Identification of Space Curves from Two-Dimensional Perspective Views

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ABSTRACT

This paper describes a new method to be used in the recognition of three-dimensional objects with curved surfaces from two-dimensional perspective views. The method requires for each three-dimensinal object a stored model consisting of a closed space curve representing some characteristic connected curved edges of the object. The input is a two-dimensional perspective projection of one of the stored models represented by an ordered sequence of points. The input is converted to a spline representation which is sampled at equal intervals to derive a curvature function. The Fourier transform of the curvature function is used to represent the shape. The actual matching is reduced to a minimization problem which is handled by the Levenberg-Marquardt algorithm [3].

index terms: scene analysis, three-dimemsional objects, space curves, perspective projections, spline fitting, Fourier tranform, minimization

I. Introduction

Recognizing three-dimensional objects from two-dimensional perspective projections is an important current problem in scene analysis. The approaches to this problem can be divided into two major categories: 1) determine a number of characteristic views of the object and match the two-dimensional perspective projection against a database of two-dimensional views, and 2) match the two-dimensional perspective projection directly against a database of three-dimensional models. We are concerned only with approach 2). The method of matching depends on the representation of the three-dimensional model. The work on three-dimensional modeling has been extensive and has recently been reported at the Workshop on Representation of Three-Dimensional Objects sponsored by the National Science Foundation [23]. Some of the major representations include surface representations (points, polygons, or surface patches) and volume representations (polyhedra, ellipsoids, generalized cylinders, spheres). For a thorough survey of these representations, see Badler and Bajcsy [2].

After standard segmentation procedures are applied to an image, the result is a partition of the image into regions of nearly uniform gray-level. Although in very simple images these regions may each correspond to an object, it is safer to assume that several adjacent regions together correspond to a single object. If certain assumptions about the structure of the three-dimensional objects are made, it is possible to label the edges between pairs of regions as interior edges or boundary edges and thus to determine possible boundaries of objects. See Huffman [13], Clowes [5], Waltz [22], and more recently Kanade [14,15], among others.

In this paper, we will assume that we are given the exterior boundary and some interior edges of the two-dimensional perspective projection of a three-dimensional object. We will further assume that we have a database of three-dimensional object models, and that this database contains a model of each object that may appear in the scene. While our matching techniques are independent of size, position, and view, the three-dimensional models are exact in that a chair without arms will not match a model of an armchair. In particular, these models each consist of a set of space curves representing important characteristic edges of the three-dimensional object. Figure 1 illustrates some of these characteristic edges for a group of chair models. Note that although shown in 2D, these characteristic edges are <u>three-dimensional</u> curves.

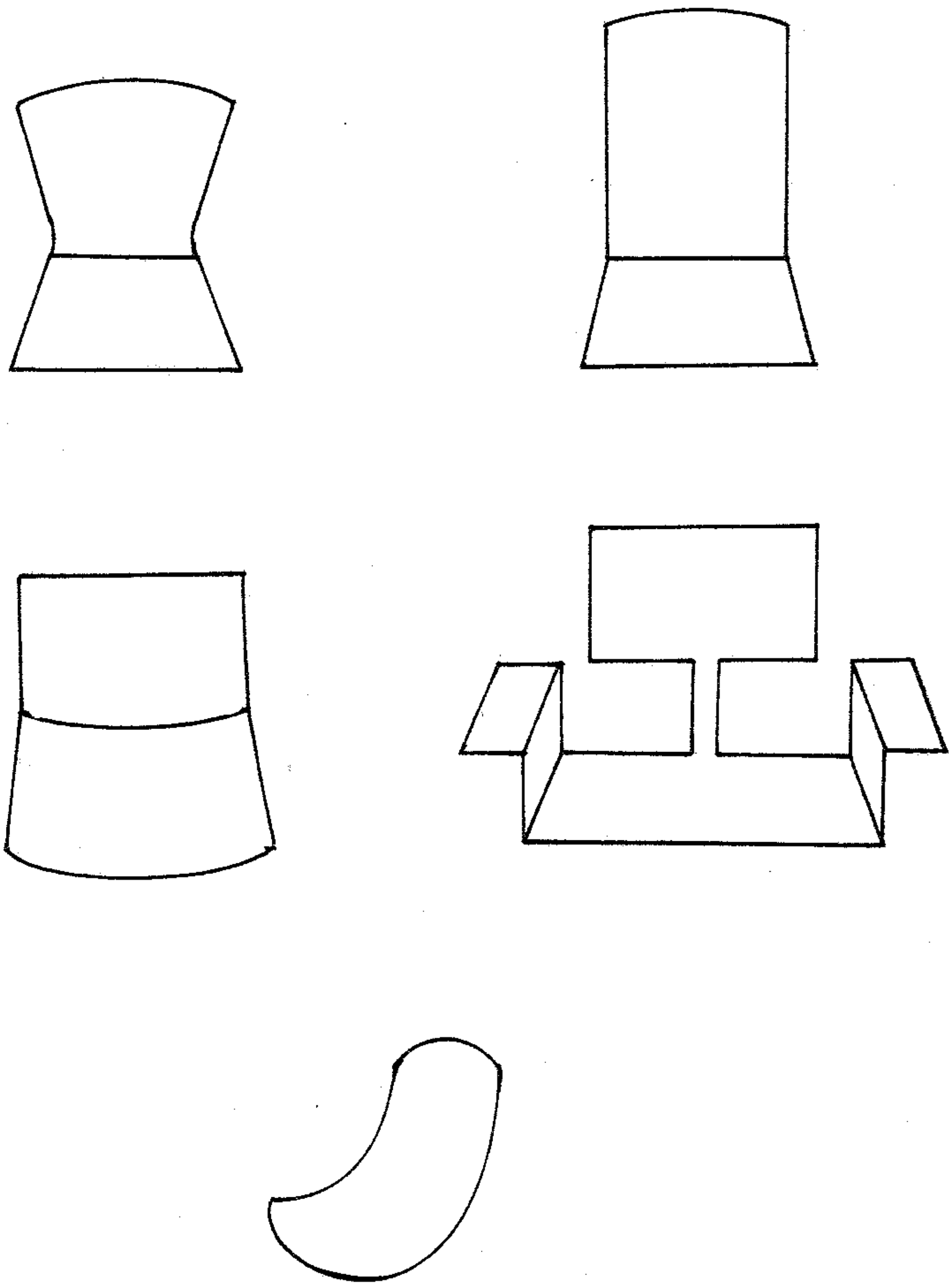

Figure 1 - Five chairs.

