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**Essays on Heterogeneity in Labor Markets**

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**Essays on Heterogeneity in Labor Markets**

by

**Gonul Sengul, B.S.; M.S.**

**DISSERTATION**

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

**DOCTOR OF PHILOSOPHY**

THE UNIVERSITY OF TEXAS AT AUSTIN

May 2009

To my parents Hanım and Adil Şengül, and to my aunt Sultan Şener.

## Acknowledgments

I am indebted to my supervisor, Russell Cooper, for his excellent guidance. Thanks to him, every obstacle I had during my dissertation turned into a source of motivation for me after our meetings.

I acknowledge my dissertation committee members for their valuable comments. I would like to thank you seminar participants at the University of Texas at Austin and at the Federal Reserve Bank of Cleveland for their insightful comments for the first chapter. I am thankful to the faculty and my classmates at the University of Texas at Austin for all I have learned from them during classes, seminars, and discussions.

I am also thankful to my friends for their support during my doctoral studies.

## Essays on Heterogeneity in Labor Markets

Publication No. \_\_\_\_\_

Gonul Sengul, Ph.D.

The University of Texas at Austin, 2009

Supervisor: Russell Cooper

My dissertation focuses on the heterogeneity in labor markets. The first chapter proposes an explanation for the unemployment rate difference between skill groups. Low skill workers (workers without a four year college degree) have a higher unemployment rate. The reason for that “... is mainly because they (low skill workers) are more likely to *become* unemployed, not because they remain unemployed longer, once unemployed” (Layard, Nickell, Jackman, 1991, p.44). This chapter proposes an explanation for the difference in job separation probabilities between these skill groups: high skill workers have lower job separation probabilities as they are selected more effectively during the hiring process. I use a labor search model with match specific quality to quantify the explanatory power of this hypothesis on differences in job separation probabilities and unemployment rates across skill groups.

The second chapter analyzes the effects of one channel of interaction (job competition) between skill groups on their labor market outcomes. Do skilled workers prefer unskilled jobs to being unemployed? If so, skilled workers compete with

unskilled workers for those jobs. Job competition generates interaction between the labor market outcomes of these groups. I use a heterogeneous agents model with skilled and unskilled workers in which the only interaction across groups is the job competition. Direct effects of job competition are reducing skilled unemployment rate (since they have a bigger market) and increasing the unskilled unemployment rate (since they face greater competition). However number of vacancies respond to job competition in equilibrium. For instance, unskilled firms have incentives to open more vacancies since filling a vacancy is easier if there is job competition. Thus how unskilled unemployment and wages are affected by job competition depends on which effect dominates. The results for reasonable parameter values show that job competition does reduce the average unemployment rate. It reduces the skilled unemployment rate more, generating an increase in unemployment rate inequality. However, the employment rate at skilled jobs is unaffected.

The third chapter focuses on skill biased technological change. Skill biased technological change is one of the explanations for the asymmetry between labor market outcomes of skill groups over the last few decades. However, during this time period there were also skill neutral shocks that could contribute to these outcomes. The third chapter analyzes the effects of skill biased and neutral shocks on overall labor market variables. I use a model in which skilled and unskilled outputs are intermediate goods, and final good sector receives all the shocks. A numerical exercise shows that both skilled and unskilled unemployment rates respond to shocks in the same direction. The response of unemployment rate to skill neutral shocks is bigger than the response to skill biased shocks for both skill groups. However,

the unskilled unemployment changes more than the skilled unemployment rate as a response to skill neutral shocks. Thus, skill neutral shocks reduce the unemployment rate gap between skill groups.



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# Chapter 1

## Selection, Separation, and Unemployment

### 1.1 Introduction

This paper explores unemployment rate differences across skill groups. I extend the empirical evidence on the difference in the unemployment experiences across skill groups to date using the Current Population Survey (CPS) data. As it is well documented, low skill workers have a higher unemployment rate than high skill workers. However, it is less known that the difference in unemployment rate by skill is mostly due to the difference in the job separation probabilities. This paper proposes an explanation for differences in job separation probabilities across skill groups: Firms use different selection procedures when hiring workers for vacancies with different skill requirements.

I use data from the CPS, between January 1976 and November 2007, on prime age males to document the skill differences in unemployment experiences.<sup>1</sup> Over the sample period, low skill workers had, on average, twice the unemployment rate of high skill workers. The difference in the job finding probability between skill groups is not large enough to cause the unemployment rate difference between these groups. The reason for the higher unemployment of low skill workers is *not* because they cannot find jobs. Furthermore, low skill workers, compared to high

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<sup>1</sup>The details of the data and the calculations are discussed in more detail in the next section.

skill workers, have approximately three times higher probability of becoming unemployed. The dispersion in the job separation probability (probability of becoming unemployed) by skill is the basis of the difference in unemployment rate these skill groups face.

The purpose of this paper is to further our understanding of the difference in job separation probabilities between skill groups. One should note that there are several possible reasons for this gap. One such explanation is the following: Firms with high skill vacancies have a higher opportunity cost of not hiring the best workers for the job. Consequently, firms who are looking for high skill workers follow more effective hiring strategies and screening processes than they would have done if searching for low skill workers. As a result of the more effective selection, high skill vacancies get higher quality matches, which are less likely to be terminated.

I use a discrete time infinite horizon labor matching model with heterogenous agents. There are two different skill groups in the economy. For simplicity, there is no interaction between these skill groups. For each skill group, a match between the firm and the worker can be either good or bad quality. The only difference between skill groups is that a good quality, high skill match produces a higher output than a good quality, low skill match. The true quality of the match is revealed after parties observe the output. Firms and workers learn about the probability of the match being good quality before they form the match, and they decide whether to form the match and produce or to continue searching. The probability of a match being good quality is drawn from a distribution, which is the selection technology. There are two technologies available: a costless employee selection technology, and a more

effective but costly employee selection technology. The more effective technology has a higher likelihood of delivering high quality matches.

I calibrate the model to match some data facts of the US labor markets. The model predicts that, in equilibrium, only high skill firms employ the more effective selection technology. As a result, high skill firms often get high quality matches, resulting in a lower job separation rate. The model also delivers, although not targeted directly, the observed magnitudes of the job separation probability disparities across skill groups.

To my knowledge, there are only two other papers that propose an explanation for the difference in job separation probabilities across skill. Moscarini (2002) quantifies the match specific capital and analyzes its implications for wage inequality. In his model, low skill workers have a comparative disadvantage in market work (lower wedge between productivity and opportunity cost of work). Thus, low skill workers do not tolerate mismatches and separate to unemployment more often than the high skill workers.

Nagypal (2007) focuses on the difference in the levels and the standard deviations of unemployment rate across skill. Nagypal has a different explanation for the differences in job separation probabilities. Nagypal explains that the existence of match specific capital (information about the quality of a match) for high skill workers and the lack of such capital for low skill workers make low skill employment relations more vulnerable to adverse idiosyncratic shocks. Nagypal uses a matching model with firms employing both high and low skill workers, and with uncertainty regarding match quality for high skill jobs. Firms' decisions to terminate a high



skill match, when faced with an adverse idiosyncratic shock, depend on the accumulated information about the quality of that match. Nagypal constructs a numerical example to show that differences in learning about the match quality can generate differences in unemployment rates.

This paper is complementary to Nagypal (2007) in the following way. Both papers agree that there is uncertainty regarding the match quality. There are two means through which firms and workers can learn about the quality of their match. First is that, they can extract some information regarding the quality before they form the employment relationship. The second is that, they can form the employment relationship and learn through the job tenure. This paper looks at the former means of learning while Nagypal (2007) looks at the latter form of learning.

The rest of the paper is organized as follows. The next section of the paper provides recent evidence on the hiring policies and unemployment experiences of different skill groups. Section two lays out the model. The equilibrium of the model is defined and analyzed in section three. Section four has the quantitative results of the model and is followed by concluding remarks.

## **1.2 Data**

### **1.2.1 Unemployment Facts**

This section talks about the data on unemployment experiences of skill groups in more detail. To document the skill differences in unemployment, I use

CPS data from January 1976 to November 2007<sup>2</sup>. I focus on prime age males (ages of 25 to 50), since they have the strongest labor market attachment among labor market participants. Following the standard definition of skill in the literature, I use education level as a proxy for skill. Workers without a college degree are low skill while those with at least 16 years of education (a college degree) are high skill workers.

