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Enhancing the Practical Usability of Dynamic Traffic Assignment

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Enhancing the Practical Usability of Dynamic Traffic Assignment

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Thesis

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Master of Science in Engineering

The University of Texas at Austin

December 2012

Acknowledgements

The completion of this thesis is my most involved academic accomplishment to date, and it most certainly would not have been possible without the help of many people. My advisor, Dr. Stephen Boyles, has been a continuous source of inspiration and guidance throughout my graduate career, and I cannot thank him enough. Similarly, Dr. Chandra Bhat has been a significant resource and source of encouragement, both through valuable course instruction as well as outside communication. Without these two professors, my graduate career surely would have been far less stimulating.

I would also like to thank each member of Dr. Boyles' research team for providing me with an open environment to learn, ask questions, and grow both academically and professionally. Similarly, I want to express my appreciation to the Civil, Architectural and Environmental Engineering Department here at the Cockrell School for their selection of me as a GRA. Without this award my graduate education would not have existed.

Finally, I express gratitude to my parents who have surrounded me with steadfast love and support through each of life's choices, allowing me to approach each new endeavor with confidence and determination. You have done more for me than words can describe.

I thank you all so much.

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A general framework is presented for replacing static traffic assignment with dynamic traffic assignment within the standard four step transportation planning model. Issues including model consistency and the implementation of a proper feedback loop are explored. The new model is compared with the standard four step model in order to highlight the benefits of using dynamic traffic assignment rather than static. The model is then extended to include a term for the difference between experienced and free-flow travel times, which can be used as a proxy for travel time reliability and highlights the benefits of time-dependent DTA. Additionally, a study on improving the quality of convergence for dynamic traffic assignment is conducted in order to help facilitate the usefulness of this modeling approach in practice. A variety of equilibration techniques are tested, and analysis is performed to contrast these techniques with the method of successive averages.

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CHAPTER 1: INTRODUCTION

1.1 BACKGROUND

Effectively constructing a planning tool for forecasting future demand levels is essential to transportation science. If planners are able to model future transportation conditions with a high level of accuracy, they gain access to information allowing implementation of corrective or advantageous system changes in the present. However, defining measures that constitute a good level of accuracy is not trivial, as there are a very large number of plausible future scenarios. Therefore, considerable effort must be given to the particular model intricacies that define how different transportation elements are included.

Traditionally the sequential four step transportation model has been used to accomplish the task of forecasting. The first step, trip generation, uses demographic and survey data to determine how many trips are being attracted and produced in each traffic analysis zone (TAZ). Trip distribution, the second model step, uses the attractions and productions from the trip generation step and distributes them among the TAZs in the planning area. Mode choice, the third step, converts the person trips from the trip distribution step into vehicle (or other mode) trips. Finally, traffic assignment distributes the vehicular origin-destination matrix from the previous step onto the transportation network using the principle of user equilibrium (UE).

The traffic assignment step is typically carried out through the use of a static traffic assignment (STA) model, where link performance functions represent the average or steady state travel-time on a link as a function of the volume of traffic on that link. However, there are a number of limitations induced by properties of link performance functions, the most significant of which is that there is no restriction to ensure that volume on a given roadway is less than the available capacity.

Dynamic traffic assignment (DTA), a fundamentally different method than STA, typically uses simulation-based network modeling to describe traffic movement in discrete time intervals. DTA models have three essential components: a traffic simulator, a path generator and an assignment module. The traffic simulator is used to propagate flow throughout a transportation network and find travel times. The path generator is used to find the time-dependent shortest path per origin-destination. Finally, the assignment module moves flow from other paths to the shortest path per origin-destination. The main benefit of such an approach is that it allows engineers and planners to access the patterns of traffic at specific points in time, which can lead to a greater understanding of the cause of a bottleneck or congestion within a regional transportation network. However, the use of DTA by practitioners has been relatively limited up to this point in time, though many wish to begin using DTA in the future. Noted concerns include the high computational load and long model run periods associated with DTA, as well as the amount of time and resources required to implement a new modeling scheme.

In this thesis, the primary goal is to successfully integrate DTA with the traditional four step planning model. Furthermore, the new model should benefit from the small time scale resolution provided by DTA. Finally, the new model will propose a measure of travel time reliability, using the information gained from dynamic traffic assignment. Also, a study of equilibration techniques for DTA is conducted so that the computational load and convergence gap may be as low as possible. It is hoped that this study, combined with ever-increasing computational power, will ingratiate DTA in the minds of practitioners.

1.2 Motivation

The benefits of dynamic traffic assignment – modeling traffic flows at a fine time scale across a large spatial area – and the availability of efficient software programs have made DTA a valuable tool for transportation planning agencies. According to a recent survey conducted by the Federal Highway Administration, 42% of respondents, mainly consisting of government agencies and consulting firms, want to incorporate DTA into their planning analyses as soon as possible (Chiu, 2010). Seventy percent of respondents plan to implement DTA within the next two years, and 90% want to incorporate DTA in three to four years at the latest. Sixty-five percent of the respondents planned to eventually replace their existing static traffic assignment model with DTA. Another accurate and theoretically consistent solution could be to integrate DTA with an activity-based travel demand model. However, for many agencies this is too costly to implement; the traditional four step model has been used since the early 1960s. Therefore, combining the four step model with DTA is a cost-effective approach (and may be the only approach) to add detailed temporal dynamics to existing planning processes. For these reasons, combined with the numerous issues of consistency posed by altering the fundamental structure of such a sophisticated model, it is necessary to build a framework that places DTA within the standard transportation planning processes.

The non-realistic properties of the link performance functions included within STA indicate the need for a model that more realistically captures traffic behavior. Most models using link performance functions, allow volume to capacity ratios to be greater than one, which is physically impossible. Moreover, this issue is aggregated across an entire network, meaning there are potentially a great number of links that are assigned traffic volumes that simply are not possible to achieve in reality. Link performance functions do not distinguish between different lanes on a roadway, though this issue may be of slightly lesser importance given the scale of some regional network models. Also, because link performance functions are based on a single value of link flow, it is implied that inflow is equal to outflow and there is no accumulation of traffic on the link. This results in an absence of representation of congestion spillback. However, congestion should be one of the biggest factors planners wish to examine when performing regional modeling, as it is the source of substantial losses of both time and money. For this reason it is crucial that this element of transportation be simulated correctly within modern planning models. Finally, link performance functions are arbitrary and cannot be traced to fundamental traffic flow principles.

In addition to the alleviation of the concerns presented by link performance functions, the inclusion of DTA into the four step model will provide access to time-dependent travel information, which can then be used to increase the efficacy of the other steps within the model. For example, one element that may be possible to extract from dynamic traffic assignment output is some measure of travel time reliability. From a behavioral standpoint, it is logical to assume that drivers include a factor for the reliability of a given route within their choice decision. For example, consider two paths between a given origin and destination. On path 1, drivers can consistently reach their destination in 13 minutes. On path 2, drivers sometimes experience travel times of 10 minutes while at other times they may experience travel times of 20 minutes – each case has 50% probability. It is clear that while path 2 may provide a shorter commute time, the average between the different path 2 travel times is 15 minutes, a longer travel time than the consistent 13 minutes of path 1. This example illustrates the importance travel time reliability can play in an individual's route choice selection and likewise the importance of incorporating a representation of this type of situation within modern transportation models.

Finally, given that a large portion of government agencies and consulting firms wish to incorporate DTA into their planning process within the next five years, it seems highly relevant to provide an unbiased comparison of many equilibration techniques within the same dynamic traffic assignment model. Consider the uncertainty associated with the inclusion of a new DTA component within an existing model – agencies will be concerned with questions such as "How much it will cost? How long will the new model take to run? How many combinations of various options and parameters are there to test and validate?" Finally, they will want to know "How will the DTA component give results in the least amount of computation time possible without sacrificing accuracy?" To alleviate some of these concerns, agencies can simply glance at the convergence study in this work to gain a quick understanding of how various DTA equilibration techniques compare to one another in terms of both the convergence rate and the accuracy of convergence.

1.3 Problem Statement

In this work, the first objective is to simply create a functioning four step transportation model that is able to include dynamic traffic assignment in the final step. Working toward this goal, it seems that a logical place to start is by a simple replacement of STA with DTA in step four of the traditional planning model. However, given that static assignment produces a single travel time output value for a given demand period while DTA reports travel times at each time interval within that period, it will be necessary to devise meaningful model updates in order to accurately integrate the dynamic assignment. Once the new model has been successfully implemented, it will be beneficial to analyze results in order to gain a quantitative understanding of any performance changes between the new and existing four step model. A comparison of results for both the standard and DTA four step models will be performed to highlight these disparities.

Following the initial implementation of dynamic assignment within the four step model, it will be advantageous to highlight model improvements made possible by the dynamic output provided by DTA and previously unavailable from STA. One way this may be achieved is through the inclusion of a term representing travel time reliability within the logit equation found within the third model step, mode split. Given that DTA produces travel time values for each discrete time step interval during the simulation period, it follows that a notion of travel time reliability formulated within this framework should be more descriptive than a similar measure found by utilizing static traffic assignment output. From a behavioral perspective, utilizing the difference between experienced travel times and free flow travel times within a DTA model seems a logical proxy for travel time reliability, as links or paths with great differences between these values are likely very congested and thus less reliable than those that produce travel times close to free flow.

Shifting focus toward the study of dynamic traffic assignment convergence, the first goal of this research effort is to review, compare and contrast existing methodological approaches used to achieve the equilibration of large scale simulation-based DTA models. If any possibilities for increasing the rate of convergence for these models are found, they could be particularly useful for practitioners who are interested in implementing DTA within their planning methods. In order to accurately study the effects of various techniques on the convergence pattern of the network, it is critical that all methods are tested within the same software platform and with a uniform pattern of path-generation and equilibration iterations.

Within this framework, the first method implemented will consist of a number of variations on a traditional approach: the Method of Successive Averages (MSA) which moves $\lambda = \frac{1}{i}$ vehicles from the current path onto the shortest path, where i represents the number of the current iteration. Several deviations, including more quickly or slowly decreasing λ as well as repeating certain values of λ, have been suggested in the literature. Perhaps implementing many of these variants within the same simulation framework will provide meaningful results. The second set of convergence techniques implemented in this study can be characterized as gradient-based techniques. Gradient projection methods, common in nonlinear optimization literature, have been successfully implemented for the solution of static traffic assignment problems, so it seems a logical progression to apply these same techniques to dynamic traffic assignment. It is hoped that the results of this effort may bring about the development of efficient implementations that can improve the ability of SBDTA models to handle larger networks in a more time-efficient manner.

1.4 Contributions

This work is believed to be the first attempt to fully integrate dynamic traffic assignment into the four step model planning tool. While previous studies have documented potential ideas for achieving this integration, as of yet none have achieved a working model that usesactual dynamic traffic assignment output. Furthermore, by completing the integration of DTA into the four step model, this work can serve as an excellent source of reference for those who wish to achieve such integration in the future. There are clearly a number of challenges that must be overcome in order to achieve a successful integration, and rather than starting from scratch, others may be able to use this work as a guide for troubleshooting their own integration set up.

There are additional benefits from this first implementation of an integration of DTA and the four step model; this is the first time that the value DTA brings to the current four step planning model can be analytically expressed. By utilizing the time-dependent travel times produced by DTA to incorporate travel time reliability into the mode split logit equation, a time-dependent expression can be obtained that is not possible when using STA in step 4 of the model. It is a goal of this research to be able to clearly display this advantage through an analysis of variations of the specific implementation of the measure of travel time reliability, which will be included in the second chapter.

The current study on improving the convergence of dynamic traffic assignment is the first of its kind, in that it expressly compares a wide variety of equilibration techniques side by side within a single DTA simulation software. While previous studies have compared a single technique with traditional MSA or given theoretical justification on how the method is expected to perform when implemented, no work could be found that quantitatively compared a large number of methods with MSA in a single set of experiments. The primary contribution of this research is to present results from an exploration of methods that may improve the rate or quality of convergence of large scale dynamic traffic assignment networks. This should serve as a valuable resource for any practitioner or researcher who wishes to achieve an optimal convergence approach without having to perform costly and time-intensive convergence studies of their own.

1.5 Organization

Chapter 2 presents the full study encompassing the integration of dynamic traffic assignment within the four step model framework. This includes a review of the literature associated with the topic as well as a statement on the fundamental challenges of consistency posed by this endeavor in integration. A base four step DTA model is created and implemented, and results are analytically compared to the standard static four step model. Furthermore, the chapter contains an important expansion of the base model: a method to incorporate a variable representing travel time reliability in the mode split step by utilizing travel time output from DTA. Chapter 3 is comprised of a study of several DTA equilibrium methodologies, which are analyzed and compared so that DTA convergence may be improved in practice. Finally, Chapter 4 summarizes the contributions and findings of this work and discusses possible extensions of the research.

Chapter 2: Integrating Dynamic Traffic Assignment and the Four Step Transportation Planning Model

2. 1 Introduction

This chapter will present a modification of the four step planning model to include dynamic traffic assignment (DTA) within the network assignment, or final step of the model. In this section, the traditional model is described in detail, followed by a discussion of the major differences of static traffic assignment (STA) and DTA. Finally, this section will outline the model-specific details necessary to complete a successful integration of DTA within the original planning model.

2.1.1 Four Step Model Description

The traditional four step transportation planning model, shown in Figure 2.1, consists of four sequential processes: trip generation, trip distribution, mode choice, and network assignment. Each is described in further detail below.