The problem we tackle in this paper is: given a set of n space curves representing characteristic edges of n three-dimensional objects, and given a two-dimensional curve which is part of the perspective projection of one of the n objects, determine which object is most likely to have produced the two-dimensional curve. The technique developed here is intended to be used as part of an integrated scene analysis system and not as a stand alone technique. We propose that this algorithm be used when the number of possible three-dimensional models has already been reduced to a small set by gross matching techniques and it is now desired to perform higher resolution matching in order to determine the exact object in the scene.

II. Related Literature

Our approach to the problem, which is given in Sections III, IV, V, and VI uses several well-known mathematical techniques. First, we will approximate the two-dimensional curves (initially given as ordered sets of points) by periodic cubic splines. For a comprehensive treatment of splines, see deBoor [7]. Second, in the process of finding the best three-dimensional curve for a given two-dimensional curve, we will require a rotation-invariant technique for comparing two two-dimensional curves. We have chosen to use the Fourier domain for this comparison and will dicuss a number of related papers in this section. Finally, we have reduced the matching problem to a minimization problem and will be using a relatively new technique, the Levenberg-Marquardt algorithm [3], to achieve the minimization.

One of the early and successul attempts to use Fourier descriptors in matching plane closed curves is the work of Zahn and Roskies [24]. They represent a curve by a function 0 that gives cumulative change in direction as a function of arc length. The curve is described by the coefficients of the polar Fourier series expansion of ϕ, which limits the applicability of this method since ϕ is periodic only for simple closed curves. The Fourier expansion is given by

$$\phi^*(t) = u_0 + \sum_{k=1}^{\infty} A_k \cos(kt-a_k)$$

where the pairs (A_k, a_k) are used to describe the curve. Features constructed from the first few pairs of coefficients can then be used in recognition algorithms. Granlund [10] also uses features constructed from (a different set of) Fourier descriptors to characterize two-dimensional curves for hand-printed character recognition. Carl and Hall [4] use low-pass filtered Fourier or Walsh spectral components to describe alphabet data in their character recognition experiments.

Persoon and Fu [18] also look at the problem of using Fourier descriptors to represent two-dimensional boundary curves. They propose the distance measure

$$d(\alpha,\beta) = \left[\sum_{\substack{n=-M \\ n\neq 0}}^{M} |a_n - b_n|^2\right]^{\frac{1}{2}}$$

where α and β are two planar curves, $\{a_n\}$ are the complex Fourier descriptors of α to be used and $\{b_n\}$ are the complex Fourier descriptors of β to be used. If β is the curve in the training set and α the unknown curve, then in matching, values of the scale factor s, rotation angle $\emptyset$, and starting point A must be determined so that

$$\sum_{\substack{n=-M \\ n\neq 0}}^{M} |a_n - se^{i(nA+\emptyset)}b_n|^2$$

is minimized. Persoon and Fu reduce this problem to that of numerically finding the roots of a periodic function. The method was tested on characters and on top views of machine parts.

Richard and Hemami [19] discuss a method for identifying three-dimensional objects (aircraft) using Fourier descriptors of their silhouettes. Like Persoon and Fu, they use a complex parametric representation of the two-dimensional curves, expand it to a Fourier series with complex coefficients, and define a distance measure based on the Fourier coefficients. Solid objects are represented by wire frame models. For a discrete set of orientations and positions in space, each model is projected onto the image plane, the boundary points traced, and 39 low-order Fourier coefficients stored for each view. The views are then matched against an unknown silhouette. In a related paper (Hemami, Weimer, and Advani [11]), a sequential matching algorithm for identifying silhouettes of three-dimensional objects is described. The algorithm tries to determine the best vector P of six translation and rotation parameters that produced a given view from a three-dimensional object. An initial guess of P starts the process. Then partial derivatives of an error function E with respect to each of the six parameters are computed. The parameters are iteratively changed to reduce E. Uniform random search is used to rule out local minima and to generate initial values. The authors comment that the method works well, but requires substantive computation.

Pattern recognition of shapes using moments is another area related to our work. Hu [12] and Alt [1] have presented methods for recognition of two-dimensional shapes using moment invariants. Sadjadi and Hall [20] have extended the work to three dimensions. They have developed a set of three-dimensional moment invariants which are invariant under size, orientation, and position change. They experimentally verified that these were true invariants using several solid objects: a rectangular solid, a cylinder, and a pyramid. For each object, a discrete set of three-dimensional coordinates was obtained for a small number of different positions (nine positions for the rectangular solid and four positions for the others). In each case, the values computed for the three-dimensional invariants were close to constant for a given object. They did not give any algorithms for actually recognizing the three-dimensional objects, given two-dimensional views.

We would also like to refer the reader to the work of Dirilten and Newman [8] on pattern matching under affine transformations by developing a set of invariants under the group of orthogonal transformations, and to the recent work of Freeman [9] using incremental curvature for describing two-dimensional shapes.

III. Two-Dimensional (closed) Curve Matching

Conceptually, we are given a smooth closed planar curve γ which is a perspective projection of one of the given 3-dimensional space curves. In practice, only a finite set of points $\{(X_i,Y_i)\}_{i=1}^{N}$ on γ is given. The number and placement of these points obviously depend on the fineness of the discrimination required to distinguish between the given space curves. Assume that the points (X_i,Y_i) used constitute an adequate representation of the smooth closed planar curve γ, and that γ is C^2.

