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Alexander E. Saak, Jeffrey M. Peterson

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Alexander E. Saak

Markets, Trade, and Institutions Division
International Food Policy Research Institute
2033 K Street, NW
Washington, DC 20006-1002 USA
Phone: 202 862-4543
Fax: 202 467-4439
E-mail address: asaak@cgiar.org

Jeffrey M. Peterson

Department of Agricultural Economics
Kansas State University
216 Waters Hall
Manhattan, KS 66506-4011, U.S.A.
Phone: 785 477-0924
Fax: 785 532-6925
E-mail address: jpeters@agecon.ksu.edu

Abstract

Farm size is a significant determinant of both groundwater irrigated farm acreage and groundwater irrigation application rates per unit land area. This paper analyzes the patterns of groundwater exploitation when resource users in the area overlying a common aquifer are heterogeneous. In the presence of user heterogeneity, the common resource problem consists of inefficient dynamic and spatial allocation of groundwater because it impacts income distribution not only across periods but also across farmers. Under competitive allocation, smaller farmers pump groundwater faster if farmers have a constant marginal periodic utility of income. However, it is possible that larger farmers pump faster if the Arrow-Pratt coefficient of relative risk-aversion is sufficiently decreasing in income. A greater farm-size inequality may either moderate or amplify income inequality among farmers. Its effect on welfare depends on the curvature properties of the agricultural output function and the farmer utility of income. Also, it is shown that a flat-rate quota policy that limits the quantity of groundwater extraction per unit land area may have unintended consequences for the income distribution among farmers.

Keywords: agriculture, conceptual models, groundwater management

1. Introduction

Theoretical models of groundwater extraction typically assume that the resource is non-exclusive or that the resource users are identical. This, along with the assumption of instantaneous interseasonal transmissivity, simplifies the analysis because there exists a representative user. However, this approach does not take into account the spatial distribution of users, and the dependence of individual groundwater stocks on the history of past extractions (Brozovic et al 2003, Koundouri 2004). Recently, some authors have taken into account the spatial variability in groundwater use, either by relaxing the assumption of instantaneous lateral flows (e.g., Brozovic et al. 2010) or by introducing spatial heterogeneity in the marginal value of resource use (e.g., Gaudet et al. 2001, Xabadia et al. 2004).

This article addresses another source of heterogeneity, that of variation in the size of the land area from which each user can access the resource. This is an important issue because irrigated agriculture, one of the major consumers of groundwater, is comprised of farms of widely varying sizes (Schaible 2004; Hoppe et al. 2010). Knapp and Vaux (1982), Feinerman (1988), Foster and Rosenzweig (2008), and Sekhri (2011) are among the few studies addressing variation in farm size or in pumping volume.

It is well known that, to the extent that groundwater is a common property resource, private decisions lead to inefficient allocation. This result holds unless the aquifer is relatively large in comparison to total groundwater use, users can cooperate, or hydraulic conductivities are so small that the resource is effectively private (Feinerman and Knapp 1983). However, it is not clear whether heterogeneity in farm size alleviates or exacerbates the so-called ‘tragedy of the commons’ (Hardin 1968). To the extent such effects are present, there are potentially important policy implications, because redistributive policies will then interact with policies to correct the common property externalities: policies targeting one of these domains may have unintended impacts in the other.

To understand the presence and nature of any such interactions, the following questions are posed in this article: What are the determinants of the relationship between farm size and groundwater use intensity? How does the distribution of farm sizes in the area influence the efficiency of groundwater allocation? What are the distributional impacts of farmland ownership structure and water management policies? To analyze these questions a two-period model is developed where land above an aquifer, all of which can be irrigated but is of undifferentiated quality, is gathered into farms of unequal size. The differences in pumping rates across farms of different sizes in this framework are entirely due to an endogenous interaction between common property effects and farm size inequalities.

For both methodological and policy reasons, it is helpful to distinguish between the cases where farmers' utility-of-income functions are linear and where they are concave. In the first case, marginal utility of income is constant, which is an appropriate representation of cases where small farmers supplement their incomes with off-farm sources (e.g., off farm employment of some household members). Even if the underlying utility functions are concave, in these cases there is no inherent reason that small farmers have smaller incomes than (or a marginal utility of income that differs from) large farmers. The second case presumes that income from irrigated farming activities are the sole source of income, which is more appropriate for many developing country contexts. As small farms have a smaller capacity to generate income, they have a higher marginal utility of income that raises the stakes of the tradeoffs in allocating water across farmers and across periods.

Linear utility is a helpful starting point because in that case farm size inequality, in itself, does not affect average utility (equivalently, it has no direct effect on total utility, which is taken here to be the measure of social welfare). However, as shown below, the common property nature of the resource creates differing incentives to pump water across size classes, so that an increase in inequality may either amplify or moderate the common property externalities and social welfare may either rise or fall.

In the linear utility case, the basic intuition is that large farms have greater spatial extent of resource access or "ownership," so that they perceive the resource as being more private. By the same token, a small farmer effectively owns a smaller share of the aquifer, and perceives groundwater as a more common resource. Therefore, smaller farmers tend to pump faster. In the aggregate, more water is always withdrawn in the first period compared to the efficient solution (the tragedy of the commons still applies), but the magnitude of overpumping depends on the inequality in land holdings. In an alternative distribution of farm sizes with greater inequality, aggregate pumping in the first period may change in either direction depending on the nature of the change in the distribution. Aggregate withdrawals increase if land area is shifted towards small farmers, but the converse holds if acreage is shifted towards large farmers. The direction of the change is shown to depend on specific curvature properties of the production function relating agricultural output to irrigation.

A separate but related question is how greater inequality in farm sizes affects social welfare. The model reveals that there are *dynamic* as well as *spatial* components determining this effect. The dynamic component refers to the effect of farm-size inequality on aggregate withdrawals in the first period, or the speed with which the aquifer is depleted. The spatial component refers to the effect of farm-size inequality on the distribution of pumping rates and income across farmers in each period. The direction of the overall effect depends on the magnitude and direction of both these components, which are determined by additional curvature conditions on the production function.

Sufficient conditions are derived that identify the cases where an increase in inequality leads to a reduction in social welfare. These conditions are quite restrictive, requiring specific curvature properties of the production function, suggesting that there are many cases where inequality is not welfare reducing. Indeed, in many cases inequality may actually raise social welfare because it dampens the tragedy of the commons problem. Moreover, as illustrated with a numerical example, greater farm-size inequality may imply *less* income inequality. This is because of an effect similar to that identified by Foster and Rozensweig (2008): smaller farmers have a *strategic* advantage as they are able to poach more groundwater per unit land than their larger neighbors.

When utility is concave, the analysis has another layer of complexity. The pure income redistribution effect of the land ownership structure, keeping the allocation of groundwater fixed, must be disentangled from its effects on the equilibrium average pumping rate and the spatial distribution of groundwater withdrawals across farmers. Here, it is possible that small farmers actually pump less in the first period than large famers. This will occur if the utility functions are "sufficiently" concave, so that small farmers (who have lower incomes) face a greater differential between marginal utilities of present and future income, and therefore, have a greater incentive to save groundwater for future use. With this as an additional determinant of pumping rates, the results discussed above continue to apply, however.

This paper may contribute to the continuing debate on the magnitude of the welfare difference between optimal control rules and competitive outcomes (Gisser 1983, Gisser and Sanchez 1980, Koundouri 2004). Provencher and Burt (1993) identify three sources of inefficiency associated with groundwater use in agriculture: stock, pumping cost, and risk externalities. In the presence of user heterogeneity, an *access inequality* externality is added to this list. The access inequality externality arises when the rates of groundwater extraction differ across farms of varying size overlying a common aquifer. This externality can be both positive and negative, depending on whether smaller farms appropriate, on a per unit land area basis, a greater share of the common resource. Small and large farmers can be thought of as, respectively, low and high income groups. And so, a common resource such as groundwater may become a natural vehicle for income transfer, and can either *neutralize* or *amplify* income inequality caused by the inequality in farmland holdings.

This paper also analyzes the effects of a specific but commonly implemented water management policy, namely pumping quotas, on the distribution of income across farm size classes. Using an example of a flat-rate quota policy, policy-induced gains and losses are shown to be unequally distributed across farmers. In general, the results suggest that the interactions between policies addressing farmland ownership structures and groundwater management should not be ignored. An effort to reduce inequities may worsen the common property problem, while efforts to reduce the common property problem may cause greater inequities. Of course, the directions of these impacts may be the opposite so that the policies are mutually reinforcing. However, careful empirical analysis that differentiates farmers' production relationships across size classes (e.g., Sekhri 2011) is required to determine the nature of the interactions

1.1 Literature Review

Knapp and Vaux (1982) and Feinerman (1988) are among the few studies that consider equity and distributional effects of groundwater management schemes. Knapp and Vaux (1982) consider groups of farmers differentiated by their derived demand for water, and present an empirical example that demonstrates that some users may suffer substantial losses from quota allocation policies even though the group as a whole benefits. Feinerman (1988) extends their analysis and considers a variety of management tools including pump taxes, quotas, subsidies, and markets for water rights. Using simulations calibrated to Kern County, California (USA), Feinerman concludes that while the welfare distributional effects on user groups may be substantial, the negotiations between the policy-makers and the users are likely to be difficult because the attractiveness of policies varies across users and is sensitive to the parameters. However, following Gisser and Sanchez (1980), these studies ignore the stock externality, and assume that under competition users behave myopically and base their decisions solely on the consideration of their immediate (periodic) profits. Also, there is no investigation of the effect of the extent of user heterogeneity on the properties of competitive allocation.