Trip Generation:

The first step in the standard four step model, trip generation, is used to forecast travel demand for each traffic analysis zone (TAZ) within a given study area. A single TAZ has a defined spatial area comprised of any number of residential and/or nonresidential localities. Typically residences are associated with trip

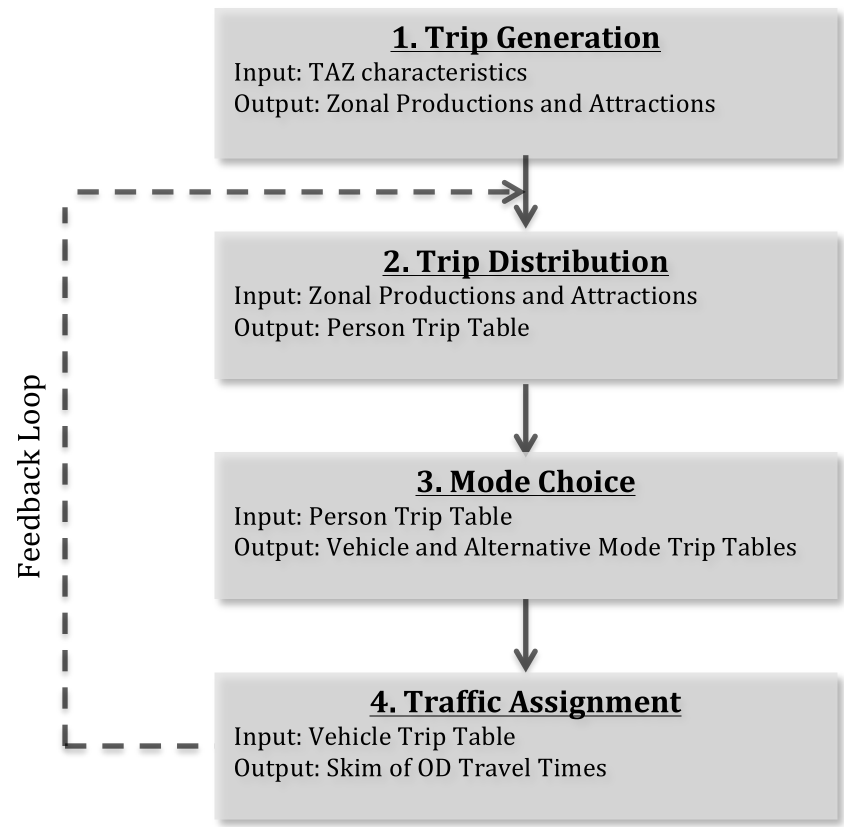


Figure 2.1 Diagram of Traditional Four Step Transportation Planning Model

productions while non-residences serve as trip attractors. Trip productions are modeled using household survey data containing demographic variables including income, vehicle ownership, household size and others. Linear regression is commonly used to relate these independent variables with produced trips. Trip attractions can be modeled using the same technique or by utilizing a standard set of Institute of Transportation Engineers *Trip Generation* handbook procedures, in which features such as the type of development, square footage, number of gas pumps, number of dwellings and other measurable qualities are considered.

Trip Distribution:

In the trip distribution step traveler's origin and destination TAZs are matched via a trip table. The trip table is a $n \; x \; n$ matrix, where n is the total number of TAZs. Typically rows are used to represent origin locations while columns correspond to destination locations. There are several techniques available for creating this matrix, though typically some variety of the gravity model is used. The basic gravity model is shown below.

$$V_{ij} = A_i O_i D_j f(c_{ij}) \qquad [2.1]$$

V_{ij} is the amount of trips originating in zone *i* and ending in zone *j*. A_i is a proportionality constant. O_i is the amount of trips originating in zone *i*, and D_j is the amount of trips ending in zone *j*. The c_{ij} term represents the cost experienced by the user while traveling from zone *i* to zone *j, and* $f(c_{ij})$ maps c_{ij} to a scaling value used in trip distribution. The shortest path travel times from the traffic assignment step are typically fed back into the model to estimate $f(c_{ij})$.

Mode Choice:

Mode choice serves the task of converting person trips from the trip distribution step into vehicle (and other mode-specific) trips. This typically requires the use of a utility function, which describes how satisfied an individual is with each available mode choice. This function normally includes statistics such as the in-vehicle and out-of-vehicle travel times (IVTT and OVTT), cost and reliability of the mode. Usually a multinomial logit or nested logit model is then estimated from the utility function at the household level with survey data. The logit results are then aggregated to the zonal level to determine the mode split for each O-D pair.

Network Assignment:

The final step of the model takes the trip distribution and, factoring the mode choice, determines the user equilibrium of travel times. User equilibrium refers to a situation where every used path per origin-destination pair OD has the same travel time, and no unused path has a shorter travel time. This is accomplished by linking origins and destinations through a network consisting of links and nodes. A flow model is used to determine travel times based on vehicle trajectories. This flow model is typically static, although a dynamic model could conceivably be used as well. A given assignment algorithm is selected to move vehicles between different paths until the network is close to equilibrium. The equilibrium yields travel times per OD and mode that can then be used as inputs back into the trip distribution and mode choice steps of the model.

2.1.2 Static vs. Dynamic Traffic Assignment Flow Models

A static flow model calculates travel time based on the time-independent flow on a given network link. Typically the travel times are computed through the use of a Bureau of Public Roads (BPR) function. The standard form of this equation is as follows for each link:

$$t = t_f\left(1 + \alpha\left(\frac{v}{c}\right)^{\beta}\right) \qquad [2.2]$$

where t_f is the free-flow travel time, α and β are calibration constants, v is flow and c is capacity. Since travel time on a link is a function of that link's flow alone, no congestion is propagated across links and static user equilibrium does not properly account for realistic congestion spillback. Also, $\frac{v}{c}$ is allowed to exceed a value of 1 to maintain algorithmic simplicity, even though this phenomenon is not possible in reality. Nevertheless, the static flow model is commonly used because of its well-behaved travel time functions, which lead to good convergence properties.

Dynamic traffic flow models were developed to avoid the issues stated previously by propagating flow over time with consideration to congestion spillback. This results in non-differentiable, non-monotone travel time functions, which makes the calculation of user equilibrium considerably more difficult. Traditional planning purposes have used static flow models for the reduced computation time and because of the reduced impact of flow propagation unrealism due to the other potential sources of error. However, Chiu (2010) found that a large percentage of planners are considering incorporating DTA in the near future. This works aims to demonstrate the feasibility of creating an integrated planning model.

The replacement of STA with DTA in the four step model presents a number of challenges. Most notable is the issue that the planning model was based upon time-invariant travel times from STA, and the output of DTA is a set of time-dependent travel times. Furthermore, STA contains time-invariant demand that must be expanded for use in DTA and then collapsed back to a single value for feedback within the four step model. There are multiple ways of handling these challenges, and it is unclear which is optimal.

The chapter is divided into multiple sections to maintain a clear organization. The first section is a discussion of research contributing to the present day transportation planning model, followed by a section which similarly describes work that has fostered the creation of modern dynamic traffic assignment. The fourth literature review section describes methods of incorporating travel time reliability within various transportation models, and the chapter concludes with a mention of the state of the art of DTA and four step model integration in order to place the contributions of this thesis within context.

2.2 Literature Review

2.2.1 Introduction

While this research is the first to derive results from the implementation of an integrated four step and DTA model, previous work exists that highlights the particular theoretical concerns presented in this integration. Also, there is a considerable body of literature on the traditional four step model as well as dynamic traffic assignment, which are relevant to the contributions made in this work. Therefore, this chapter aims to describe the existing research that is applicable to the problem of integration.

2.2.2 The Four Step Transportation Planning Model

Beginning in the early 1960s, individual states within the U.S. were compelled to adopt an individual transportation master plan in order to meet changing government regulations. To address this need, models were developed so that accurate and meaningful long-term planning could be performed. Eventually the standard modeling procedure came to be known as the four step transportation planning model. McNally (2000) provides an extensive discussion of how each step of the model developed over time to better serve ever-growing transportation needs. Also described is how growing computing power and data collection efforts have shaped the state-of-the-art of transportation planning.

Guo et al. (2010) investigate what kinds of feedback solutions should be used to obtain uniquely converged model output in an efficient manner. Two major methods of model feedback from the final network assignment step to the trip distribution step are tested: the constant weights method, which includes conventionally applied direct feedback as a special case, as well as the method of successive averages. Empirical results suggest that the application of direct feedback within the constant weights method converges most efficiently.

Maerivoet and Moor (2008) place the four step model alongside other planning models, such as activity-based planning models and even various traffic flow models, in order to see how the various data requirements and model outputs compare to one another. The study places particular emphasis on the fact that relatively little work has been explored pertaining to using these techniques side-by-side, as most practitioners instead choose to adopt a particular framework and work solely with the models they have included. Lin et al. (2008) encourage the notion of more advanced models, suggesting that four step modeling does not take advantage of modern computation power and is less precise when compared to activity-based modeling. The work proposes theory on how activity-based modeling could be used alongside DTA to create a more realistic transportation planning model.

2.2.3 Traffic Assignment

There has been a substantial amount of research conducted in the area of traffic assignment, including many model formulations and extensions for both static and dynamic assignment. In this section, the focus will be only on the research that has directly influenced this work.

A standard network model represents a transportation system with the purpose of being able to predict route choice and macroscopic traffic flow as well as to evaluate various planning alternatives or policies. This is typically achieved through the use of links, which represent roadway segments, and nodes, which are used as intersection points so that vehicles may move from one link to another. Traffic networks are essential for the completion of traffic assignment, and in this study the focus is on utilizing dynamic traffic assignment (DTA) within a framework that has commonly used static traffic assignment (STA). Chiu (2010) advocates the need for increased use of DTA via a survey of transportation practitioners, which indicates that an overwhelming percentage of city and regional planners and transportation consultants wish to incorporate DTA into their practice within the next five years.

The DTA model used in this paper is the Visual Interactive System for Transport Algorithms (VISTA) (Ziliaskopoulos and Waller, 2000) based on the cell transmission model (CTM) introduced by Daganzo (1994, 1995). CTM divides links into a series of cells based on link length and free flow speed, such that a vehicle can traverse at most one cell during each simulation time interval. Limitations on in/out flow and cell capacity restrict vehicle movement and increase travel time in traffic congestion. Vehicles are discretized and individual vehicle path and arrival times are reported. The default method of determining the traffic assignment alternates between sequences of path generation, which finds new shortest paths and moves a proportion of vehicles onto it, and dynamic user equilibrium iterations, which modify vehicle routing on the set of existing paths (see Figure 2.2). For the purpose of finding shortest paths, link travel times are averaged per assignment interval (typically 15 minutes). Individual vehicle travel times vary due to the specific experienced network conditions.

2.2.4 Travel Time Reliability

A number of studies have been conducted to assess how the reliability of travel time affects the decisions made by a traveler, such as which route to take or which mode to choose. Sweet and Chen (2011) explicitly explore the question of whether or not regional travel time reliability has a significant impact on mode choice selection. Results suggest that reliability of travel time is particularly important for home-based work trips, and a one standard deviation change in travel time reliability is associated with approximately a 23% reduction in the chance that a traveler will choose to drive a car. It is also worth noting that this impact is likely dependent on the type of area being modeled. Perhaps drivers in metropolitan areas are likely to have a higher sensitivity to travel time reliability than those in more rural locations.

Dong and Mahmassani (2009) propose a method for the online prediction of travel time reliability based on real-world measurements through the use of a discrete time Markov chain that predicts the probability of flow breakdown or recovery along a given traffic facility. Guo et al. (2011) suggest a multistate model in order to accurately model and report travel time. This model hopes to advance travel time modeling by providing improved model fitting as compared with single-mode models, as well as by providing a connection between travel time distributions and the underlying travel time state.

Finally Martchouk (2009) presents a full study of the inclusion of travel time variability in modeling and suggests a number of ways in which this can be achieved. One method suggests using the difference of experienced and free-flow travel times as a proxy to represent congestion and therefore travel time reliability. Also included is an experiment evaluating the importance of the reliability of travel time for different periods throughout the day (AM and PM peak and off-peak periods) and a discussion of how this reliability should be incorporated within a logit equation for mode choice. This work was particularly beneficial in that it provided a strong framework that suggested how travel time reliability could be properly included in the integrated DTA and four step model.

2.2.5 State of the Art: Integration of Dynamic Traffic Assignment and the Four Step Transportation Model

While the previous sections describe some of the research pertaining to traffic assignment, planning models and travel time reliability, there currently are very few studies that have approached the problem of integrating these ideas within a common framework. Tung et al. (2010) describe one possible approach for integrating DTA with the four step model by a simple replacement of static with dynamic assignment within the final model step. Also included are statements on potential consistency concerns, such as how to derive a single meaningful travel time value from the dynamic assignment output to use for feedback to trip distribution or mode choice. However, the study lacks results and has no analysis of the described integration.

Melson et al. (2012) likewise describe in theory how DTA could replace STA within the four step model. Also included in the study are potential extensions to the model, such as the implementation of time-of-day measures and the incorporation of a representative term for travel time reliability, that further highlight the benefits DTA provides over STA. Finally, the work includes a study of key links that represent fundamental differences between static and dynamic traffic assignment. Lin et al. (2008) describe the problem of integrating DTA with an activity-based planning model rather than the four step planning model. However, the discussion of potential convergence criteria is very relevant to this study. The two suggested measures for convergence include trip table and travel time convergence, and deciding which is more useful depends on the particular application.

This thesis attempts to advance the state of the art by providing a completed integration of DTA and the four step model along with an analysis of the results. Also included is an analysis of how travel time reliability may be incorporated within this framework. While there are a number of uncertainties that will require further study, this work can serve as the framework for the development of a new and fundamentally different transportation planning approach.

2.3 Methodology

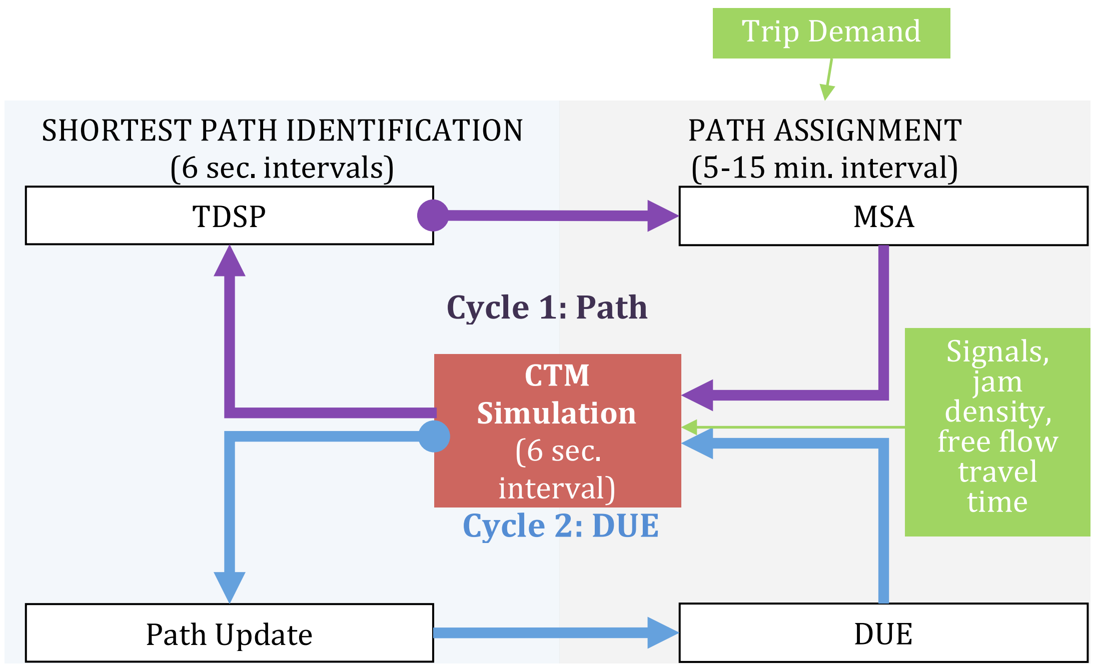


Figure 2.2: VISTA Convergence Algorithm Structure

The particular dynamic traffic assignment software that was used to replace static traffic assignment in the final step of the four step model for this study was VISTA - Visual Interactive System for Transport Algorithms (Ziliaskopoulos and Waller, 2000). This simulation-based model utilizes the Cell Transmission Model

(CTM) (Daganzo, 1994 and 1995) for representing vehicular movement, which is a discrete extension of the DTA concept presented by Lighthill and Whitman (1955) and Richards (1956). The essential component of CTM is the division of network links into discrete sections called cells and time into distinct intervals such that a vehicle can traverse exactly one cell in one time interval at free flow conditions. Instead of tracking continuous flows, VISTA further discretizes demand into individual vehicles and tracks paths and arrival times per vehicle. While this is a further discretization of the original CTM, it is perhaps more realistic in modeling path and link flows.