Let $(X_0,Y_0) = (X_N,Y_N)$, $s_0 = 0$, and

$$s_k = \sum_{i=0}^{k-1} \left[(X_{i+1}-X_i)^2 + (Y_{i+1}-Y_i)^2\right]^{\frac{1}{2}}, \quad k = 1,2,\ldots,N.$$

Let $\sigma(t)$, $\tau(t)$ be the unique periodic cubic splines interpolating the given data points, i.e.,

$$\sigma(s_i) = X_i, \quad \tau(s_i) = Y_i, \quad i = 0,1,\ldots,N$$

Periodic cubic splines are C^2 periodic piecewise cubic polynomials, uniquely determined by the above interpolation conditions, and are efficiently and accurately calculated as a linear combination of B-splines [7]. The parametric equations

$$x = \sigma(t), \quad y = \tau(t), \quad 0 \leq t \leq s_N \tag{3.1}$$

define a smooth closed curve Γ in the plane, intended as a sufficiently good approximation to γ. (This spline curve approaches γ as $\max_i(s_{i+1}-s_i)$ --> 0.) The curvature of Γ is given by

$$\tilde{\phi}(t) = \frac{d\theta}{ds} = \frac{d\theta}{dt} \Bigg/ \frac{ds}{dt} = \frac{\sigma'\tau'' - \tau'\sigma''}{(\sigma'^2 + \tau'^2)^{3/2}} \tag{3.2}$$

where θ is the tangent angle and s is arc length (note that the variable t is not arc length along Γ, but approaches the arc length as N --> ∞). Note that the normalized curvature function

$$\phi(\xi) = \frac{s_N}{2\pi}\tilde{\phi}(s_N\xi/2\pi)$$

is

1) continuous and 2π-periodic;
2) invariant under rotation of Γ;
3) invariant under translation of Γ;
4) invariant under dilation of Γ.

Now consider two closed (spline) curves Γ and Δ in the plane, with normalized curvature functions $\phi(\xi)$ and $\psi(\xi)$ respectively. If Δ is a rotation, translation, and dilation of Γ, then

$$\psi(\xi) = \pm\phi(a \pm \xi) \tag{3.3}$$

for some a, $0 \leq a \leq 2\pi$. (The minus sign occurs if Γ and Δ have opposite orientation. For other than closed Jordan curves, the orientation is nontrivial to determine.) Let c_n, $\tilde{c}_n$ denote the (complex) Fourier coefficients of ϕ, ψ, respectively. If (3.3) holds with the plus sign, then

$$\tilde{c}_n = \frac{1}{2\pi}\int_0^{2\pi} \psi(\xi)e^{-in\xi}d\xi = \frac{1}{2\pi}\int_0^{2\pi} \phi(\xi+a)e^{-in\xi}d\xi = e^{ina}\frac{1}{2\pi}\int_0^{2\pi}\phi(u)e^{-inu}du = e^{ina}c_n. \tag{3.4}$$

If (3.3) holds with the minus sign, then

$$\tilde{c}_n = e^{-ina}c_{-n} = e^{-ina}\bar{c}_n. \tag{3.5}$$

Thus if Γ and Δ are really the same curve, the Fourier coefficients of their normalized curvature functions are simply related by (3.4) or (3.5). This leads naturally to the distance measure

$$\omega(\Gamma,\Delta) = \min\{\inf_a \sum_{n=-\infty}^{\infty} |\tilde{c}_n - e^{ina}c_n|^2, \inf_a \sum_{n=-\infty}^{\infty} |\tilde{c}_n - e^{-ina}\bar{c}_n|^2\} \tag{3.6}$$

for any two smooth closed planar curves Γ and Δ, where c_n, $\tilde{c}_n$ are the Fourier coefficients of the curvature functions ϕ, ψ, of Γ, Δ, respectively. Note that $\omega(\Gamma,\Delta) = 0$ if and only if Δ is a rotation, translation, and dilation of Γ. ω is a pseudo-metric (a metric except that $\omega(\Gamma,\Delta) = 0$ does not imply $\Gamma = \Delta$) on the set of C^2 closed curves in the plane, and a metric on equivalence classes of curves (where two curves are equivalent if one is a translation, rotation, and dilation of the other).

In practice the Fourier coefficients c_n are approximated by a discrete Fourier transform, and $\omega(\Gamma,\Delta)$ is calculated as

$$\Omega(\Gamma,\Delta) = \min\{\min_a ||(\tilde{c}_n)-(e^{ina}c_n)||^2,$$

$$\min_a ||(\tilde{c}_n)-(e^{-ina}\bar{\tilde{c}}_n)||^2\}, \tag{3.7}$$

where (x_n) denotes a vector with nth component x_n. The one-dimensional minimizations with respect to a pose no theoretical difficulties, but are very expensive computationally. Appendix 1 outlines an alternative to (3.7) which is much more efficient computationally, but less mathematically rigorous and elegant than (3.7).

IV. Two-Dimensional (open) Curve Matching

The main difficulty in matching closed curves is obtaining a functional representation of the curve which does not depend on a "starting point". This difficulty disappears if the curve has end points, because the end points constitute the only logical choices for the "starting point". If the 3-dimensional space curve has end points, then with probability one a 2-dimensional perspective view of it also has end points. In other words, perspective projections preserve end points, which can therefore be used as uniquely determined "starting points" in a functional representation of the curve. The phrase "probability one " means that the set of open space curves whose perspective projections are closed planar curves is nowhere dense in the set of all open space curves. Equivalently, with probability zero an open curve will project to a closed curve.

For the case of open curves, the entire discussion in Section 3 applies with the following exceptions: the interpolating cubic splines $\sigma(t)$, $\tau(t)$ in (3.1) are not periodic. The end conditions recommended by deBoor [7] are used. The curvature function $\phi(\xi)$ is not periodic. (3.3) is replaced by

$$\psi(\xi) = \phi(\xi) \text{ or } -\phi(2\pi-\xi). \tag{4.1}$$

(3.4) is replaced by

$$\tilde{c}_n = c_n. \tag{4.2}$$

(3.5) is replaced by

$$\tilde{c}_n = \bar{c}_n. \qquad (4.3)$$

(3.7) is replaced by

$$\Omega(\Gamma,\Delta) = \min\{||(\tilde{c}_n) - (c_n)||^2 , ||(\tilde{c}_n) - (\bar{c}_n)||^2\} \qquad (4.4)$$

which is trivial compared to (3.7).

