Credibility Assessment of Simulation Results: The State of the Art

Osman Balci

TR 86-31

Technical Report TR-86-31

OF SIMULATION RESULTS: THE STATE OF THE ART†

Osman Balci

Department of Computer Science Virginia Polytechnic Institute and State University Blacksburg, Virginia 24061

1 November 1986

To appear in the Proceedings of the Conference on Simulation Methodology and Validation to be held as part of the 1987 Eastern Simulation Conferences (Orlando, Fla., Apr. 6-9). The Society for Computer Simulation, San Diego, Calif.

ABSTRACT

The purpose of this paper is to provide a state-of-the-art survey of credibility assessment of simulation results and suggest some future research directions. A hierarchy of the credibility assessment is introduced and the state-of-the-art survey is presented with respect to this hierarchy. A glossary is provided to alleviate the lack of standard terminology. The future research calls upon looking at the "global picture" when conducting a simulation study and being concerned with all of the eleven credibility assessment stages not just model validation and programmed model verification.

CR Categories and Subject Descriptors: I.6.4 [Simulation and Modeling]: Model Validation and Analysis

Additional Key Words and Phrases: Credibility assessment, model evaluation, model verification.

1. INTRODUCTION

In a report to the U.S. Congress, the U.S. General Accounting Office (U.S. GAO) [1976] reviewed 57 federally funded models in detail, each costing over \$100,000 to develop, and found that many model development efforts experienced large cost overruns, prolonged delays in completion, and total user dissatisfaction with the information obtained from the model. The U.S. GAO report initiated a sequence of significant events in promoting research on model/credibility assessment.

Under the leadership of Saul I. Gass, the National Bureau of Standards organized several symposia and produced three special publications [Gass 1979, 1980, 1981]. The Society for Computer Simulation established a technical committee on model credibility which published a terminology for model credibility [Schlesinger et al. 1979]. The U.S. GAO [1979] published guidelines for model evaluation.

A uniform, standard terminology is yet nonexistent. A recent literature review [Balci and Sargent 1984a] indicated the usage of 16 terms: acceptability, accuracy, analysis, assessment, calibration, certification, confidence, credibility, evaluation, performance, qualification, quality assurance, reliability, testing, validation, and verification. Except some early papers which appeared between 1966 and 1972, model verification and model validation have been most of the time consistently defined reflecting the following differentiation:

model verification refers to building the model right; and model validation refers to building the right model.

To alleviate the lack of standard terminology, a glossary is provided in Section 5.

The purpose of this paper is to provide a state-of-the-art survey of credibility assessment of simulation results and suggest some future research directions. A hierarchy of the credibility assessment is introduced in Section 2 and the state-of-the-art survey is presented with respect to this hierarchy in Section 3. Section 4 contains the conclusions and future

research directions.

2. A HIERARCHY OF THE CREDIBILITY ASSESSMENT

To provide a proper framework for the state-of-the-art survey, it is convenient to introduce the hierarchy of the credibility assessment of simulation results as depicted in Figure 1 [Balci 1986]. Each branch of the hierarchy represents a credibility assessment stage (CAS) or an indicator. Figure 1 reveals the effect of a CAS upon the other. For example, model validity can be assessed in terms of several indicators each being a subjective or an objective test. Model validity affects the quality of experimental model which in turn affects the credibility of simulation results.

There are two more CASs not shown in Figure 1: presentation verification and acceptability of simulation results (see [Balci 1986] for details). The credibility assessment and presentation verification affect the acceptability of simulation results.

3. THE STATE OF THE ART

Recently, Banks et al. [1986a,1986b,1987] provided an excellent overview of modeling processes, validation, and verification and proposed a methodology. Gass [1983], in his feature article, presented an excellent review of the issues related to the credibility assessment. Ören [1981] proposed a frame of reference for the concepts and criteria to assess acceptability of simulation studies.