In order to achieve a converged traffic assignment, VISTA uses a simplicial decomposition approach. Outer cycles of path generation add the shortest path per origin-destination time interval (ODT) to the path set, and move a proportion of vehicles onto it. Inner cycles of equilibration shift vehicles among the existing paths (see Figure 2.2). It should be noted that vehicles were loaded within VISTA utilizing a partial demand loading technique. In this manner, $\left(\frac{1}{n}\right) * 100$ percent of the total demand is loaded onto the network over n iterations rather than loading all of the vehicles at once. This avoids gridlock and generally improves early iteration network performance. For more information on partial demand loading and a numerical validation of the technique, please see Chapter 3 of this thesis.

In order to properly implement DTA within the four step framework, some measure of aggregation is necessary to convert time-dependent travel times into a single travel time per OD for input into the trip distribution and mode choice steps. While a number of techniques may be suitable, this study proposes a simple average value of travel time taken across all departure time intervals with no weighting. In this way travel time may be used as a feedback measure into the standard four step trip distribution. Differing from STA, these travel times are the result of more accurate congestion propagation provided by the CTM. Once the average travel times are returned as input to the earlier model stages, feedback occurs in much the same way as traditional four step modeling. Multiple iterations of the four step process are conducted until the changes occurring in the trip distribution and mode choice steps fall beneath a desired cutoff value.

To gain an understanding of how the new four step model performs over multiple iterations, it was necessary to develop some criteria to measure convergence. In accordance with the Capitol Area Metropolitan Planning Organization's Travel Demand Model, the first measure of convergence included in this work is the root mean square error for travel times, which is computed for each OD as follows:

$$R_T = \frac{\sqrt{\sum_{OD}(\tau_{OD}^{i+1} - \tau_{OD}^{i})^2}}{N} \qquad [2.3]$$

where N is the number of ODs and τ_{OD}^{i} is the travel time for OD at iteration i. The same equation was applied to the demand as follows:

$$R_D = \frac{\sqrt{\sum_{OD}(d_{OD}^{i+1} - d_{OD}^{i})^2}}{N} \qquad [2.4]$$

where d_{OD}^{i} is the demand for OD at iteration i. Also included in the four step convergence evaluation is the cost gap from DTA after a fixed number of traffic assignment problem (TAP) convergence iterations, which reflects how close a given trip distribution is to an equilibrium solution. The cost gap is defined as follows:

$$G_{OD}^{t} = \sum_{t} \sum_{k \in K_{OD}^{t}} (r_k^t - \rho_{OD}^t) \times V_k^t \qquad [2.5]$$

where G_{OD}^{t} is the total ODT gap, K_{OD}^{t} is the set of all paths for OD, r_k^t is the cost of path k at time t, ρ_{OD}^t is the shortest path cost for ODT and V_k^t is the number of vehicles departing in assignment interval t and assigned to path k.

One of the challenges when integrating DTA into the four step model concerns how the transit mode is represented within dynamic traffic assignment. DTA software vary greatly in the methods by which transit systems are incorporated within the traffic assignment. VISTA includes an option for public transportation, such as buses, which allows the mode split model step to reduce the number of passenger vehicles as more travelers choose public transit. However, it is worth noting that the inclusion of other modes is possible depending on the DTA software involved.

2.4 Static vs. Dynamic Model Comparison

To evaluate the differences between static and dynamic assignment steps within the four step model, both were compared by evaluating a number of different metrics. Travel time root-mean squared error (RMSE) is defined as:

$$RMSE_{tt} = \frac{\sqrt{\sum_{OD}\left(\rho_{OD}^{t,i+1} - \rho_{OD}^{t,i}\right)^2}}{N} \quad [2.6]$$

where $\rho_{OD}^{t,i}$ is the shortest path travel time for ODt for iteration i of the four step model. This method shows the average change in shortest path travel time from the converged traffic assignment solution in step four. Within each four step iteration, the equilibrium convergence algorithm was run with constant stopping conditions. It is expected that $RMSE_{tt} \rightarrow 0$ as $i \rightarrow \infty$ because the change in trip distribution and mode choice should decrease as the four step model reaches a stable solution. The shortest path time was used, because for a well-converged solution all used path travel times should be close to the shortest path time.

Demand RMSE is defined similarly as follows:

$$RMSE_{V} = \frac{\sqrt{\sum_{OD}\left(V_{OD}^{t,i+1} - V_{OD}^{t,i}\right)^2}}{N} \quad [2.7]$$

where $V_{OD}^{t,i}$ is the demand for OD at time t for iteration i of the four step model. This directly measures the change in the trip distribution step between each model iteration, and similarly $RMSE_{V} \rightarrow 0$ as $i \rightarrow \infty$. Both $RMSE_{tt}$ and $RMSE_{V}$ were evaluated because the metrics might show different rates of stabilization.

Finally, G_{OD}^{t} was evaluated after a fixed number of assignment iterations as well to analyze how finding equilibrium could be affected by the stability of trip distribution and mode choice.

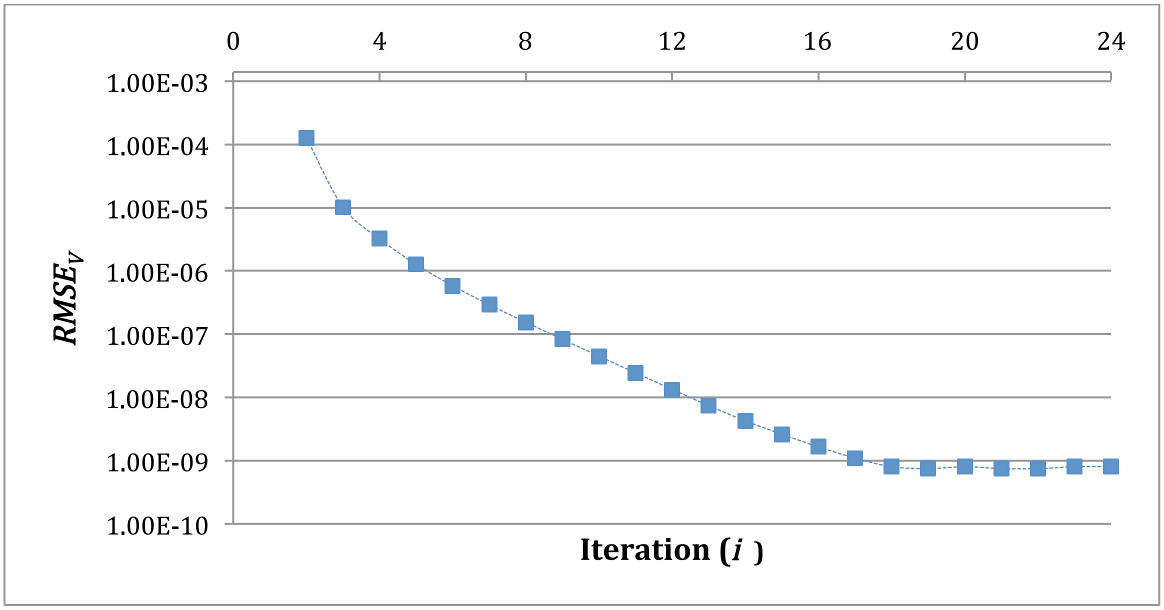


Figure 2.3a: $RMSE_V$ vs i when using the STA flow model within the network assignment step

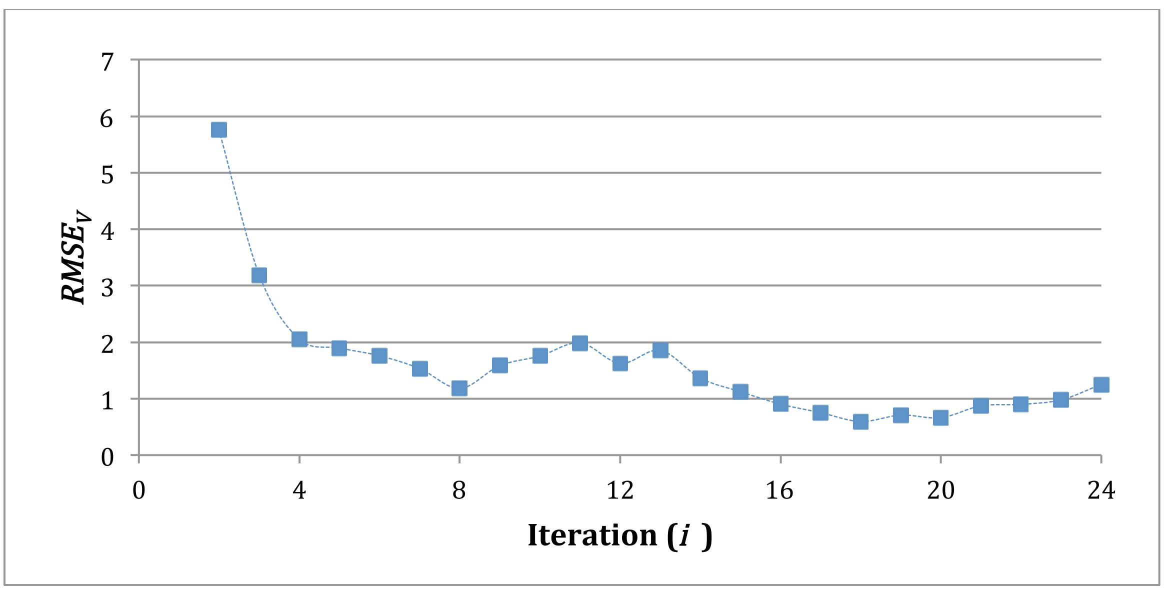


Figure 2.3b: $RMSE_V$ vs i when using the DTA flow model within the network assignment step

$RMSE_V$ has a generally decreasing trend when using a DTA flow model, which indicates movement towards a stable trip distribution and mode choice. However, the result is significantly different than that found with a STA flow model, indicating that the feedback from DTA is causing substantial changes in the demand table used as input for step four. $RMSE_V$ for the static flow model decreases very quickly and approaches 0 within 4 iterations of the four step model, while the same measure for the dynamic flow model takes a minimum value near 1. However, considering the convergence properties of both static and dynamic traffic assignment, the increase in error from the dynamic model is to be expected. Nevertheless, the downward trend present in Figure 2.3b is encouraging, as it indicates the integrated four step-DTA planning model is performing as expected.

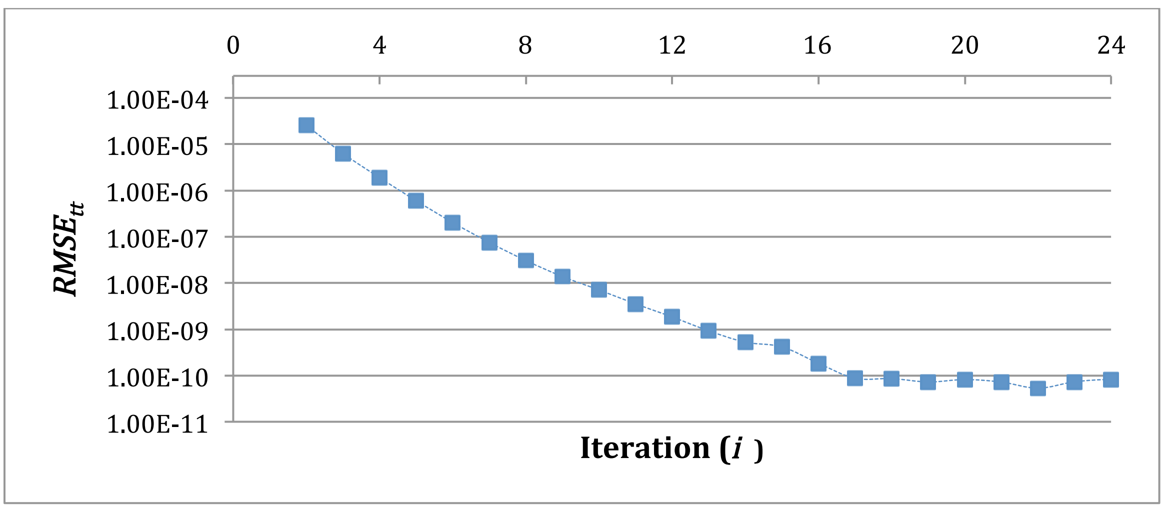


Figure 2.4a: $RMSE_{tt}$ vs i when using the STA flow model within the network assignment step

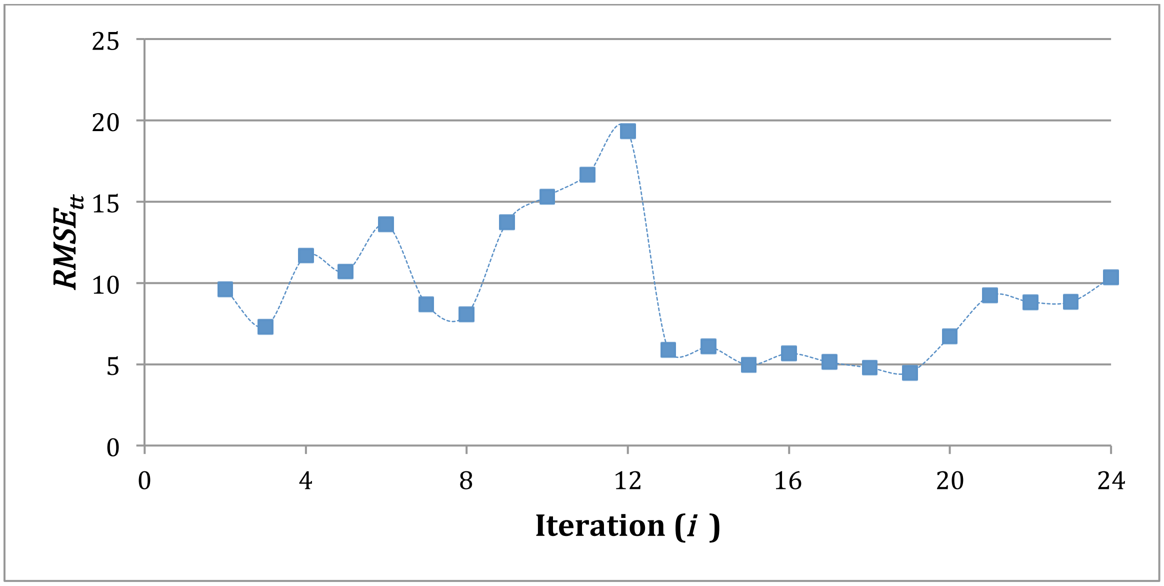


Figure 2.4b: $RMSE_{tt}$ vs i when using the DTA flow model within the network assignment step

$RMSE_{tt}$ in the model utilizing DTA had substantial differences in the trend per iteration that did not reflect $RMSE_V$. At iteration 12, there is a steep decline which then stabilizes around $RMSE_{tt} = 5$. However, there is an increase in $RMSE_{tt}$ at iteration 19. The lack of stability present in the $RMSE_{tt}$ for DTA can likely be explained by the fact that travel time for an OD is going to be affected by changes in the demand for other ODs. However, the $RMSE_V$ is a measure of demand change per OD, which is less dependent on demand changes in other ODs. Therefore, $RMSE_V$ appears more stable.