There is a rather thin literature base in development economics that is concerned with the effect of inequality in land holdings on groundwater exploitation. Motivated by the role of groundwater in sustaining the Green revolution and developing agrarian economies, Foster and Rosenzweig (2008) consider the patterns of groundwater extraction in rural India. They develop a dynamic model of groundwater extraction that captures the relationships between growth in agricultural productivity, the distribution of land ownership, water table depth, and tubewell failure. Using data on household irrigation assets including tubewell depth as a proxy for irrigation intensity, they find that large landowners are more likely to construct tubewells, but their tubewells tend to be less deep than those dug by smaller landowners. Foster and Rosenzweig conclude that this is indicative of a free-riding effect in the sense that large farmers are less able to effectively poach the water from neighboring farmers by lowering the water-table under their own lands. They also find evidence of land consolidation as a way to improve efficiency of groundwater exploitation.

This paper captures some of the same effects through a simple model where wells of equal depth are already in place and each farmer faces an irrigation application rate decision. A two-period framework with a “quasi-bathtub” aquifer is particularly well suited to fully work out the equilibrium effects of farm-size inequality on the welfare difference between the competitive and efficient allocations. By assuming an initial stock that is scarce enough to impose tradeoffs between the two periods, both the pumping cost externality and stock externality naturally arise in the model, which are then either amplified or moderated by the farm size inequalities. The pumping cost and stock externalities are the costs that one user imposes on others through higher future pumping costs and reduced groundwater availability, respectively. Following Gisser and Sanchez (1980), groundwater economic studies in multiperiod settings typically consider only the pumping cost externality; Provencher and Burt (1993, 1994) are notable exceptions.

Given the seasonality of production in irrigated agriculture, a groundwater resource can be regarded as a “quasi-bathtub” with features of a common property resource over time. The quasi-bathtub property means that the resource at each extraction point is private within each period, but the aquifer becomes a “bathtub” or purely common pool across periods. This happens when the time period during which groundwater is extracted is relatively short, and does not allow for seepage from one point in the aquifer (such as a well or a pool) to another. However, the water level tends to be more uniform throughout the aquifer in the long run. The quasi-bathtub assumpition is appropriate if (a) the irrigation season is considerably shorter than the time that elapses between the two seasons, and (b) wells are spaced so that the localized cones of depression caused by pumping from neighboring wells do not overlap within each irrigation season.

The analysis also assume no time discounting, although farmers’ time preferences of income are captured in the concave utility model. These assumptions ensure that the results are not an artifact of any other source of spatial or temporal heterogeneity other than that introduced by size inequality. However, the main insights and policy implications obtained in this framework carry on to more realistic settings.

From here, the paper presents a simple two-period model of groundwater extraction in the presence of farm-size heterogeneity. The social planner’s solution is considered. Then the paper analyzes the equilibrium allocation and the effect of farm-size inequality on the pumping rates and farm income when farmers' marginal periodic utility of income is constant. Consideration is given to equilibrium allocation when farmers' marginal periodic utility of income is decreasing. Lastly, before the conclusions, consideration is given to a flat-rate quota policy that illustrates political economy issues that arise in the presence of user heterogeneity.

2. Model

For simplicity, the model focuses on the stock, cost, and access inequality externalities. It considers the decisions of water application per acre taking the distribution of irrigated acres across farmers as exogenous. With slight modifications, the model can be extended to include decisions about the share of farm acreage allocated to irrigated crops. Farmers are identical except for the distribution of land ownership, and irrigation technology is constant returns to scale. All profits are derived from agricultural outputs using groundwater for irrigation on a fixed land area, and farmers hold exclusive pumping rights on their land. The individual groundwater stocks are private during each irrigation season because there is no *intra-seasonal* well interference. However, the groundwater is an *inter-seasonal* common property resource based on the groundwater hydrology over a longer time interval. The following assumptions are standard (e.g., Negri 1989):

1. (**Fixed land ownership**) The distribution of farmland ownership does not change over time.
2. (**Constant returns to scale and homogenous land quality**) The agricultural production function has the property of constant returns to scale (output is proportional to farm size). Land quality is identical across all farms. Inputs other

than groundwater, including the choice of irrigation technology, fertilizer, crops, etc., are optimized conditional on the rate of water extraction. Output and input prices, including energy costs, are exogenous.

3. (**Pumping cost**) The total cost of groundwater extraction per acre increases with the pumping rate and decreases with the level of the water table (or the stock of groundwater).
4. (**User location is irrelevant**) The aquifer is confined, non-rechargeable, homogenous, and isotropic. The groundwater basin has parallel sides with a flat bottom.
5. (**Quasi-bathtub**) There are no intra-seasonal lateral flows of groundwater across farms. However, inter-seasonal changes in groundwater level are transmitted instantaneously to all users (i.e., the groundwater has an infinite rate of transmissivity during the time elapsed from one irrigation season until next). Brozovic et al (2003) provide a detailed discussion of the consequences of this assumption.
6. (**Two periods**) There are only two periods (irrigation seasons), and farmer preferences over income are additively separable across periods.

Provencher and Burt (1994) and Saak and Peterson (2007) also consider and provide justifications for a two-period framework. The assumption that the aquifer is non-renewable is for expositional convenience, and a positive rate of recharge can be easily incorporated. The groundwater extractions are the net quantity of water withdrawn if some fraction of the water percolates back to the stock. Next the model notation is introduced.

2.1 Aquifer

The total stock of groundwater stored in the aquifer in the beginning of period 1 is $x_1 = Ah_1$, where h_1 is the height of the water table in period 1, and A is the size of the area measured in acres (1 acre = 0.4047 ha). Let $L = \{1,..., A\}$ denote the set of acres. The hydraulic heads of the water table under each acre are the same in the beginning of each period, $h_{i,t} = h_{j,t} = h_t \ \forall i, j \in L$ and $t = 1,2$. Let $u_{i,t}$ denote the quantity of groundwater applied in period t on acre i. By the quasi-bathtub assumption, the per acre quantity of groundwater withdrawn in each period cannot exceed the per acre stock or h_t

$$u_{i,t} \le h_t \text{ for all } i \in L \text{ and } t = 1,2. \tag{1}$$

Let $u_1 = A^{-1}\sum_{i=1}^{A} u_{i,1}$ denote the average pumping in period 1. Since there is no recharge, the stock of groundwater in the aquifer in period 2 is $x_2 = x_1 - Au_1$, and the level of the water table is

$$h_2 = h_1 - u_1. \tag{2}$$

2.2 Land ownership

There are n farmers (users of groundwater) who are located in the area overlying the aquifer and grow irrigated crops. Farmer k farms acres $L_k \subseteq L$, and let $A_k = |L_k|$ denote the number of irrigable acres owned by farmer k, where $\sum_{k=1}^{n} A_k = A$. In what follows, the set of acres L_k will be referred to as "farm k" or "farmer k". For concreteness, the farm indices are assumed to be ordered by farm size, $A_1 \le A_2 \le ... \le A_n$. Throughout, the first symbol in doubly subscripted variables identifies the acre and the second identifies the period, $t = 1,2$. Variables with one subscript typically refer to the aggregate values in the specified period, unless they are farm-specific and invariant across periods. The letters i, j will index acres, and letters k, l will index farmers.

2.3 Production technology

The periodic per acre benefit of water consumption net of all costs including groundwater pumping cost is

$$g(u_{i,t}, h_t), \tag{3}$$

where g is strictly increasing and concave. While irrigation increases yield, a higher groundwater stock decreases the cost of pumping due to a decrease in pumping lift, and increases the efficiency of irrigation by permitting a more flexible application schedule. Land quality is assumed to be homogeneous so that total farm income is proportional to farm size (i.e., technology exhibits constant returns to spatial scale). For simplicity, the rainfall and surface water supply are the same on all farms in both periods. For example, (3) can take the following form:

$$g(u,h) = \max_z \; py(u,h,z) - c(u,h) - qz,$$

where p is the per unit price of the crop, y is yield, and c is the cost of pumping groundwater, z is the vector of other inputs, and q is the price vector of other inputs. For notational convenience, let

$$f(h) = g_u(h,h) + g_h(h,h) \tag{4}$$

denote the marginal per acre benefit of water consumption evaluated at the point of depletion of an individual groundwater stock. (Here and throughout, subscripts on functions denote differentiation with respect to the lettered arguments.) By concavity of g, $f'(h) < 0 \;\; \forall h \in (0, h_1)$. All of the results that follow will also hold under weaker technical conditions, namely $g_{uu} < 0$, $g_{hh} < 0$, and $f'(h) = g_{uu}(h,h) + g_{hh}(h,h) - 2g_{uh}(h,h) < 0$, which are implied by concavity of g.

Let v denote the periodic utility of farm income, $v' > 0, v'' \le 0$. Each farmer maximizes the sum of utilities of the whole-farm revenue in each period:

$$\pi_k = \max_{\{u_{i,t}\}_{i \in L_k}} \sum_{t=1,2} v\left(\sum_{i \in L_k} g(u_{i,t}, h_t)\right) \text{ subject to (1) and (2).} \tag{5}$$

For simplicity, there is no discounting of future income.