3.1 Formulated Problem Verification

Problem formulation and its verification which greatly affect the credibility and acceptability of simulation results, have not received the attention that they deserve in a simulation study. This is an educational problem. Educators usually emphasize how to solve a given problem rather than how to formulate one. As a result, people tend to jump

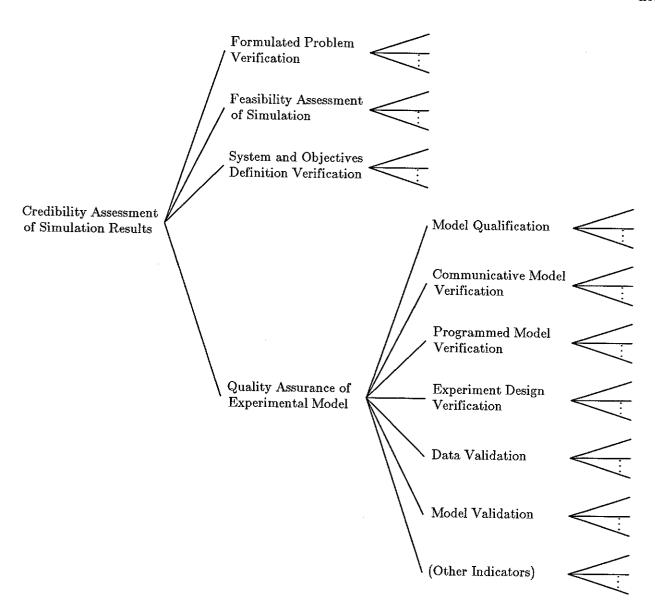


Figure 1. A Hierarchy of the Credibility Assessment.

into the solution of the communicated problem without spending sufficient time and effort in formulating the *real* problem. The consequence of this practice is frequently the type III error.

Balci and Nance [1985] introduced the formulated problem verification as an explicit

requirement of model credibility. They provided a high-level procedure for problem formulation and proposed 38 indicators for evaluating a formulated problem.

3.2 Feasibility Assessment of Simulation

It is well to remember the dictum that if a hammer is the only tool you have, you may tend to view each problem as a nail. One should not jump into simulation without assessing its feasibility for solving the problem under study. On the other hand, the statement "when all else fails, use simulation" is misleading if not invalid. Another technique may provide a less costly solution, but it may not be as useful. See [Balci 1986] for some indicators of the feasibility of simulation.

3.3 System and Objectives Definition Verification

The system of concern here is the one which contains the whole formulated problem. Although study objectives are specified within the formulated problem, it is extremely important to explicitly define and verify them since the rest of the simulation study are based upon those objectives. System definition should be verified in terms of the system characteristics identified by Shannon [1975]: (1) change, (2) environment, (3) counterintuitive behavior [Forrester 1971], (4) drift to low performance, (5) interdependency, and (6) organization. None of the energy models could predict the oil embargo in 1973; because, at the time, it was a counterintuitive behavior. Incorrect identification of system characteristics may result in type II or type III error.

3.4 Model Qualification

A model, by definition, is an abstraction of the reality. Many assumptions are made with respect to the study objectives in abstracting the reality (system). These assumptions define the underpinnings of the model and their reasonableness must be assessed as early as

possible in the model development life cycle. Using a model without knowing or understanding its underlying assumptions is absurd.

Model qualification has been studied by Gass and Thompson [1980] under the name of theoretical validity and by Sargent [1985] under the name of conceptual model validity.

3.5 Communicative Model Verification

How well the communicative model can be verified is dependent upon how much its form of representation lends itself to formal analysis and verification. Balci [1986] identified 21 forms of representation suggested in the literature.

Nance and Overstreet [1986] proposed several diagnostics which are based on analysis of graphs constructed from a particular form of model specification called condition specification [Overstreet 1982; Overstreet and Nance 1985]. Data-Flow Analysis and Control-Flow Analysis [Adrion et al. 1982] are the other two graph-based analysis techniques applicable for communicative model verification. Desk Checking [Adrion et al. 1982] and Model Review [Balci 1986] are also useful.

3.6 Programmed Model Verification

Graph-based analysis, desk checking, and model review can also be used for the verification of a programmed model. In addition, Balci [1986] proposed the use of Instrumentation-Based Testing and Functional Testing.