2.5 Incorporation of Travel Time Reliability

De Palma (2005) found that travel choices are affected as much by variance in travel time as by the average travel time itself. Therefore, the variability in travel time for vehicles on the same path was incorporated into the logit model. The variability in this case represents the variation in travel time due to time-dependent congestion under fixed demand. Since VISTA tracks individual vehicles, each vehicle can experience different travel times on the same path even with similar departure times. However, since the traditional four step model is time-invariant in nature, travel time variance was measured per OD. A weight of 6.058 was given as suggested by Martchouk (2009). To avoid the effect of outliers, an inner range of the set of experienced travel times was selected. Values of the inner 70%, 80%, and 90% of experienced travel times were tested. For example, the inner 70% travel time range refers to the difference between the 85th percentile travel time and the 15th percentile travel time.

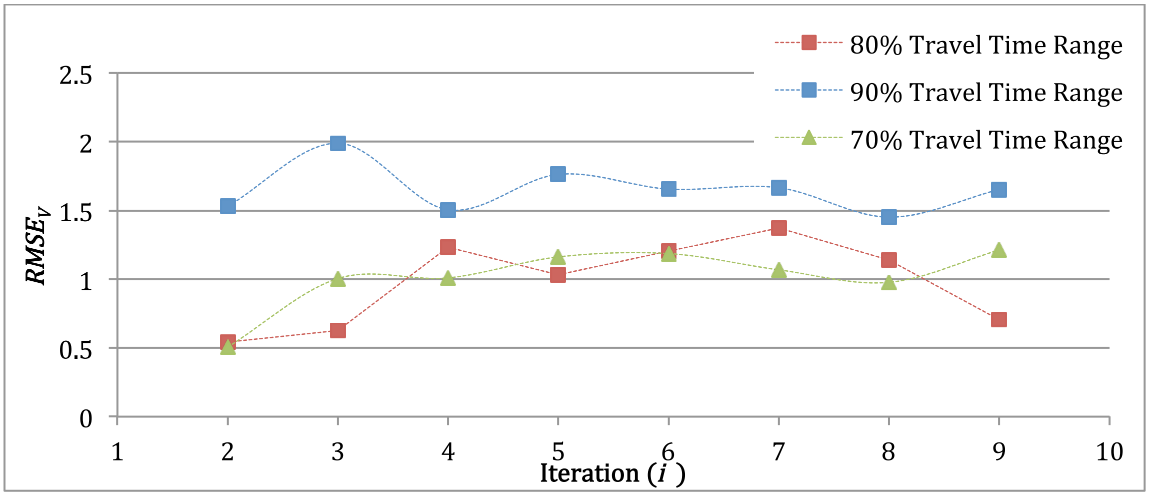


Figure 2.5a: Demand RMSE vs. Iteration for Multiple Travel Time Ranges

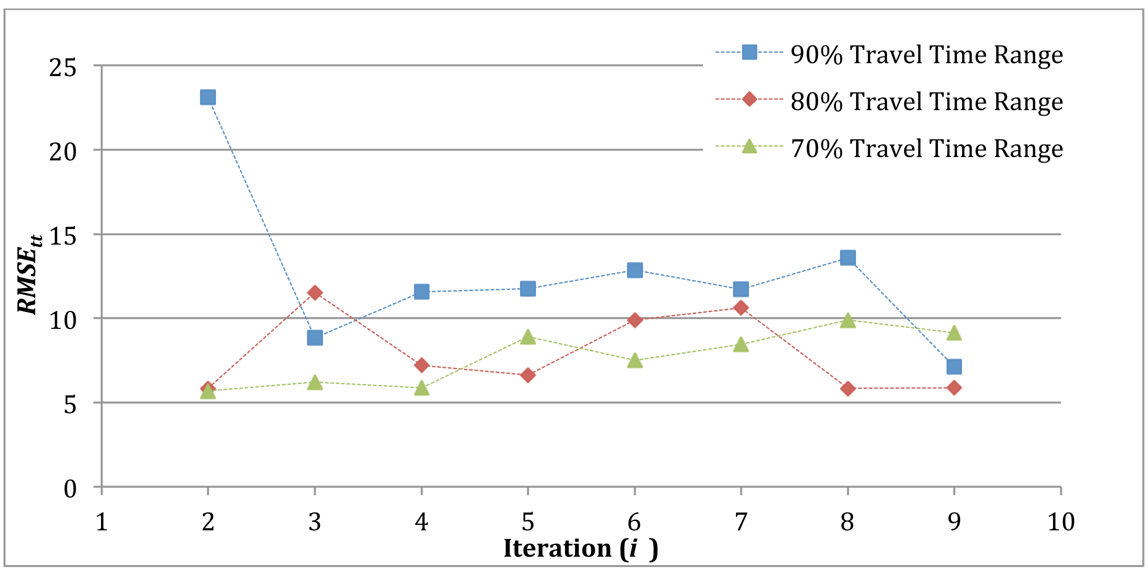


Figure 2.5b: Travel Time RMSE vs. Iteration for Multiple Travel Time Ranges

The addition of reliability significantly affected the decreasing trend seen in the $RMSE_V$ in Figure 2.1b. From Figure 2.5b, none of the different ranges of travel time used as input to the logit model demonstrated a decreasing trend. The $RMSE_{tt}$ in Figure 2.5b generally showed the same pattern. The 90% travel time range decreases significantly at several iterations, but does not appear asymptotic overall. This indicates that trip distribution and mode choice are influenced by the variance in travel time, and are likely too heavily impacted.

The 90% travel time range showed significantly higher error than the 70% and 80% ranges, which were more similar. This suggests that a higher variance combined with a large weight will increase the change in demand and therefore travel times between iterations of the four step model and thus slow convergence of trip distribution and mode choice. As will be discussed further below, calibrating the weight and variance range should improve this convergence.

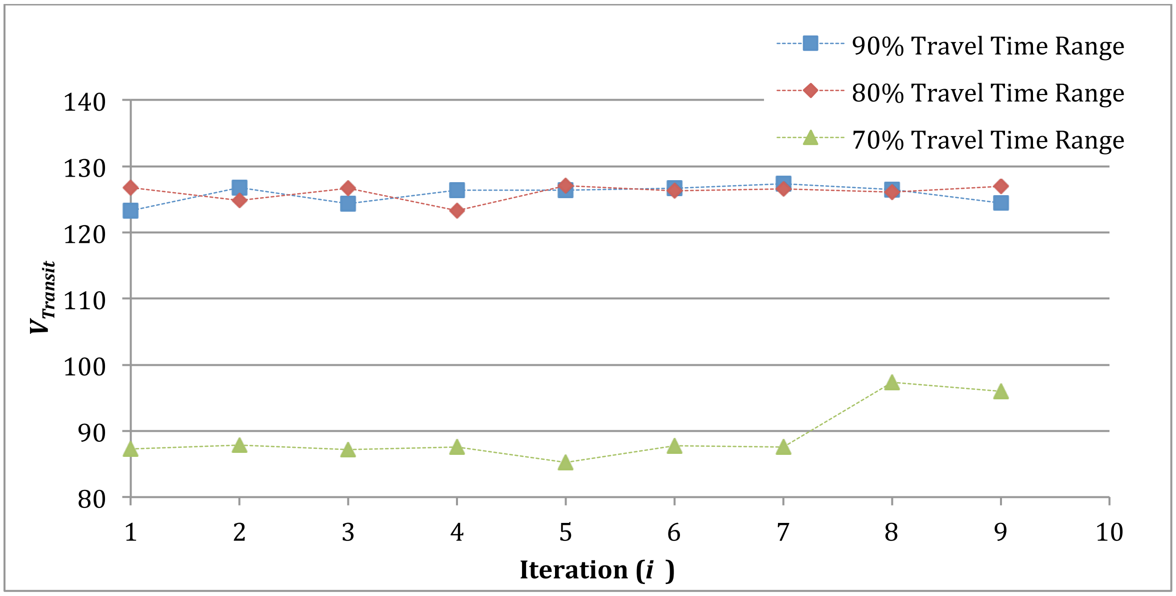


Figure2.5c: Transit Demand vs. Iteration for Multiple Travel Time Ranges where $V_{Transit}$ is the total demand moved to transit.

The most noticeable feature of Figure 2.5c is the difference in transit demand between the different travel time ranges for the reliability input. A travel time range of 70% produced far less transit demand than the 80% and 90% ranges, which both had very similar transit demand. The most extreme 20% of travel times are not equal as discussed above, but they may be high enough that most travelers are choosing transit if given the opportunity. However, when the travel time range was reduced to 70%, the transit demand significantly decreased, indicating that the range of travel times was much lower for this cutoff value. It is possible that these trends may vary by network. In terms of usefulness to practitioners, this means that the selection of travel time range must affect the weight given to the variance in travel time. Real-world data should be collected to correctly choose the correct travel time range and reliability weighting for different types of networks (i.e. urban or rural).

It should be noted that the minimal variance in transit demand for a single reliability level can likely be explained by the small number of bus routes encoded within the model. Adding more bus routes will give more travelers the option of mode choice, and thus increase the total transit demand and variation.

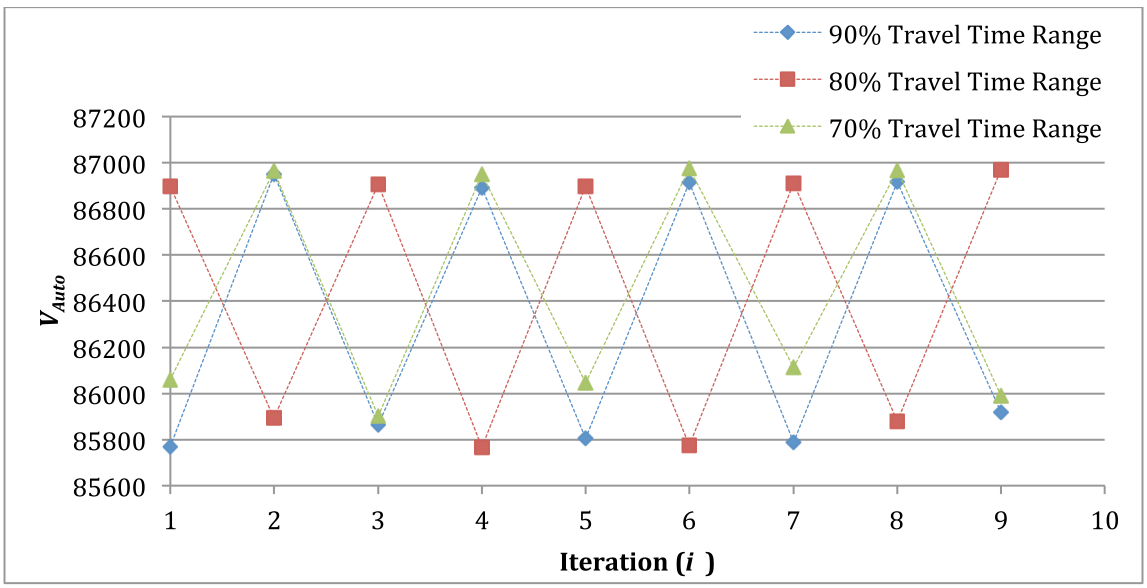


Figure 2.5d: Total Auto Vehicles vs. Iteration for Multiple Travel Time Ranges where V_{Auto} is the automobile demand after mode choice.

The total number of automobiles on the network, shown in Figure 2.5d, appears sinusoidal with little indication of convergence. This is explained by too much weight being placed on the variance in travel time for trip distribution and mode choice. Low demand ODs have smaller variance, so travelers are first shifted to those. Then those ODs have increased variance due to the additional trips, and as a result vehicles are shifted off. The number of vehicles moved is high because of the large weight placed on the variance in travel time. The decreasing amplitude in the 70% travel time range reliability provides evidence for this hypothesis. As mentioned earlier, the variance in the 70% travel time range is significantly lower. Again, using data to choose correct variability weighting will yield more accurate results.

2.6 Conclusion

The results of this work shows that integrating DTA into the four step model framework is a viable solution and will most likely lead more accurate predictions of trip distribution and mode choice due to the more realistic propagation of traffic congestion in DTA. Reliability, which contains parameters that should be tested and calibrated further, could potentially produce increased accuracy of predictions. One potential drawback of this integration is that DTA requires significantly more computation time to approach an equilibrium solution than STA. However, advances in DTA algorithms and heuristics will reduce this difference. This topic is explored in detail in Chapter 3 of this work.

This study indicates that complete integration of DTA into the four step model still has many unresolved questions. As previously discussed, the calibration of the weight on reliability requires work. Also, all results expressed in this work are potentially affected by network characteristics, so it would be useful to conduct similar experiments on a variety of networks. Time-dependent DTA information was compressed into a single average travel time that was fed back into earlier model steps. However, a different statistic, such as median travel time, might better model traveler decisions. Furthermore, the time-dependent travel time output from DTA was not fully utilized because the traditional four step model performs trip distribution and mode choice independent of departure time. Separation of trips into assignment intervals, as is done in VISTA for the trips, will more realistically model how travelers account for changes in traffic during different periods (such as rush hour). In fact, these time-dependent travel times could be extended over an entire day for predictive modeling of trip distribution and mode choice.