To do the following calculations, I construct flow data by merging every two consecutive months of the CPS and seasonally adjust the flow data<sup>3</sup>.

The unemployment rate is calculated as the ratio of the number of the unemployed in a particular month to the size of the labor force in that month for each skill group. Figure 1.1 illustrates the skill differentials in unemployment rates<sup>4</sup>. Although unemployment rates of both skill groups follow similar trends, the high skill unemployment rate is always below the low skill unemployment rate.

There are two determinants of unemployment. The first one is the job finding probability whereas the second one is the job separation probability. The job finding probability in a given month is the fraction of unemployed in the previous month that have become employed in the current month. Figure 1.2 shows the job finding probabilities of skill groups. Low skill workers, on average, have higher job finding probabilities than high skill workers. Job finding probabilities are not

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<sup>2</sup>CPS is a monthly survey of about 50,000 households. It is the primary source of information on the labor force characteristics of the U.S. population. The CPS is conducted by the Census Bureau for the Bureau of Labor Statistics. The web site for the Survey is: <http://www.census.gov/cps>. The data can be downloaded from [http://www.nber.org/data/cps\\_basic.html](http://www.nber.org/data/cps_basic.html)

<sup>3</sup>In doing so, I use the Stata codes made available by Robert Shimer. Codes can be downloaded from <http://robert.shimer.googlepages.com/flows>

<sup>4</sup>Data in all figures are quarterly averages of monthly values.

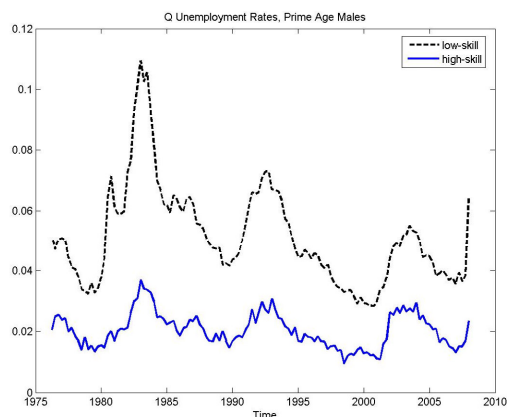


Figure 1.1: Unemployment Rates by Skill Groups

the main reasons for unemployment rate differences between skill groups. One can also look at unemployment duration across skill groups. Although duration data reveal a slightly longer duration for low skill workers (which would imply a lower job finding probability for this group), the difference is still not big enough for job finding probabilities to be the main actor of unemployment rate disparity. Moreover, Tristao (2007) uses National Longitudinal Survey of Youth-79<sup>5</sup> data and looks at the unemployment duration for detailed occupation groups. Low skill occupations have durations close to the overall average. Thus job finding probabilities are not different enough to cause the differences in unemployment rates.

The second determinant, the job separation rate, is also from merged CPS data. The job separation probability in a given month is the fraction of workers who were employed in the previous month, but are reported to be unemployed in the

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<sup>5</sup>For more information see <http://www.bls.gov/nls/nlsy79.htm>.

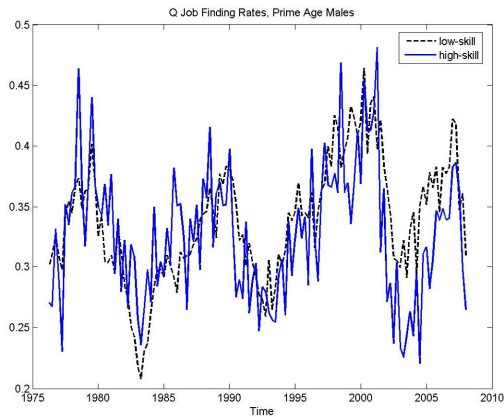


Figure 1.2: Job Finding Probabilities by Skill Groups

current month. Figure 1.3 illustrates the job separation probabilities of skill groups over time. High skill workers have substantially lower rates than low skill workers. Note that the graph for separation rates resembles the graph for unemployment rates.

These facts are robust to several different specifications of the data. I analyze the difference between skill groups in unemployment that is not due to lay-offs. The CPS collects data on the reason for unemployment after 1994. Over the period of 1994 to 2007, the average unemployment rate is 4.9 percent for low skill workers, and it is 2.1 percent for high skill workers. If we look only at unemployment for reasons other than lay-offs, the average unemployment rate is 3.9 percent and 1.9 percent for low and high skill workers, respectively. Clearly, lay-offs affect low skill workers more; however, higher lay-offs of low skill workers are not significant enough to account for the difference in unemployment rates.

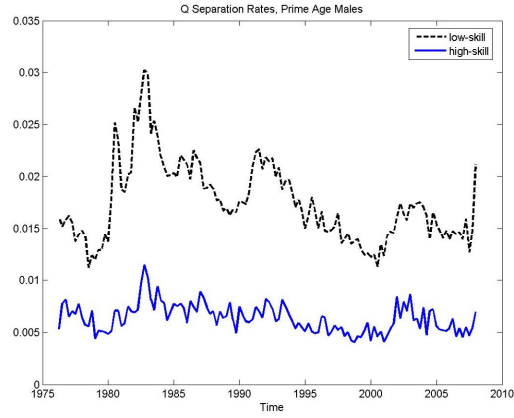


Figure 1.3: Job Separation Probabilities by Skill Groups

These data facts are also robust to the cutoff for high and low-skill. Counting workers with some college education as high skill does change quantitative results but not enough for the gap to vanish or decrease significantly. Moreover, the gap in the unemployment rate and the job separation probability is significant for more detailed categories of skill.

Table 1.1: Data Summary

	<b>Unemployment Rate</b>	<b>Job Finding Probability</b>	<b>Separation Probability</b>
All	.042 (.01)	.34 (.05)	.014 (.003)
Low-skill*	.051 (.02)	.34 (.06)	.017 (.004)
High-Skill	.02 (.006)	.33 (.07)	.006 (.002)

Monthly averages of prime age male data between 1976 and 2007

\* Workers with less than a college degree

Observe that the separation rates given above are for the separations to unemployment. It is also informative to look at job separations that did not result in unemployment: job to job transitions. The CPS started, in 1994, to ask employed workers whether they are with the same employer as they were the previous month. During the 1994 -2007 period, less than two percent of employed workers in each skill group changed employers. Although there is a slight difference in numbers by skill, low skill workers have substantially larger job separation probabilities (regardless of the destination of the separation).

Table 1.1 summarizes unemployment experiences by skill group. Between January 1976 and November 2007, low skill workers experienced an average unemployment rate of five percent whereas high skill workers faced two percent. Moreover, low skill workers had a slightly better chance of finding jobs than their high skill counterparts. The data reveal that low skill workers have higher unemployment rates for the reason that they have higher job separation probabilities.

The skill difference in unemployment experiences is also documented by other authors. Mincer (1991) uses the Panel Study of Income Dynamics data on the male labor force. He finds that higher levels of education reduce the risk of unemployment. The difference in the incidence of unemployment is more important than the difference in unemployment durations for educational differences in unemployment. Nagypal (2007) uses March Supplements and Displaced Worker Supplements of the CPS. She also finds differences in unemployment by education. Moreover, the differences in unemployment duration by education are not large enough to account for the differences in unemployment rates. Layard, Nickell, and Jackman (1991) use

managerial vs. manual occupations, and Juhn, Murphy, and Topel (2002) use wages as a proxy for skill, and find a higher unemployment rate for low skill workers.

### **1.2.2 Hiring Processes**

This section of the paper presents the data facts on employee selection and its differentials by skill. Van Ours and Ridder (1992) analyze the selection process of employers using Dutch establishment data. Van Ours and Ridder show that firms form an applicant pool shortly after (within the first two weeks) the vacancy has been posted, and the rest of the vacancy duration is used to select a new employee from that pool of applicants. They show that 80 percent of vacancies are filled with applicants who applied for the job within the first two weeks of the vacancy opening. Hence, they conclude, vacancy durations should be interpreted as selection periods.

There are differences in the intensity of search for employees, which firms conduct by the skill requirement of the vacancy. Van Ours and Ridder (1993) report that the mean selection period increases with the required level of education and experience. Moreover, Barron, Berger, and Black (1997) find that employers search more when hiring workers with more education and prior experience and hiring for jobs with higher training requirements. They show that as the education requirement of the vacancy increases, the number of interviews per offer and number of applicants per offer goes up (extensive search) as well as the number of hours per interview and per applicant (intensive search). Barron and Bishop (1987) report that total time spent on hiring is longer for high skill occupations in comparison to low skill occupations.