3.7 Experiment Design Verification

Since all simulation models are descriptive, it is the responsibility of the simulation analyst to correctly interpret the model results. To aid the analyst in this interpretation, experiments are designed and incorporated into the programmed model producing the experimental model with which the experiments are conducted and results are obtained.

Incorrect design of experiments may result in inaccurate interpretation of model results.

It is well to remember the dilemma of the scientific method as pointed out by Blyth [1973]: "The scientist needs to be objective, but the way he [or she] makes progress is through following up subjective insights." When a statistical procedure is used, we think that we are using an objective method. When it comes to satisfying the assumptions underlying the procedure, however, we sometimes use our subjective insights, intuitions, and guesses.

Balci [1986] proposed some indicators for verifying the design of simulation experiments.

3.8 Data Validation

U.S. GAO [1979] proposed a two-step approach for data validation: (1) establish the accuracy, completeness, impartiality, and appropriateness of the original data, and (2) verify the manner in which the model deals with the transformation of the original data. U.S. GAO [1979] also provided some indicators for data validity. Emphasizing the validation of input data models, Balci [1986] proposed some indicators as well.

3.9 Model Validation

The existing literature on simulation model validation [Balci and Sargent 1984a] generally falls into two broad areas: subjective validation techniques and statistical techniques proposed for validation. Tables 1 and 2 list these techniques and contain the related reference(s). The applicability of the techniques in Tables 1 and 2 depends upon the following cases where the system being modeled is: (1) completely observable—all data required for validation can be collected from system, (2) partially observable—some required data can be collected, and (3) nonexistent or completely unobservable. The statistical techniques in Table 2 are applicable only for case 1.

Table 1. Subjective Validation Techniques.

Event Validation[Hermann 1967]
Face Validation[Hermann 1967]
Field Tests[Shannon 1975; Van Horn 1971]
Graphical Comparisons[Cyert 1966; Forrester 1961; Miller 1975; Wright 1972]
Historical Methods[Naylor and Finger 1967]
Hypothesis Validation[Hermann 1967]
Internal Validation[Hermann 1967]
Multistage Validation[Naylor and Finger 1967; Law and Kelton 1982]
Predictive Validation[Emshoff and Sisson 1970]
Schellenberger's Criteria[Schellenberger 1974; U.S. General Accounting Office 1979]
Sensitivity Analysis[Hermann 1967; Miller 1974a, 1974b; Van Horn 1971; Shannon 1975]
Submodel Testing[Balci 1981]
Turing Test[Mitroff 1969; Schruben 1980; Turing 1963; Van Horn 1971]

3.10 Quality Assurance of Experimental Model

The quality of experimental model is assured by way of integrating the six CASs and other indicators shown in Figure 1. The other indicators are given by Balci [1986] as follows: accessibility, accountability, accuracy, augmentability, communicativeness, completeness, conciseness, consistency, device-independence, efficiency, legibility, self-containedness, self-descriptiveness, structuredness, and robustness.

3.11 Credibility Assessment of Simulation Results

The credibility of simulation results is assessed by way of integrating the following four CASs: formulated problem verification, feasibility assessment of simulation, system and objectives definition verification, and quality assurance of experimental model.

Table 2. Statistical Techniques Proposed for Validation.

Analysis of Variance[Naylor and Finger 1967]
Confidence Intervals/Regions
Factor Analysis[Cohen and Cvert 1961]
Hotelling's T ² Tests[Balci and Sargent 1981, 1982a, 1982b, 1983; Shannon 1975]
Multivariate Analysis of Variance[Garratt 1974]
 Standard MANOVA Permutation Methods Nonparametric Ranking Methods
Nonparametric Goodness-of-fit Tests[Gafarian and Walsh 1969; Naylor and Finger 1967]
 Kolmogorov-Smirnov Test Cramer-Von Mises Test Chi-square Test
Nonparametric Tests of Means[Shannon 1975]
 Mann-Whitney-Wilcoxon Test Analysis of Paired Observations
Regression Analysis[Aigner 1972; Cohen and Cyert 1961; Howrey and Kelejian 1969]
Theil's Inequality Coefficient
Time Series Analysis
— Spectral Analysis
t-Test[Shannon 1975; Teorey 1975]

4. CONCLUSIONS AND RESEARCH DIRECTIONS

As illustrated by the survey, most work has concentrated on model validation and very little has been published on the other ten CASs. However, as indicated by the hierarchy in Figure 1, model validity is a necessary but not a sufficient requirement for the credibility of simulation results. Future research should concentrate on all of the CASs.

