm: presentation to the Chess Endgame Study Circle.

Haworth, G. McC., Bleicher, E. and van der Heijden, H.M.J.F. (2011). Uniqueness in chess studies. ICGA Journal, Vol. 34, No. 1, pp. 22-24.

Haworth, G. McC., Karrer, P., Tamplin, J.A. and Wirth, C. (2001). 3-5 Man Chess: Maximals and Mzugs. ICGA Journal, Vol. 24, No. 4, p. 225-230.

Nunn, J. (1992). *Secrets of Rook Endings*. B.T. Batsford, London. ISBN 0-7134-7164-6.

Nunn, J. (1994). *Secrets of Pawnless Endings*. B.T. Batsford, London. ISBN 0-7134-7508-0. Expanded Edition 2 including 6-man endgames (2002). Gambit. ISBN 1-9019-8365-X.

Nunn, J. (1995). *Secrets of Minor-Piece Endings*. B.T. Batsford, London. ISBN 0-7134-7727-X.

Tamplin, J. and Haworth, G.McC. (2001). Ken Thompson's 6-man Tables. ICGA Journal, Vol. 24, No. 2, pp. 83-85.

van der Heijden, H.M.J.F. (2010). http://www.hhdbiv.nl/. HHDBIV, ENDGAME STUDY DATABASE IV.

Valois, P. (2011). Index to the Endgame Studies magazine, *EG*. http://www.arves.org/egindex.txt.