GENERAL NOTES ON PROCESSES AND THEIR SPECTRA

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Abstract. *The frequency spectrum performs one of the main characteristics of a process. The aim of the paper is to show the coherence between the process and its own spectrum and how the behaviour and properties of a process itself can be deduced from its spectrum. Processes are categorized, and general principles of their spectra calculation and recognition are given. The main stress is put on signal power spectra, as they also perform a kind of processes. These spectra can be directly measured, observed and examined by means of spectral analysers and they are very important characteristics which cannot be omitted at transmission techniques in telecommunication technologies. Further, the paper also deals with non-electric processes, mainly with processes and spectra at mass servicing and how these spectra can be utilised in praxis. All processes analysed in this paper are supposed to be in a stable state and ergodic.*

Keywords

Auto correlation function, dispersion, Fourier transforms, mean value, power spectrum, probability distribution, random process, sampling, scanning, stochastic process, Wiener-Kchintchin transform.

1. Introduction

In general, a process is events (actions) running in the course of time. It can be represented by a time depending deterministic or random function. The harmonic signal, clock pulses, a trajectory of a flying airplane, electric or optic signals conveying information (physical processes), values of shares on a stock exchange, financial flows in a company (economic processes), meteorological temperature or wind observations, or a number of service (communication) channels occupied are the typical examples of processes.

Processes can be categorised from various points of view. The next categorisation appears the most important for the purpose of mathematical description:

- deterministic processes (Section 2),
- stochastic processes (Section 3),
- random processes (Section 4).

The stochastic processes can further be categorised as follows:

A. Baseband stochastic processes (Section 3.1):

a) Almost periodic (Section 3.1.1):

- with uncorrelated periods (Section 3.1.1.1),
- with correlated periods (Section 3.1.1.2);

b) Non-periodic (Section 3.1.2):

- with discrete amplitude changes in certain random time instants (Section 3.1.2.1),
- with continuous amplitude changes in any time (Section 3.1.2.2).

B. Bandpass stochastic processes (Section 3.2).

2. Deterministic Processes

Non modulated and un-coded signals as a pilot frequency, a clock synchronization signal or transient events in linear electric circuits are the good examples of deterministic (non-random) processes. They can be anticipated with certainty at any past or future time instant and therefore, they cannot convey any piece of information. If these signals are periodic, they have discrete spectra composed of one or more sharp demarcated spectral lines. Spectral components of deterministic processes can be calculated using the Fourier analysis or the Fourier transform. These tools are well known and thus it is not further necessary to discuss them. The main concern will be put on stochastic and random processes.

3. Stochastic Processes

The word “stochastic” was introduced to express the knowledge that even a chance has also its rules, so the whole scale of dynamic events and changes can be a consequence of a unique hidden deterministic rule [1]. Unlike of deterministic processes, future values of stochastic processes can be anticipated only with a certain probability $0 < p < 1$ acceptable for a shorter time which is the consequence of good defined statistical laws and hidden deterministic parameters.

As the values of a stochastic process are random variables, the spectral lines cannot be sharp demarcated as it is at a deterministic process. The sharp spectral lines become uncertain, get blurred, and they will be spread in a continuous curve.

Due to the uncertain behaviour, a stochastic process can only be described by parameters that have the statistical character and have not only a numeric but also a physical meaning which is useful at physical processes. The statistical parameters describing a stochastic process are: the mean value, the dispersion, the power and the more complex characteristics – the auto correlation function and the power spectral density (shortly power spectrum).

The power spectrum as a very important characteristic of a process (deterministic or non-deterministic) can always be calculated by means of the auto correlation function. However, the utilisation of the auto correlation function for this purpose is not necessary at stochastic processes with uncorrelated periods.

3.1. Baseband Stochastic Processes

As it has already been mentioned, stochastic processes are characterised by the fact they always contain:

- an apparent or hidden deterministic component,
- a correlation coupling among various time values,
- the both above features.

Therefore, they cannot be random in any case from these reasons.

1) Baseband Almost Stochastic Processes

Almost periodic stochastic processes can be:

- discrete both in time and in amplitude or,
- discrete in time but continuous in amplitude.

The discrete time indicates the exact instants at which the same deterministic process course, the am-amplitude or the phase of which is a random variable, regularly starts to repeat. These discrete time instants performing periods are the non-random variables. Dis-continuities or sudden changes usually happen in the time course of a stochastic process at these time instants.

Except of the almost periodic stochastic process it is always discrete in time, it can be discrete or continuous in amplitude. The amplitude is always a random variable that can be discrete when it only gains certain discrete values, or continuous when it gains any value from its possible range.

Digitally encoded and digitally modulated signals conveying information are the typical representatives of almost periodic stochastic processes. The periods are represented by a train of pulses of a certain shape the height of which is a random variable A that gains discrete or continuous values from a certain range. Periodically repeating symbols of the same shape and a non-zero mean value performs the apparent deterministic component. A correlation coupling among particular symbols performs the hidden deterministic component.

- *Uncorrelated Periods*

When there is no correlation among periods, the power spectrum can be calculated without the necessity to know the autocorrelation function as [2], [3]:

$$S(f) = \frac{\sigma_\alpha^2 \left|G(f)\right|^2}{T_0} + m_\alpha^2 \sum_{n=-\infty}^{\infty} \left|c_n\right|^2 \delta(n.f_0). \quad (1)$$

Here are:

- $G(f)$ – the Fourier transform of the unit pulse shape (with the amplitude $A = 1$),
- c_n – complex Fourier coefficients,
- $\delta(nf_o)$ – the pulse function,
- $T_o = 1/f_o$ – the repeating period,
- σ_α^2 – the amplitude dispersion of the process,
- m - the mean level of the process.

$$m = \frac{m_\alpha}{T_0} \int_{-\frac{v}{2}}^{\frac{v}{2}} g(t)\,dt, \quad (2)$$

where:

- m_a - the amplitude mean value,
- $g(t)$ - the shape of the unit pulse,
- v - the width of the unit pulse within the period T_0.