Subjectivity is and will always be part of the credibility assessment for a reasonably complex simulation study. The reason for subjectivity is two-fold: modeling is an art and credibility assessment is situation dependent. The approach using the concept of indicators proposed by Balci [1986] is promising; however, future research is needed to determine more indicators for the CASs especially for specific areas of application (e.g., combat system simulation, manufacturing system simulation, missile system simulation, etc.).

We apparently lack good quality education on the art of modeling. It is not uncommon to find people who use the results of a simulation model without any idea about the underlying model assumptions. The dictum stated by Elmaghraby [1968] has not been fully appreciated: "Nobody solves the problem. Rather, everybody solves the model that he [or she] has constructed of the problem."

5. GLOSSARY

Calibration. An iterative process in which a probabilistic characterization for an input variable or a fixed value for a parameter is tried until the model is found to be sufficiently valid.

Communicative Model. A model representation which can be communicated to other humans and can be judged or compared against the system and the study objectives by more than one human [Nance 1981].

Communicative Model Verification. Ensuring that the communicative model is correctly constructed as intended and confirming the adequacy of the communicative model to provide an acceptable level of agreement for the domain of intended application.

Conceptual Model. The model which is formulated in the mind of the modeler [Nance 1981].

Data Validation. Substantiating that each input data model used possesses satisfactory accuracy consistent with the study objectives and confirming that the simulation model parameter values are accurately identified and used.

Descriptive Model. A model which describes the behavior of a system without any value judgment on the "goodness" or "badness" of such behavior [Elmaghraby 1968].

Domain of Applicability. The set of prescribed conditions for which the experimental model has been tested, compared against the system to the extent possible, and judged suitable for use [Schlesinger et al. 1979].

Domain of Intended Application. The prescribed conditions for which the model is

Λ

intended to match the system under study [Schlesinger et al. 1979].

Experiment Design. The process of formulating a plan to gather the desired information at minimal cost and to enable the analyst to draw valid inferences [Shannon 1975].

Experiment Design Verification. Substantiating that the experiments are correctly designed as intended.

Experimental Model. The programmed model incorporating an executable description of an experiment design.

Formulated Problem Verification. Substantiating that the formulated problem contains the actual problem in its entirety and is sufficiently well structured to permit the derivation of a sufficiently credible solution.

Indicator. An indirect measure of a concept, that can be measured directly.

Level of Agreement. The required correspondence between the model and the system, consistent with the domain of intended application and the study objectives [Schlesinger et al. 1979].

Model Builder's Risk. The probability of committing type I error.

Model Certification. Confirmation (usually by a third party) that a simulation model, within its domain of applicability, can produce results which are sufficiently credible with respect to the study objectives.

Model Qualification. Justifying that all assumptions underlying the conceptual model are appropriate and the conceptual model provides an adequate representation of the system under study with respect to the study objectives.

Model User's Risk. The probability of committing type II error.

Model Validation. Substantiating that the experimental model, within its domain of applicability, behaves with satisfactory accuracy consistent with the study objectives.

Peer Assessment. The assessment of the acceptability/credibility of simulation results by a panel of expert peers.

Prescriptive Model. A model which describes the behavior of a system with a value judgment on the "goodness" or "badness" of such behavior [Elmaghraby 1968].

Programmed Model. A model representation that admits execution by a computer to produce results [Nance 1981].

Programmed Model Verification. Substantiating that the programmed model represents the communicative model within an acceptable range of accuracy consistent with the study objectives.