V. Generating Projections of Rigid Motions

The space curves, one of which produced the given 2-dimensional perspective view, are specified in some standard position. The most difficult part of identifying the space curve is that the given 2-dimensional curve is a perspective view of the space curve, rotated and translated from its (known) standard position. If the rotations and translations were known, identification would simply consist of computing perspective projections of all the space curves and then doing 2-dimensional curve matching. Computing the unknown rotations and translations is discussed in Section 6, but first an arbitrary rigid motion must be characterized.

A rigid movement in 3-dimensional space (finite composition of translations and rotations) is given by

$$Rx + q \tag{5.1}$$

where R is a 3 x 3 orthogonal matrix (producing a single rotation about some axis) and q is a 3-vector (producing a single translation). This fact follows from the classical geometry of rigid symmetries [6]. Let

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix}, \quad B = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix},$$

$$C = \begin{pmatrix} \cos\nu & -\sin\nu & 0 \\ \sin\nu & \cos\nu & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Then

$$R = B\,A\,C\,A^t\,B^t \tag{5.2}$$

where BA is the transition matrix from an orthonormal basis ϕ_1, ϕ_2, ϕ_3 to the standard basis e_1, e_2, e_3, and C is a rotation around the axis direction ϕ_3.

Note that a rigid motion (5.1) is completely determined by six parameters -- α, β, ν and the vector q.

Assume that the space curve (in some convenient standard position) is given by (the vector)

$$v(\zeta) \quad , \quad 0 \leq \zeta \leq Z. \tag{5.3}$$

A rigid motion then produces the space curve

$$\tilde{v}(\zeta) = Rv(\zeta) + q, \ 0 \leq \zeta \leq Z. \tag{5.4}$$

Take the origin as the center of perspectivity (focal point), and project $\tilde{v}(\zeta)$ onto the plane $z = 1$. This produces the planar curve γ, which is perspective with (corresponds to) the space curve $\tilde{v}(\zeta)$. γ is given parametrically by

$$x = \tilde{v}_1(\zeta)/\tilde{v}_3(\zeta), \ y = \tilde{v}_2(\zeta)/\tilde{v}_3(\zeta), \ 0 \leq \zeta \leq Z.$$

Approximating γ by a cubic interpolatory spline curve Γ leads to the normalized curvature function $\phi(\xi)$, $0 \leq \xi \leq 2\pi$. Note that $\phi(\xi)$ is completely determined by the parameters α, β, ν, and q of (5.1) and (5.2): α, β, ν, q determine the rigid movement $\tilde{v}$ in (5.4), $\tilde{v}$ determines its perspective projection γ, which determines the spline curve Γ (given by (3.1)) with normalized curvature function $\phi(\xi)$ (given by (3.2)).

It is now clear how curvature functions corresponding to perspective projections of arbitrary rigid motions of the given space curves can be generated.

VI. Nonlinear Least Squares Optimization

Let $\vartheta^{(1)}(\xi),\ldots,\vartheta^{(m)}(\xi)$ be the given space curves,

$$p = \begin{pmatrix} \alpha \\ \beta \\ \nu \\ q \end{pmatrix}$$

be the parameters in (5.1) and (5.2), and

$\phi(\xi;\ p,\ k)$

be the normalized curvature function representing the spline approximation $\Gamma(t;p,k)$ to the perspective projection of the rigid movement of $\vartheta^{(k)}(\xi)$ specified by p. Let $\psi(\xi)$ be the curvature function of the spline curve Δ approximating the given 2-dimensional perspective view δ. The problem then is to find a vector p and index k which minimize

$$\Omega(\Gamma(t;p,k),\Delta) \tag{6.1}$$

defined in (3.7). p may not be unique since dilations and rotations of Γ may correspond to translations and rotations of $\vartheta^{(k)}$, and $\Omega(\Gamma,\Delta)$ is computed modulo dilation and rotation. Also k may not be unique since several $\vartheta^{(k)}$ may produce the same view Δ. The point is to find <u>some</u> p and k which minimize $\Omega(\Gamma,\Delta)$, because then δ is identified as $\tilde{v}^{(k)}$ (assuming enough data points have been used to discriminate between the $\tilde{v}^{(i)}$).

Thus the overall problem is

$$\min_k \min_p \Omega(\Gamma(t;p,k),\Delta) =$$

$$\min_k \min_p \left[\min \left\{ \min_a \sum_{n=0}^{M} |\tilde{c}_n - e^{ina} c_n|^2, \min_a \sum_{n=0}^{M} |\tilde{c}_n - e^{-ina} \bar{c}_n|^2 \right\} \right] =$$

$$\min_k \min \left\{ \min_{a,p} \sum |\tilde{c}_n - e^{ina} c_n|^2, \min_{a,p} \sum |\tilde{c}_n - e^{-ina} \bar{c}_n|^2 \right\}. \quad (6.2)$$

The two outer minimizations are discrete and trivial. The inner minimizations

$$\min f(a,p) = \min \sum |\tilde{c}_n - e^{ina} c_n|^2, \quad (6.3)$$

$$\min g(a,p) = \min \sum |\tilde{c}_n - e^{-ina} \bar{c}_n|^2 \quad (6.4)$$

are highly nonlinear and very difficult. (6.3-4) are solved by the following iterative algorithm:

1) Choose starting values $a^{(0)}$, $p^{(0)}$ for the scalar a and vector p.

2) For $k = 0,1,2,\ldots$ until convergence do:

   3) With $p^{(k)}$ fixed, find the global minimum of $f(a,p^{(k)})$ and $g(a,p^{(k)})$ with respect to a, $0 \le a \le 2\pi$. Let $a^{(k+1)}$ be the point corresponding to the smaller of these two minima.