When the process is discrete in amplitude, the amplitude mean, ma and the amplitude dispersion, σ_a^2 are calculated as:

$$m_\alpha = \sum_{j=-N}^{N} a_j p_j, \quad (3)$$

$$\sigma_\alpha^2 = \sum_{j=-N}^{N} a_j^2 p_j - m_\alpha^2, \quad (4)$$

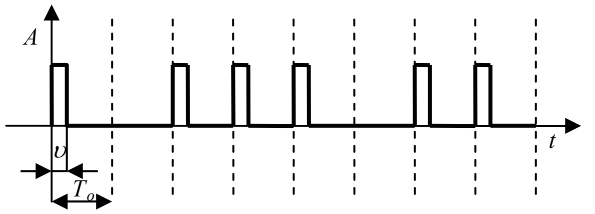
where a_j are amplitudes which the random variable, A gains with probabilities p_j, $j = 0, 1, 2,...,N$. It is desirable at the almost periodic stochastic signal that the probabilities p_j are equipment's probable, e.g. $p_j = p$ for all j.

When the process is continuous in amplitude, the amplitude mean, m_a and the amplitude dispersion, σ_a^2 are calculated as:

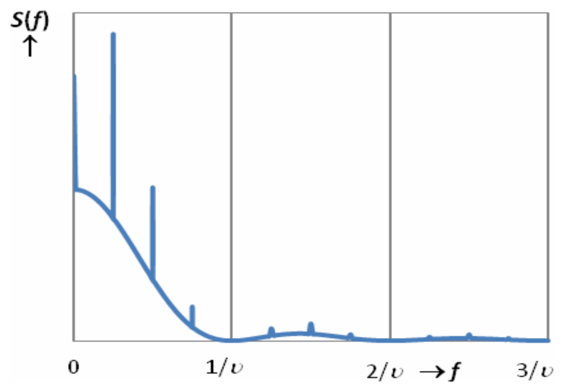
$$m_\alpha = \int_{-\infty}^{\infty} x f(x)\, dx , \tag{5}$$

$$\sigma_\alpha^2 = \int_{-\infty}^{\infty} x^2 f(x)\, dx - m_\alpha^2 , \tag{6}$$

where x is a value which the stochastic process $x(t)$ can gain in a time t and $f(x)$ is the probability distribution of values x. It is the Gaussian distribution in most cases. No matter of the stochastic process is discrete or continuous in amplitude; this has no influence on the spectrum shape of the process.



(a)



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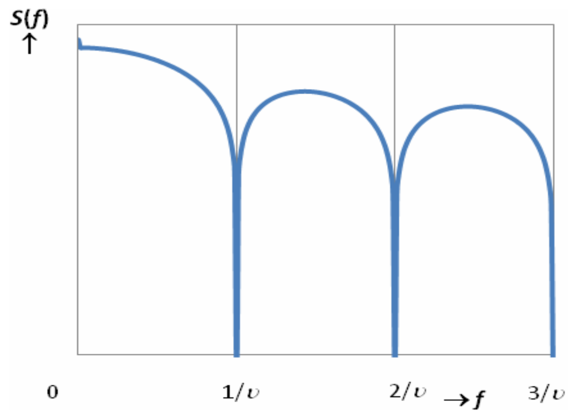


(c)

Fig. 1: Unipolar stochastic digital signal (M = 2, To = 4) (a) and its spectrum in the linear (b) and in the logarithmic (c) scales.

In the case when the mean level of the process, $m = 0$, the power spectrum can be easily calculated as:

$$S(f) = \frac{\sigma_\alpha^2 \left| G(f) \right|^2}{T_0} . \tag{7}$$

The spectrum of a train of randomly occurring rectangular pulses creating a digital stochastic signal can be taken as the template of the almost stochastic process with uncorrelated periods. The continuous power spectrum is shaped according to the function $sinc^2\ x$. Generally, the spectrum of the unipolar digital stochastic signal can be mathematically written as [3]:

$$S(f) = \left(\frac{A}{2} \frac{\upsilon}{T_0} \right)^2 \left\{ \frac{M+2}{3M} T_0 \left[\frac{\sin(\pi \upsilon f)}{\pi \upsilon f} \right]^2 + \right. , \tag{8}$$
$$\left. + \sum_{n=-\infty}^{\infty} \left[\frac{\sin(n \pi \upsilon f_0)}{n \pi \upsilon f_0} \right]^2 \delta(n f_0) \right\}$$

and that of the bipolar digital stochastic signal [3]:

$$S(f) = \frac{M+1}{M-1} \frac{A^2}{3} \frac{\upsilon^2}{T_0} \left[\frac{\sin(\pi \upsilon f)}{\pi \upsilon f} \right]^2 , \tag{9}$$

where M denotes the number of the discrete amplitude states.

There is an example of the almost periodic stochastic process in Fig. 1 – the unipolar two-state stochastic digital signal. In comparison to the non-periodic stochastic processes (see Chapter 3.1.2), the curve of the power spectrum does not decrease monotonically to 0 at the infinite frequency, on the contrary, it has zero values at frequencies $f = n/\upsilon$, $n = 1, 2, 3, \ldots$ The presence of lobes and zero values are the main features how the periodicity manifests as the apparent deterministic component. The occurrence of peaks indicates the existence of a deterministic component which can be for example a non-zero mean level in the case when the peak also occurs at zero frequency.

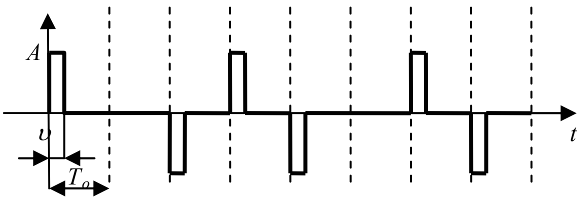


Fig. 2: Bipolar stochastic digital signal (M = 2, T_o = 4).

There is the bipolar two-state stochastic digital signal in Fig. 2. Its spectrum is the same as that in Fig. 1b, 1c, but without the spectral lines because the signal does not have any mean level as a deterministic component.

Further, there are 3 examples of the spectra of the bipolar stochastic digital signal with the pulse shaping as in Fig. 3. Figure 3a presents 3 particular pulse shapes with the same two-state $(M = 2)$ random amplitude A, the repeating period T_o and the same pulse width $\upsilon = T_o/2$.