Chapter 3: Improving the Convergence of Simulation-Based Dynamic Traffic Assignment

3.1 INTRODUCTION

Dynamic traffic assignment models have become a widely accepted tool to support a variety of transportation network planning and operation decisions. The ability of these models to produce stable and meaningful solutions is crucial for practical applications, particularly for those involving the comparison of modeling results across multiple scenarios. Although the literature presents a fairly unified approach to define the conditions that characterize equilibrium in the context of simulation-based DTA (SBDTA) (e.g. Lo et al., 2000; Lu et al., 2009; Chiu et al., 2009), practical implementations differ in the methodology used to attain these.

The first goal of this research effort is reviewing and contrasting existing methodological approaches for the equilibration of large-scale SBDTA models. Increased convergence in SBDTA models may improve viability for practitioners. In order to study the convergence pattern of different methodologies under comparable conditions, this work implements several of the surveyed techniques and some novel variations within a common SBDTA platform.

There are two main processes which are repeated multiple times during the solution of a SBDTA framework: the simulation of traffic conditions for a given assignment of vehicles to paths, and the search for new shortest paths based on the simulated traffic conditions. Both may involve significant computational effort, depending on the characteristics of specific SBDTA implementations. This paper is focused on providing a better understanding of the characteristics of the convergence process of different algorithms. The results of the numerical experiments, conducted on real networks with up to 200 000 trips, are described in terms of the number of simulations runs and time-dependent shortest path computations required to achieve an equilibrium solution. The analysis includes a discussion of the properties of the solutions obtained through different methodologies intended to reveal the cause for the observed convergence rate. These properties may play a role in the selection of an acceptable convergence level for practical applications. The computational efficiency of the analyzed techniques is not explicitly described, as it will highly depend on implementation-stage decisions that involve other components of a SBDTA model. Instead, this paper is focused on providing a better understanding of the characteristics of the convergence process of different algorithms. The results of this effort may motivate the development of efficient implementations that can improve the ability of SBDTA models to handle larger networks more efficiently.

3.2 Literature Review

3.2.1 Introduction

The typical solution framework for SBDTA models, described in Section 2.1, seeks to attain equilibrium conditions as defined in the literature (e.g. Chiu et al., 2011). To this end, early implementations of DTA models mostly relied on the method of successive averages (MSA) as described in Sheffi (1985), which has been shown to converge to the equilibrium solution in static traffic assignment problems with well-behaved link-cost functions (Powell et al., 1982). The framework used for the static case may be easily extended to the solution of simulation-based DTA problems, although convergence is not guaranteed in the dynamic case due to the complex nature of link costs when traffic dynamics are accounted for. Furthermore, the typically slow convergence rate of MSA (Sheffi, 1985) is particularly detrimental in large-scale DTA applications, where the computation cost per iteration can be very high. The limitations of MSA approaches have spurred research aimed at both, heuristically improving the efficiency of MSA for DTA, and developing more advanced solution algorithms. Both types of methodological approaches are described in the following sections.

3.2.2 Simulation-Based DTA Models: Solution Framework

SBDTA models are typically chosen for practical applications over their analytical counterparts, which are typically suitable only for the study of very small networks. Moreover, SBDTA models are appealing because they can realistically capture the impact of a variety of traffic control devices, network operation strategies, and time-dependent changes in traffic conditions. Typical SBDTA frameworks include three main components: a traffic simulator, a path generator, and an assignment module, searching for equilibrium conditions using an iterative approach. A traffic simulator is used to evaluate the network performance based on a specific assignment of vehicles to paths. The path generator uses simulation results to find the time-dependent least-cost path under prevalent conditions per origin-destination pair and assignment interval combination. The assignment module adjusts the allocation of vehicles to paths with the goal of attain dynamic equilibrium conditions.

Convergence criteria are assessed and the assignment of vehicles to paths is adjusted based on some pre-defined logic. The process is repeated until an acceptable solution is found. In order to evaluate convergence most SBDTA applications define a "gap" which measures the proximity of a given solution to the equilibrium conditions. SBDTA models differ mostly in the type and refinement of the selected traffic simulator, and on the rationale behind the assignment adjustments, which are the focus of this paper. Various techniques are proposed in the literature and presented in the following sections.

3.2.3 MSA-Based Techniques

In the context of DTA, MSA algorithms involve finding the time dependent shortest paths under prevalent conditions and shifting a pre-determined fraction of vehicles to such routes. The fraction of vehicles to be re-assigned, called the step size, decreases as the algorithm progresses, and is equal to $\frac{1}{n}$ (where n is the iteration number) for all ODTs. Sbayti et al. (2007) and Chiu et al. (2009) notice that the use of a global step-size is a source of inefficiencies, as some ODT pairs may be closer to convergence than others at any point in the process. Further, later assignment intervals are typically further away from convergence (Mahut et al., 2007). Based on these observations several heuristic approaches have been proposed, aimed at making more efficient selection of the vehicles to be re-assigned.

Sbayti et al. (2007) propose two techniques based on MSA which differ in the criterion used to select the re-assigned vehicles for each ODT. The first method, aimed at reducing memory requirements, makes a random selection. The second approach implements a criterion-based selection that gives priority to the vehicles experiencing the highest travel time within each ODT. When implemented on a real network both methodologies were observed to converge, with the criterion-based technique producing the lowest gap.

Mahut et al. (2007) suggest using a larger time step for later assignment intervals by offsetting the MSA step size by a fixed quantity for increasing time intervals. The technique is observed to clearly accelerate the convergence of MSA in a network with 580 links and 47,000 trips between 62 OD pairs.

In combination with some of the previous techniques, Florian et al. (2008) also incorporate a partial demand loading scheme that progressively adds vehicles to the system over a pre-specified number of iterations. This method avoids the high congestion resulting from an initial assignment to one path from ODT.

3.2.4 Gradient Based Techniques

Gradient projection and reduction methods, common in non-linear optimization literature, have been successfully implemented for the solution of static traffic assignment problems (Bertsekas et al., 1983) and analytical DTA models (Szeto et al., 2006). Simulation-based DTA models do not meet the conditions under which gradient-based methods can be directly applied; the use of simulation typically prevents the formulation of these models as an optimization problem with a differentiable objective function. However, Lu et al. (2009) propose a re-formulation of DTA via a gap function that provides a sound theoretical basis for the development of gradient-based heuristics. These methods typically involve an iterative process similar to that followed by MSA techniques, but use a step size selected based on endogenous data, such as simulated path costs, in an attempt to approximate the missing gradients. Lu et al. (2009) define a per-path step size that is proportional to the difference between the path cost and the corresponding ODT shortest path cost. Their method, embedded within a column generation framework (Section 2.4), is observed to outperform MSA in several experiments conducted on small and medium size networks.

Using the same step-size definition, Tong and Wong (2010) compare the convergence of the gradient-based procedure to that of MSA on a small network under different demand scenarios. Their results do not show significant differences among methodologies, although the gradient-based approach is observed to lead to a slightly lower gap in less congested scenarios. Chiu and Bustillos (2009) and Mahut et al. (2007) propose step sizes calculated based on individual path costs, but aggregated at the ODT level. Both research efforts report faster and smoother convergence patterns for gradient-based heuristics than for MSA and MSA-based heuristic methods.

Jayakrishnan et al. (1994) proposed a gradient-projection method which improves the performance of static traffic assignment by choosing the path flows of the optimal descent direction. New path flows are a function of path costs as well as the derivative of the link delays of certain links. Although the derivative of link cost is not well defined for a SBDTA model, the remainder of the formulation inspired one of the gradient-based heuristics.

3.2.5 Column Generation Techniques

The generation of the set of paths to which vehicles may be assigned is the focus of the third type of heuristic approaches considered in this paper. MSA methodologies start from an empty set and (potentially) augment it every iteration. The optimization literature suggests that a more efficient path set can be created if column-generation principles are applied (Patriksson, 1992). The latter is appealing as a means to improve convergence and reduce memory-requirements. The column generation approach involves two nested cycles: an outer loop that augments the path set at every iteration, and an inner cycle during which trips are distributed among the available routes seeking to equalize travel costs. Lu et al. (2009) present results in which the column generation approach outperforms other algorithms, particularly when combined with a gradient-based equilibration in the inner loop.

3.2.6 Summary

The research efforts reviewed in this section propose interesting methodologies and report satisfactory results. However, aside from a few exceptions (Mahut et al., 2007; Tong and Wang., 2010), the performance of each methodology is compared only to that of MSA. The experiments presented in this chapter assess the relative performance of the surveyed techniques in terms of convergence rate and stability of the results. Additionally, Section 3.4 also proposes and tests some novel variations of the existing methodologies.

3.3 METHODOLOGY

This section describes the algorithms to be implemented in Section 3.4, including methods proposed in the literature and original variations. All the methodologies are presented using the same notation (Table 3.1) to facilitate the understanding of their similarities. The selection of the techniques to be tested followed two criteria: the success of the methods in previous studies, and the compatibility of the approaches with the platform used in this study.

Table 3.1: Notation

Symbol	**Description**
$\alpha(i)$	Step size at iteration i for MSA-based heuristics
d	Destination index
$c_v^{o,d,t}(i)$	At iteration i, cost experienced by vehicle v departing from origin o to destination d at assignment interval t
$c_p^t(i)$	Cost experienced by vehicles on path p departing in time interval t at iteration i
D	Number of partial demand loading iterations
$\delta_{v,k}(i)$	Vehicle-path incidence, equal to 1 if vehicle v is assigned to path k at iteration i (zero otherwise)
$f_{o,d}^t(i)$	Modified step size per ODT at iteration i
$f_p^t(i)$	Modified step size per path at iteration i for path k in interval t
F	User defined cutoff factor for the OD gap sort methodology
$g_v(i)$	Gap for vehicle v at iteration i
$G_{o,d}^t(i)$	Total ODT gap at iteration i
$\gamma(i)$	Gap per iteration expressed as a percentage of total travel time
i	Iteration index
k	Path index
$n_{o,d}^t$	Demand from origin o to destination d at assignment interval t
N	Total number of trips to be assigned

Table 3.1, cont.

o	Origin index
$P_{o,d}^{t}(i)$	Set of paths for an ODT at iteration
$\rho_{o,d}^{t}(i)$	Shortest path cost for an ODT combination at iteration i
$R_{o,d}^{t}(i)$	Average vehicle gap per ODT at iteration i
$S(i)$	Total number of vehicles swapped to the corresponding ODT shortest path at iteration i
$s_{o,d}^{t}(i)$	Number of vehicles swapped to the shortest path for each ODT
t	Assignment interval index
v	Vehicle index
$V_{k}^{t}(i)$	Number of vehicles departing in assignment interval t and assigned to path k

3.3.1 MSA-Based Heuristics

The methodologies in this category are aimed at improving the performance of the Method of Successive Averages (MSA) when applied to the solution of simulation-based DTA (SBDTA) problems. While they all implement a pre-fixed sequence of decreasing step sizes, they differ on the selection of specific vehicles shifted to the new ODT shortest path.

3.3.1.1 Partial Demand Loading

The partial demand loading scheme, described in Florian, Mahut, and Tremblay (2008) is an initialization procedure that involves the incremental assignment of the demand over a fixed number of iterations (D). In the first D iterations the algorithms proposed in this work assign a fraction ($1/D$) of the total ODT demand to the corresponding shortest path $\rho^t_{o,d}(i)$, which is recomputed at each iteration. By spreading out the demand among a larger number of paths in the initial stages, this heuristic approach is designed to prevents an artificial oversaturation of the network during early iterations.

3.3.1.2 ODT Gap Sorting (ODT Sort)

This technique is a simple heuristic adjustment aimed at addressing the inefficiencies derived from applying the same step size to all ODT combinations, regardless of how far from or close to equilibrium they may be. ODTs are sorted based on their total gap (Equation 3.1), and assignment adjustments are applied only to the first T ODTs, where T is such that $\sum_j G_j\,(i) = F, 1 < j < T$.

$$G^t_{o,d}(i) = \sum_t \sum_{k \in K^t_{o,d}} \left(r^t_k(i) - \rho^t_{o,d}(i) \right) \times V^t_k \qquad [3.1]$$

In this statement j is the index of the sorted ODTs, and F is a user defined cutoff factor. The number of vehicles to be re-assigned is given by equation 3.2. This technique is embedded in the VISTA SBDTA platform (Ziliaskopoulos & Waller, 2000).

$$s_{o,d}^{t}(i) = \alpha(i) \times n_{o,d}^{t} \qquad [3.2]$$

3.3.1.3 ODT-Based Vehicle-Path-Cost Sorting (Path Sort)

In this approach, which is also described in Sbayti et al. (2007), the same fraction of vehicles is re-assigned to the new shortest path for every ODT ($a_{o,d}^{t}(i) = \alpha(i) \forall_{ODt}$). A total of $s_{o,d}^{t}(i)$ vehicles are selected by sorting all paths $k \in K_{o,d}^{t}$ based on the cost gap of vehicles on the path $r_k^t(i)$, and choosing vehicles such that $\delta_{v,k}(i) = 1$ until the desired quota is met. The prioritization of vehicles in higher-cost paths is expected to lead to a faster convergence rate than traditional MSA approaches.

3.3.1.4 Vehicle-Cost Sorting (Vehicle Sort)

This approach sorts vehicles based on their experienced cost $c_v^{o,d,t}(i)$ without grouping by ODT. This is hoped to improve convergence by moving the vehicles that contribute most to the gap first.

3.3.1.5 ODT-Based Vehicle-Cost Sorting (ODT Vehicle Sort)

This approach is a variation of the previous method, in which the first $s_{o,d}^{t}(i)$ (Equation 3.1) are selected for each ODT combination from a list of vehicles sorted by $c_v^{o,d,t}(i)$. Notice that in general $c_v^{o,d,t}(\mathrm{i}) \neq r_k^t(\mathrm{i})$ for $\delta_{v,k}(i) = 1$, and this approach is more likely to select fewer vehicles from a larger set of paths when compared to the previous one.