4) With $a = a^{(k+1)}$ fixed, minimize $f(a^{(k+1)},p)$ and $g(a^{(k+1)},p)$ with respect to p. Let $p^{(k+1)}$ be the point corresponding to the smaller of these two minima.

Step 3) is done by a simple one-dimensional search (as in [9]), and 4) is done by the Levenberg-Marquardt algorithm [3] developed at Argonne National Laboratory [17]. Note that (6.3) and (6.4) are in terms of the 2-norm in M-dimensional complex space. Since all norms are equivalent (induced topologies are identical) in finite dimensional spaces, the 2-norm can be replaced by the 4-norm, yielding

$$f(a,p) = \sum_{n=0}^{M} |\tilde{c}_n - e^{ina}c_n|^4, \tag{6.5}$$

$$g(a,p) = \sum_{n=0}^{M} |\tilde{c}_n - e^{-ina}c_n|^4. \tag{6.6}$$

This is necessary because of technical requirements in the Levenberg-Marquardt algorithm. (See Appendix 1 for an alternative to 6.5-6.6.)

VII. Computational Results

The data base of three-dimensional curves consisted of a characteristic edge curve from each of the five chairs in

Figure 1. These chairs were motivated by the collection of chairs illustrated in [16]. Spline fits to two-dimensional perspective views of these five space curves are shown in Figures 2-6. The experiment was to take each space curve, move it from its standard position, compute its perspective projection, and then match this two-dimensional curve against all five given space curves. On each trial, a starting point was chosen at random, and if the program reported a successful match, the process terminated. If the program reported failure due to converging to a local minimum, a new starting point was randomly chosen, and the process was repeated. This was done 50 times for each chair. The computer code used for the minimization was subroutine LMDIF1 from Argonne National Laboratory [17], and the spline codes are from deBoor [7].

The results of our experiments are shown in Table 1. The first number in row i, column j of Table 1 indicates the number of times that a perspective projection of chair i successfully matched the three-dimensional model of chair j. The second number (in parentheses) in row i, column j gives the number of times that, for some starting point, the minimization routine failed when trying to match a view of chair i to the model of chair j. As shown in the table, all the views of chairs 1 and 4 successfully matched chairs 1 and 4, respectively. Forty-eight out of fifty of the views

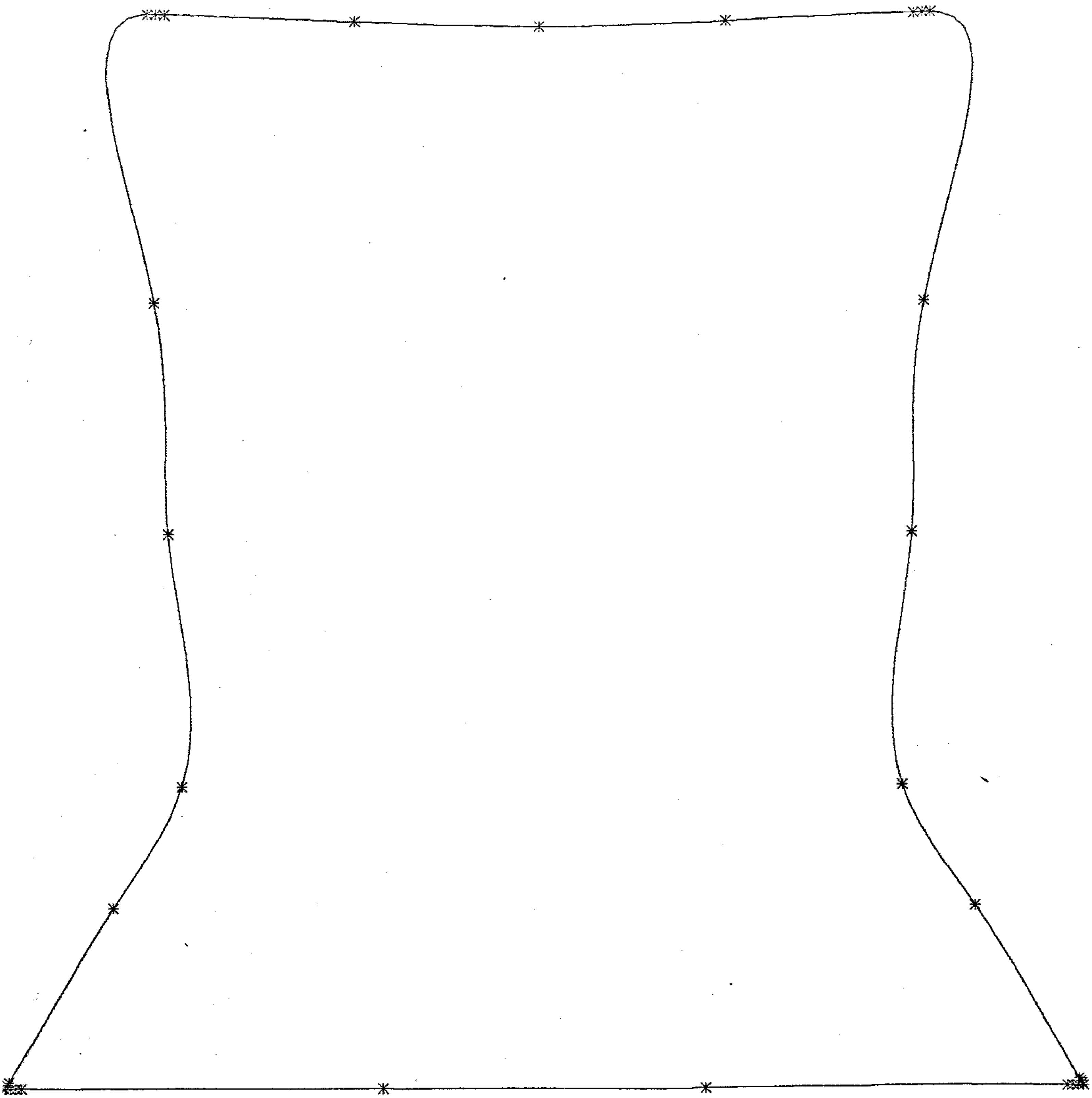

Figure 2 - Spline fit of characteristic
edges of chair 1.