The first pulse type (A) is the referencing rectangular pulse with the real power spectrum [3]:

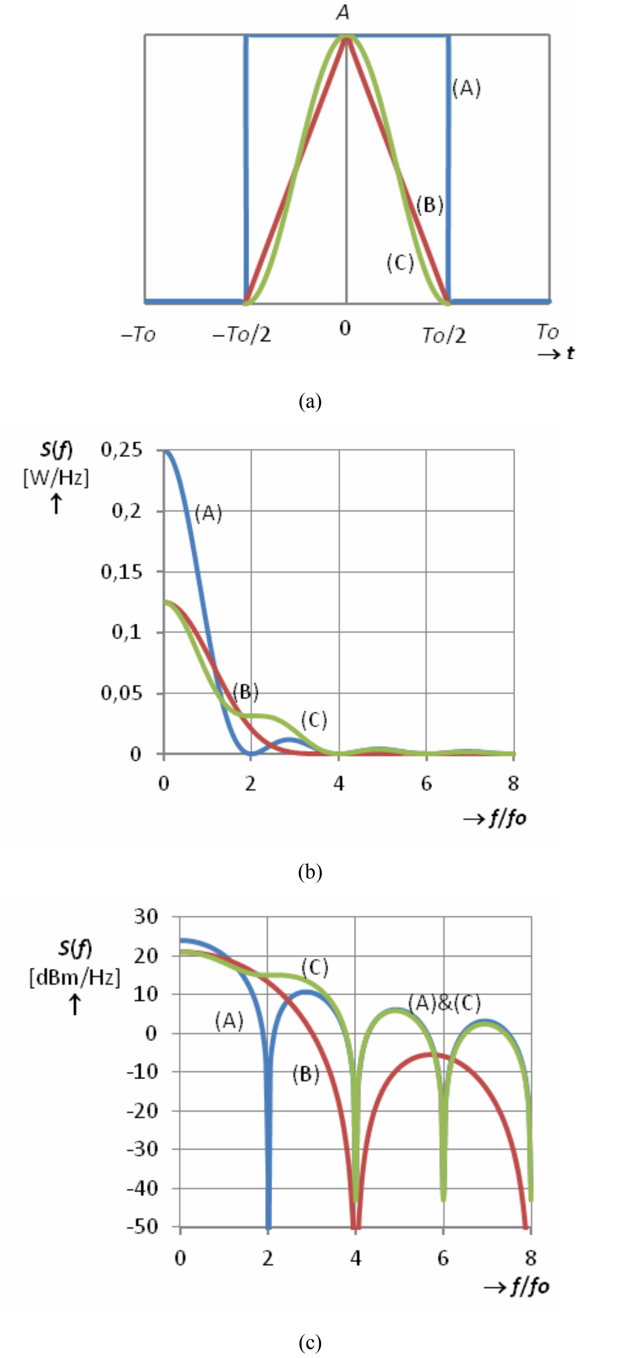


Fig. 3: Spectral comparison of the almost periodic stochastic digital signals composed of the rectangular, the triangular and the raised cosine pulse shapes.

$$S(f) = \frac{1}{4} A^2 T_0 \left(\frac{\sin \frac{\pi f}{2 f_0}}{\frac{\pi f}{2 f_0}} \right)^2 . \quad (10)$$

The second pulse type (B) is the triangular pulse with the real power spectrum:

$$S(f) = \frac{1}{8} A^2 T_0 \left(\frac{\sin \frac{\pi f}{4 f_0}}{\frac{\pi f}{4 f_0}} \right)^4 . \quad (11)$$

And the third pulse type (C) is the raised cosine pulse with the real power spectrum:

$$S(f) = \frac{1}{32} A^2 T_0 \left\{ \left[2 \frac{\sin \frac{\pi f}{2 f_0}}{\frac{\pi f}{2 f_0}} \right]^2 + \left[\frac{\sin \frac{\pi (f - 2 f_0)}{2 f_0}}{\frac{\pi (f - 2 f_0)}{2 f_0}} \right]^2 + \left[\frac{\sin \frac{\pi (f - 2 f_0)}{2 f_0}}{\frac{\pi (f - 2 f_0)}{2 f_0}} \right]^2 \right\}, \quad (12)$$

The power spectra corresponding to the given formulae are plotted in Fig. 3b and 3c in the linear and in the logarithmic scales, respectively. As it can be seen from these figures, when comparing the template power spectrum (A) with the others two (B) and (C), the pulse shaping may shift the zero points out of the frequencies

$$f = \frac{n}{\upsilon} = \frac{n}{\frac{T_0}{k}} = nkT_0 \rightarrow n, k = 1, 2, 3, \ldots , \quad (13)$$

where k denotes the ratio of the repeating period, T_o, to the pulse width, υ (see the horizontal axis in Fig. 3b, 3c). Hereby, the power spectrum may get wider [see (B) and (C)] or get "distorted" [see (C)]. Such distortion may sometimes look like a doubled number of the lobes in the logarithmic scale. Also, the peaks indicating the existence of a hidden deterministic component, Fig. 2c, are hardly visible in the logarithmic scale that is exclusively used in power spectra analyzers. This can cause false conclusions at the stochastic signal evaluation.