There is also evidence on search for high skill workers being more effective. Bagger and Henningsen (2008) use Danish and Norwegian data and look at the job ending hazard rates by skill. Bagger and Henningsen find that for all skill levels the likelihood of a job ending decreases with tenure<sup>6</sup>. At low levels of tenure, low skill workers are more likely to separate from their jobs, and the difference diminishes as the tenure increases. The difference in levels of the hazards indicate that high skill hires are more likely to be good matches.

Hiring policies are used to answer some other facts regarding labor markets. Pries and Rogerson (2005) analyze differences in labor and job turnovers between the US and Europe. The differences in labor market policies of these economies generate differences in hiring policies of firms. Tasci (2006) looks at firms' hiring policies over the business cycle, and proposes changes in hiring behavior over the cycle as another mechanism to increase the response of the key aggregate labor market variables to productivity shocks.

### 1.3 Model

This section introduces the model used to answer the question of interest. The economy is inhabited by workers with two skill types; high skill and low skill workers. The skill distribution is exogenous and skill type of a worker is observable. There is also a continuum of firms with heterogenous jobs. Upon entering the market, firms choose the type of job they want to have. A firm can employ at

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<sup>6</sup>It is well documented that job separations have negative hazards with tenure. See, for example, Farber (1999).



most one worker. Firms and workers that search for an employment relationship are brought together via a matching function. I assume no interaction between high and low skill sectors, i.e., they are *segregated*.<sup>7</sup> Having no interaction between skill groups allows me to solve for the equilibrium for each skill group separately. In consequence, the model explained below applies to each of the skill groups.

There is a continuum of homogenous workers, with total mass equal to one. There is also a continuum of ex-ante identical firms. All agents are risk neutral, and they discount future at rate  $\beta$ . A worker can be either unemployed or employed. When unemployed, workers consume unemployment benefits  $b$ .

The production unit in the economy is a firm-worker pair. The pair produces  $y = y^i$  amount of output, where  $i$  is the quality of the match.  $y^i$  takes on the value  $y^g(y^b)$  if the match is good (bad) quality<sup>8</sup>, where  $y^g > y^b$ . I assume that the amount of bad quality output is the same across sectors ( $y_{hs}^b = y_{ls}^b$ ) while good quality output in the high skill sector is higher than the good quality output in the low skill sector ( $y_{hs}^g > y_{ls}^g$ ). This is the only exogenous difference between skill groups. Wages are outcomes of Nash Bargaining, and  $\mu$  is the worker's bargaining power.

Production units that are active (i.e. that produced in the current period)

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<sup>7</sup>Although there is empirical evidence in favor of interaction between labor markets, this simplifying assumption allows me to focus on the interaction between the unemployment rate and hiring policies of firms. Interactions across labor markets can take on many forms, affecting the unemployment rate disparity between sectors in different directions. One commonly modeled interaction across skill groups is that high skill workers can look for both high and low skill jobs while unemployed and can continue to search for high skill jobs if they exit from unemployment into a low skill job. Granting such interaction will affect the unemployment rate of high skill workers through affecting both their probability of finding a job and separating from a job in this model. Since this paper focuses on the causes of the job separation disparity between skill groups, which is the main factor of the unemployment rate gap, abstracting away from such interaction is plausible.

<sup>8</sup>Observe that the quality of the match is not contingent on skill per se.

are subject to an exogenous destruction at rate  $\delta$ , which is the same across sectors. Bagger and Henningsen (2008) estimate the monthly job hazard function for different education groups using data on Danish and Norwegian workers. At low levels of tenure, less educated workers have higher hazard rates. However, the difference closes after five years of tenure. Based on this evidence, it is plausible to assume the same exogenous destruction rate across groups.

Unemployment stems from frictions in the labor markets. These frictions are modeled via a matching function. The matching function provides a mapping between the number of vacancies ( $v$ ) and the unemployed ( $u$ ) and the number of total matches across firms and worker. Thus, it determines the matching probabilities of firms and workers with each other endogenously<sup>9</sup>. The matching function,  $M$ , is constant returns to scale. Consequently, it only depends on the vacancy-unemployment ratio  $v/u = \theta$ , which is called the market tightness. A worker meets with a firm with probability  $f(\theta) = M/u$ , and a vacant firm meets with a worker with probability  $q(\theta) = M/v$ . The matching function also satisfies the following boundary conditions:  $f \rightarrow 0$  and  $q \rightarrow 1$  as  $\theta \rightarrow 0$ , and  $f \rightarrow 1$  and  $q \rightarrow 0$  as  $\theta \rightarrow \infty$ . Moreover, the probability that a worker meets with a vacancy,  $f$ , is an increasing function of  $\theta$  while the probability of firms meeting with workers,  $q$ , decreases with  $\theta$ , i.e.,  $f'(\theta) > 0$  and  $q'(\theta) < 0$ .

The quality of a match is ex-ante uncertain. The true quality of the match

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<sup>9</sup>The model does not have on-the-job search. Thus, all separations in the model result in unemployment. CPS data reveal that both high and low-skill workers experience job-to-job transitions. However, low-skill workers have substantially higher job separation rates even after accounting for job-to-job transitions. Hence, abstracting away from on the job search yields simplification without distorting the validity of the hypothesis of the model.

is revealed after the first period of production. Hence, a worker-firm pair does not know the quality of the match unless they produce. However, parties draw the probability of being in a good quality match,  $\pi$ , when they meet. After observing  $\pi$ , the worker and the firm decide whether to form the match. A higher value of  $\pi$  means that it is more likely that the match is good quality.

I model the employee selection procedure as a technology that randomly delivers a value of  $\pi$  to the firm-worker pair when they first match.  $\pi$  can be drawn from either a costless distribution  $\Gamma$ , or from a more effective distribution  $\Omega$ , which comes at a cost ( $\kappa$ ). The more effective the technology is, the more likely it is that the technology delivers higher values of  $\pi$ , i.e.  $\Omega$  first order stochastically dominates  $\Gamma$ . More formally,  $\Omega(\pi) \leq \Gamma(\pi) \forall \pi$ . One can think of more effective employee selection procedures as firms using more effective recruitment channels (i.e. advertising the job opening more extensively and intensively), better employee assessment, etc. that will result in a better quality match between firm and worker as opposed to the quality of a match had the firm not used the more effective technology. Another way of thinking of the more effective selection is as if firms sample from the same distribution multiple times and choose the highest  $\pi$  level. In this case the empirical distribution of  $\pi$  for firms that do multiple draws will first order stochastically dominate the original distribution.

I assume that  $y^b \leq b$  and  $y^g > b$ . Under this assumption, bad quality matches are undesirable in equilibrium. Firm and worker pairs terminate such matches. If all the separations in this economy were to be exogenous, then the job separation rates in the equilibrium would be the same across skill groups. However, undesirability

of bad quality matches generates endogenous separations, which is the source of the difference in job separation rates across sectors.

### 1.3.1 Timing of Events

- Each period begins with a number of unemployed workers; a number of worker-firm matches with known quality; and a number of worker-firm pairs that met in the previous period the first time and observed their  $\pi$ .
- All parties decide whether to produce or to detach.
- All workers that do not produce in the current period consume unemployment benefits.
- Worker-firm pairs that have decided to stay attached produce, and workers consume their wages.
- All the matches with unknown quality learn the match quality (Observe that separation decisions will be made at the beginning of the next period).
- Vacant firms decide whether to post a vacancy or not.
- The vacant firms choose a selection technology to use.
- Job markets open; unemployed workers and vacant firms meet.
- Firm-worker pairs that met learn the probability that their match will be good quality. They will decide at the beginning of next period whether to form the match or not.

- Job markets close.
- Exogenous destructions occur;  $\delta$  fraction of worker-firm pairs that produced in the current period are destroyed.
- New period begins.