3.3.2 Gradient-Based Heuristics

Following the principles proposed by Lu et al. (2009) and Chiu and Bustillos (2009), the proposed algorithm seeks to circumvent some of the inefficiencies of the traditional MSA approach by utilizing an endogenous step-size, computed based on the level of convergence at each ODT, as given by the average ODT vehicle gap (Equation 3.3) or the total ODT gap (equation 3.1)

$$R_{o,d}^{t}(i) = \frac{\sum_{k \in K_{o,d}^{t}} \left(r_k^t(i) - \rho_{o,d}^{t}(i)\right) * V_k^t}{n_{o,d}^{t}} \qquad [3.3]$$

Based on either of the formerly defined gap measures, two ODT step scaling factors may be computed, which originate two possible heuristics: lambda average gap (equation 3.4) and lambda total gap (equation 3.5).

$$f_{o,d}^{t}(i) = \frac{R_{o,d}^{t}(i)}{\sum_{o,d,t} R_{o,d}^{t}(i)} \qquad [3.4]$$

$$f_{o,d}^{t}(i) = \frac{G_{o,d}^{t}(i)}{\sum_{o,d,t} G_{o,d}^{t}(i)} \qquad [3.5]$$

Scaling factors are used to define the local step size $a_{o,d}^{t}(i) = min\left[f_{o,d}^{t}(i) \times \alpha(i), \alpha(i)\right]$. Following the step size selection, $s_{o,d}^{t}(i) = a_{o,d}^{t}(i) \times n_{o,d}^{t}$ vehicles are re-assigned to the new shortest path for each ODT, chosen from a sorted list as described in Section 3.1.5.

Mahut et al. (2007) suggested that later assignment intervals cannot be truly converged until previous assignment intervals are stabilized. They propose implementing a cascading pattern of step sizes by time interval, with higher values at later time intervals, such as the one presented in Table 3.2. Lambda is initially constant with respect to time period up to a pre-specified time interval, then at some N begins to gradually shift into a cascade pattern. The shift finalizes into a pattern described by the reset parameter n, which is the iteration difference in the lambda value between period T and $T + 1$. In practice, the optimal parameters are unknown, and effectiveness is likely to vary with respect to the combination of cascade parameters and network.

Table 3.2: Time-Varying Cascade of Lambda Values (reset parameter $n = 2$ in this example)

Assignment interval

Iteration	**1**	**2**	**3**	**4**	**5**	**6**
20	$\frac{1}{20}$	$\frac{1}{18}$	$\frac{1}{16}$	$\frac{1}{14}$	$\frac{1}{12}$	$\frac{1}{10}$
21	$\frac{1}{21}$	$\frac{1}{19}$	$\frac{1}{17}$	$\frac{1}{15}$	$\frac{1}{13}$	$\frac{1}{11}$
22	$\frac{1}{22}$	$\frac{1}{20}$	$\frac{1}{18}$	$\frac{1}{16}$	$\frac{1}{14}$	$\frac{1}{12}$

Two further heuristics relying on the difference in costs between the shortest paths and others. Inspired by Lu et al. (2009), Lambda Relative Path Cost applies a path-specific step size $f_p^t(i)$ (equation 3.6)

$$f_p^t(i) = \min\left(1.5\times\alpha(i), \frac{c_p^t(i)-\rho_{o,d}^t(i)}{\rho_{o,d}^t(i)}\right) \qquad [3.6]$$

where c_p is the cost of a path and p^t is the shortest path. This method is similar to a STA gradient-projection method by Jayakrishnan et al. (1994) without the incorporation of the second derivatives (the derivative of link travel times). Further work using an approximation of the second derivatives may be beneficial, but was left out here because the second derivatives are not well defined for SBDTA.

Lambda Relative Gap Sum aggregates path-based gap per ODT in order to compute the adjustment factor $f_{o,d}^t(i)$ (equation 3.7).

$$f_{o,d}^t(i) = \min\left(1.5\times\alpha(i), \sum_{p\in P_{o,d}^t(i)} \frac{c_p^t(i)-\rho_{o,d}^t(i)}{\rho_{o,d}^t(i)}\right) \qquad [3.7]$$

Due to the variation in vehicle travel times even on the same path in DTA, this could be more effective.

3.3.3 Column Generation Approach

The column-generation framework described in some of the reviewed works (e.g. Lu et al., 2009) was implemented in this work in combination with the path sort technique. The approach involves two nested loops, an outer loop that augments the path set $K_{o,d}^t(i)$, and an inner loop, which applies an MSA-based heuristic to find the equilibrium solution within the augmented path set. The model parameters include the number of paths added per iteration of the outer loop and the level of convergence required from the path-swapping algorithm used in the inner cycle. The aim of this technique is to generate a more efficient path set, which should ideally lead not only to a faster convergence, but also to more stable and better equilibrated solutions.

3.4 Numerical Experiments

The numerical experiments presented in this work were conducted using the traffic simulator and data structures embedded in the VISTA SBDTA platform (Ziliaskopoulos & Waller, 2000), which implements Daganzo's cell-transmission-model (Daganzo, 1994, 1995) for traffic simulation. The adopted mesoscopic simulation framework captures traffic dynamics, such as queue formation and dissipation, and has been extended to account for traffic signals and other characteristics of urban intersections.

The various assignment techniques compared in this effort were programmed outside of the SBDTA platform and designed to access individual modules as needed. As a result, the computational efficiency of the studied methods is not optimized. The latter does not affect the conclusions of this study, which are based on comparing the convergence patterns of assignment methodologies.

The following sections describe the networks considered for the numerical experiments and provide a detailed description of the analyzed scenarios and corresponding assumptions.

3.4.1 Test Networks

The networks used in this study (Table 3.3) include downtown Austin, with 90 thousand trips, and the Williamson County network, with more than 200,000 trips during the peak period. Figure 3.1 presents the Austin regional planning network and highlights the selected sub-networks. The Downtown network, although geographically smaller, incorporates a much larger number of local streets and signalized intersections. Most of it also exhibits a grid structure, resulting in paths that share many links, while the Williamson County network represents a suburban configuration. Individual models were built and ran separately for each analyzed sub area. In all experiments the modeled demand spans the 2-hour peak period, and is profiled across using 10-minute intervals. The simulation is allowed to run for a longer period in order to allow all vehicles to complete their trips.

Table 3.3: Test Networks

Network	Links	Signals	OD pairs	Trips
Downtown Austin	1590	168	3,518	89,078
Williamson County	2,184	69	45,586	201,588

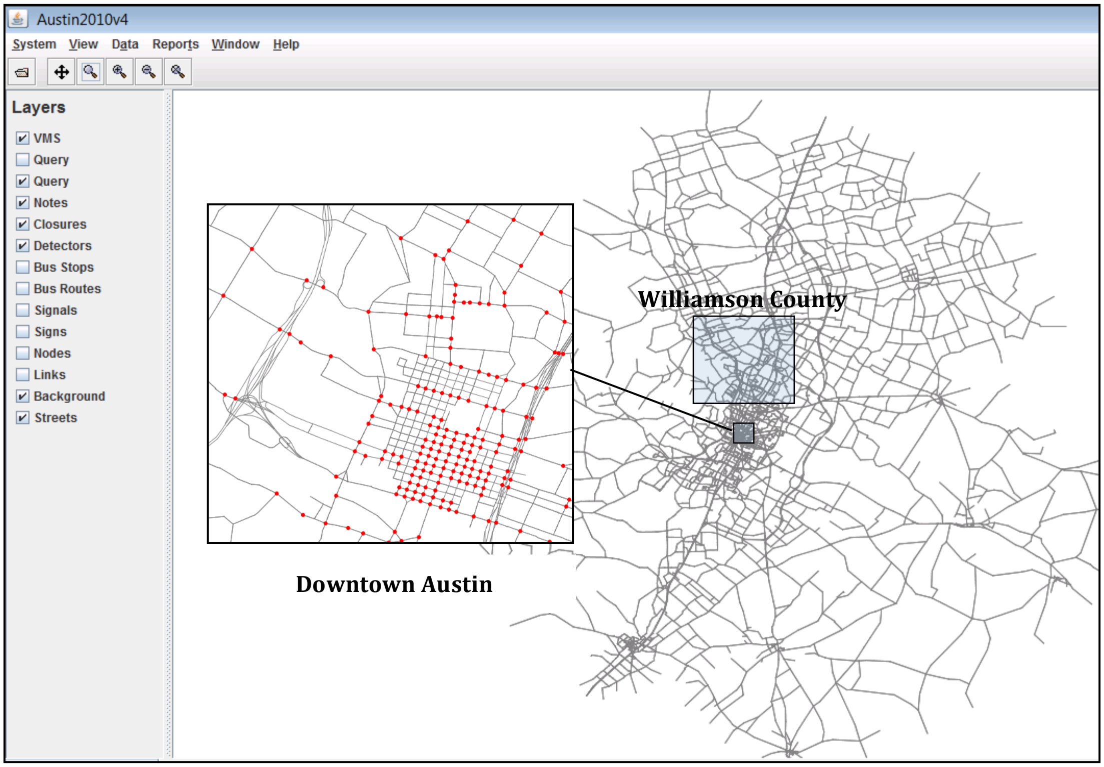


Figure 3.1:Austin Sub-Networks

3.4.2 Experimental Design

Table 3.4 provides a list of the experiments conducted in this work. All methods were tested on both, the Downtown and Williamson County networks, to ascertain convergence behavior on varying network structure. All implemented algorithms include the partial demand loading scheme described in Section 3.2.2. The MSA-based heuristics are set to generate a maximum of 35 paths per ODT (5 during the partial demand loading phase and 30 during the first iterations), after which they behave as path-swapping methodologies. The proximity between relative and true gap (Section 3.5) in all analyzed cases suggests that the selected threshold is sufficient.

Upon examining initial results, the best performing technique for the first 30 iterations was selected to be implemented with the column generation framework. The number of inner loop iterations for this methodology was set to a maximum of 10, pending on an early switch back to the outer loop if the gap reduction from a given iteration to the next falls below a user-defined parameter. A final set of experiments was run to test the stability of the results obtained using different methodologies.

Table 3.4: Experimental Design

Methodology	**Parameters**
Partial Demand Loading	$D \in \{5,10\}$
ODT sort	$F \in \{20,50,75\}$
Path sort	35 paths max.
Vehicle sort	35 paths max.
ODT vehicle sort	35 paths max.
ODT-gap-based step-size scaling	35 paths max.
Column Generation	Used 3.1.3 for assignment, 10 iter. max.

3.5 EXPERIMENTAL RESULTS

A set of preliminary experiments was conducted to assess the advantages of implementing a partial demand loading initialization along with other methodologies. In this table (and in the remainder of this section) the reported gap value $\gamma(i)$ (Equation 3.8) is expressed as a percentage of the total system travel time.

The results of the preliminary tests, presented in Table 3.5, suggest that the partial demand loading heuristic has very favorable impacts on the overall algorithmic performance. The number of iterations in which the system was overly congested was reduced by more than 50%, while satisfactory convergence levels were attained after as little as 20 iterations in some cases.

$$\gamma(i) = \frac{\sum_{o,d}\sum_t \sum_{k\in K_{o,d}^t}\left(r_k^t(i) - \rho_{o,d}^t(i)\right)\times V_k^t}{\sum_{o,d}\sum_t \sum_{k\in K_{o,d}^t}\left(r_k^t(i)\right)\times V_k^t} \qquad [3.8]$$

Table 3.5: Partial Demand Loading Results

Network	**Performance Measure**	**Scenario** D=1 (no initialization)	**Scenario** D=5	**Scenario** D=10
Downtown Austin	Iterations with overflow	9	3	3
	$\gamma(20)$	8.774	4.395	3.078
	$\gamma(50)$	-	1.912	1.724
Williamson County	Iterations with overflow	13	5	3
	$\gamma(20)$	11.939	9.118	8.568
	$\gamma(50)$	-	4.907	4.492

Table 3.6 presents convergence metrics for the remaining methodologies tested in this study. The results show that for most methodologies, acceptable gap levels were attained in both networks after 50 iterations (pre-defined stopping criteria). Exception is given to the ODT sort methods, which consistently performed worse than MSA. Figure 3.3 displays the convergence level as a function of the iteration number for several methodologies on the Williamson County network, suggesting that the selected number of iterations is enough to achieve a stable gap. In this figure most of the methodologies exhibit very similar performance. The ODT vehicle sort approach presents a slightly smoother behavior than others, while the ODT path sort approach is less smooth than most. The column generation implementation did not perform as well as expected, which may be related to the relative low gap reduction attained within the inner loop.

In general, none of the techniques was observed to significantly outperform MSA, which is in agreement with the findings of Tong and Wong (2010). However, further analyses of the stability and overall quality of the results obtained through different methodologies indicate that the altered vehicle reassignment has a significant impact, which might result in an improvement in convergence performance after further research. The following sections describe the results for each methodology in further detail.

Table 3.6: Convergence Metrics for Tested Heuristics (gap as a percentage of total travel time)

	Downtown Austin			Williamson County		
	$\gamma(10)$	$\gamma(20)$	$\gamma(50)$	$\gamma(10)$	$\gamma(20)$	$\gamma(50)$
MSA	11.579	4.459	2.053	16.197	8.700	4.366
ODT sort F=20	25.494	26.017	25.991	39.928	38.652	37.994
ODT sort F=50	17.603	17.287	16.900	29.845	27.159	25.189
ODT sort F=75	8.525	7.909	6.819	21.417	15.406	11.910
Path sort	14.006	2.712	1.129	15.055	10.457	5.029
Vehicle sort	10.549	8.390	8.043	34.411	34.493	33.863
ODT vehicle sort	7.325	2.438	1.692	15.547	8.555	3.892
Lambda Average Gap	7.165	2.933	1.808	16.739	10.534	5.312
Lambda Total Gap	10.818	2.759	1.858	17.826	9.117	4.349
Lambda Relative Path Cost	27.603	7.474	1.644	21.935	21.280	10.143
Lambda Relative Gap Sum	8.578	2.657	1.995	12.376	7.711	4.438
Lambda Time Varying	10.778	3.913	1.817	10.472	10.555	4.998
Column Generation	3.961	3.026	1.839	19.483	7.502	7.332

3.5.1 MSA-Based Heuristics Results

3.5.1.1 ODT sort

Each of the three ODT sort experiments produced gaps, which were consistently worse than MSA for both the Downtown and Williamson County network scenarios. The fact that the gap increased as F decreased suggests that F = 100, or equivalently ODT vehicle sort, should be the best performing cutoff value. It is worth noting that the gap decreases almost linearly with increasing F, suggesting that a local minima in gap for F less than 100 is unlikely. This technique may be useful to refine the results of a nearly converged network, but not to be applied throughout the process.