- *Correlated Periods*

When a correlation exists among periods, first it is necessary to find the auto correlation series and only after that the power spectrum can be derived. In case of a stochastic process with correlated periods, the auto correlation series may be derived as [4]:

$$R_n = \overline{A_k A_{k+n}} = \sum_{k=0}^{N-n-1} \sum_{i=0}^{M} \sum_{j=0}^{M} a_{k,i} a_{(k+n),j} p_{k,ij} , \quad (14)$$

for $n = 0, 1, 2, \ldots, N-1$, where $p_{k,ij}$ is the occurrence probability of a *j-th* amplitude in a *(k + n)-th* pulse, $\alpha_{(k+n),j}$ on condition that an *i-th* amplitude, $\alpha_{k,i}$ has occurred in a *k-th* pulse:

$$p_{k,ij} = p_{k,j} p_{(k+n),i} . \quad (15)$$

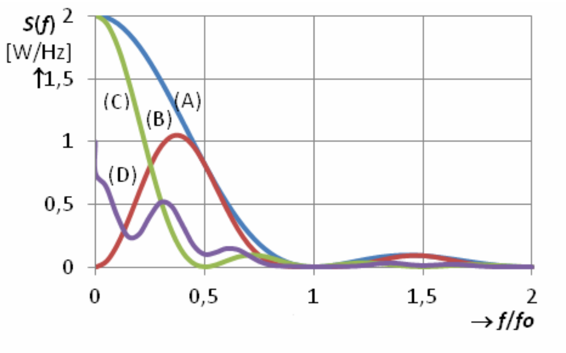
The N denotes the correlation range, i.e. the number of pulses that may have a correlation coupling among each other within the correlation range:

$$T = NT_0 . \quad (16)$$

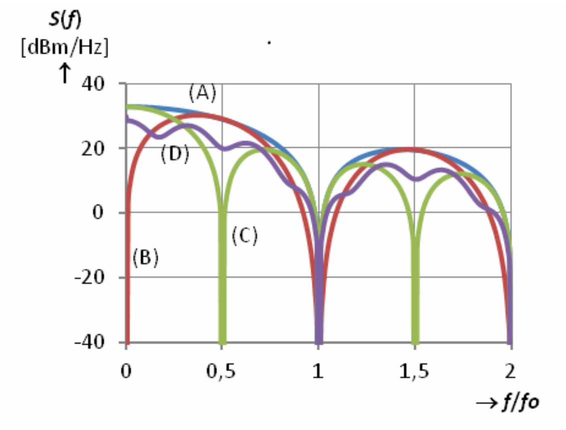
Then the power spectrum will be calculated as [2]:

$$S(f) = \frac{|G(f)|^2}{T_0} \sum_{n=-\infty}^{\infty} R_n e^{-jn2\pi T_0}, \qquad (17)$$

Again, no matter of the correlated almost periodic stochastic process is discrete or continuous in amplitude, this has no influence on the spectrum shape of the process.



(a)



(b)

Fig. 4: Spectral comparison of the stochastic digital signals with correlated periods (B), (C), (D) with the reference stochastic digital signal with the uncorrelated periods (A) in the linear (a) and the logarithmic (b) scales.

Let´s consider 4 almost periodic stochastic signals, all with the rectangular shapes of the random pulses filling the whole repeating period ($\upsilon = T_0$): the basic bipolar digital stochastic signal with uncorrelated periods (A) as a template one and 3 other signals with correlated periods – the AMI-NRZ code (Alternate Mark Inversion – Non Return to Zero) (B), the MLT-3 code (Multilevel Threshold with 3 levels) (C) and a convolution coded signal (D). Their real power spectra are, respectively [4]:

$$S(f) = 2A^2 T_0 \left(\frac{\sin \frac{\pi f}{f_0}}{\frac{\pi f}{f_0}} \right)^2, \qquad (18)$$

$$S(f) = 2A^2 T_0 \left(\frac{\sin \frac{\pi f}{f_0}}{\frac{\pi f}{f_0}} \right)^2 \sin^2 \frac{\pi f}{f_0}, \qquad (19)$$

$$S(f) = 2A^2 T_0 \left(\frac{\sin \frac{2\pi f}{f_0}}{\frac{2\pi f}{f_0}} \right)^2, \qquad (20)$$

$$S(f) = \frac{1}{2} A^2 T_0 \left(\frac{\sin \frac{\pi f}{f_0}}{\frac{\pi f}{f_0}} \right)^2 \left(\frac{1}{2} + \cos^2 \frac{3\pi f}{f_0} \right) + \frac{A^2}{4} \delta(0), \qquad (21)$$

Figure 4 shows the power spectra of these signals in the linear (a) and in the logarithmic (b) scales. As it is evident, the correlation among periods also changes the power spectral curve like pulse shaping. The power spectrum may get narrower [see (C)] or get “distorted” [see (D)].

2) Baseband Non Periodic Stochastic Processes

There is always a correlation coupling among values which processes of this type gain in a time. To obtain the power spectrum, the auto correlation function must be determined as the first step. Unlike of the correlation series (1), the auto correlation function is moreover time dependent. The auto correlation function may be determined similarly as in equation (1):

$$R(\tau) = \overline{A(t)A(t+\tau)} = \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} a_i(t) a_j(t+\tau) p_{i,j}. \qquad (22)$$

Here, p_{ij} denotes the probability that a discrete amplitude state, a_j occurs in a time $t + \tau$ when a discrete amplitude state, a_i has occurred before in a time t, where τ is a time difference between these two events; M is the number of possible discrete amplitude states.

When continuous amplitude changes happen in any time, the general formula for calculation of the auto correlation function is only applicable [5]:

$$R(\tau) = \frac{1}{T-\tau} \int_0^{T-\tau} x(t) x(t-\tau) dt, \qquad (23)$$

where T denotes the correlation range.

As the correlation coupling is getting weak when the time difference, τ increases, the auto correlation function exponentially decreases, in general.

The power spectrum can be determined by the Fourier transform of the auto correlation function (Wiener-Kchintchin transform):

$$S(f) = \int_{-\infty}^{\infty} R(\tau) e^{-j2\pi f \tau} d\tau. \qquad (24)$$

Due to the exponential decrease of the auto correlation function, Fig. 5a, the real power spectrum will have the power spectrum as on Fig. 5b which is given by the Fourier transform of the exponentially decreasing auto correlation function [3]:

$$R(\tau) = \sigma^2 e^{-\frac{|\tau|}{c}}, \tag{25}$$

$$S(f) = 2|S(f)| = 4\sigma^2 c \frac{1}{1+(2\pi c f)^2} + m_a^2 \delta(0) \cdot \tag{26}$$

The constant c defines a measure of the correlation coupling that shall be either explicitly derived or empirically determined.