### 1.3.2 Bellman Equations

Let  $\lambda$  be the probability that firms choose  $\Omega$  as their employee selection technology. Moreover, let  $U(\lambda)$  be the value of unemployment to a worker, and  $W(\pi, \lambda)$  be the value of being in a match with a firm to a worker where  $\pi$  is the probability that the match is good quality.

$$U(\lambda) = b + \beta(1 - f(\theta))U(\lambda) + \beta f(\theta) \left\{ \lambda \int_0^1 W(\pi, \lambda) d\Omega + (1 - \lambda) \int_0^1 W(\pi, \lambda) d\Gamma \right\} \quad (1.1)$$

If a worker is unemployed, she gets the unemployment benefit,  $b$ . The worker does not meet with any firms, thus continuing to get the value of being unemployed, with probability  $1 - f(\theta)$ . The worker meets with a firm with probability  $f(\theta)$ , and gets an expected value from being in a match with a firm. Note that the expected value the worker gets from being in a match with the firm depends on the selection technology the firm has chosen.

$$W(\pi, \lambda) = \max \left\{ U(\lambda), w(\pi, \lambda) + \beta \left[ \pi \{ \delta U(\lambda) + (1 - \delta) W(1, \lambda) \} + (1 - \pi) U(\lambda) \right] \right\} \quad (1.2)$$

If a worker is in a match with a firm and they have the probability  $\pi$  of the match being good quality, the worker decides whether to stay in the match or separate to unemployment. If she stays in the match, the worker gets the wage in the current period. If the match survives to the next period and it is revealed to be

good quality, the worker will get the value of being in a good quality match with a firm,  $W(1, \lambda)$ . If the match is revealed to be bad quality, which will happen with probability  $1 - \pi$ , the worker will get the value of being in a bad quality match with a firm,  $W(0, \lambda)$ . Recall that, in equilibrium, a match will be terminated if it is bad quality, therefore  $W(0, \lambda) = U(\lambda)$ .

Let  $V(\lambda)$  be the value of a firm with a vacancy, and  $J(\pi, \lambda)$  be the value of a firm in a match which is good quality with probability  $\pi$ . The value of a vacancy is the discounted value of expected profits, net of cost of the vacancy.

$$V(\lambda) = -c + \max \left\{ \begin{array}{l} -\kappa + \beta(1 - q(\theta))V(\lambda) + \beta q(\theta) \int_0^1 J(\pi, \lambda) d\Omega, \\ \beta(1 - q(\theta))V(\lambda) + \beta q(\theta) \int_0^1 J(\pi, \lambda) d\Gamma \end{array} \right\} \quad (1.3)$$

The first term of equation (1.3) is the per period cost of having a vacancy,  $c$ . The second term of (1.3) has a maximum operator since the firm will decide which selection procedure to implement. If the firm chooses the more effective distribution, then it pays the extra cost of using that distribution ( $\kappa$ ). Regardless of the choice of the employee selection procedure, the firm will match with a worker with the probability  $q(\theta)$ . If the firm matches with a worker, then it will get the expected value of being in a match which is good quality with probability  $\pi$ . If the firm does not match with any worker, which happens with probability  $1 - q(\theta)$ , then it will stay vacant and continue to get the value  $V$ .

Equation (1.4) formalizes the decision problem of a firm which is in a match with a worker with the match being good quality with probability  $\pi$ .

$$J(\pi, \lambda) = \max \left\{ V(\lambda), E(y|\pi) - w(\pi, \lambda) + \beta\delta V(\lambda) + \beta(1 - \delta)(\pi J(1, \lambda) + (1 - \pi)J(0, \lambda)) \right\} \quad (1.4)$$

where  $E(y|\pi) = \pi y^g + (1 - \pi)y^b$ . The firm compares the expected discounted value of profits from producing output with the current worker to the discounted present value of separating (being vacant). The value the firm gets from producing with a worker with probability  $\pi$  of having a good quality match is the sum of the current period profit, which is the expected value of output produced minus the wage paid to the worker, and the discounted value of being in a match with the same worker the subsequent period, if the match survives. Observe that a surviving match is revealed to be good quality with probability  $\pi$ . In this case the firm will get the discounted present value of being in match with a worker with the match quality being good with probability one,  $J(1, \lambda)$ . However, with probability  $1 - \pi$  the match quality will be revealed to be bad, and the firm will get the value  $J(0, \lambda)$ . With the assumption that  $y^b \leq b$ , firms and workers will separate in equilibrium if the match is bad quality, i.e.  $J(0, \lambda) = V(\lambda)$ .

Although an equilibrium object,  $\lambda$  appears as an argument in value functions of agents. This is because, beliefs about  $\lambda$  can potentially affect firms' decision whether to adopt the effective technology. The value of lambda affects the value of unemployment for workers, because it affects the wages at other firms. Thus, a firm needs to take into account what the  $\lambda$  is, and which technology it uses for given  $\lambda$  (wages). This point is discussed in more detail in section 3.2.

Wages are determined according to a Nash bargaining rule, where worker's bargaining power is  $\mu$ . The wage rate that solves the bargaining problem is such that the worker gets a constant ( $\mu$ ) fraction of the net value generated by the worker-firm

union<sup>10</sup>.

Nash assumption guarantees the unanimity of the separation or match formation decision. That is because parties bargain over the net surplus of forming the production unit, and if the surplus is positive (negative) they decide to form the match (separate). Hence there is no inconsistency across parties in decision making.

### 1.3.3 Worker Flow Across Employment States

Let  $u_t$ ,  $e_t^g$ , and  $e_t^n$  be the unemployment rate, good quality employment, and unknown quality employment at period  $t$ , respectively. Let  $X(\pi) = 1(0)$  be the decision of a worker-firm pair who observe  $\pi$  (not) to form the match. Moreover, let  $E(\pi|X(\pi) = 1)$  denote the expected value of the match being good quality, conditional on the match being formed. Recall that  $\lambda$  is the probability that  $\Omega$  in the equilibrium selection technology.

The number of good quality matches in the subsequent period is the sum of the good quality matches of the current period and the current period unknown quality matches which are revealed to be good quality in the subsequent period that survive exogenous destruction.

$$e_{t+1}^g = (1 - \delta)e_t^g + e_t^n(1 - \delta)E_\lambda(\pi|X_\pi = 1)$$

Since all matches reveal their quality after the first period of production, the number

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<sup>10</sup>Let the second argument of the maximization operation in 1.2 (1.4) equation be  $\tilde{W}(\pi, \lambda)$  ( $\tilde{J}(\pi, \lambda)$ ). Then, the wage is such that

$$\tilde{W}(\pi, \lambda) - U(\lambda) = \mu \left\{ \tilde{W}(\pi, \lambda) - U(\lambda) + \tilde{J}(\pi, \lambda) - V(\lambda) \right\} \quad (1.5)$$



of unknown quality matches in any period are the same as the number of matches formed in that period.

$$e_{t+1}^n = u_t f_t E_\lambda(X_\pi)$$

The number of the unemployed workers in the subsequent period is the sum of the unemployed in the current period who did not meet with a firm, the current period unemployed who met with a firm but drew a low  $\pi$  (thus stayed unemployed), the current period employed who were hit by an exogenous destruction shock, and the current period employed with unknown match quality who learned their match was of bad quality.

$$u_{t+1} = u_t(1 - f_t E_\lambda(X_\pi)) + \delta(e_t^g + e_t^n) + e_t^n(1 - \delta)(1 - E_\lambda(\pi|X_\pi = 1))$$

## 1.4 Equilibrium

The steady state equilibrium, for each of the sectors, is a list  $\{e^g, e^n, v, u, \lambda, \pi^*, w(\pi, \lambda), X(\pi), J(\pi, \lambda), V(\lambda), W(\pi, \lambda), U(\lambda)\}$  such that

- $\{J(\pi, \lambda), V(\lambda), W(\pi, \lambda), U(\lambda)\}$  satisfy equations 1.4, 1.3, 1.2, and 1.1.
- $V(\lambda) = 0$ .
- $w(\pi, \lambda)$  is the solution to the Nash bargaining, i.e., satisfies equation 1.5.
- $X(\pi) = 1$  iff

$$w(\pi, \lambda) + \beta\delta U(\lambda) + \beta(1 - \delta)(\pi W(1, \lambda) + (1 - \pi)W(0, \lambda)) - U(\lambda) \geq 0$$

$$E(y|\pi) - w(\pi, \lambda) + \beta\delta V(\lambda) + \beta(1 - \delta)(\pi J(1, \lambda) + (1 - \pi)J(0, \lambda)) - V(\lambda) \geq 0$$

- Selection technology is chosen optimally

$$\lambda(\lambda^-) = \begin{cases} 1 & \text{if } -\kappa + \beta q(\theta) \int_0^1 J(\pi, \lambda^-) d\Omega > \beta q(\theta) \int_0^1 J(\pi, \lambda^-) d\Gamma \\ 0 & \text{if } -\kappa + \beta q(\theta) \int_0^1 J(\pi, \lambda^-) d\Omega < \beta q(\theta) \int_0^1 J(\pi, \lambda^-) d\Gamma \\ \in [0, 1] & \text{if } -\kappa + \beta q(\theta) \int_0^1 J(\pi, \lambda^-) d\Omega = \beta q(\theta) \int_0^1 J(\pi, \lambda^-) d\Gamma \end{cases}$$

where  $\lambda^-$  is the decision rule for the rest of the firms.