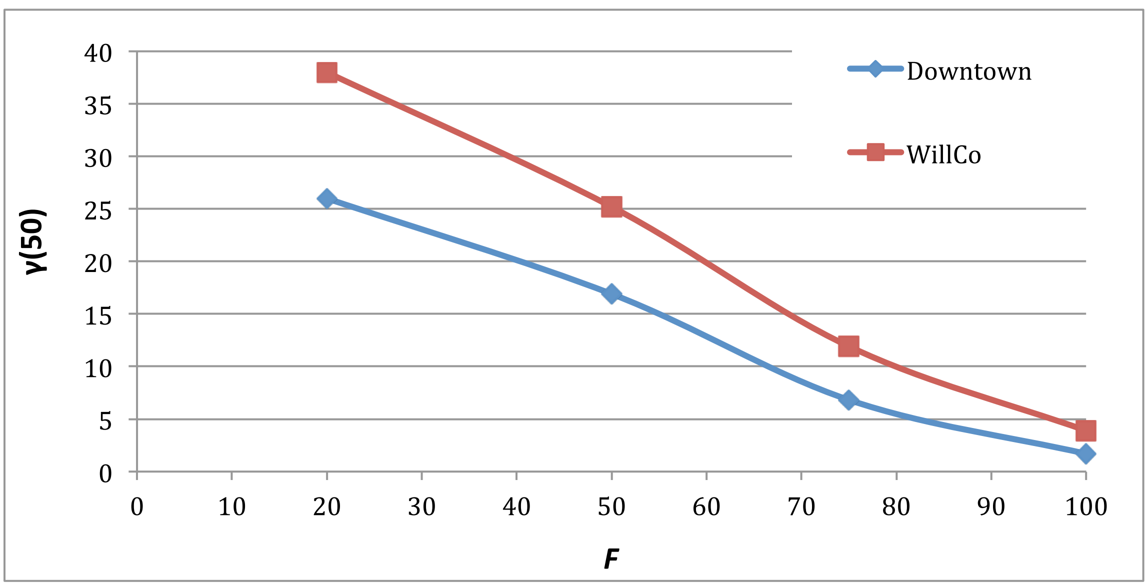
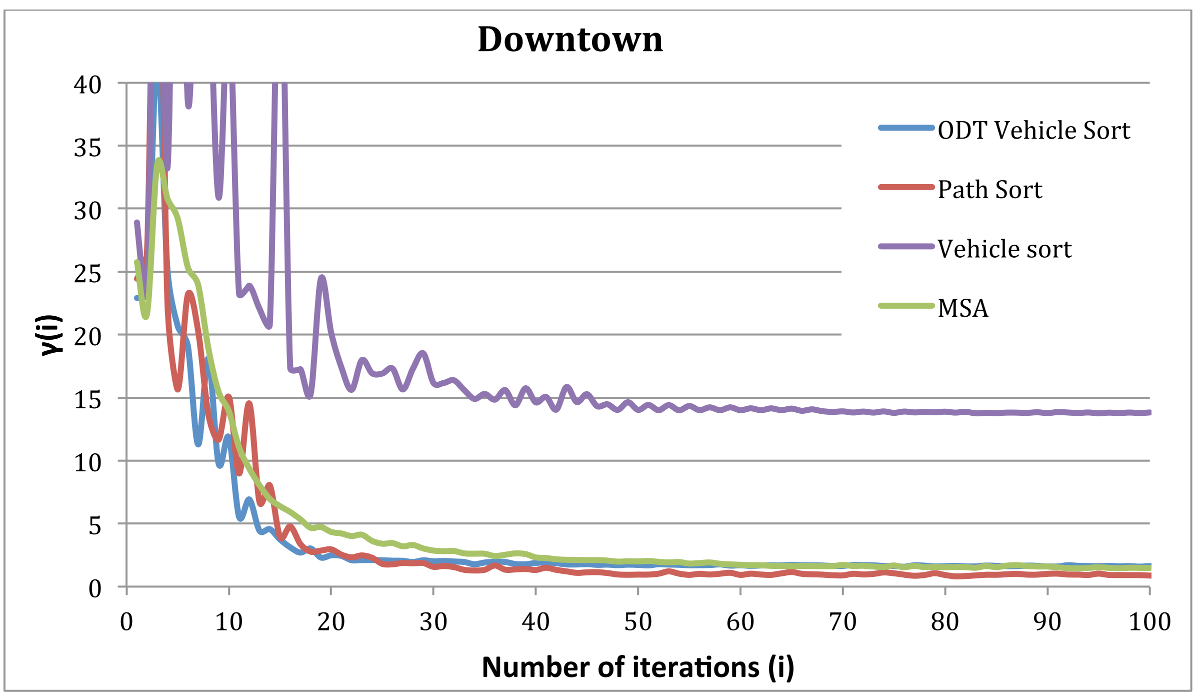


Figure 3.2: Vehicle Cost Gap vs. Cutoff Value in ODT sort

3.5.1.2 ODT Path Sort

Path sort performed slightly better than MSA in Downtown but slightly worse in Williamson County. This is most likely related to the network size and structure. The Downtown network is a small, grid-type network, with a high level of path overlap. Congestion in the heart of the region is high, and the worst paths probably over-utilize several links. Moving vehicles from all paths, as MSA does, may not quickly alleviate that congestion since many paths share the same links. Path sort, on the other hand, focuses on removing vehicles from high travel time paths, alleviating artificial bottlenecks faster and promoting a quicker gap reduction. On Williamson County, the spread out network structure results in paths sharing fewer links. Therefore, moving vehicles onto the shortest path will have a larger impact on high congestion links.



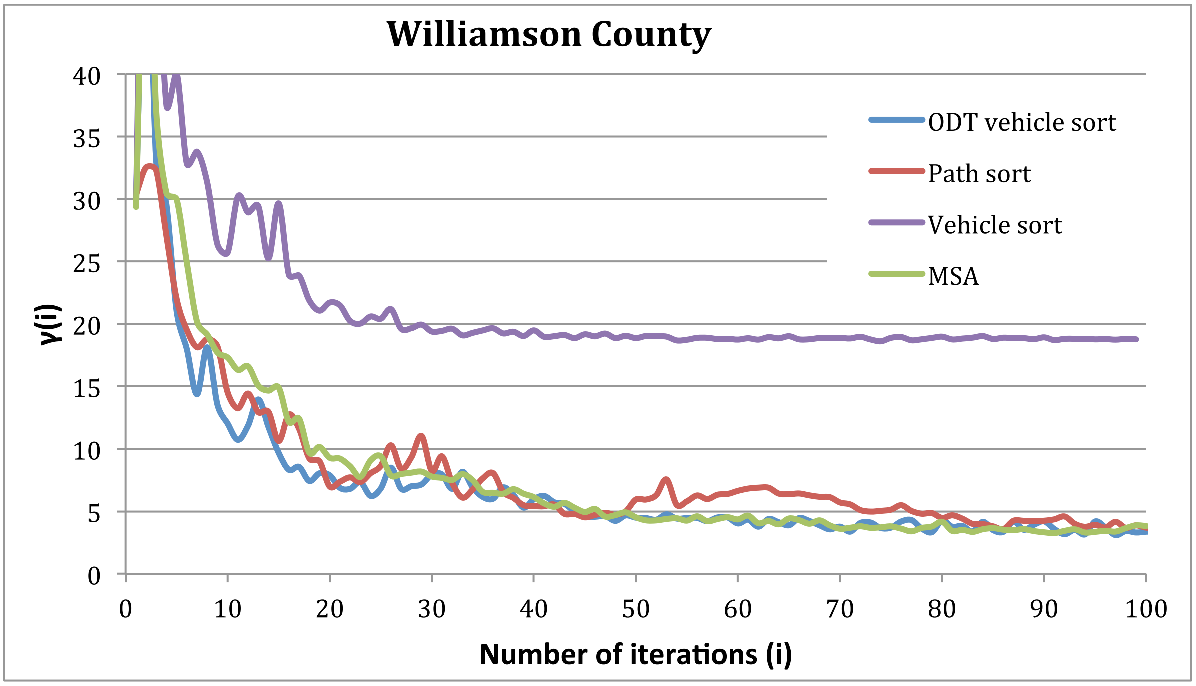


Figure 3.3: Convergence from Vehicle Sort Based Heuristics

3.5.1.3 Vehicle Sort

Vehicle sort performed poorly, most likely due to an undesirable selection of vehicles to be re-assigned. After around 15 iterations in Downtown and 12 iterations in Williamson County the gap essentially flatlines, and no gap improvements are made in the remaining iterations. This method is likely ignoring the better performing vehicles after the number of vehicles moved becomes too low at later iterations. As a result the gap remains constant, similar to the pattern experienced in ODT sort.

3.5.1.4 ODT Vehicle Sort

ODT vehicle sort performed slightly better than MSA in both Downtown and Williamson County. Moving the worst performing vehicles is expected to provide a better convergence rate by focusing on the vehicles contributing most to the cost gap. However, the results were not as substantial in terms of improvement in the cost gap. Nevertheless, the overall gap does not provide complete information on the quality of the solution. ODT vehicle sort was successful in creating a more even convergence across OD pairs. In MSA results, a smaller number of vehicles was found to be responsible for a large portion of the total gap (see Figure 3.4). Downtown had a 17.58% reduction in cost gap percentage, and Williamson County had a 10.86% reduction in gap. The number of vehicles moved by ODT vehicle sort is surprisingly comparable to MSA despite the emphasis on moving the worst vehicles per ODT. ODT path sort, as expected, moves more vehicles because it sorts by the worst path (see Figure 3.5). That might also help explain the better performance by ODT path sort on Downtown. At later iterations, few vehicles on Downtown are moved by MSA and ODT vehicle sort. More vehicles are moved on the larger Williamson County network, in which the methods performed more similarly.

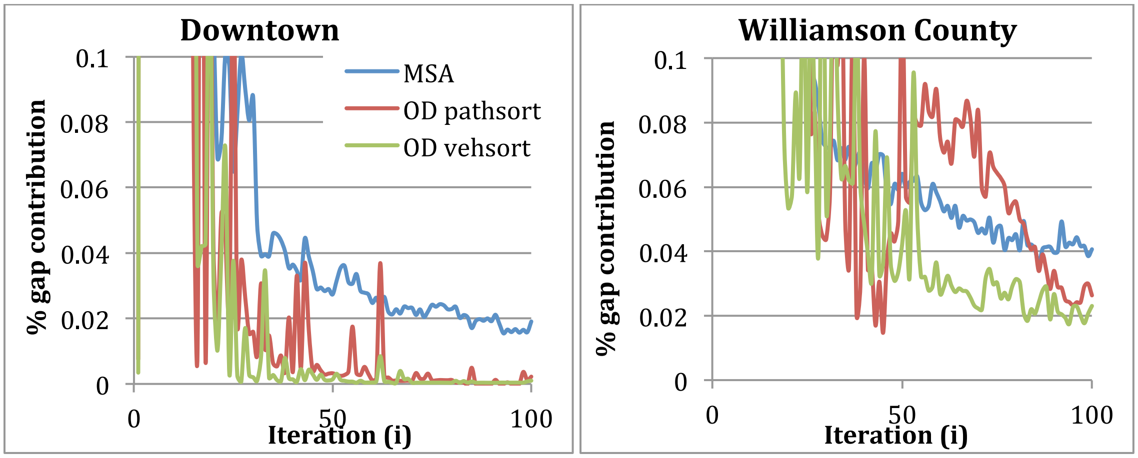


Figure 3.4: Percentage of Total Gap from Vehicles with a Cost Gap Greater than Shortest Path Travel Time

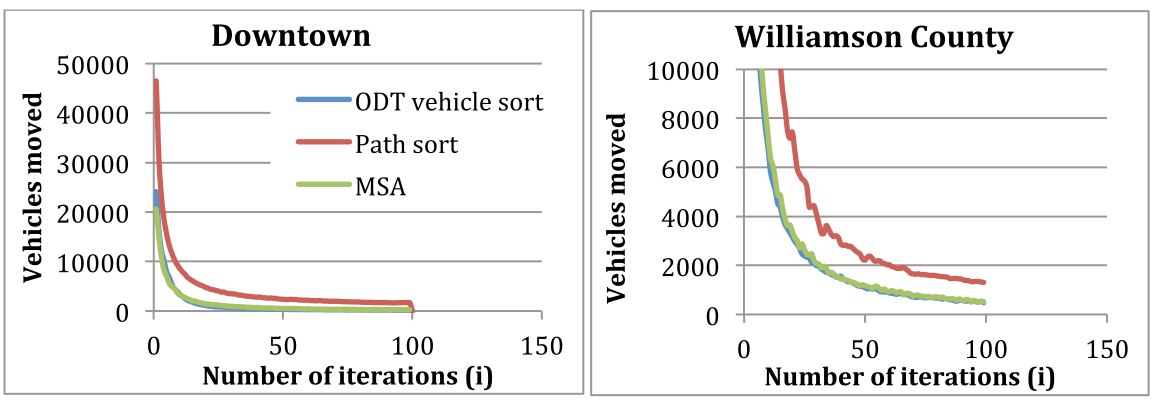


Figure 3.5: Vehicles Moved by Vehicle Sort Heuristics

3.5.2 Gradient-Based Heuristics Results

3.5.2.1 Lambda Cost Gap

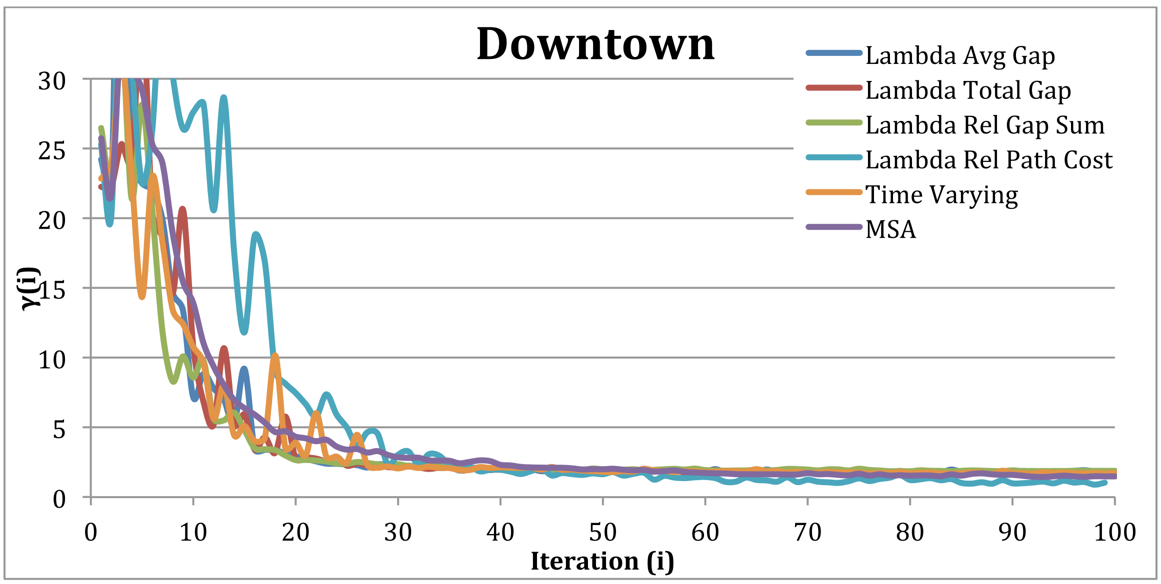
Little difference was observed between MSA and either Lambda Cost Gap methodology. Lambda average gap produced a relatively smooth rate of convergence; however, performance was comparable to MSA. Perhaps this behavior may be explained by insufficient scaling of the lambda value per ODT. In other words, the modified lambda was too similar to the base lambda. The lambda total gap methodology resulted in spikes in the convergence graph for both Downtown and Williamson County. The temporary increase in gap may be acceptable if followed by a reduction in gap. Unfortunately this is not the case, and again no improvement from MSA is noticeable. Perhaps prevention of significant increases in gap may avoid this issue.