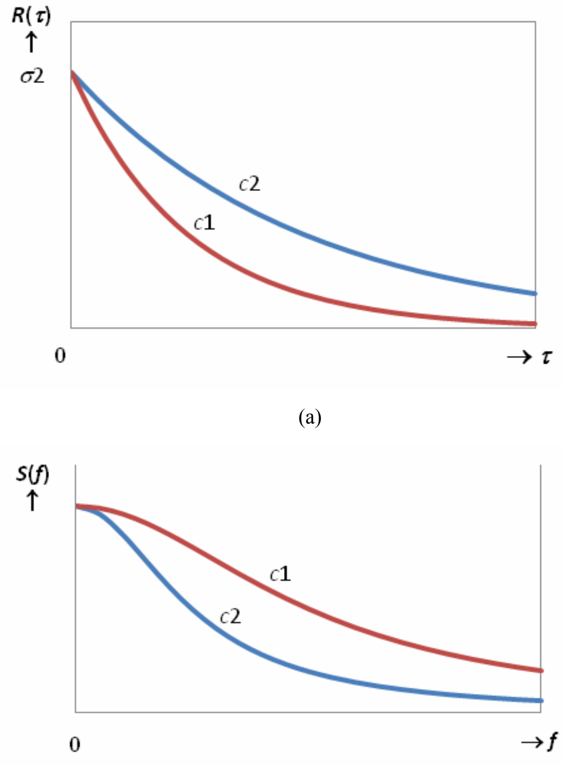


Fig. 5: Auto correlation function (a) and belonging power spectrum (b) without mean level component ($c_1 < c_2$).

As there is no periodicity at this type of process, its power spectrum perishes monotonically (Fig. 5). The repeating period, T_o can be considered to be infinite *(*$T_o \rightarrow \infty$*)*. Therefore, no zeros in the power spectrum curve indicating the presence of periods in a stochastic process can occur (as if they were shifted to infinite frequencies).

- *Cases with Discrete Amplitudes*

There are two types of the non-periodic stochastic process:

- the two-state one and,
- the multi-state one.

Two-State Non Periodic Stochastic Process

This type of the process can be explained on these practical examples: Data communication in wideband packet networks realizes by means of transmission of data bursts. Similarly, when optic network terminations of gigabit passive optic network subscriber's communicate with their optic network node, they send transmission containers [6] that are data bursts, too. Their lengths and their positions on the time axis on the shared communication channel (the optic fiber) are random. These communication types perform the non-periodic stochastic process with 2 states – 0 or 1 when a data burst is or is not present as it is depicted in Fig. 6.

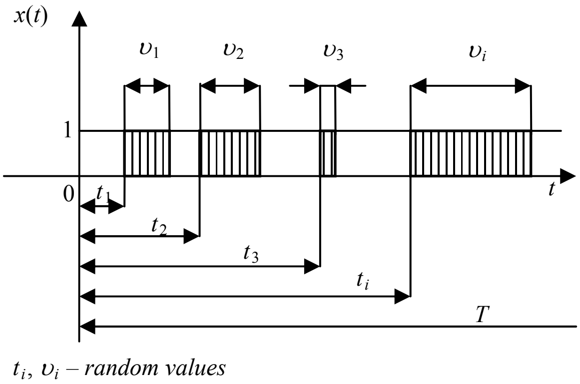


Fig. 6: Two-state non periodic stochastic process.

The dispersion σ^2 and the constant c necessary for spectrum calculation, *S(f)* as in equation (2) were derived in [7]:

$$\sigma^2 = p(1-p), \tag{27}$$

$$c = (1-p) m_\upsilon \cdot \tag{28}$$

Here, *p* denotes the probability a data burst occurs on the communication channel:

$$p = \lim_{T \to \infty} \frac{1}{T} \sum_{i=1}^{\infty} \upsilon_i, \tag{29}$$

and m_υ is the mean duration of data bursts. Supposing the stochastic process has the Markov´s properties, the value m_υ also means the occurrence frequency of data bursts. If the data bursts durations are the same, then $m_\upsilon = \upsilon$.

The real power spectrum of this process is:

$$S(f) = \frac{4p(1-p)^2 m_\upsilon}{1+\left[2\pi(1-p) m_\upsilon f\right]^2} + p^2 \delta(0) \cdot \tag{30}$$

To make a practical use of this power spectrum, let´s only take the continuous part of the spectrum that represents variable random changes in the process without the deterministic constant component $p^2.\delta(0)$ and let´s transform it into the probability distribution $f(\omega)$[7]:

$$f(\omega) = \frac{c}{\pi} \frac{1}{1+(c\pi)^2}, \tag{31}$$

which is the Cauchy distribution. Limiting (truncating) the frequency range $\omega \in (-\infty, \infty)$ to

$$\omega_s = \pm 2\pi f_s , \quad (32)$$

the sampling frequency, f_s can be determined:

$$f_s = \frac{1}{2\pi(1-p)m_\upsilon} tg \frac{\pi P}{2} , \quad (33)$$

This result tells us how often the communication channel shall be scanned in order to catch-up each channel occupation by a data burst with a given probability P. This is the practical issue how to utilize the spectral theory and thus save the working capacity of control processors in digital devices [7].

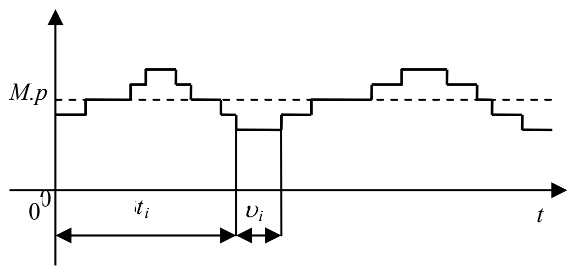
Multi-State Non-Periodic Stochastic Process

When there are M communication channels that are occupied by statistically independent traffic streams which can be data burst trains or telephone calls or, in general, various service lines, we get multi-state non periodic stochastic process with $M + 1$ state (Fig. 7). It can be shown using [5], [8], [9] that the power spectrum

$$S(f) = \frac{4p(1-p)^2 m_\upsilon}{1 + \left[\frac{2\pi(1-p)m_\upsilon}{M} f\right]^2} + M^2 p^2 \delta(0) , \quad (34)$$

and the necessary scanning frequency shall be M-times higher as before:

$$f_s = \frac{M}{2\pi(1-p)m_\upsilon} tg \frac{\pi P}{2} , \quad (35)$$



t_i, υ_i – random values
$M.p$ – mean level of the process
$M \in \langle 0, \infty)$

Fig. 7: Multi-state non periodic stochastic process.