- $\lambda$  is consistent, i.e.,  $\lambda(\lambda^-) = \lambda^-$
- The reservation probability  $\pi^*$  satisfies

$$E(y|\pi^*) - w(\pi^*, \lambda) + \beta\delta V(\lambda) + \beta(1 - \delta)(\pi^* J(1, \lambda) + (1 - \pi^*) J(0, \lambda)) = V(\lambda)$$

$$w(\pi^*, \lambda) + \beta\delta U(\lambda) + \beta(1 - \delta)(\pi^* W(1, \lambda) + (1 - \pi^*) W(0, \lambda)) = U(\lambda)$$

- The flows among employment and unemployment states are constant

$$u = 1 - e^g - e^n$$

$$\delta e^g = e^n(1 - \delta)E_\lambda(\pi|X)$$

$$e^n = ufE_\lambda(X_\pi)^{11}$$

The existence of equilibrium is discussed in the appendix. Following subsections talk about employee selection technology in equilibrium, and the model's implications for labor market outcomes.

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<sup>11</sup>There is also a flow equation that equates flows out of unemployment to flows into unemployment.

$$ufE_\lambda(X_\pi) = \delta e^g + e^n [\delta + (1 - \delta)(1 - E_\lambda(\pi|X))]$$

Observe that given the two other flow equations, this equation is redundant.

### 1.4.1 Employee Selection in the Equilibrium

Firms choose the more effective selection technology if

$$\beta q(\theta_\lambda) \left[ \int_{\pi_\lambda^*}^1 J(\pi_\lambda) d\Omega - \int_{\pi_\lambda^*}^1 J(\pi_\lambda) d\Gamma \right] > \kappa$$

One can rewrite expected profits of firm from having an employment relationship as

$$\int_{\pi_\lambda^*}^1 J(\pi_\lambda) d\cdot = (1 - \mu) \frac{(y_g - y_b)}{1 - \beta(1 - \delta)(1 - \pi_\lambda^*)} \int_{\pi_\lambda^*}^1 [\pi - \pi_\lambda^*] d\cdot$$

This equation uses the fact that what firm gets is a constant fraction of net match surplus, and match surplus depends on the difference between good and bad quality outputs as well as the reservation probability. This equation is derived in the appendix for existence of equilibrium.

Substituting for expected profits of firm from having an employment relationship into the equation for selection technology yields

$$\frac{\beta q(\theta_\lambda)(1 - \mu)(y_g - y_b)}{1 - \beta(1 - \delta)(1 - \pi_\lambda^*)} \left[ \int_{\pi_\lambda^*}^1 [\pi - \pi_\lambda^*] d\Omega - \int_{\pi_\lambda^*}^1 [\pi - \pi_\lambda^*] d\Gamma \right] > \kappa \quad (1.6)$$

Observe that, firms are more likely to select  $\Omega$  if, everything else being the same,  $y^g - y^b$  is higher. Since high skill firms have a higher uncertainty associated with the match quality, they are more likely to use the effective employee selection technology in equilibrium. Note that this is an ex-ante condition, it does not guarantee that high skill firms use that technology.

How the equation above reacts to changes in  $\lambda$  determines whether there can be multiple equilibria. Suppose the left hand side of the equation (1.6) is decreasing

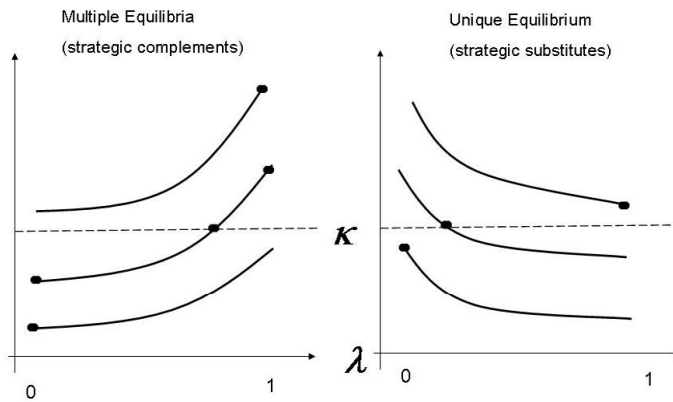


Figure 1.4: Illustration of Equation (1.6)

in  $\lambda$ . If there are many (few) firms using the effective selection technique (i.e.,  $\lambda$  is high [low]), then using the same technique, as opposed to the costless technique, is less (more) profitable. In other words, firms' actions are strategic substitutes. In this case the equilibrium is unique.

However, if left hand side of equation (1.6) increases in  $\lambda$ , then firms' actions are strategic complements and it is possible to have multiple equilibria. Note that, if a higher fraction of firms use the effective selection technology, then using the effective technology, as opposed to the costless technology, yields higher profits.

Why does this happen? How the deviation condition (equation (1.6)) responds to changes in  $\lambda$ , depends on how  $\theta$  changes with  $\lambda$ .<sup>12</sup> Using effective selection

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<sup>12</sup>Although qualitatively ambiguous, numerical exercises show that the response of left hand side

technology increases firms profits, condition that they produce. However, how the expected profits change depends how  $\theta$  changes. If  $\theta$  increases (decreases) with  $\lambda$ , then, when there are many firms using effective technology, expected profits go down (up). This happens because now a firm is less (more) likely to find a worker to realize the profits<sup>13</sup>. Whether market tightness increases or decreases with  $\lambda$  depends on the primitive parameters. This is because, for given market tightness, as the fraction of the firms using the effective selection technology increases, the reservation probabilities go up as workers get a higher probability of forming good quality matches elsewhere. That increases expected profits from having an employment relationship, however whether this increase is high enough to cover the higher expected cost of selection is ambiguous. Thus how market tightness reacts is ambiguous.

Quantitative analysis reveals that market tightness, for these specific parameter values, decreases with  $\lambda$ . Thus, as will be discussed later, there is possibility of multiple equilibria.

#### 1.4.2 Model Implications for Labor Market Outcomes

An unemployed worker's probability of finding a job, which I denote by  $p$ , has two components. The first component is the probability that the worker matches with a firm,  $f(\theta)$ . This probability is endogenously determined in the model, and it is a function of the equilibrium vacancy unemployment ratio. The second component

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follows the response of  $\theta$ .

<sup>13</sup>Change in  $\theta$  also affects surpluses through affecting workers' outside option. If, for instance, increase in  $\lambda$  decreases  $\theta$  and if many firms choose effective technology, then market tightness is low, so is the workers' outside option. That allows the firm have enough profits from the match to cover the cost of effective technology. Thus, the firm is more likely to choose the effective selection technology.

is the expected probability of a worker-firm pair drawing a high enough probability of a match being good quality so that the parties decide to form the production unit. The reservation probability, together with the selection technology a firm chooses, determines the acceptance probability. Then, the job finding probability of a worker is

$$p = f(\theta)(1 - \lambda\Omega(\pi^*) - (1 - \lambda)\Gamma(\pi^*))$$

Note that the difference between the job finding probabilities of high and low skill workers can come from the difference in market tightness, the difference in the selection technologies implemented, or the difference in the reservation probabilities.

Similarly, the vacancy filling probability, denoted by  $h$ , is determined by market tightness, which determines the probability that the vacancy meets with a worker, and the acceptance probability of a match. Formally:

$$h = q(\theta)(1 - \lambda\Omega(\pi^*) - (1 - \lambda)\Gamma(\pi^*))$$

The other important moment is total job separation rate. Total separation rate is determined by the exogenous and endogenous separations.

$$s(e^n + e^g) = (e^n + e^g)\delta + e^n(1 - \delta)(1 - E_\lambda(\pi|X_\pi = 1))$$

The first term in equation above is the exogenous separations, which affects all of the matches. Endogenous separation, on the other hand, occurs only for matches with unknown match quality which have survived the exogenous destruction. Among

those, the ones that are revealed to be bad quality get destroyed. Using the relationship between good (or known) quality matches and unknown quality matches, which is given via the flow equation in definition of equilibrium, I derive that

$$\frac{e^n}{e^n + e^g} = \frac{\delta}{\delta + (1 - \delta)E_\lambda(\pi|X_\pi = 1)}$$

Substituting for  $\frac{e^n}{e^n + e^g}$  in equation for separations, we get

$$s = \frac{\delta}{\delta + (1 - \delta)E_\lambda(\pi|X_\pi = 1)} \tag{1.7}$$

Observe that the separation rate is the same as the fraction of employees that have unknown quality matches. This is because, every period, workers with unknown quality matches either separate to unemployment, or they replace workers with good quality matches that separated to unemployment. The total number of job separations depends on the exogenous destruction rate and the conditional expected value of the match being good quality.