3. Social planner

Before turning to the analysis of the competitive allocation by non-cooperating users, the efficient allocation is first characterized. The social planner chooses $\{u^s_{i,t}\}$ to maximize producer welfare conditional on the land ownership distribution:

$$W^s = \max_{\{u^s_{i,t}\}} \sum_{t=1,2} \sum_{k=1}^{n} v\left(\sum_{i \in L_k} g(u^s_{i,t}, h_t)\right) \text{ subject to (1) and (2).} \tag{6}$$

The following result shows that the efficient allocation of groundwater compensates for income inequality caused by the inequality in farm sizes. The common resource may serve as a vehicle to decrease income inequality by redistributing income from larger farmers to smaller farmers. This effect is absent if either farm sizes are identical, or farmers' periodic utility functions are linear in income. Note that optimal groundwater consumption in the final period exhausts the remaining stock on each farm, and hence, must be identical on all acres, $u^s_{i,2} = u^s_{j,2} = h_2 \;\; \forall i, j \in L$, because the income utility and water benefit functions are strictly increasing. And so, the focus is solely on period 1 pumping. All proofs that are not in the text are in the Appendix.

Proposition 1. (Efficient pumping) *Efficient allocation of groundwater is*

a) invariant across acres, $u^s_{i,1} = u^s_{j,1} \;\; \forall i, j \in L$ *, and is determined by*

$$g_u(u^s_{i,1}, h_1) - f(h_1 - u^s_{i,1}) = 0, \qquad (7)$$

if either farmers have linear utility, $v'' = 0$ *, or acreage is uniformly distributed across farmers,* $A_k = A/n$ for $k = 1,...,n$;

(b) characterized by smaller farmers pumping groundwater faster, $u^s_{j,1} \geq u^s_{i,1}$, for $j \in L_k$, $i \in L_l$, $k < l$, *if* $v'' \leq 0$ *(decreasing marginal utility of income).*

(7) is easiest to interpret for the special case when the water benefit depends only on water use, *u*. In this case, it is efficient to equalize the marginal benefits of water use in the two periods: $g_u(u^s_{i,1}) = g_u(h_1 - u^s_{i,1})$, which implies that $u^s_{i,1} = h_1/2 \;\; \forall i \in L$. This is equivalent to the assertion that, in the absence of a pumping cost externality and inequality of income across farmers, the efficient solution distributes the available water equally across the two periods on each farm.

It is convenient to differentiate between the case when farmers' per period marginal utility of income is (1) constant (i.e., utility is linear), and (2) decreasing (i.e., utility is concave). In the former case, from the social planner's point of view, a non-uniform distribution of acreage across farmers has no effect on either the optimal allocation of water either spatially or temporally. However, as demonstrated in the next section, such differences may still arise in competitive equilibrium. In the latter case, as is demonstrated in Part (b) of Proposition 1, the social planner faces a trade-off between dynamic and distributional sources of inefficiencies.

From a policy perspective, an important insight of the analysis to follow is that, in the presence of farmer heterogeneity, competitive allocations go beyond the *tragedy of the commons,* and affect *income inequality* as well. The welfare difference between the optimal and competitive allocations may be particularly large, when, from the societal point of view, the income distribution matters. This happens when the equilibrium distribution of pumping rates across heterogeneous farmers *amplifies* the income inequality caused by size inequality. However, the competitive allocation may also *moderate* the inherent inequality in income distribution caused by the inequality in land ownership, or even change its sign, whereas total incomes over two periods earned by smaller farmers exceed that of larger ones.

4. Linear utility

This section considers the case of linear utility functions, $v'' = 0$. The competitive equilibrium is first characterized, followed by an analysis of the effect of inequality in farm sizes on the groundwater stock and the distribution of income.

4.1. Equilibrium

Farmers are non-cooperative, and each farmer takes the quantity of water pumped by others in each period as given. In period 2, all farmers exhaust the available stocks of groundwater on each acre, so that $u_{i,2}^{*} = h_2$ for $\forall i \in L$. By (5), in period 1 farmer k's payoff is

$$\pi_k = \max_{\{u_{i,1}\}_{i\in L_k}} \sum_{i\in L_k} g(u_{i,1},h_1) + g(h_2,h_2) \text{ subject to (1) and (2).} \tag{8}$$

The competitive allocation can now be characterized. Differentiating (8), the best response by farmer k on acre $i \in L_k$, $u^*_{i,1}$, satisfies

$$g_u(u^*_{i,1},x) - a_k f(h_2) = 0 \text{, if } u^*_{i,1} \le h_1 \text{, and } u^*_{i,1} = h_1 \text{, if otherwise} \tag{9}$$

where $a_k = A_k / A$ is the share of the aquifer that can be captured by farmer k. (9) can be written in a more compact form

$$u^*_{i,1} = \min[h_1,\, g_u^{-1}(a_k f(h_2);h_1)], \quad \forall i \in L_k \tag{10}$$

where $g_u^{-1}(.;h)$ is the inverse of $g_u(u,h)$ obtained by treating h as a parameter. Note that per acre pumping rates on each farm are identical $u^*_{i,1} = u^*_{j,1}$ $\forall i, j \in L_k$. Summing pumping rates (10) over all $k = 1,...,n$ and $i \in L_k$, and substituting (2), yields

$$u^*_1 = \sum_{k=1}^n a_k \min[h_1,\, g_u^{-1}(a_k f(h_1 - u^*_1);h_1)], \tag{11}$$

where $u^*_1 = (1/A)\sum_{i=1}^A u^*_{i,1}$ is the equilibrium average pumping in period 1. By concavity of g, (11) uniquely determines the aggregate pumping in period 1, u^*_1. Together (10) and (11) prove the existence and uniqueness of equilibrium.

Proposition 2. (Competitive allocation) *Suppose that farmers' utility is linear in income. Competitive equilibrium exists, it is unique, and is given by (10) and (11). The average pumping rate is higher than the socially efficient average rate,* $u^*_1 \ge u^s_1$. *Also, smaller farmers pump faster than larger farmers,* $u^*_{i,1} \ge u^*_{j,1}$, for any $i \in L_k$, $j \in L_l$, $k < l$.

Comparing the first-order conditions that characterize the efficient and competitive allocations, (7) and (9), respectively, shows that the discrepancy between them arises along both *spatial* and *temporal* dimensions. That is, the competitive allocation leads to an inefficiently high *aggregate* pumping in period 1, which entails an inefficient allocation of groundwater across periods. Nonetheless, it is possible that *individual* farmers extract groundwater at a *slower* rate than the socially efficient average rate, i.e. $u_{i,1}^{*} \leq u_1^{s}$ for some i (see Section ***Small and large farms: an example*** and Figure 1b). Also, unless all farmers are identical, the competitive allocation results in inefficient pumping rates *across* farmers in period 1. Recall that, by Proposition 1(a), efficiency requires that the per acre irrigation application rates be identical when farmers have linear utility.

Under linear utility, smaller farmers always deviate more from the socially efficient allocation. However, it is not clear whether the non-uniformity of the distribution of land ownership, in and of itself, leads to a loss or gain of total farm income. As shown in the next section, the effects of the inequality in farm sizes on the groundwater stock and farm income depend on rather subtle properties of the agricultural production function.

4.2. Inequality in farm sizes

The measure of inequality that is used to model an increase in the concentration of land ownership (a smaller share of farmers owns a larger share of land) is introduced next. The rest of this section analyzes the effect of inequality in farm sizes on the remaining groundwater stock and on total income. An example is presented that illustrates the findings.

4.2.1. Measuring inequality

To model the effect of increased inequality in land holdings a precise measure of inequality is needed. The analysis here relies on the Lorenz measure, which is widely used to measure wealth inequality more generally. Let $\overrightarrow{W}$ = (W_1, ..., W_n) denote a vector of wealth (in this paper, wealth is measured by the area of land owned) by *n* individuals, where $W_1 \leq W_2 \leq ... \leq W_n$ and $\sum_{k=1}^{n} W_k = W$. The Lorenz measure of $\overrightarrow{W}$ is defined as $\lambda(l/n, \overrightarrow{W}) = \sum_{k=1}^{l} W_k \big/ W$; its interpretation is the share of land held by the smallest $100(l/n)$ percent of farmers. If $\overrightarrow{W}$ is a perfectly equal wealth distribution (i.e., $W_k = W/n \ \forall k$), then the Lorenz function is linear in $x = l/n$ with a slope of 1; for all other distributions it is a (weakly) convex curve that never lies above this line. In general, increasing inequality implies more curvature of the Lorenz curve, so that the value of λ at a given value of *x* will be smaller.

The effect of inequality in farm size is modeled by comparing the equilibrium under the given distribution of land holdings, $A_1 \leq A_2 \leq ... \leq A_n$, to an alternative distribution, $B_1 \leq B_2 \leq ... \leq B_n$ ($\sum_{k=1}^{n} B_k = A$). Where distribution $\vec{B}$ is more unequal distribution $\vec{A}$ based on the Lorenz measure: $\lambda(l/n, \vec{A}) \geq \lambda(l/n, \vec{B}) \ \forall l = 1, \ldots, n$. The proofs of several of the propositions below rely on the majorization order, a general tool to compare the dissimilarity within the components of vectors that is closely related to the Lorenz measure. Marshall and Olkin (1979) provide a comprehensive treatment of majorization.