Remark: Only 1 step up or 1 step down or no change is allowed in the multi-state non-periodic stochastic process with Markov´s properties.The power spectra corresponding to the two-state and the multi-state non periodic stochastic process with discrete amplitudes are plotted in Fig. 8a and 8b in the linear and in the logarithmic scales (without their mean level components). As the processes are non-periodic, their spectra do not have any lobes as it is evident from Fig. 8a, 8b. Also, the difference be-tween the linear and the logarithmic presentation of the spectral curves is insignificant.

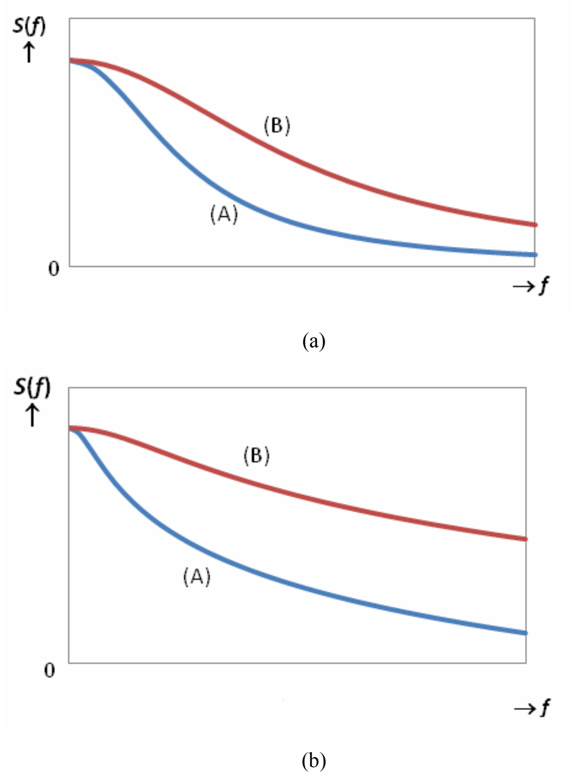


Fig. 8: Spectra of the two-state (A) and the multi-state (B) non periodic stochastic process in the linear (a) and in the logarithmic (b) scales.

- *Case with Continuous Amplitudes*

This type of processes are the most uncertain among stochastic ones (Fig. 9a) as the auto correlation function needed for spectra calculation cannot be derived explicitly and thus the spectrum cannot be determined explicitly, too. The only way how to determine the auto correlation function is to find it statistically by periodic sampling the process in regular time intervals

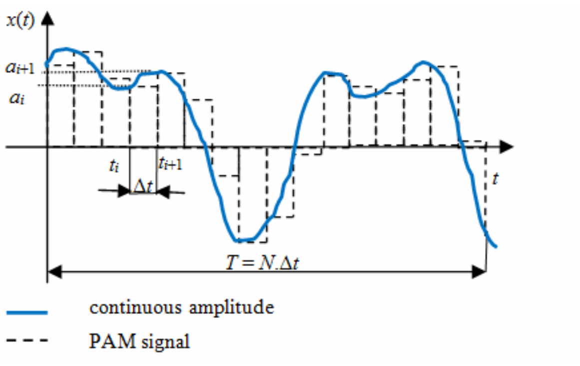
$$\Delta t = T_0 = \frac{T}{N} = t_{i+1} - t_i \rightarrow i = 0,1,2,\ldots,N , \quad (36)$$

where N is the number of samples taken within the observation time, T. In this way, a statistical file of N amplitude values $x(t_i) = a_i$ will be collected. The correlation range, N can be considered for a discrete range period. The continuous auto correlation function, $R(\tau)$ (3) will be replaced approximately by the series of correlation coefficients:

$$R(\tau) \approx R_n = \frac{1}{N+1-n} \sum_{i=0}^{N-n} a_i a_{i+n} , \quad (37)$$

for $n = 0, 1, 2, \ldots, K$ where K is the number of correlation coefficients calculated. The correlation range, N shall be as large as possible in order to collect as many of correlation coefficients, R_n as possible, whereby

$N - K \gg 0$ in order to achieve reliable values R_n, calculated from a sufficient number of the summations through $i \in \langle 0, N - n \rangle$.



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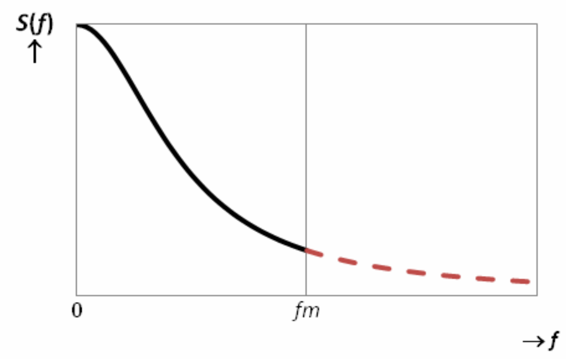

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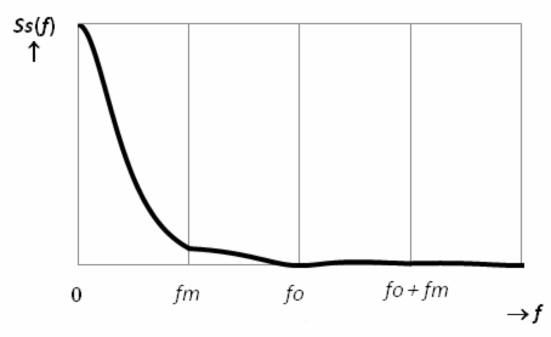

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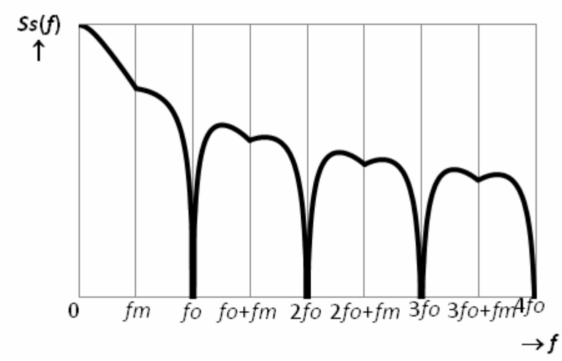


(d)

Fig. 9: Stochastic process with continuous amplitude (full line) and its sampled PAM equivalent (dashed lines) (a); spectrum of the original process in the linear scale (b); spectrum of the sampled process in the linear (c) and the logarithmic (d) scales (without the mean level component).