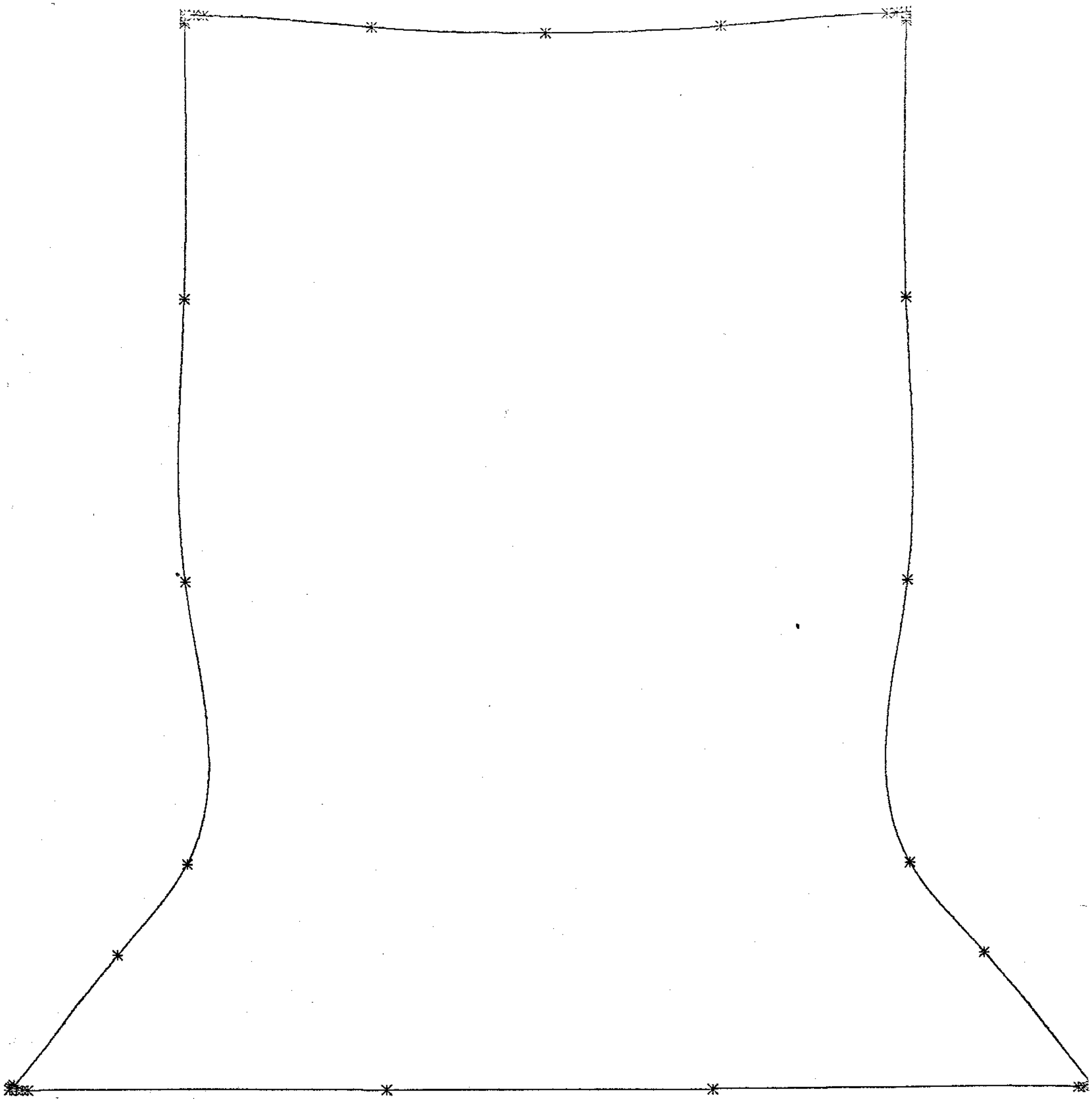

Figure 3 - Spline fit of characteristic
edges of chair 2.