Since the exogenous destruction rate  $\delta$  is the same across sectors, the only source of a discrepancy between total job separation rates across skill groups is the differences in the conditional expected value of a match being good quality. The higher the  $E_\lambda(\pi|X_\pi = 1)$ , the lower the total separations. Observe that, the assumption that  $\Omega$  first order stochastically dominates  $\Gamma$  does not guarantee a higher separation rates, since separation rate also depends on an endogenous value  $\pi^*$ .

After some algebra on the equilibrium flow equations, one can show that the unemployment rate is:

$$u = \frac{\delta}{\delta + p(\delta + E_\lambda(\pi|X(\pi) = 1)(1 - \delta))}$$

Substituting  $\delta$  from equation (1.7) and rearranging terms gives the familiar equation of

$$u = \frac{s}{s + p}$$

The unemployment rate depends on the probability of workers finding a job and the probability of them separating from their jobs to unemployment. To see whether the proposed model can generate labor market outcomes for high and low-skill workers that are consistent with the data, I utilize the following quantitative exercise.

## 1.5 Quantitative Analysis

I assign values to the parameters of the model to match some facts of the U.S. labor markets. The period length of the model is one month. I set  $\beta = 0.9967$ , to get an annual interest rate of 4 %. The bargaining power of the workers is generally set to a number between 0.3 and 0.5 in the literature<sup>14</sup>. This number corresponds to the empirical measures of match elasticity of unemployment<sup>15</sup>. I set the bargaining power of the workers in Nash bargaining to 0.36, a value commonly used in the literature.

Observe that multiplying  $c$ ,  $\kappa$ ,  $y_g$ ,  $y_b$ , and  $b$  by the same number does not change the solution to the equation system. Thus, I normalize  $b$  in both sectors to 1. I also set  $y_b = b$ . This is sufficient for bad quality matches to be terminated in

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<sup>14</sup>See Petrangola and Pissarides (2001) for an excellent literature survey.

<sup>15</sup>Bargaining power is set to be the same as match elasticity of unemployment to get efficient outcomes in equilibrium (Hosios Condition).



the equilibrium. Moreover, this assumption implies that only the difference between high and low quality output matters for the equilibrium.

I assume the matching function to be  $M(u, v) = \frac{uv}{(u^\alpha + v^\alpha)^{1/\alpha}}$ . This functional form naturally bounds workers' and firms' matching probabilities to be in the unit interval. Employee selection distributions are assumed to be Beta distributions. Although there is not direct evidence on the distributional form of the selection technologies, I choose the Beta distribution because it has support in the unit interval. Moreover, I assume the first parameter of both Beta distributions to be 1<sup>16</sup>.

Table 1.2: Parameter Values

Parameter	Value		Parameter	Value	
$\beta$	0.996	4% interest	$c_{ls} = c_{hs}$	2.7	Match
$b$	1	Normalization	$\kappa$	6.66	Match
$y_b$	b	Restriction	$y_{ls}^g$	2.11	Match
$\mu$	0.36	Petrongolo et al. (2000)	$y_{hs}^g$	3.63	Match
$\alpha$	0.98	Match	$\beta_\Gamma$	3.8	Match
$\delta$	0.0039	Match	$\beta_\Omega$	0.67	Match

Remaining parameters of the model are the exogenous job destruction rate  $\delta$ , matching function parameter  $\alpha$ , the output a good quality low skill match produces  $y_{ls}^g$ , the cost of effective selection technology  $\kappa$ , and parameter values of  $\Gamma$  and  $\Omega$  distributions. I estimate these parameters so that the distance between the values of the chosen moments (described below) in the data and the values of these moments

<sup>16</sup>In this model's equilibrium  $\pi^*$  values will be low. Thus I need distributions with some mass in the lower tail. I normalized the first, not the second, parameter of the Beta distribution, because setting the second parameter of the Beta distribution to 1 gives low cumulative densities for low values of  $\pi$ .

in the model is minimized.

I use the following data moments. Davis et al. (1996) report that 23 percent of all annual job destruction is due to plant shutdowns in the manufacturing industry. I target 23 percent of all separations being exogenous in the low skill sector in the steady state. The literature widely assumes the match elasticity of unemployment to be 36 percent (Shimer 2005). I match the unemployment rate of 0.05 and job finding probability of 0.323 for low skill workers. To have the unemployment rate of 0.05 and the job separation rate of 0.017 requires a job finding probability of 0.323 in this model. This is because in the model the relationship between unemployment rate and the job finding probability is  $\frac{s}{s+p}$ <sup>17</sup>. I also target the value of .294 for the steady state job finding probability of high skill workers. I match per worker output produced to be 79 percent higher in high-skill jobs. Acemoglu and Zilibotti (2001) look at value added in high and low skill sectors and report a difference of 79 percent between the sectors. The expected value of wages of high skill workers are 90 percent higher, and the highest wage low skill workers can earn is 25 percent higher than the lowest wage in the steady state. Heathcote et al. (2008) report the college premium to be around 90 percent. Topel and Ward (1992) report that the cumulative change in wages over the first 10 years of work history that is associated with job change is around 33 percent. I set the ratio of the highest wage to the lowest in the low skill sector to be 25 percent, the number that is also used by Pries and Rogerson (2005).

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<sup>17</sup>Due to fluctuations over the sample period I have, the correlation between actual unemployment rate and steady state unemployment rate implied by matching model ( $\frac{s}{s+p}$ ), is not 1.

Table 1.3: Targeted Moments

<b>Moment</b>	<b>Value</b>	<b>Source</b>
low-skill job finding probability	0.323	Monthly average, 76-07
low-skill unemployment rate	0.005	Monthly average, 76-07
high-skill job finding probability	0.294	Monthly average, 76-07
Output difference across skill	1.79	Acemoglu et al (2001)
Expected wage gap across skill	1.9	Heatchote et al (2008)
Highest to lowest wage ratio; low skill	1.25	Topel et al (2002)
Fraction of annual job destruction due to plant shutdowns	.23	Davis et al. (1996)
Match elasticity of Unemployment	.36	Petrangolo et al(2000)

Observe that I have a system of eight equations with eight unknowns. The parameter values will be the minimizers of the distance between the data moments and their counterparts in the model. Recall that, if firms' actions are strategic complements, then there is a possibility of multiple equilibria. In computational exercise, I find the values of  $\pi^*$  and  $\theta$  for  $\lambda = 1$  and  $\lambda = 0$  and check whether the firms want to deviate from the distribution all other firms are assumed to use. If deviations are not profitable for either  $\lambda$  values, i.e., if there are multiple equilibria, then I choose the equilibrium in which  $\lambda = 1$ . I do not search over an interior solution for  $\lambda$  in this case, since the equilibrium for such  $\lambda$  is not stable. The economy reaches to this  $\lambda$  value only if it starts there. I choose the  $\lambda = 1$  equilibrium in computations because i) multiplicity happens only for skilled firms, and ii) this is the equilibrium we see in data.

### 1.5.1 Results

The estimated parameter values are reported in Table 1.2 along with the other parameter values. The value of good quality output for high skill workers is

more than 70 percent higher than that of low skill workers. The vacancy cost of a low skill job is higher than the amount of good quality output the job can produce. Although the vacancy cost of a high skill job is less than the amount of good quality output, the total cost, vacancy cost and the cost of more effective technology, is more than twice the good quality output the high skill job can produce. The total cost of a high skill vacancy is more than 3 times higher than the vacancy cost of low skill jobs. Although there is not direct evidence on the value of  $\kappa$ , Barron and Bishop (1985) report that total time spent in recruiting managerial, professional, and technical occupations is more than twice that of the blue collar and service occupations, and more than 50 percent higher than the clerical and sales occupations. Taking into account that the actual cost will be higher than the time cost, since the opportunity cost of workers who hire for high-skill occupations is expected to be higher as well, the value of  $\kappa$  is reasonable.