The length of time intervals Δt shall be chosen according to the frequency of changes of the stochastic process. When these changes are relatively slow, the interval Δt can be longer, in opposite to the short interval when the stochastic process changes often and quickly. The more the waves creating the stochastic process have high frequency components, the more often the sampling of the stochastic process shall be. The Shannon theorem may be used as a guideline for the time interval Δt:

$$\Delta t = T_0 \leq \frac{1}{2 f_m}, \tag{38}$$

where f_m denotes the highest assumed frequency component occurring in a stochastic process.

On the other hand, the sampling frequency does not often depend on an observer. It can be governed by the timing of an observation instrument, or it directly yields from the stochastic process nature when it performs a time series.

There are two ways how to obtain the spectrum for this stochastic process type:

The first way is to explicitly presuppose the auto correlation function exponentially decreases. Approximating the calculated correlation series consisting of coefficients R_n by the exponential function, the constant c can be determined which is necessary for the spectrum calculation through Wiener Kchintchin transform.

The second way is to apply the discrete Fourier transform on the before calculated correlation series:

$$S(k) = \frac{1}{K+1} \sum_{n=0}^{K} R_n e^{-j2\pi \frac{n}{N} k} + m_a^2 \delta(0), \tag{39}$$

This term can be rewritten as [10]:

$$S(k) = \sigma^2 \left| A(z) \right|^2 = \sigma^2 \left| a_z + a_{1z}^{-1} + a_{2z}^{-2} + \ldots + a_{Kz}^{-K} \right|^2, \tag{40}$$

for $k = 0, 1, 2, \ldots, N$ where

$$z = e^{j\frac{2\pi}{N}k} \rightarrow k = 0, 1, 2, \ldots, N. \tag{41}$$

This method is more complicated than the first one. It requires many computation operations. The fast Fourier transform could be used [11].

If a sampled value is held-on until the next sample, the spectrum of the almost periodic stochastic process with correlated periods and continuous in amplitude (PAM signal) will be obtained.

The discussed stochastic processes and their spectra are depicted in Fig. 9. The full line on Fig. 9a represents the stochastic process itself and the dashed lines represent its sampled variant. Figure 9b shows the spectrum of the original process (in the linear scale) which is truncated by frequency f_m. The amplitude spectrum of a continuous process sampled with sampling pulses with width υ and with repeating frequency

$f_o = 1/T_o$ is known as the sampling of the 2nd type and it is generally given as [12]:

$$\left|S_s(f)\right| = \frac{\upsilon}{T_0}\left|\frac{\sin(\pi\upsilon f)}{\pi\upsilon f}\right|\left[\left|S(f)\right| + \sum_{n=1}^{\infty}\left|S(f \pm nf_0)\right|\right] . \quad (42)$$

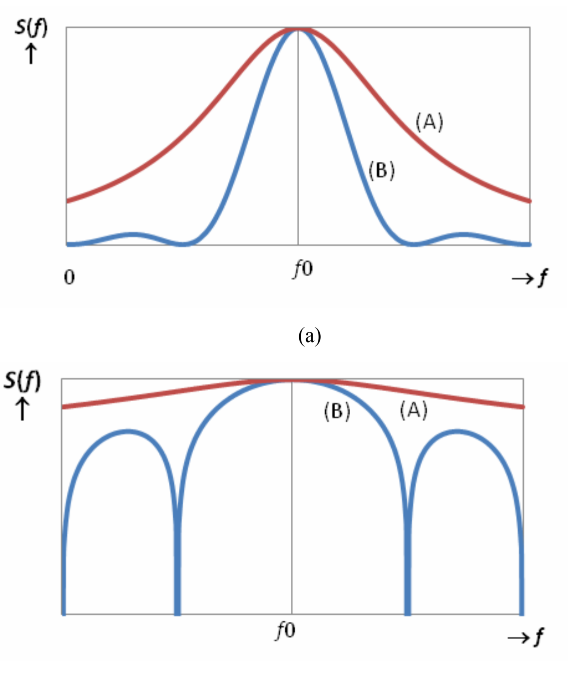
For this case when $\upsilon = T_o = 1/f_o$ and $f_o = 2f_m$ the real power spectrum will be:

$$S_s(f) = \frac{4}{3}\sigma_a^2 c A^2 T_0 \left(\frac{\sin\frac{\pi f}{f_0}}{\frac{\pi f}{f_0}}\right)^2 \left\{\frac{1}{1+(2\pi c f)^2} + \right. . \quad (43)$$

$$\left. + \sum_{n=1}^{\infty}\frac{1}{1+\left[2\pi c(f \pm nf_0)\right]^2} + m_a^2\delta(0)\right\}$$

The spectrum of the sampled process contains the basic spectral component which is almost the same as the spectrum of the original continuous process. Moreover, the basic spectral component is augmented in other spectral components that are regularly spread around the multiplies of sampling frequency f_o which can be defined by means of the Shannon theorem. The height of these components is determined by the sinc^2x function (see Fig. 9c and 9d). The sampling introduces a non-random component (originally non periodic stochastic process becomes almost periodic one). Therefore, the original smooth spectral curve becomes undulated

3.2. Band Pass Stochastic Processes



(a)

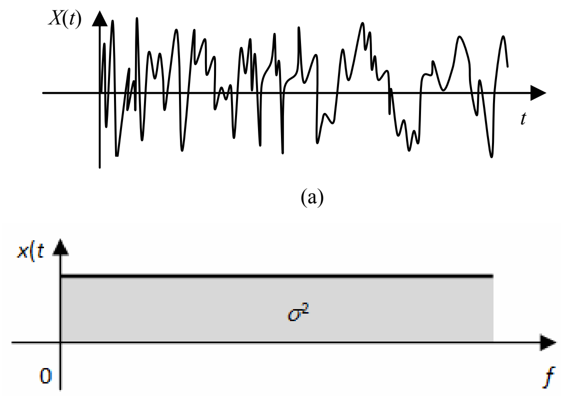
(b)

Fig. 10: Spectrum of a band pass modulated analogue signal (A) and a digital modulated signal (B) in the linear (a) and in the logarithmic (b) scales.