3.5.2.2 Lambda Relative Gap Sum

Early iterations of this methodology produce significant gap reduction in both network scenarios. Even more encouraging is the fact that gap is consistently decreasing at almost every iteration. Performance at later iterations, though, is again comparable to MSA. One potential improvement in this methodology is a better approximation of the gradient. Simulation-based DTA has no analytical formulation of the gradient, so any method used is a heuristic approximation.

3.5.2.3 Lambda Relative Path Cost

Lambda Relative Path Cost had distinctly different performance on Downtown and Williamson County. As seen in Figure 3.6, on Downtown it was initially slower but resulted in a lower gap than any other method. On Williamson County, however, it was considerably worse than the other gradient-based heuristics. This is likely due to neglecting the second derivatives (see Jayakrishnan et al.,1994) which include the derivatives of link costs of links that are not shared between the old and the new shortest path. On the grid Downtown network, many links are shared and so the difference is smaller than on Williamson County which is more spread out. That likely explains the difference in performance on the two networks, and suggests that a suitable approximation of link cost derivatives could further improve this method.



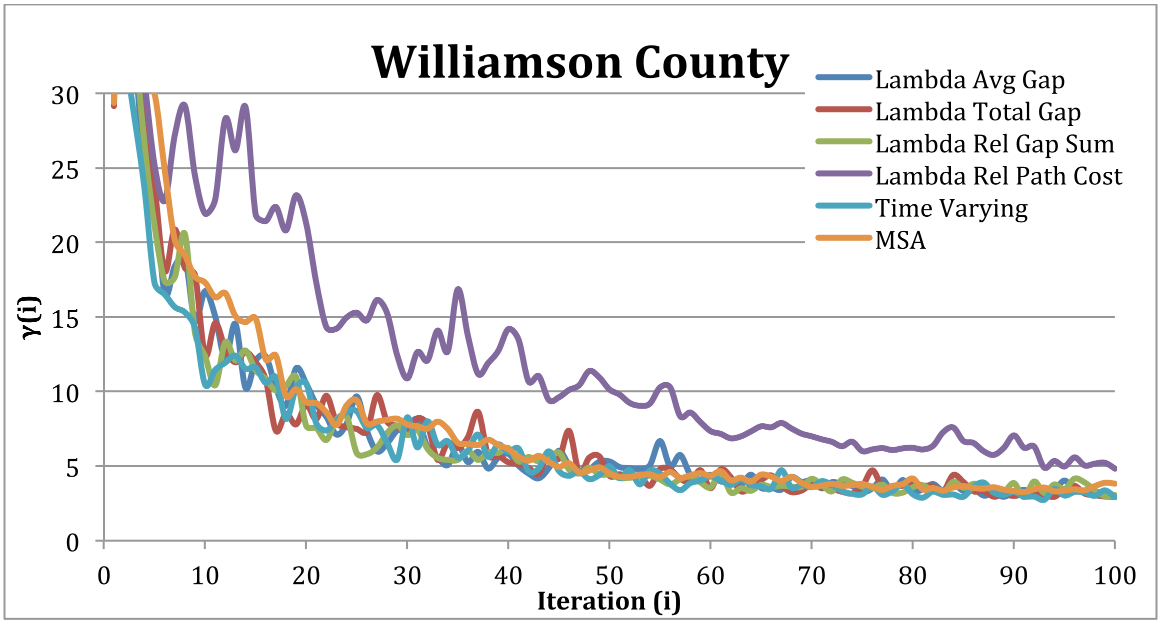


Figure 3.6: Gradient-based Heuristics

3.5.2.4 Lambda Time Varying

The time-varying method with reset parameter $n = 2$, starting at the 10th iteration, produced little difference on Williamson County, but significant spikes in the convergence rate were observed on Downtown near the start of the lambda reset. It is possible that a different reset parameter may have yielded better results, but it is not clear why the same was not observed on Williamson County. A possible explanation is that the increased number of vehicles moved coupled with a greater path overlap led to more instability.. Considering that Mahut et al. (2007) achieved good results, this indicates that perhaps a significantly higher reset parameter may be needed, or that a small change in the cascade pattern / reset parameter could create a disproportionate effect in the convergence rate.

3.5.3 Column Generation Results

There are many possibilities in the alternating cycles of least-cost path search and equilibration. The experiments implemented a 5 path generation / 5 equilibration split for the first 60 iterations, followed by 40 iterations of path swapping. This is consistent with the total number of path search and equilibration iterations used in all previous methods. The 5 iterations of equilibration after every 5 iterations of path generation are hoped to improve the quality of future paths found.

A second set of tests was conducted to further assess the stability of the results on the Williamson County network. Three of the methodologies were run until a stable gap was achieved (40 iterations). The peak hour link volumes (at 26 locations) and average travel times (along 28 selected routes) were compared at two different points during the process (iteration 20 and 40). Figure 3.5 displays the frequency distribution of the observed oscillation, defined as the percent change between the measurements taken at iterations 20 and 40. It is interesting to notice that even though the gap is fairly stable in the selected range, link volumes and route travel times at some of the locations of interest are still fairly volatile. Furthermore, the volatility varies across approaches, with MSA exhibiting the highest. These stability results provide an additional incentive to further our understanding of the convergence of SBDTA methodologies.

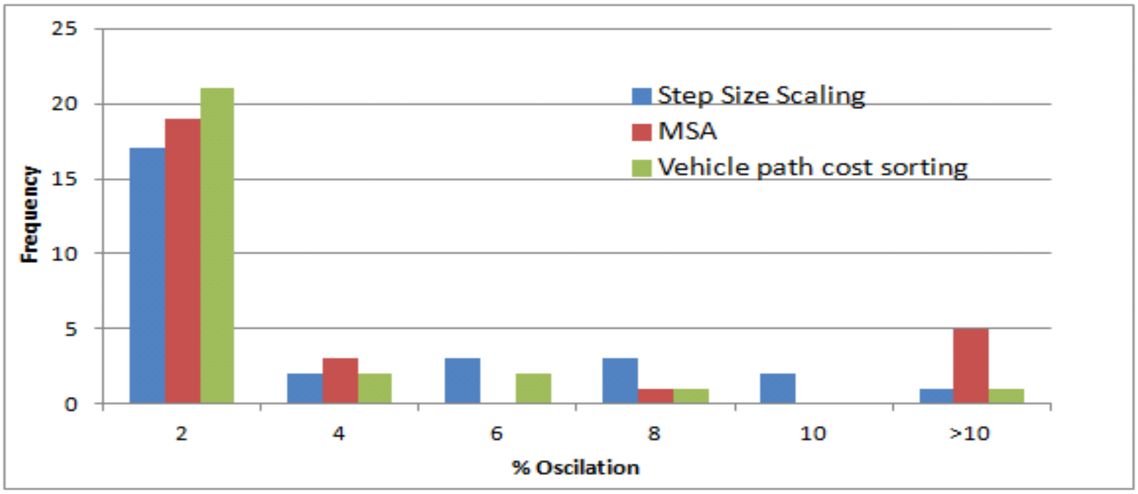


Figure 3.7: Stability of Selected Link Volumes

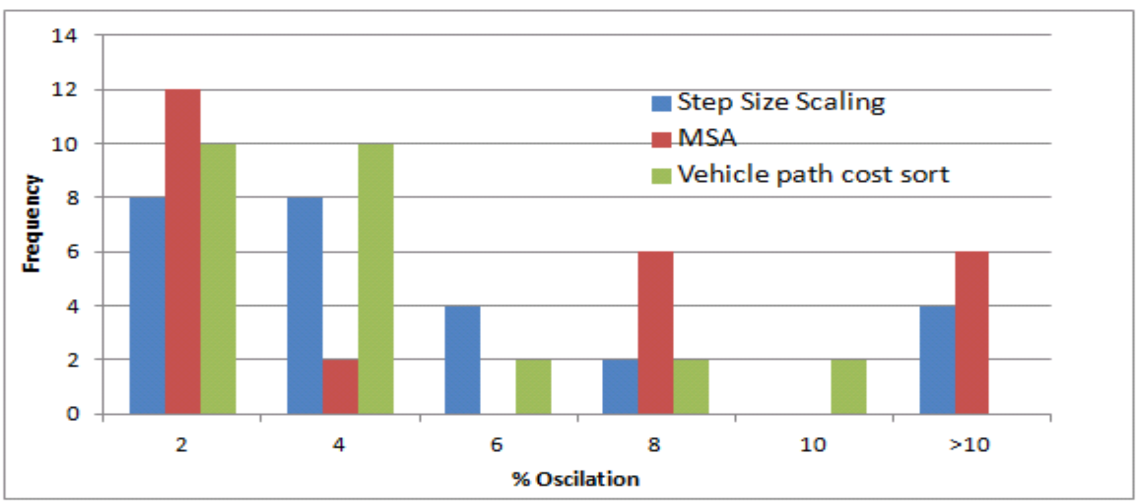


Figure 3.8: Stability of Travel Times

3.6 Conclusion

This work reviewed, implemented, and compared existing and novel techniques for finding equilibrium solutions in simulation-based dynamic traffic assignment problems (SBDTA). The considered methodologies include MSA-based sorting heuristics, gradient-based heuristics, column generation frameworks and partial demand loading schemes.

The algorithms studied in this research effort were implemented in a common SBDTA platform, which uses a cell transmission model (CTM) for traffic simulation. Numerical experiments were conducted on two networks of varying sizes. The performed tests clearly suggest that implementing a partial demand loading scheme favorably impacts convergence, reducing the congestion during earlier iterations and leading to a faster gap stabilization. None of the remaining methodologies tested in this work was observed to consistently outperform the others (or MSA) in terms of the number of required simulation iterations. However, analyses of the stability of the results obtained through different approaches suggest that there may be significant differences among methodologies not revealed by the cost gap. For instance, MSA may be more volatile due to the distribution of the total gap among fewer vehicles. Further research is needed to explore the observed trends, which may have significant implications for practical applications.

Even analysis of the cost gap demonstrates spikes in the convergence per iteration at later iterations. Observation of convergence over time, or a method that converges smoothly without spikes yet without sacrificing average convergence rate, is important for practical applications to quickly find a good solution. One possible improvement is heuristically preventing gap increase after a certain stability is achieved. From a research perspective, comparison of methods across few iterations may not reveal trends in the cost gap. Some methods were observed to converge quickly initially but reach a similar cost gap as MSA after many iterations. Nevertheless, selecting a method that converges quickly may reduce the number of iterations necessary to achieve an acceptable solution. End behavior, however, may be significant when a highly converged solution is required. Most methods tend to converge around a similar gap, suggesting that in practice further improvement may be difficult. Some of the methodologies may be sensitive to the parameters selected for implementation, and that generating those parameters endogenously would be of help for practical applications purposes.

The research presented in this paper provides valuable reference for implementation of new SBDTA methodologies and the development of more efficient assignment methodologies. Future research will also implement additional gradient-based approaches with better approximations and the potential to reduce the number of simulations needed to achieve stable equilibrium solutions.

Chapter 4: Conclusion

With the hope of aiding both planners and practitioners to feel more confident with dynamic traffic assignment, this work first investigates the possibility of integrating DTA with the standard four step transportation planning model. Also, a study of different DTA convergence techniques was conducted in order to create a meaningful comparison of a number of different network assignment algorithms within a single network modeling software. Both studies have yielded promising results, indicating that the future of DTA within the realm of engineering practice is very bright.

In Chapter 2, a model for integrating DTA into the four step model by replacing static traffic assignment in the network assignment step provided positive results. Each of the convergence measures introduced showed a generally decreasing trend. Moreover, the computation time for these runs was particularly low on a smaller city-sized network (~1 hour), suggesting that repeating the procedure with a larger, regional network model is conceivable. Also, travel time reliability was incorporated into the model via the addition of a representative term within the logit model equation present in the third model step, mode choice. While the results for this process were not ideal, this is explained by an incorrect selection of model parameters. Ideally these parameters should be estimated from real data, and this process is one possible extension of this work.

There are a number of other studies that could further contribute to the integration of DTA and the four step model. For instance, this work suggested taking the average travel time for all DTA departure time interval travel times as a single output to be feedback into trip distribution and mode choice. However, another measure, such as the median or a given percentage value of travel time may be more appropriate. Likewise, perhaps the four step model could be modified to accept a number of different travel time values, essentially creating a model that captures travel behavior over different periods of the day. Finally, an important improvement that could be made to further advance this integration is an enhancement of modal representation within DTA. Currently many DTA software packages only model automobile vehicles, and the others likely include only cars and buses. However, it would be very useful to incorporate a number of other travel modes within DTA to achieve a more meaningful integration with the four step model.

In Chapter 3, a number of different MSA-based and Gradient-based traffic assignment techniques were investigated within the same DTA software. Results were analytically compared with the Method of Successive Averages, and while not all performed better than MSA, many showed promising results. What was particularly interesting about this study was the difference in results noted between the downtown (grid) network and the Williamson County (sparse/rural) network. It appears that a number of different assignment approaches vary with the type of network being modeled, which provides an excellent starting point for extensions of this research. Other future work could include the inclusion of other assignment approaches as well as modeling on different network types, including a large-scale regional network.

It is hoped that this work can serve as a resource for practitioners who are currently on the fence about DTA. Given modern computing power, many transportation application should certainly be able to benefit from this more advanced and accurate form of network modeling.

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