While searching for parameter values, I do not impose the restriction that the more effective distribution,  $\Omega$ , should first order stochastically dominate the less effective selection distribution,  $\Gamma$ . As Figure 3 show, estimated values of  $\Omega$  and  $\Gamma$  distributions are such that  $\Omega$  first order stochastically dominates  $\Gamma$ .

The equilibrium values of various labor market outcomes for high and low skill workers are displayed in Table 1.4. The model delivers targeted data moments. The unemployment rate and the job finding probability of low skill workers are the same as in the data. Observe that the model has the following relationship between the unemployment rate and job finding and job separation probabilities:  $u = \frac{s}{s+p}$ . Thus, the job separation rate of low-skill workers, although not directly targeted, is

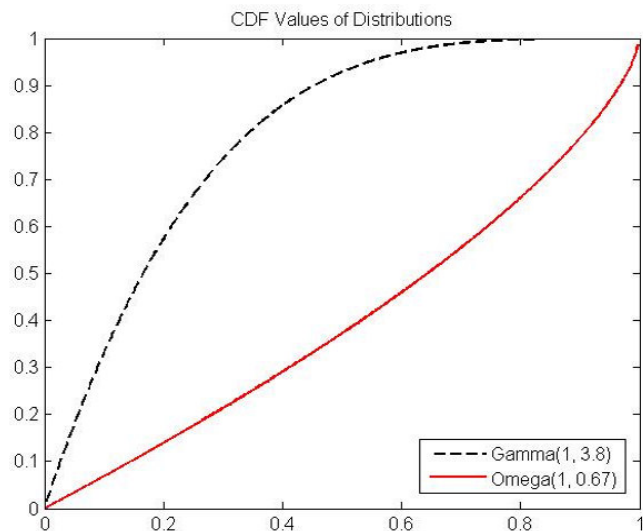


Figure 1.5: CDF Values of Employee Selection Distributions

the same as in the data.

For high skill workers, the model successfully delivers a job separation rate that is as low as it is in the data. The model also replicates the discrepancy between high and low skill unemployment rates. Observe that neither the unemployment rate nor the job separation rate of high skill workers is targeted. Moreover, the difference in job separation rates across skill groups is due to the more effective selection procedure high skill firms employ. The market tightness skill groups experience is 0.56 and 0.45 for low and high skill workers, respectively. Although there is no data on market tightness of skill groups, these numbers imply an overall market tightness in between these two values. Market tightness for the US since 2001 is on average

0.46<sup>18</sup>.

Table 1.4: Results

	High Skill		Low Skill	
	Data	Model	Data	Model
unemployment rate	0.02	0.021	0.05	0.05 <sup>+</sup>
job finding probability	0.294	0.294 <sup>+</sup>	0.323	0.323 <sup>+</sup>
job separation probability	0.006	0.0063	0.017	0.017
vacancy filling probability	-	0.65	-	0.58
$\theta$	-	0.45	-	0.56
$\pi^*$	-	0.059	-	0.024
$E(\pi \pi^*)$	-	0.63	-	0.23
$w(1)/w(\pi^*)$	-	1.35	1.25	1.25 <sup>+</sup>
$w(1)$	-	3.46	-	1.94
$w(\pi^*)$	-	2.57	-	1.55

<sup>+</sup>: Targeted moments.

Although there is no data for vacancy duration by skill for the US, Danish data suggest that (Van Ours and Ridder (1993)) high skill vacancies have higher durations. The model, however, predicts a lower duration for high skill vacancies, compared to low skill vacancies. This is not because high skill workers do not spend more time in recruitment, it is because the market tightness in high skill sector is low. As a result of low market tightness (low numbers of vacancies), high skill vacancies fill up more quickly than low skill vacancies. The market tightness high skill firms face is low because cost of opening a vacancy (including the cost of the selection technology) is too high.

The results are robust to different plausible values of  $\mu$ . Re-estimating the model parameters with  $\mu = 0.4$ , and  $\mu = 0.5$  delivers targeted moments and changes

<sup>18</sup>The data can be found at <http://www.bls.gov/jlt/>

Table 1.5: A Counterfactual Exercise

<b>High Skill</b>	<b>Experiment</b>			
	<b>Data</b>	<b>Model</b>	<b>Only <math>\Gamma</math></b>	<b>Only <math>\Omega</math></b>
unemployment rate	0.02	0.021	0.031	0.011
job finding probability	0.294	0.294	0.508	0.567
job separation probability	0.006	0.0063	0.0162	0.0061
vacancy filling probability	-	0.65	0.349	0.347
$\theta$	-	0.45	1.452	1.634
$\pi^*$	-	0.059	0.037	0.109
CDF value at $\pi^*$	-	0.04	0.133	0.074
$E(\pi \pi^*)$	-	0.63	0.237	0.642
$w(1)/w(\pi^*)$	-	1.35	1.373	1.313
<b>Low Skill</b>				
unemployment rate	0.05	0.05	0.05	0.0162
job finding probability	0.323	0.323	0.323	0.375
job separation probability	0.017	0.017	0.017	0.0062
vacancy filling probability	-	0.58	0.58	0.563
$\theta$	-	0.56	0.56	0.665
$\pi^*$	-	0.024	0.024	0.074
CDF value at $\pi^*$	-	0.088	0.088	0.05
$E(\pi \pi^*)$	-	0.23	0.23	0.628
$w(1)/w(\pi^*)$	-	1.25	1.25	1.219

other moments of interest slightly. For  $\mu = 0.4$ , the model predicts a job separation probability and an unemployment rate of 0.0059 and 0.0196 for high skill workers, respectively. The job separation rate and the unemployment rate of high skill workers are predicted to be 0.0053 and 0.0177, respectively, for  $\mu = 0.5$ . The results do not change if I normalize the first parameter of the distributions to 1.1 or 1.2, instead of 1.

To clarify the effects of employee selection technology, I carry out the following exercises. In the first exercise, I let firms use only the less effective selection tech-

nology,  $\Gamma$ . In this case, the only difference between skill groups in their equilibrium outcomes is due to productivity differences. The third column of Table 1.5 presents the results of the model for parameter values given in Table 2. The unemployment rate of low skill workers is 60 percent higher than the high skill unemployment rate. Although the difference in unemployment rates is high, it is not as high as it is in the data. Moreover, the reason for the unemployment rate discrepancy is mainly the difference in job finding probabilities between skill groups. High skill workers experience a high job finding probability because their labor market is too tight. Higher market tightness in the high skill sector generates a longer vacancy filling duration for high skill firms, compared to low skill firms. This is because opening a vacancy costs the same for all firms, and high skill jobs are expected to be more productive.

The last column of Table 1.5 shows the equilibrium of the model if  $\Omega$  were the only selection distribution that was available without any cost. The difference between high and low skill unemployment rates is close to 50 percent. Like the previous case, the difference in unemployment rates is due to the difference in job finding probabilities. The reason that columns two and four are not the same for high skill workers is that, in the case where  $\Omega$  is the only selection technology (column four), there is no cost ( $\kappa$ ) for using  $\Omega$ . Observe that it is the high cost of using effective selection technology that draws market tightness for high skill firms down.



## 1.6 Concluding Remarks

This paper proposes a new explanation for the unemployment rate disparity between skill groups. It is well documented that high skill workers have lower unemployment rates. Data also show that the reason for the lower unemployment rate of high skill workers is their lower probability of job separation. High skill workers are less likely to separate from their jobs because they are selected more effectively. Firms do more intensive and extensive employee search when hiring for high skill vacancies in the data.

This paper uses a matching model with uncertainty about match quality and with two employee selection technologies that differ in their cost and effectiveness. In the equilibrium, high skill firms, which are the firms with higher productivity, self-select themselves into using more effective technology. As a result of the choice of more effective technology, a higher fraction of high skill firms end up with good quality matches, thus a lower fraction experience endogenous match termination. Consequently, high skill workers have an unemployment rate in equilibrium that is as low as the data, compared to low skill workers, as they have substantially low job separation rates.

There is more work that needs to be done to explore skill bias in job separation rates. This paper focuses on the role of employee selection on job separation rates. In this paper, the driving force for high skill firms employing more effective technology is the productivity difference between high and low quality matches. Another possible reason is that learning about the match quality is slower for high skill jobs. This will not only directly contribute to the lower job separation of high skill,

but also affect the employee selection procedures firms use. Whether the difference in productivity or in the speed of learning about the match quality is more influential on the choice of employee selection technologies of firms is an open question.