System and Objectives Definition Verification. Substantiating that the system characteristics are correctly identified and the study objectives are explicitly defined with sufficient accuracy.

Type I Error. The error of rejecting the results of a simulation study when in fact they are sufficiently credible.

Type II Error. The error of accepting the results of a simulation study when in fact they are *not* sufficiently credible.

Type III Error. The error of solving the wrong problem.

ACKNOWLEDGMENTS

This research was sponsored in part by the Naval Sea Systems Command and the Office of Naval Research under Contract N60921-83-G-A165 through the Systems Research Center at VPI&SU.

REFERENCES

- Adrion, W.R., M.A. Branstad, and J.C. Cherniavsky (1982), "Validation, Verification, and Testing of Computer Software," Computing Surveys 14, 2 (June), 159-192.
- Aigner, D.J. (1972), "A Note on Verification of Computer Simulation Models," Management Science 18, 11 (Nov.), 615-619.
- Balci, O. (1981), "Statistical Validation of Multivariate Response Simulation Models," Ph.D. Dissertation, Syracuse University, Syracuse, N.Y., Aug.
- Balci, O. (1986), "Guidelines for Successful Simulation Studies: Part I and II", Technical Report TR-85-2, Department of Computer Science, Virginia Tech, Blacksburg, Va., Sept.
- Balci, O. and R.E. Nance (1985), "Formulated Problem Verification as an Explicit Requirement of Model Credibility," Simulation 45, 2 (Aug.), 76-86.
- Balci, O. and R.G. Sargent (1981), "A Methodology for Cost-Risk Analysis in the Statistical Validation of Simulation Models," Communications of the ACM 24, 4 (Apr.), 190-197.
- Balci, O. and R.G. Sargent (1982a), "Some Examples of Simulation Model Validation Using Hypothesis Testing," In *Proceedings of the 1982 Winter Simulation Conference* (San Diego, Calif., Dec. 6-8). IEEE, New Jersey, pp. 620-629.
- Balci, O. and R.G. Sargent (1982b), "Validation of Multivariate Response Models Using Hotelling's Two-Sample T² Test," Simulation 39, 6 (Dec.), 185-192.
- Balci, O. and R.G. Sargent (1983), "Validation of Multivariate Response Trace-Driven Simulation Models," In *Performance '83*, A.K. Agrawala and S.K. Tripathi, Eds. North-Holland Publ., Amsterdam, pp. 309-323.
- Balci, O. and R.G. Sargent (1984a), "A Bibliography on the Credibility Assessment and Validation of Simulation and Mathematical Models," Simulation 15, 3 (July), 15-27.
- Balci, O. and R.G. Sargent (1984b), "Validation of Simulation Models via Simultaneous Confidence Intervals," American Journal of Mathematical and Management Sciences 4, 3&4, 375-406.
- Banks, J., D.M. Gerstein, and S.P. Searles (1986a), "The Verification and Validation of Simulation Models: A Methodology," Technical Report, School of Industrial and Systems Engineering, Georgia Tech, Atlanta, Ga., Sept.