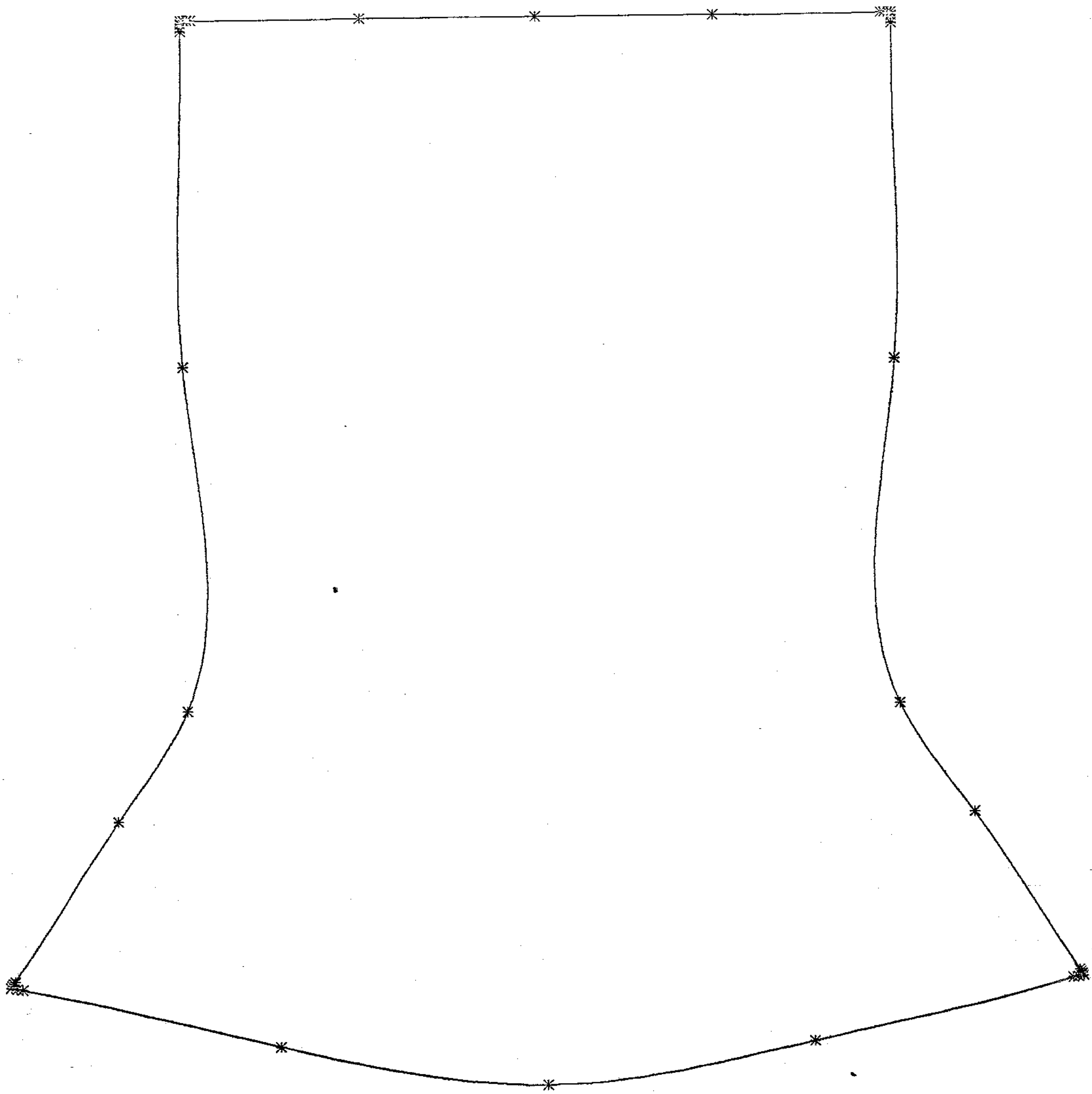

Figure 4 - Spline fit of characteristic edges of chair 3.

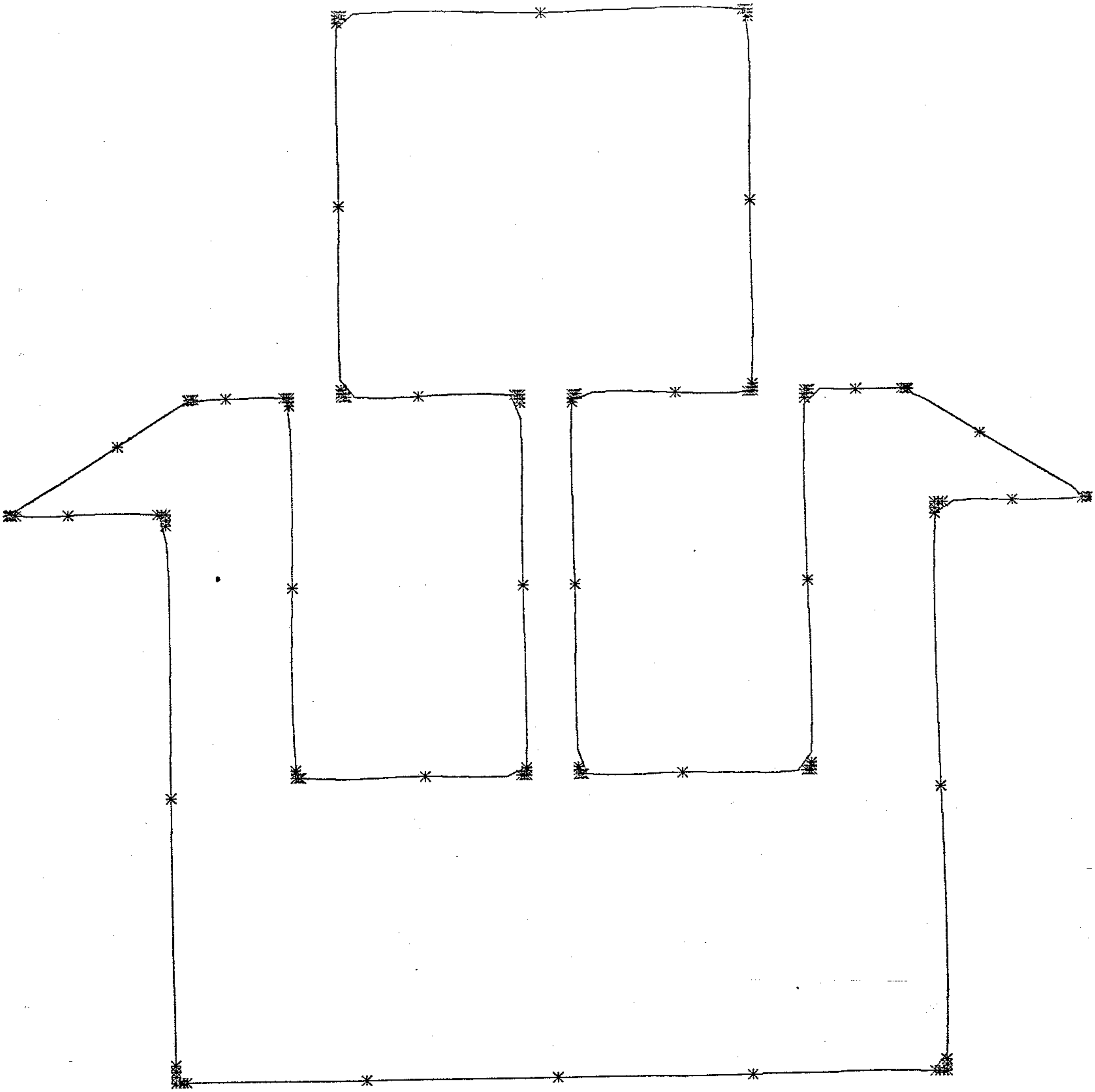

Figure 5 - Spline fit of characteristic
edges of chair 4.