Definition. Real vector $\vec{A}$ is majorized by $\vec{B}$, denoted $\vec{A} \leq^m \vec{B}$, if $\sum_{k=1}^{l} A_k \geq \sum_{k=1}^{l} B_k$ for $l = 1,...,n$, and $\sum_{k=1}^{n} A_k = \sum_{k=1}^{n} B_k$.

Thus, the comparison of interest can be expressed as the majorization $\vec{A} \leq^m \vec{B}$. A related notion of Schur-concave and Schur-convex functions will also be needed. A real-valued function $y(\vec{A})$ is called Schur-concave if $\vec{A} \leq^m \vec{B}$ implies $y(\vec{A}) \geq y(\vec{B})$, and $y(\vec{A})$ is Schur-convex, if $-y(\vec{A})$ is Schur-concave. Schur-concavity might be more intuitively called "Schur-monotonicity" because it simply requires function *y* to always decrease in response to a perturbation that induces more dissimilarity in its arguments. The Lorenz function itself is an example of a Schur-concave function. The analysis to follow will appeal to the following important property of Schur-concave functions. Suppose that $y(\vec{A}) = \sum_{k=1}^{n} z(A_k)$. Then $y(\vec{A})$ is Schur-concave if and only if z is concave.

4.2.2. Measuring concavity

The analysis that follows will also depend on the curvature properties (specifically the degree of concavity) of the agricultural production function, *g*. Even though there is no uncertainty in this model, it is convenient to derive its results using well-known measures of curvature from the literature on decisionmaking under uncertainty. Let $R = -g_{uu}(u,h_1)/g_u(u,h_1)$ denote the index of concavity of agricultural output function, and $P = -g_{uuu}(u,h_1)/g_{uu}(u,h_1)$ denote the index of concavity of the marginal output function of a farmer with technology $g(u,h_1)$ in period 1. If $g(u,h_1)$ were a utility of income function, then R would be interpreted as the Arrow-Pratt coefficient of absolute risk aversion, and P would be the coefficient of absolute prudence.

As g represents technology and not preferences in the model here, these indexes are employed simply as measures of the curvature of the physical relation between output and water. In this non-stochastic framework, they are indicators of the strength of the motive to smooth water extraction over time (i.e., the diminishing marginal productivity of water). Adding uncertainty will not change the qualitative nature of the results. There is an empirical literature on the relationship between farmers' risk preferences and their dynamic use of groundwater (e.g., Antle (1983, 1987) and Koundouri et al. 2006) as well as on the effects of risk preferences on farmer's reaction to water quota policies (e.g., Groom et al. 2006).

4.2.3. Inequality of farm sizes and groundwater stock

With the definitions above, the relationship between inequality and the residual water stock in period 2 can now be analyzed.

Proposition 3. *Suppose that farmers' utility is linear in income. Then under more unequal distribution of farm sizes,* $\vec{A} \leq^m \vec{B}$*, the groundwater stock in period 2*

(a) increases, $h_2^*(\vec{A}) \leq h_2^*(\vec{B})$*, if* $2R \geq P$*;*

(b) decreases, $h_2^*(\vec{A}) \geq h_2^*(\vec{B})$*, if* (i) $B_1 / A \geq g_u(h_1, h_1) / f(h_2^*(\vec{A}))$*, i.e. the smallest farm under the new land ownership distribution is not "too small" and* (ii) $2R \leq P$.

The inequality in land ownership creates a trade-off in terms of its effect on the pumping decisions in period 1. A heavier left tail of the acreage distribution implies that there are more farmers who own a smaller share of the aquifer and tend to pump faster than the average farmer. However, a heavier right tail implies the opposite. Therefore, ascertaining the effect of *any* increase in acreage inequality on the competitive allocation requires structure on the *farm-size sensitivity* of the difference in pumping rates between small and large farmers, $u_{i,1}^* - u_{j,1}^*$, where $i \in L_k$, $j \in L_l$, $A_k < A_l$. The *farm-size sensitivity* of the difference in pumping rates across farms is $a_k u''(a_k)/u'(a_k)$, where $u(a_k) = g_u^{-1}(a_k f(h_2); h_1) < h_1$. If the pumping rate differential, u', is increasing (decreasing), the sensitivity is negative (positive).

Condition (a) states that, when the aquifer is full, the agricultural output, $g(., h_1)$, is in a sense more concave than the marginal output, $g_u(., h_1)$. Then the perceived benefit from a more stable inter-seasonal groundwater use pattern increases with size at an accelerating rate, and a greater inequality stimulates, on average, a slower pumping rate. Note that condition $2R \leq (\geq) P$ is equivalent to log-concavity (log-convexity) of the first derivative of the demand for water with respect to output when the aquifer is full, $g_y^{-1}(y; h_1)$, where $g^{-1}(y; h_1) = \{u : y = g(u; h_1)\}$ is the inverse of agricultural output function obtained by treating the stock of groundwater, h_1, as a parameter.

To guarantee that the average pumping rate increases, the additional condition (i) in Part (b) is needed because the aquifer is a quasi-bathtub (see constraint (1)). This condition puts a limit on the increase in the size of large farms. It implies that, under the new distribution of land ownership, the number of farmers who grow irrigated crops is the same, $B_1 > 0$, and that, under the initial distribution of land ownership, no farmer depleted his/her stock of groundwater in period 1, $u^*_{i,1}(\vec{A}) < h_1$ for all $i \in L_1$, where 1 is the index of the smallest farmer.

4.2.4. Farm-size inequality and farm income

The effect of farm size inequality on total farm income is now considered. In the case of linear utility, (6) becomes

$$W^c(\vec{A}) = \sum_{k=1}^{n} \pi_k = \sum_{k=1}^{n} A_k \{ g(\min[h_1, \, g_u^{-1}(a_k f(h_2^*); h_1)], h_1) + g(h_2^*, h_2^*) \}, \quad (12)$$

where $h_2^* = h_1 - u_1^*$ is given by (11), and $W^c(\vec{A})$ symbolizes the dependence of total farm income (agricultural output) on the distribution of land ownership among farmers.

The farm-size inequality affects both the groundwater stock in period 2 (*dynamic allocation*) and the distribution of groundwater application rates across farms in period 1 (*spatial allocation*). Keeping everything else equal, a more stable inter-seasonal pattern of groundwater use increases total farm income. The distributional effect of farm-size inequality on farm income is more difficult because a higher variability in farm sizes may or may not lead to a higher variability in the per acre pumping rates (see Proposition 3).

Proposition 4. *Suppose that farmers' utility is linear in income. Then under more unequal distribution of farm sizes,* $\vec{A} \leq^m \vec{B}$*, total farm income*

(a) decreases, $W^c(\vec{A}) \geq W^c(\vec{B})$, *if* (i) $3R \geq P$ and (ii) $h_2^*(\vec{A}) \geq h_2^*(\vec{B})$*;*

(b) increases, $W^c(\vec{A}) \le W^c(\vec{B})$, *if* (i) *the smallest farm under the new land ownership distribution is not "too small",* $B_1 / A \ge g_u(h_1, h_1) / f(h_2^*(\vec{A}))$, (ii) $3R \le P$, and (iii) $h_2^*(\vec{A}) \le h_2^*(\vec{B})$.

Conditions in (a) guarantee that the unequal distribution of farm acreage aggravates both the distributional (a(i)) and dynamic (a(ii)) inefficiencies, that are associated with the competitive allocation. Condition a(i) requires that the net benefit of irrigation when the aquifer is full, $g(u, h_1)$, is in a sense more concave than the marginal benefit, $g_u(u, h_1)$. Then a greater inequality in farm sizes stimulates a greater variability in (acreage-weighted) pumping rates and lowers total output. Observe that a(i) is less stringent than (a) in Proposition 3. This is because the net benefit of irrigation, $g(u, h_1)$, is concave in u, which adds additional curvature, and thus, on average, a smaller (or positive) *farm-size sensitivity* of the spatial pumping rate differential suffices to cause a total output loss.

Part (b) has a similar interpretation. Condition b(i) is the same as in Proposition 3. But now sufficient condition b(ii) is more stringent compared with b(ii) in Proposition 3. This is because a negative and "sufficiently" large (in absolute value) *farm-size sensitivity* of the spatial pumping rate differential is required in order to assuredly raise total output. Note that condition $3R \le (\ge) P$ is equivalent to concavity (convexity) of the first derivative of the inverse output function (i.e., demand for water as a function of output) when the aquifer is full, $g_y^{-1}(y; h_1)$.

Combining Propositions 3(b) and 4(a) yields

Corollary. *Suppose that farmers utility is linear in income. Then under more unequal distribution of farm sizes,* $\vec{A} \leq^m \vec{B}$, *total farm income decreases,* $W^c(\vec{A}) \geq W^c(\vec{B})$, *if* $2R \leq P \leq 3R$.

Sufficient conditions under which more unequal distribution of farm sizes has an unambiguously positive effect on total farm income cannot be obtained in this way. To guarantee a lesser inequality in pumping rates, the pumping rate spatial differential, $u'(a_k)$, must be "sufficiently" decreasing (in absolute value) with farm size. In contrast, to guarantee a more stable average pumping rate, the pumping rate spatial differential must be increasing or "slightly" decreasing (in absolute value) with farm size.