- Banks, J., D.M. Gerstein, and S.P. Searles (1986b), "The Verification and Validation of Simulation Models: Unresolved Issues," Technical Report, School of Industrial and Systems Engineering, Georgia Tech, Atlanta, Ga., Oct.
- Banks, J., D.M. Gerstein, and S.P. Searles (1987), "Modeling Processes, Validation, and Verification of Complex Simulations: A Survey," In *Proceedings of the Conference on Simulation Methodology and Validation* (Orlando, Fla., Apr. 6-9). SCS, San Diego, Calif.
- Blyth, C.R. (1973), "Subjective vs. Objective Methods in Statistics," The American Statistician 26, 3 (June), 20-22.
- Cohen, K.J. and R.M. Cyert (1961), "Computer Models in Dynamic Economics," Quarterly Journal of Economics 75, 1 (Feb.), 112-127.
- Cyert, R.M. (1966), "A Description and Evaluation of Some Firm Simulations," In Proceedings of the IBM Scientific Computing Symposium on Simulation Models and Gaming (White Plains, N.Y.), IBM, White Plains, N.Y., pp. 3-22.
- Damborg, M.J. and L.F. Fuller (1976), "Model Validation Using Time and Frequency Domain Error Measures," ERDA Report 76-152, available from NTIS, Springfield, Va.
- Elmaghraby, S.E. (1968), "The Role of Modeling in IE Design," Industrial Engineering 19, 6 (June), 292-305.
- Emshoff, J.R. and R.L. Sisson (1970), Design and Use of Computer Simulation Models, Mac-Millan, New York.
- Fishman, G.S. and P.J. Kiviat (1967), "The Analysis of Simulation Generated Time Series," Management Science 13, 7 (July), 525-557.
- Forrester, J.W. (1961), Industrial Dynamics. MIT Press, Cambridge, Mass.
- Forrester, J.W. (1971), "Counterintuitive Behavior of Social Systems," *Technology Review* 73, 3 (Jan.), 1-16.
- Gafarian, A.V. and J.E. Walsh (1969), "Statistical Approach for Validating Simulation Models by Comparison with Operational Systems," In *Proceedings of the 4th International Conference on Operations Research*, John Wiley & Sons, New York, pp. 702-705.
- Gallant, A.R., T.M. Gerig, and J.W. Evans (1974), "Time Series Realizations Obtained According to an Experimental Design," J. American Statistical Association 69, 347 (Sept.), 639-645.
- Garratt, M. (1974), "Statistical Validation of Simulation Models," In *Proceedings of the 1974 Summer Computer Simulation Conference* (Houston, Tex., July 9-11). Simulation Councils, La Jolla, Calif., pp. 915-926.
- Gass, S.I., Ed. (1979), Utility and Use of Large-Scale Mathematical Models, Special Publication 534, Nat. Bur. of Standards, Washington, D.C.
- Gass, S.I., Ed. (1980), Validation and Assessment Issues of Energy Models, Special Publication 569, Nat. Bur. of Standards, Washington, D.C., Feb.

- Gass, S.I., Ed. (1981), Validation and Assessment of Energy Models, Special Publication 616, Nat. Bur. of Standards, Washington, D.C., Oct.
- Gass, S.I. (1983), "Decision-Aiding Models: Validation, Assessment, and Related Issues for Policy Analysis," *Operations Research 31*, 4 (July-Aug.), 603-631.
- Gass, S.I. and B.W. Thompson (1980), "Guidelines for Model Evaluation: An Abridged Version of the U.S. General Accounting Office Exposure Draft," Operations Research 28, 2 (Mar.-Apr.), 431-439.
- Hermann, C.F. (1967), "Validation Problems in Games and Simulations with Special Reference to Models of International Politics," *Behavioral Science 12*, 3 (May), 216-231.
- Howrey, P. and H.H. Kelejian (1969), "Simulation Versus Analytical Solutions," In *The Design of Computer Simulation Experiments*, T.H. Naylor, Ed. Duke University Press, Durham, N.C., pp. 207-231.
- Hunt, A.W. (1970), "Statistical Evaluation and Verification of Digital Simulation Models Through Spectral Analysis," Ph.D. Dissertation, The University of Texas at Austin, Austin, Tex.
- Kheir, N.A. and W.M. Holmes (1978), "On Validating Simulation Models of Missile Systems," Simulation 30, 4 (Apr.), 117-128.
- Law, A.M. and W.D. Kelton (1982), Simulation Modeling and Analysis, McGraw-Hill, New York.
- Miller, D.K. (1975), "Validation of Computer Simulations in the Social Sciences," In Proceedings of the Sixth Annual Conference on Modeling and Simulation (Pittsburg, Pa.), pp. 743-746.
- Miller, D.R. (1974a), "Model Validation Through Sensitivity Analysis," In *Proceedings of the* 1974 Summer Computer Simulation Conference (Houston, Tex., July 9-11). Simulation Councils, La Jolla, Calif., pp. 911-914.
- Miller, D.R. (1974b), "Sensitivity Analysis and Validation of Simulation Models," J. Theoretical Biology 48, 2 (Dec.), 345-360.
- Mitroff, I.I. (1969), "Fundamental Issues in the Simulation of Human Behavior: A Case in the Strategy of Behavioral Science," Management Science 15, 12 (Dec.), B635-B649.
- Nance, R.E. (1981), "Model Representation in Discrete Event Simulation: The Conical Methodology," Technical Report CS81003-R, Department of Computer Science, Virginia Tech, Blacksburg, Va., Mar.
- Nance, R.E. and C.M. Overstreet (1986), "Diagnostic Assistance Using Digraph Representations of Discrete Event Simulation Model Specifications," Technical Report TR-86-8, Department of Computer Science, Virginia Tech, Blacksburg, Va., Mar.
- Naylor, T.H. and J.M. Finger (1967), "Verification of Computer Simulation Models," Management Science 14, 2 (Feb.), B92-B101.
- Ören, T.I. (1981), "Concepts and Criteria to Assess Acceptability of Simulation Studies: A Frame of Reference," Communications of the ACM 24, 4 (Apr.), 180-189.