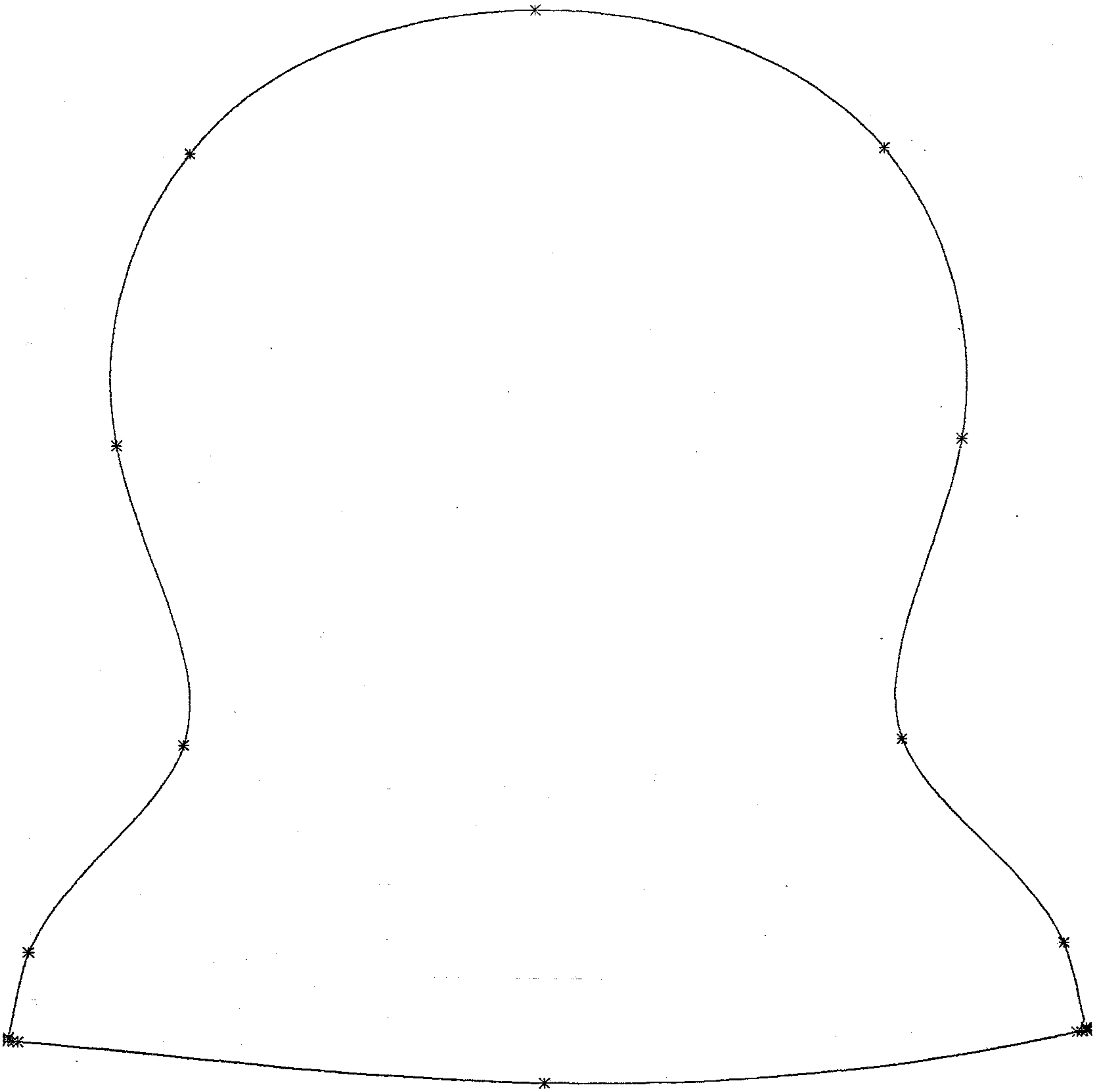

Figure 6 - Spline fit of characteristic
edges of chair 5.

of chairs 2 and 5 successfully matched 2 and 5, respectively. However, only seventeen views of chair 3 successfully matched chair 3, and fifteen views of chair 3 were identified as chair 5. We believe these errors are due to the program converging to a local minimum, but not recognizing the fact.

Most recognition experiments of this type have been run using very dissimilar objects. We purposely chose to use five very similar objects in our experiments. For instance, in chairs 1 and 2, the only difference is the height of the back and the curvature of the sides of the back. Under these circumstances, we feel that the program has done very well. While not a solution to all object recognition problems, our method should be useful in applications where distinguising between shapes on the basis of curvature is required. We expect, in future work, to tackle the local minima problem in more depth.

Matches Chair

2D Curve From Chair	1	2	3	4	5
1	50 (12)	0 (4)	0 (5)	0 (3)	0 (8)
2	0 (2)	48 (7)	1 (0)	1 (0)	0 (3)
3	7 (0)	3 (2)	17 (2)	8 (1)	15 (9)
4	0 (0)	0 (10)	0 (0)	50 (0)	0 (10)
5	1 (3)	1 (1)	0 (0)	0 (1)	48 (0)

Table 1 shows the number of times that a two-dimensional perspective projection of chair i successfully matched the three-dimensional model of chair j and (in parentheses) the number of times that, for some starting point, the minimization routine failed when trying to match a view of chair i to the model of chair j.

Appendix 1.

A necessary condition for the two planar curves and ,
to match is that their power spectra match,

$|c_n| = |\tilde{c}_n|$ for all n.

This suggests using the distance measure (which is at best a pseudo-metric)

$$\Omega(\Gamma,\Delta) = \sum_{n=0}^{M} (|\tilde{c}_n|^2 - |c_n|^2)^2$$

which involves no minimizations at all. Note that $\Omega(\Gamma,\Delta) = 0$ is not a <u>sufficient</u> condition for Γ and Δ to match.

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