This type of processes is exclusively represented by keyed (periodic) or analogue (non-periodic) signals modulated on a carrier frequency f_0. It can be shown [2], [3] that a spectrum of a band pass signal is the shifted version of the equivalent baseband signal (Fig. 10).

4. Random Processes

Real random processes do not show any deterministic or statistical laws. Such process is the white noise. The white noise is an example of the process that has the right to be called the random process because it does not contain any apparent or hidden deterministic component and it has no correlation among its values in various time instants, even between closest ones. The future behaviour of such process is absolutely unpredictable. The spectrum is smooth and constant in the whole frequency range in a given bandwidth.



(a)

(b)

Fig. 11: White noise (a) and its spectrum (b).

5. Summary

The next general conclusions can be formulated for spectra recognition, estimation and evaluation:

The random process without any correlation and without any non-random (deterministic) component has the smooth, constant spectrum in the whole frequency range (white noise). The more the spectrum of a process approaches this, the less stochastic and the more random a process is.

The stochastic process with correlation and without any non-random (deterministic) component has the smooth, to zero continuously decreasing spectrum like in Fig. 8.

The stochastic process with a non-random (deterministic) component may have an undulated (waving) spectrum.

Whatever type of a stochastic process would be, the presence of spectral lobes points on its periodicity.

The presence of the mean value component in a process may manifest as periodically repeating peaks. However, there is always the peak at the zero spectral frequency.

When the process contains a periodic component, the peaks may also occur at certain discrete spectral frequencies.

All random and stochastic processes have the continuous spectra on which peaks can be superposed when they have the mean level and/or they contain a deterministic periodic component.

Deterministic periodic processes have discrete spectra. When a deterministic process is not periodic, its energetic spectrum is continuous.

The presence of a hidden or apparent deterministic component in a stochastic process, or no correlation coupling among periods in an almost periodic stochastic process makes the spectrum calculation easier.

Knowledge of spectra in theory of mass servicing makes possible to find out how often service (communication) channels shall be scanned in order to catch-up each channel occupation with a given probability. This may be utilized when saving the working capacity of control processors in digital devices.

The correlation function is useful for estimation of features of such stochastic processes at which the spectrum cannot be physically observed (all non-physical processes).

Knowledge of signal spectra is very important in telecommunication at spectral compatibility assessment of various transmission technologies used for signal transmissions through metallic cable networks and for identification of disturbing sources in radio communication environment.

REFERENCES

[1] STEWART, Ian. *Hraje Buh kostky?: Nova matematika chaosu*. Praha: Argo, 2009. ISBN 978-80-257-0024-2.

[2] XIONG, Fuqin. *Digital Modulation Techniques*. Norwood: Artech House Print on Demand, 2006. ISBN 978-1580538633.

[3] CEPCIANSKY, Gustav and Martin VACULIK. *Spektralna analyza digitalnych signalov*. Zilina: EDIS - vydavatelstvo ZU, 2005. ISBN 80-8070-406-6.

[4] CEPCIANSKY, Gustav and Ladislav SCHWARTZ. Methods of Calculation of Digital Signals Spectra. *Advances in Electrical and Electronic Engineering*. 2011, vol. 9, no. 3, pp. 118-126. ISSN 1336-1376.

[5] VENTCEL, Jelena Sergejevna. Teoria pravdepodobnosti. Bratislava: Alfa, 1973.

[6] CEPCIANSKY, Gustav and Ladislav SCHWARTZ. *Technologia GPON*. Zilina: EDIS, 2011. ISBN 978-80-554-0323-6.

[7] CEPCIANSKY, Gustav and Ladislav SCHWARTZ. Sampling of Random Data Streams. *Advances in Electrical and Electronic Engineering*. 2011, vol. 9, no. 1, pp. 1-6. ISSN 1336-1376.

[8] SKRASEK, J. and Z. TICHY. *Zaklady aplikovane matematiky*. Praha: SNTL, 1986. ISBN 0451386.

[9] GRESAK, Pavol and Pavol MARUSIAK. *Matematika III: zaklady funkcie komplexnej premenej, integralne transformacie*. Bratislava: Alfa, 1987.

[10] BOX, G. E. P., and G. M. JENKINS. *Time Series Analysis: Forecasting and Control*. San Francisco, CA: Holden-Day, 1970. ISBN 978-0470272848.

[11] KOTULIAKOVA, Jana and Gregor ROZINAJ. *Cislicove spracovanie signalov*. Bratislava: FABER, 1996. ISBN 80-967503-2-1.

[12] SKOP, Miroslav. *Prenosove systemy s casovym sdruzovanim kanalu*. Praha: Nakladatelstvi dopravy a spoju, 1980.

[13] LEVIN, B. R.: *Teorie nahodnych procesu a jeji aplikace v radiotechnice*. Praha: SNTL, 1965.

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Gustav CEPCIANSKY was born on 1947. 1971 – Master of sciences (M.Sc.) in telecommunications on University of Zilina. 1971 – 1976 – employed at Slovak Telecom as a technical development specialist. 1976 – 2002 – employed at Slovak Telecom as the head of the operational research center. Actually as an external lecturer on the Telecommunication and Multimedia Department of the Electrical Faculty, University of Zilina. Next formation: 1984 – the 1st doctor degree (CSc). 1995 – the study stay at Technical University Supelec in Paris. 1998 – the 2nd doctor degree (Ph.D.). In 2005 – the associated professor of Electrical Faculty, University of Zilina.

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