Furthermore, as clear from the proof of Proposition 4 (see (21) in Appendix), the sign of $\partial\pi_k / \partial A_k$ is ambiguous. Therefore, it is possible that smaller farmers earn more total income than larger farmers, $\pi_k \geq \pi_l$ for $k < l$. Of course, larger farmers always have higher total revenues in period 2. But smaller farmers have more intensive-margin operations and higher per acre revenues in period 1. The differential in total revenues between small and large farmers in period 1 can be positive, and even exceed the magnitude of the negative differential in total revenues in period 2. Intuitively, smaller farmers will earn higher profits from being in a better *strategic* position to take advantage of the common property resource; they are able to steal more groundwater *per unit* of land than their larger neighbors. The following example illustrates.

4.2.5. Small and large farms: an example

Let $g(u,h)=(u+z)^{\gamma}$, $\gamma\in(0,1)$, $z\geq -0.5h_1$, and $v''=0$. By Proposition 1, the efficient allocation of groundwater across acres and seasons is invariant to the distribution of land ownership, and is given by $u^{s}_{i,1}=0.5h_1$ for $i\in L$. The maximal regional farm income is $W^{s}=2A(0.5h_1+z)^{\gamma}$.

For simplicity, all farms fall in one of the two categories: small and large. The size of small farms is s acres, $A_k=s$ for $k=1,...,m$, and the size of large farms is l acres, $A_k=l$ for $k=m+1,...,n$, where $s\leq l$. The number of small farms is m, and the number of large farms is $n-m$, where $ms+(n-m)l=A$. By (10) and (11) equilibrium pumping in period 1 is

$$u^{*}_{i,1}=\min[h_1,(\frac{s}{A})^{1/(\gamma-1)}(h_1-\frac{E+z(1-E)}{1+E})+z(1-(\frac{s}{A})^{1/(\gamma-1)})] \text{ for } i\in L_k,\ k=1,...,m,$$

$$u^{*}_{i,1}=\frac{h_1-smu^{*}_{m,1}/A+z((l/A)^{1/(1-\gamma)})-1)}{(l/A)^{1/(1-\gamma)}+l(n-m)/A} \text{ for } i\in L_k \text{ and } k=m+1,...,n$$

where $E=m(s/A)^{\gamma/(\gamma-1)}+(n-m)(l/A)^{\gamma/(\gamma-1)})]$.

For concreteness, this example consider a special case of an increase in farm size inequality whereas small farms get uniformly smaller and large farms get uniformly larger. Note that $\vec{A}(s';m,l(s'))\leq^{m}\vec{A}(s'';m,l(s''))$ for $s'>s''$, where $l(s)=(A-ms)/(n-m)$. Clearly, a uniform shift of acreage from small farms to large farms, keeping the number of farms in each size category fixed, constitutes an increase in farm size inequality. Inequality can then be measured simply as the gap between the acreage on small and large farms, $\Delta=l-s\geq 0$, keeping the number of each type of farms, m, fixed.

In Figure 1, parameters are: $\gamma = 0.8$, $z = -0.3$, $n = 100$, $m = 50$, $h_1 = 1$, and $A = 100{,}000$. Then the maximal farm income per acre is $W^s / A = 10 \times 0.2^{1.8}$. At $\Delta = 0$ (i.e., $s = l = 1000$), small and large farms are the same, and the distribution of land ownership is uniform across farmers. The effects of an increase in farm size inequality on the equilibrium groundwater stocks, pumping rates, and incomes are analyzed next.

As shown in Figure 1(a), when the difference in farm sizes is relatively small, $\Delta \le 280$, the difference in the *pumping rates* increases until the small farmers deplete their wells in period 1, $u^*_{i,1} = h_1 = 1$ for $i \in L_k$ and $k = 1,...,50$. This limits the ability of small farmers to "steal" groundwater from their neighbors, and therefore, establishes an upper bound on the difference in the pumping rates. Curiously, the large farmers pump ***less*** than the efficient quantity, $u^*_{i,1} \le 0.5h_1 = 0.5$ for $i \in L_k$ and $k = 51,...,100$, when $\Delta \in [\,220, 400]$. In this range, the gain in the dynamic efficiency for the large farmers outweighs the loss associated with letting the small farmers steal their groundwater. However, as the size of each large farm, and hence the total share of the aquifer farmed by large farms, increases, large farmers are able to more effectively "push" the aggregate groundwater use towards the efficient allocation. Even though the incentive to pump groundwater efficiently for each individual large farmer declines, the aggregate groundwater usage in period 1 decreases. This is because the distribution of total acreage is skewed more (less) heavily towards large (small) farmers, who pump slowly (who deplete their wells in period 1).

Figure 1(b) illustrates the non-monotone relationship between the *stock* of groundwater in period 2 and farm-size inequality. As explained earlier, when the gap between small and large farms is small, $\Delta \in [\,0, 280]$, the large farmers are relatively ineffective in raising the dynamic efficiency. This is because, even though they decrease their pumping rates in order to compensate for the higher pumping rates by small farmers, their weight in aggregate pumping is relatively light. And so, the negative effect of the aggressive pumping by small farms dominates, and the groundwater stock in period 2 falls. As the share of total acreage owned by small farmers declines, but their pumping rates remain constant ($u^*_{i,1} = h_1 = 1$ for $i \in L_k$ and $k = 1,...,50$), the large farmers need to give up less of period 1 pumping to push the region towards more dynamically efficient allocation. From the perspective of a large farmer, the groundwater resource is more private, which reinforces the diminished influence of aggressive pumping by small farmers. As a result, the average stock in period 2 increases, and the region moves towards a more dynamically (and spatially) efficient allocation.

Figure 1(c) shows the non-monotone effect of the inequality in farm sizes on *total income*. Proposition 4 shows that, in general, an increase in size inequality affects the total farm income in two distinct ways. First, it affects the groundwater stock in period 2. Second, it affects the variability of the pumping rates among farmers in period 1. When the gap is small, $\Delta \in [0, 280]$, both the "stock" and "pumping rate variability" effects work in the same direction. When the gap is "sufficiently" large, any further increase in farm-size inequality raises the total farm income. Note that the dip in the total income in Figure 1(c) has a rather pointed peak. This is because for $\Delta \geq 280$ there is an additional income gain associated with the gain in the *spatial efficiency* due to the *decline* in the heterogeneity of pumping rates. The period 1 pumping on large farms increases, while pumping on small farms remains constant (as they deplete their wells in period 1).

As shown in Figure 1(d), total *per farm* incomes are also non-monotone in the extent of farm-size inequality. Surprisingly, the total small farm income *increases* when the acreage on small farms *decreases* in the range $\Delta \in [0, 280]$. The converse holds for large farms. This is because small farms are in a better position to steal groundwater from their neighbors operating on large farms. However, the cap on the pumping in period 1, $u_{i,1}^* \leq 1$, eventually annuls this effect. Consequently, a further increase in farm-size inequality affects farm incomes in the expected direction because, keeping everything else equal, a smaller (larger) acreage entails a smaller (larger) whole-farm income.

5. Concave utility

So far, the analysis has considered the effect of farm-size heterogeneity on welfare in the case of farmers with linear utility functions (constant marginal utility of income). As shown next, relaxing this assumption may lead to rather different conclusions. Even the result that smaller farmers pump faster under the competitive allocation may no longer hold. This section considers the case of farmers with (strictly) concave per period utility functions, $v'' < 0$. To highlight the role of concavity of utility, profit per unit of land area (e.g., yield) is now assumed to be a linear function of the amount of water applied per acre, and that pumping costs do not depend on the hydraulic head, $g(u,h) = u$.

Following the same steps as before, it can be shown that the equilibrium best response of farmer k on acre $i \in L_k$, $u_{i,1}^*$, is

$$u_{i,1}^* = \min[h_1,\ (1/A_k)v_1^{-1}(a_k v'(A_k(h_1 - u_1^*)))],\quad \forall i \in L_k \tag{13}$$

where $v_1^{-1}(.)$ is the inverse of v', and the average pumping in period 1, u_1^*, solves

$$u_1^* = (1/A)\sum_{k=1}^{n} \min[A_k h_1,\ v_1^{-1}(a_k v'(A_k(h_1 - u_1^*)))]. \qquad (14)$$

Let $r(u) = -uv''(u)/v'(u)$ denote the Arrow-Pratt coefficient of relative risk-aversion of a farmer with the periodic utility of income v.

Proposition 5. *Suppose that farmers' utility is strictly concave in income . Then the average pumping rate is higher than the socially efficient average rate,* $u_1^* \geq u_1^s$, *and for all* $i \in L_k$, $j \in L_l$, $k < l$

a) smaller farms pump faster, $u_{i,1}^* \geq u_{j,1}^*$, *if* $r' \geq 0$.

b) smaller farms pump slower, $u_{i,1}^* \leq u_{j,1}^*$, *if* $1 + r(v_1^{-1}(av'(ahA))) \leq r(ahA)$ $\forall a \in [a_k, a_l]$ *and* $h \in (0, 0.5h_1)$.

Farm size has two effects on the farmer's pumping decision. On the one hand, larger farmers view their stock of groundwater as a relatively more private resource. This provides them with a greater incentive to push the regional use towards a dynamically more efficient allocation. On the other hand, larger farmers may have a smaller (negative) difference in marginal utilities of income in periods 1 and 2. This diminishes their incentive to push the region towards a dynamically more efficient allocation compared with smaller farmers. The "private resource" effect dominates if the coefficient of relative risk-aversion is increasing in income. The "income scale" effect dominates if the coefficient of relative risk-aversion is "sufficiently" large and decreasing in income (in the sense of condition in Part (b)).