- Overstreet, C.M. (1982), "Model Specification and Analysis for Discrete Event Simulation," Ph.D. Dissertation, Virginia Tech, Blacksburg, Va., Dec.
- Overstreet, C.M. and R.E. Nance (1985), "A Specification Language to Assist in Analysis of Discrete Event Simulation Models," Communications of the ACM 28, 2 (Feb.), 190-201.
- Rowland, J.R. and W.M. Holmes (1978), "Simulation Validation with Sparse Random Data," Computers and Electrical Engineering 5, 3 (Mar.), 37-49.
- Sargent, R.G. (1985), "An Expository on Verification and Validation of Simulation Models," In Proceedings of the 1985 Winter Simulation Conference (San Francisco, Calif., Dec. 11-13). IEEE, Piscataway, N.J., pp. 15-22.
- Schellenberger, R.E. (1974), "Criteria for Assessing Model Validity for Managerial Purposes," Decision Sciences 5, 4 (Apr.), 644-653.
- Schlesinger, S., et al. (1979), "Terminology for Model Credibility," Simulation 32, 3 (Mar.), 103-104.
- Schruben, L.W. (1980), "Establishing the Credibility of Simulations," Simulation 34, 3 (Mar.), 101-105.
- Shannon, R.E. (1975), Systems Simulation: The Art and Science, Prentice-Hall, Englewood Cliffs, N.J.
- Teorey, T.J. (1975), "Validation Criteria for Computer System Simulations," Simuletter 6, 4 (July), 9-20.
- Theil, H. (1961), Economic Forecasts and Policy, North-Holland Publ., Amsterdam.
- Turing, A.M. (1963), "Computing Machinery and Intelligence," In Computers and Thought, E.A. Feigenbaum and J. Feldman, Eds. McGraw-Hill, New York, pp. 11-15.
- Tytula, T.P. (1978), "A Method for Validating Missile System Simulation Models," Technical Report E-78-11, U.S. Army Missile R&D Command, Redstone Arsenal, Ala., June.
- U.S. General Accounting Office (1976), "Report to the Congress: Ways to Improve Management of Federally Funded Computerized Models," LCD-75-111, U.S. G.A.O., Washington, D.C., Aug.
- U.S. General Accounting Office (1979), "Guidelines for Model Evaluation," PAD-79-17, U.S. G.A.O., Washington, D.C., Jan.
- Van Horn, R.L. (1971), "Validation of Simulation Results," Management Science 17, 5 (May), 247-258.
- Watts, D. (1969), "Time Series Analysis," In *The Design of Computer Simulation Experiments*, T.H. Naylor, Ed. Duke University Press, Durham, N.C., pp. 165-179.
- Wright, R.D. (1972), "Validating Dynamic Models: An Evaluation of Tests of Predictive Power," In *Proceedings of the 1972 Summer Computer Simulation Conference* (San Diego, Calif., July 14-16). Simulation Councils, La Jolla, Calif., pp. 1286-1296.