While not reported here due to space constraints, the counterparts of Proposition 3-4 carry over to the case of concave utility as well. Competitive allocations may either exacerbate or alleviate income inequality associated with the distribution of land holdings among farmers. If the coefficient of relative risk-aversion is increasing in income, small farmers pump more groundwater per acre than large farmers. This lessens the income inequality caused by an unequal distribution of acreage. The converse is true if larger farmers pump more aggressively (on a per acre basis), which is possible if the coefficient of relative risk-aversion is "sufficiently" large and decreasing.

Note that, in the absence of the effect of farm-size inequality on the disaggregated pumping rates, from the societal point of view, the heterogeneity in land holdings is immaterial if farmers are *risk-neutral* (i.e., they value marginal income in both periods independently of the number of acres they farm). When farmers are *risk-averse*, the heterogeneity in the pumping rates can be welfare-increasing, given that the per acre irrigation rates increase on smaller farms and decrease on larger ones, so that in period 1 income is redistributed from rich to poor farmers (see Proposition 1). However, because of the decreasing marginal per acre benefits of water, total income always decreases under a greater variability of the pumping rates. This may create a tension between the effects of farm-size inequality on *income distribution* and *total income (output)*. The next section takes a policy perspective and investigates the workings of a very simple groundwater use policy in the presence of farmer heterogeneity.

6. Policy analysis: an example of flat-rate quota policy

The analysis now considers some political economy aspects of implementing a simple policy that allocates per period per farm pumping quotas. Suppose that the policy takes the form

$$\sum_{i\in L_k} u_{i,1}^* \le A_k q \text{ and } \sum_{i\in L_k} u_{i,2}^* \le A_k q + \max[A_k q - \sum_{i\in L_k} u_{i,1}^*, 0] \text{ for } k = 1,...,n, \qquad (15)$$

where $q \in (0, h_1]$ is the per acre quota (measured in acre-feet), and the quota allocated to each farm is proportional to its size. The quota limits the quantity of groundwater extracted in each period, but allows farmers to carry over unused portions of their quota into the next period. There is no market for water rights, and the unused quotas cannot be bought or sold.

For concreteness, the case of risk-neutral farmers and a strictly concave agricultural output function (analyzed in Section ***Linear utility***) is considered. The following result establishes that, while this policy always slows the rate of the aquifer depletion, the effect on farmer incomes is likely heterogeneous. The setting is assumed to be such that the equilibrium pumping rates decrease with time $u_{i,1}^* \ge u_{i,2}^* \;\; \forall i \in L$, so that $u_1^* \ge 0.5h_1 \ge u_2^*$. For example, this is always true if all farmers are sufficiently small relative to the aquifer, $a_n \le \inf_{u\in(0,h_1)}\{g_u(h_1 - u, h_1) / f(h_1 - u)\}$. Then, under quota policy (15), farmers do not transfer the unused portion of their quotas from period 1 to period 2: $q \ge u_{i,1}^* \ge u_{i,2}^*$, if $q \ge h_1/2$, and $u_{i,1}^* = u_{i,2}^* = q \;\; \forall i \in L$ if $q < h_1/2$. Hence, for $q \ge h_1/2$ equilibrium is given by

$$u_{i,1}^*(q) = \min[q,\, g_u^{-1}(a_k f(h_1 - u_1^*(q)); h_1)], \;\; \forall i \in L_k,\; k = 1,...,n \qquad (16)$$

$$u_1^*(q) = \sum_{k=1}^n a_k \min[q,\, g_u^{-1}(a_k f(h_1 - u_1^*(q)); h_1)]\;. \qquad (17)$$

The income of farmer k under the quota policy is

$$\pi_k(q) = A_k\{g(\,q,\ h_1) + g(q, h_1 - q)\}\text{, if } q < h_1/2\text{, and} \tag{18}$$

$$\pi_k(q) = A_k\{g(\,\min[q,\ g_u^{-1}(a_k f(h_1 - u_1^*(q)); h_1)], h_1) \tag{19}$$

$$+\, g(h_1 - u_1^*(q), h_1 - u_1^*(q))\}\text{, if } q \geq h_1/2\,.$$

From (18) it follows that all farmers lose (gain) from a more restrictive quota, if the initial quota is sufficiently small and the marginal benefit of a higher stock is "small" ("large") relative to the marginal benefit of water consumption: $\partial\pi_k(q)/\partial q = A_k$ $\{g_u(q, h_1) + g_u(q, h_1 - q) - g_h(q, h_1 - q)\} \geq (\leq) 0$ for all $k = 1,...,n$. On the other hand, from (19) it follows that the income of large farmers, who are not bound by the quota, increases because the quota policy slows down the average pumping rate in period 1.

Let $m(q) = \sup\{k : a_k \leq g_u(q, h_1)/f(h_2^*(q)),\ 1 \leq k \leq n\}$. Note that $m(q)$ is a non-increasing function. Then farmers $k = 1,..., m(q)$ are bound by the quota in period 1. Also, farmers $k = 1,...,\ m(q = h_1)$ deplete their wells in period 1, where $q = h_1$ symbolizes the absence of the quota policy.

Proposition 6. *Suppose that the quota is applicable,* $u_{1,1}^*(q = h_1) > q'$. *Then under the groundwater quota policy* $q = q' < h_1$

a) the groundwater stock in period 2 increases, $h_2(q = h_1) < h_2(q = q')$ $\forall q' < h_1$.

Suppose that the period 2 quota is not binding, $q' \geq h_1/2$. *Then*

b) large farmers gain, $\pi_k(q = h_1) \leq \pi_k(q = q')$ *for* $k = m(q') + 1,...,n$ *;*

c) small farmers lose, $\pi_k(q=h_1) \geq \pi_k(q=q')$ *for* $k=1,...,m(h_1)$, *if (i)* $g_{uuu} \geq 0$, $g_{uuh} \geq 0$, $2g_{uh}(h,h)+g_{hh}(h,h) \leq 0$, *and (ii)* $a_z \geq \sum_{k=1}^{z-1} a_k / \sum_{k=z+1}^{n} a_k^2$ *for all* $z=m(h_1),...,m(q')$.

Farmers in the medium size range, $m(h_1) \leq k \leq m(q')$, may lose or gain from a quota. The intuition for this result is very clear: Small farmers, who pump faster than the average farmer, stand to lose the most from a quota policy. Large farmers, who are not restricted by the policy, strictly gain from the quota because of the more stable inter-seasonal allocation of groundwater induced by this policy.

This illustrates that policies that do not account for user heterogeneity, are likely to affect not only the inter-seasonal but also the spatial distribution of incomes among farmers. The ensuing political economy issues and the relative weight of small and large farmers in the policy-making process pose additional constraints on the design of efficient groundwater management policies.

7. Conclusions and policy implications

This article has analyzed the economic inefficiencies that arise when farmers controlling operations of varying sizes withdraw irrigation water from a common aquifer. Farm size inequality was shown to affect the degree of inefficiency because small farmers are more strongly influenced by common property externalities than large farmers, who have an incentive to internalize inter-well costs within their operations. This insight alone has the policy implication that the gains from groundwater management are likely to be greater in regions populated by small farms, such as in developing nations.

The overall effect of an increase in inequality on social welfare was shown to be ambiguous and dependent on the agricultural production function as well as on the differences in marginal utility between large and small farmers. To the extent that these relationships vary across regions, it is one explanation for wide gaps in the prosperity of groundwater-dependent agricultural regions.

Sufficient conditions were established to identify the cases where increased inequality reduces aggregate welfare, and these conditions which appear to be quite restrictive. This finding suggests that in many regions, there is a meaningful, if not recognized, policy tradeoff between common property distortions and inequality. Wealth disparities within the farm population is a concern in both high and low income countries, particularly as it relates to the incomes of small farmers (Hoppe et al. 2010). However, in the case of access to a common aquifer, a reduction in inequality may have the unintended effect of accelerating the depletion of the resource. Moreover, the analysis reveals that the common aquifer can, in effect, become a conduit to transfer income from large to small farmers.

Finally, water management policies designed to correct common property externalities were demonstrated to have potentially significant and undesirable distributional impacts. In particular, it was shown that a quota policy may well reduce the speed of aquifer depletion as intended, but the welfare gains from groundwater conservation will not be evenly distributed; in general irrigators in certain size classes will incur welfare losses.

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Appendix

Proof of Proposition 1: First, note that in period 2, the planner optimally exhausts the remaining stock on each farm because g and v are strictly increasing. This implies that constraint (1) binds for $t = 2$ (i.e., $u^s_{i,2} = h_2 \ \forall i \in L$), so that (6) can be written

$$W^s = \max_{\{u^s_{i,1}\}} \sum\nolimits_{k=1}^{n} (v(\sum\nolimits_{i \in L_k} g(u^s_{i,t}, h_t)) + v(A_k g(h_2, h_2))).$$

Because $\sum_{i \in L_k} g(u^s_{i,t}, h_t)$ is symmetric and concave in $u^s_{i,1}$, and W^s is symmetric in $v(.)$, optimality requires that $u^s_{i,1} = u^s_{j,1}$ for any $i \in L_k$ and $j \in L_l$ if $A_k = A_l$. Additionally, corner solutions are ruled out because v and g are increasing and concave in each argument. The first-order conditions for a maximum are

$$v'(A_k g(u^s_{i,1}, h_1)) g_u(u^s_{i,1}, 1) - \frac{f(h_1 - u^s_1)}{A} \sum\nolimits_{l=1}^{n} A_l v'(A_l g(h_1 - u^s_1, h_1 - u^s_1)) = 0, \qquad (20)$$

if $u^s_{i,1} \le h_1$, and $u^s_{i,1} = h_1$, otherwise, for all $i \in L_k$ and $k = 1,...,n$. Part (a) follows by observing that (20) reduces to (7) when $v'' = 0$ because $\sum_{l=1}^{n} A_l = A$. Part (b) follows by observing that only the first term in (20) depends on farm size A_k, and, by concavity of utility function, v, it decreases with A_k. Then by concavity of yield function, g, this implies that $u^s_{i,1}$ is a non-increasing function of farm acreage.

Proof of Proposition 2: Suppose that $u^s_1 > u^*_1$. Then, by (11)

$$u^*_1 = \sum\nolimits_{k=1}^{n} a_k \min[h_1, \ g_u^{-1}(a_k f(h_1 - u^*_1); h_1)] \ge \sum\nolimits_{k=1}^{n} a_k \min[h_1, \ g_u^{-1}(a_k f(h_1 - u^s_1); h_1)]$$

$$\ge \sum\nolimits_{k=1}^{n} a_k \min[h_1, \ g_u^{-1}(f(h_1 - u^s_1); h_1)] = g_u^{-1}(f(h_1 - u^s_1); h_1) = u^s_1.$$

The inequalities follow by concavity of g. The equality follows by (7). And so, a contradiction was obtained. Also, $u_{i,1}^* = \min[h_1,\ g_u^{-1}(a_k f(h_2);h_1)] \geq \min[h_1,$ $g_u^{-1}(a_l f(h_2);h_1)] = u_{j,1}^*$ for any $i \in L_k$, $j \in L_l$, $k < l$.

Proof of Proposition 3:

Part (a). Suppose that $h_2^*(\vec{A}) > h_2^*(\vec{B})$. Then, by (11),

$$u_1^*(\vec{A}) = \sum_{k=1}^{n} a_k \min[h_1,\ g_u^{-1}(a_k f(h_1 - u_1^*(\vec{A}));h_1)]$$

$$\geq \sum_{k=1}^{n} b_k \min[h_1,\ g_u^{-1}(b_k f(h_1 - u_1^*(\vec{A}));h_1)]$$

$$\geq \sum_{k=1}^{n} b_k \min[h_1,\ g_u^{-1}(b_k f(h_1 - u_1^*(\vec{B}));h_1)] = u_1^*(\vec{B}).$$

The first inequality follows because the sum of compositions of two concave functions (here $\min[a_k h_1, a_k g_u^{-1}(a_k f(.);h_1)]$), is Schur-concave in $a_1,...,a_n$. To show this, it must be demonstrated that $a g_u^{-1}(af)$ is concave in a. Differentiating twice yields

$$\frac{\partial^2 [a g_u^{-1}(af)]}{\partial a^2} = \frac{f}{R(u) g_{uu}(u,h_1)} (2R(u) - P(u)) \leq 0,$$

where the inequality follows by condition (a) stated in Proposition 3. The second inequality follows by concavity of g. And so, a contradiction was obtained.

Part (b). Suppose that $h_2^*(\vec{A}) < h_2^*(\vec{B})$. Then, by (11),

$$u_1^*(\vec{A}) = \sum_{k=1}^{n} a_k\ g_u^{-1}(a_k f(h_1 - u_1^*(\vec{A}));h_1) \leq \sum_{k=1}^{n} b_k\ g_u^{-1}(b_k f(h_1 - u_1^*(\vec{A}));h_1)$$

$$= \sum_{k=1}^{n} b_k \min[h_1,\ g_u^{-1}(b_k f(h_1 - u_1^*(\vec{A}));h_1)]$$

$$\leq \sum_{k=1}^{n} b_k \min[h_1,\, g_u^{-1}(b_k f(h_1 - u_1^*(\vec{B})); h_1)] = u_1^*(\vec{B}).$$

The equalities follow because, by condition b(i) in the statement of Proposition 3 and concavity of g, $g_u^{-1}(a_k f(h_2^*(\vec{A}); h_1) \leq h_1$ and $g_u^{-1}(b_k f(h_2^*(\vec{A}); h_1) \leq h_1$ for all $k = 1,\ldots,n$, since $\vec{A} \leq^m \vec{B}$ implies $a_1 \geq b_1$. The first inequality follows because, by condition b(ii) in the statement of Proposition 3, $\sum_{k=1}^{n} a_k \; g_u^{-1}(a_k f(h_1 - u_1^*(\vec{A})); h_1)$ is Schur-convex (see Part (a)). The second equality follows by assumption. And so, a contradiction was obtained.

Proof of Proposition 4:

To show parts (a) and (b), we need two facts.

Fact 1. (i) $\pi_k(a_k) = A a_k g(\min[h_1, u(a_k)])$ is concave in a_k when $3R \leq P$.

(ii) $\pi_k(a_k)\big|_{u(a_k)<h_1}$ is convex in a_k when $3R \geq P$, where $u(a_k) = g_u^{-1}(a_k f(h_2^*(\vec{A}); h_1)$.

Proof of fact 1: To verify, differentiate twice with respect to $a_k = a$:

$$\left.\frac{\partial \pi_k(a)}{\partial a}\right|_{u(a)<h_1} = A\frac{\partial[ag(u(a),h_1)]}{\partial a} = A(g(u,h_1) + \frac{(af)^2}{g_{uu}(u,h_1)}), \text{ and} \tag{21}$$

$$\left.\frac{\partial^2 \pi_k(a)}{\partial a^2}\right|_{u(a)<h_1} = A\frac{\partial^2[ag(u(a),h_1)]}{\partial a^2} = A\frac{af^{\,2}}{R(u)g_{uu}(u,h_1)}(3R(u) - P(u)) \leq (\geq) 0\,. \tag{22}$$

depending on whether $3R \leq (\geq) P$. This proves Fact 1(ii). To show Fact 1(i), note that $a_k g(\min[h_1, u(a_k)]) = \min[a_k g(h_1, h_1), a_k g(u(a_k), h_1)]$ by monotonicity of g. Hence, $a_k g(\min[h_1, u(a_k)])$ is concave in a_i when $3R \leq P$ as a composition of concave functions.

Fact 2. $\partial W^c / \partial h_2^* > 0$.

Proof of fact 2: $\partial W^c / \partial h_2^*$ inherits the sign of $\partial\{g(g_u^{-1}(af(h_2);h_1),h_1) + g(h_2,h_2)\}/\partial h_2$ $= af'(h_2)/g_{uu}(u,h_1) + f(h_2) > 0$, where the inequality follows by concavity of g. Keeping everything else equal, as the extent of dynamic inefficiency of the competitive allocation increases, welfare falls.

Part (a). By (12),

$$W^c(\vec{A}) = \sum_{k=1}^{n} A_k\{g(\min[h_1,\ g_u^{-1}(a_k f(h_2^*(\vec{A}));h_1)],h_1) + g(h_2^*(\vec{A}),h_2^*(\vec{A}))\}$$

$$\geq \sum_{k=1}^{n} B_k\{g(\min[h_1,\ g_u^{-1}(b_k f(h_2^*(\vec{A}));h_1),h_1)] + g(h_2^*(\vec{A}),h_2^*(\vec{A}))\}$$

$$\geq \sum_{k=1}^{n} B_k\{g(\ \min[h_1, g_u^{-1}(b_k f(h_2^*(\vec{B}));h_1)],h_1) + g(h_2^*(\vec{B}),h_2^*(\vec{B}))\} = W^c(\vec{B}).$$

The first inequality follows because function $W(\vec{A})$ is Schur-concave as the sum of concave functions by condition a(i) in the statement of Proposition 4and Fact 1(i). The second inequality follows by condition a(ii) in the proposition statement and Fact 2.

Part (b). By condition b(i) in the proposition statement, $u_{i,1}^*(\vec{A}) < h_1$ for all $i \in L$ because $\vec{A} \leq^m \vec{B}$ implies that $a_1 \geq b_1$ so that $g_u^{-1}(a_k f(h_2^*(\vec{A});h_1) \leq h_1$ and $g_u^{-1}(b_k f(h_2^*(\vec{A});h_1) \leq h_1$ for all $k = 1,...,n$. Then, by (12),

$$W^c(\vec{A}) = \sum_{k=1}^{n} A_k\{g(\ g_u^{-1}(a_k f(h_2^*(\vec{A}));h_1)],h_1) + g(h_2^*(\vec{A}),h_2^*(\vec{A}))\}$$

$$\leq \sum_{k=1}^{n} B_k\{g(\ g_u^{-1}(b_k f(h_2^*(\vec{A}));h_1),h_1) + g(h_2^*(\vec{A}),h_2^*(\vec{A}))\}$$

$$= \sum_{k=1}^{n} B_k\{g(\min[h_1,\ g_u^{-1}(b_k f(h_2^*(\vec{A}));h_1),h_1)] + g(h_2^*(\vec{A}),h_2^*(\vec{A}))\}$$

$$\leq \sum_{k=1}^{n} B_k\{g(\ \min[h_1, g_u^{-1}(b_k f(h_2^*(\vec{B}));h_1)],h_1) + g(h_2^*(\vec{B}),h_2^*(\vec{B}))\} = W^c(\vec{B}).$$

The first inequality follows because function $W(\vec{A})$ is Schur-convex by Fact 1(ii). The equality follows by condition b(ii) in the statement of Proposition 4. The second inequality follows by condition b(iii) in the proposition statement and Fact 2.

Proof of Proposition 5: Suppose that $u_1^s \geq u_1^*$. Then, by (20) and (14) in the text,

$$u_1^s = (1/A)\sum_{k=1}^n \min[A_k h_1, v_1^{-1}(\sum_{l=1}^n a_l v'(A_l(h_1 - u_1^s)))]$$

$$< (1/A)\sum_{k=1}^n \min[A_k h_1, v_1^{-1}(\sum_{l=1}^n a_l v'(A_l(h_1 - u_1^*)))]$$

$$\leq (1/A)\sum_{k=1}^n \min[A_k h_1, v_1^{-1}(a_k v'(A_k(h_1 - u_1^*)))] = u_1^*.$$

The inequalities follow by concavity of v. And so, a contradiction was obtained.

Part (a). Let $i \in L_k$. First, consider $u_{i,1}^*(A_k) < h_1$. By (13), differentiation yields

$$\partial u_{i,1}^* / \partial A_k = v'(A_k h_2)/(A_k v''(A_k u_{i,1}^*))[1 + R(A_k u_{i,1}^*) - R(A_k h_2)] \leq 0$$

The inequality follows because, by (13), $u_{i,1}^* \geq h_1 - u_1^*$, and so $1 + R(A_k u_{i,1}^*) - R(A_k(h_1 - u_1^*)) \geq 1 > 0$. If $u_{i,1}^* = h_1$ then $u_{j,1}^* \leq h_1$ for $j \in L_l$, $k < l$.

Part (b). Proof is analogous.

Proof of Proposition 6:

Part (a). Note that this is trivially true when the quota is binding in period 2, $q' < h_1/2$, because then $u_{i,1}^* = q$, and $u_{i,2}^* = h_2 = h_1 - q \quad \forall i \in L$. So consider the case when $q' \geq h_1/2$ and suppose that $u_1^*(q = h_1) < u_1^*(q = q')$. Then, by (17),

$$u_1^*(q = h_1) = \sum_{k=1}^n a_k \min[h_1, g_u^{-1}(a_k f(h_1 - u_1^*(q = h_1)); h_1)]$$

$$\geq \sum_{k=1}^{n} a_k \min[q',\ g_u^{-1}(a_k f(h_1 - u_1^*(q = h_1)); h_1)]$$

$$\geq \sum_{k=1}^{n} a_k \min[q',\ g_u^{-1}(a_k f(h_1 - u_1^*(q = q')); h_1)] = u_1^*(q = q'),$$

where the last inequality follows by concavity of g. And so, a contradiction was obtained.

Part (b). By (19), farmer k's income for $k = m(q') + 1, ..., n$ is

$$\pi_k(q = q') = A_k\{g(\ g_u^{-1}(a_k f(h_1 - u_1^*(q')); h_1), h_1) + g(h_1 - u_1^*(q'), h_1 - u_1^*(q'))\}$$

$$\geq A_k\{g(g_u^{-1}(a_k f(h_1 - u_1^*(h_1)); h_1), h_1) + g(h_1 - u_1^*(h_1), h_1 - u_1^*(h_1))\} = \pi_k(q = h_1),$$

where the inequality follows by Part (a), and monotonicity and concavity of g.

Part (c). By (19), farmer k's income is $\pi_k(q') = A_k\{g(\ q'\ , h_1) + g(h_1 - u_1^*, h_1 - u_1^*)\}$ for $k = 1, ..., m(h_1)$. Differentiation yields

$$\frac{\partial \pi_k(q')}{\partial q} = A_k\{g_u(q, h_1) - f(h_1 - u_1^*)\frac{\partial u_1^*}{\partial q}\} \geq A_k f(h_1 - u_1^*)\{a_{m(q')} - \frac{\partial u_1^*}{\partial q}\} \qquad (23)$$

$$\geq A_k f(h_1 - u_1^*)\{a_{m(q')} - \frac{\sum_{l=1}^{m(q')} a_l}{1 + \sum_{l=m(q')+1}^{n} a_l^2}\} \geq 0.$$

The first inequality follows because $m(h_1) \leq m(q')$, which follows by concavity of g. The second inequality follows because, by (17), $u_1^*(q) = q\sum_{l=1}^{m(q')} a_l + \sum_{l=m(q')+1}^{n} a_l$ $g_u^{-1}(a_l f(h_1 - u_1^*(q)); h_1)$, and implicit differentiation yields

$$\frac{\partial u_1^*(q)}{\partial q} = \frac{\sum_{l=1}^{m(q')} a_l}{1 + \sum_{l=m(q')+1}^{n} a_l^2 f'(h_1 - u_1^*(q)) / g_{uu}(u_{i,1}^*(A_l); h_1)} \leq \frac{\sum_{l=1}^{m(q')} a_l}{1 + \sum_{l=m(q')+1}^{n} a_l^2},$$

since, by c(i),

$$f'(h_1 - u_1^*) = g_{uu}(h_1 - u_1^*, h_1 - u_1^*) + 2g_{uh}(h_1 - u_1^*, h_1 - u_1^*) + g_{hh}(h_1 - u_1^*, h_1 - u_1^*)$$

$$\leq g_{uu}(u^*_{i,1};h_1)\,.$$

The third inequality in (23) follows by c(ii). Hence, $\pi_k(q = q') \leq \pi_k(q = h_1)$ for $k = 1,\ldots,m(h_1)$ because $\partial \pi_k(q)/\partial q \geq 0$ for all $q \in [q', h_1]$.

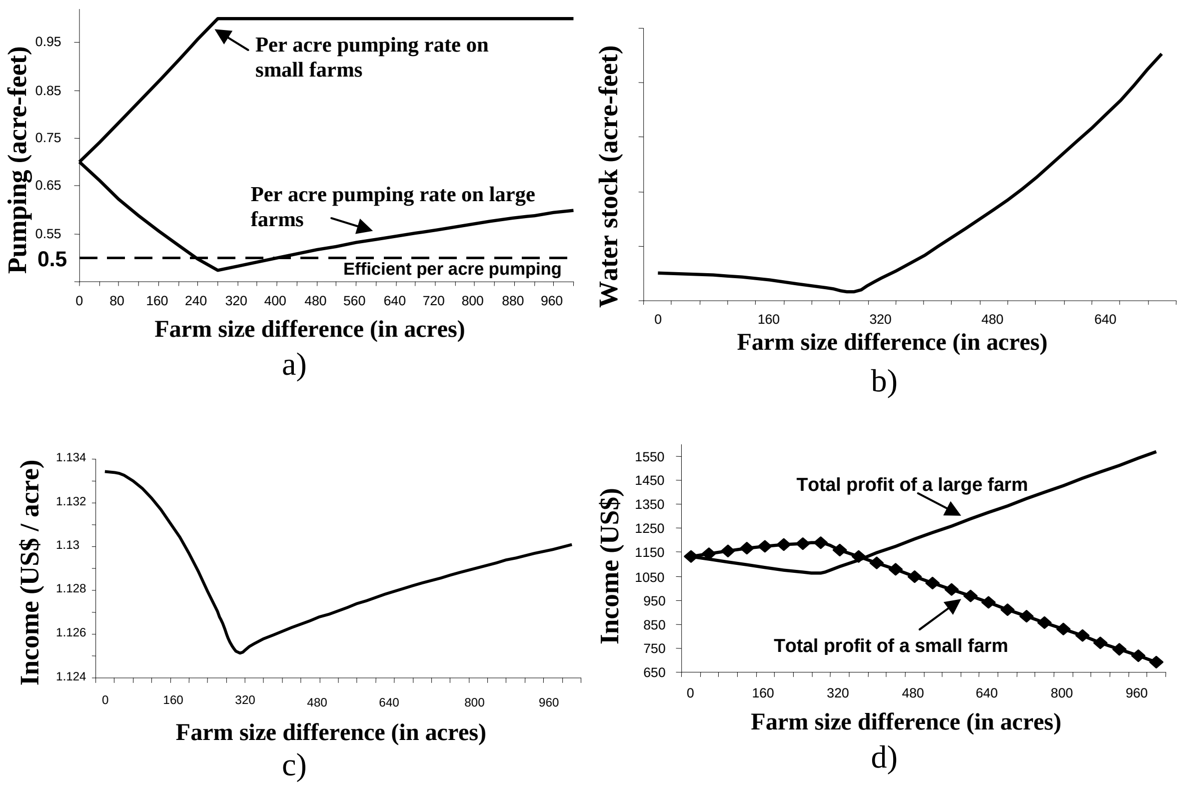

Figure 1. Inequality in farm sizes, pumping rates, and income. (a) Per acre pumping rates (b) Groundwater stock in period 2 (c) Average income per acre (d) Income for small and large farms (1 acre = 0.4047 ha = 4047 m^2)