The model abstracts from interaction across skill groups. For future work, it will be interesting to explore the effects of interactions across markets on employee selection technologies of firms. There are also other possible contributors to the bias, such as firm specific training, that should be explored.

## Chapter 2

### Interaction Between Labor Market Outcomes through Matching Markets

#### 2.1 Introduction

This paper analyzes effects of job competition between different skill groups on their labor market outcomes. Skill groups interact with each other through different channels. They are hired by the same firms, products they produce may be substitutes, and they may look for the same type of jobs. Thus, labor market outcomes of these skill groups are interrelated. Understanding the importance and effects of these channels will guide us in developing models and understanding the overall labor markets.

Job competition occurs if skilled workers compete with unskilled workers for unskilled jobs. Job competition is one of the possible explanations for the higher unemployment rate of unskilled workers compared to skilled workers. To my knowledge, there are no other papers that look specifically at effects of job competition. Moreno-Galbis and Sneessens (2007), Albrecht and Vroman (2002), Stadler and Wapler (2004), and Pierrard and Sneessens (2003) study skill biased technological change in models with job competition.

There are limited studies that look at job competition empirically. Van Ours and Ridder (1995) use Dutch labor market data from the 1980s. They find that job

competition takes place among higher educated workers.

Job competition has a testable implication: Participation of skilled workers at unskilled job search should create thinner markets on the unskilled workers' side, resulting in a decrease in their job finding probability. Hence, changes in skilled unemployment rate should negatively affect the probability of employment for unskilled workers. Observe that unskilled unemployment rate will affect the employment probability of a skilled worker as well. A probit regression of the probability of being employed on skilled and unskilled unemployment rates should reflect that interaction.

I use the Current Population Survey data to run the probit regression. The data and the regression are discussed in more detail in the following section. The regression shows that the employment probability of both skilled and unskilled workers is affected by the unemployment rate of the other skill group. Although that regression is silent about the source of interaction, it proves that an interaction exists, which makes this paper worth pursuing.

The purpose of this paper is to study effects of job competition on labor market outcomes of skilled and unskilled workers systematically. I use a labor matching model with two skill groups. Unskilled workers can only do unskilled jobs, whereas skilled workers can work at any type of job. Matching markets are segregated by the skill requirements of jobs. Consequently, workers can direct their search by the skill type of the job (job search is not directed within skill). Moreover, job markets operate sequentially, with the skilled job market operating first. This timing assumption allows skilled workers to participate in unskilled search if they

cannot find a skilled job. They do not need to choose between a skilled and an unskilled job.<sup>1</sup> The only source of interaction between groups is through matching markets. The decision of skilled workers to search for unskilled jobs is endogenous to the model. Skilled workers who are employed at unskilled jobs can search on the job.

Direct effects of job competition are reducing skilled unemployment rate (since they have bigger market) and increasing the unskilled unemployment rate (since they face higher competition). Since skilled workers are expected to have higher outside options, their wages at skilled jobs is expected to go up. Similarly, unskilled wages are expected to go down. However the number of vacancies responds to job competition. Unskilled firms have incentives to open more vacancies since filling a vacancy is easier in the presence of job competition. Thus how unskilled unemployment and wages are affected by job competition depends on which effect dominates. Moreover, since skilled workers search on the job while employed at unskilled jobs, one would expect the vacancy filling rate for skilled vacancies to be not affected by job competition. It will only affect skilled workers' outside option, thus wages. However, it is not obvious whether this is a strong effect. The results for reasonable parameter values show that job competition does reduce the average unemployment rate. It reduces the skilled unemployment rate more, generating an increase in unemployment rate inequality. However, employment rate at skilled jobs is unaffected.

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<sup>1</sup>There is no consensus in the literature in modeling matching within a heterogeneous agents framework.

The following section of the paper provides evidence on interaction between labor market outcomes of skill groups. Section 2 describes the model. Computing the equilibrium of the model and calibration of parameter values are discussed in section 3. The next section documents results and section 5 concludes..

## 2.2 Interaction Between Labor Markets

If there is job competition between different skill groups, then their unemployment rates would reflect such information. Interaction across skill groups through matching markets suggests that skilled workers prefer employment at an unskilled job to staying unemployed. Participation of skilled workers in unskilled job markets will create thinner markets on the unskilled workers' side. Thus, an increase in skilled unemployment rate will reduce the job finding probability for unskilled workers, and increase their unemployment rate as well. Equivalently, changes in skilled unemployment rate should negatively affect the probability of employment of unskilled workers. Observe that unskilled unemployment rate will affect the probability of employment of skilled workers as well.

To substantiate this relationship, I run a probit regression in order to calculate the effects of skilled and unskilled unemployment rates on the employment probability of workers of a particular skill group. Let  $y_i$  be equal to one if the worker  $i$  of a specific skill group is employed, and zero if she is not. Then, we can express the probability of observing  $y_i = 1$  as a function of individual characteristics and some labor market indicators.

$$P(y_i = 1|x) = \Phi(x\eta), \tag{2.1}$$

where  $x$  is  $1 \times K$ ,  $\eta$  is  $K \times 1$ , and the first element of  $x$  is unity. The vector  $x$  contains personal characteristics, skilled and unskilled unemployment rates for the metropolitan statistical area in which the person lives, and some control variables for the area, month and year.

I use the monthly CPS data from January 1996 to December 2006 to estimate the model above. I use the working age population that is not self employed, not in the armed forces, and lives in one of the largest twelve consolidated metropolitan statistical areas (CMSA). I restrict the sample to the biggest metropolitan areas because in smaller CMSAs, there are too few skilled unemployed workers. Individual characteristics are age, gender, marital status, experience, squares of age and experience, race, and citizenship status.<sup>2</sup>

Labor market variables are skilled and unskilled unemployment rates. I calculate unemployment rates for each month of the sample period for each CMSA.<sup>3</sup> Unemployment rates vary by geography and time.

The first four columns of Table ?? report the marginal effects of skilled and unskilled unemployment rates on the probability of an unskilled worker being employed, under different specifications. First of all, unemployment rates of both skilled and unskilled workers negatively affect the probability of an unskilled worker being employed. These effects are significant for all model specifications. Changes in the unskilled unemployment rate affects the probability of an unskilled worker

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<sup>2</sup>CPS do not have direct data on work experience. I constructed the series by subtracting years of education plus 6 from the age.

<sup>3</sup>This is because smaller metropolitan areas will have smaller sample sizes.

being employed more. These results hold true if we control for only geography, only seasonal changes, both geography and seasonal changes, and geography, seasonal, and year to year changes.

The probability of a skilled worker being employed is also affected by her own skill group's unemployment rate. The effect of unskilled unemployment rate on this probability is relatively much smaller.

One should interpret these results with caution, since they only suggest the presence of interaction. If skilled and unskilled workers were complements in the production process, then we would have seen the same effect of unemployment rates on the probability of employment.

### 2.3 Model

I use a search model of unemployment with heterogenous agents and firms to investigate the effects of job matching on labor market outcomes. There are two types of agents: skilled (s) and unskilled (u). Worker population is normalized to 1. Skilled workers form a fraction  $p$  of the population. All agents have linear utility and maximize expected earnings.

There are two types of firms: skilled and unskilled. Each firm has one job. Skilled jobs can only be done by skilled workers while unskilled jobs can be filled by workers of any skill type. Each firm chooses the skill type of the job and employs at most 1 worker. A filled job produces output  $y_i(k)$  where  $i \in \{s, u\}$  and  $k \in \{1, 2, \dots, n\}$ . Without loss of generality, assume  $y_i(1) < y_i(2) < \dots < y_i(n)$ .



$y_i(k)$  follows a Markov process. Let  $\Pi_i$  be the transition matrix for the Markov process of firms in sector  $i$ .  $y_s(\cdot)$  and  $y_u(\cdot)$  are independent processes. Moreover, let  $z = \{y_u(\cdot), y_s(\cdot)\}$  be the state of the economy.

The cost of posting a vacancy is  $c_i$ . A match survives to the next period with probability  $\delta_i$ . Firms and workers have the same discount rate  $\beta$ .

A worker can be employed or unemployed. All workers earn wage while employed and are entitled to unemployment income while unemployed. Wage is determined through Nash bargaining. Skilled workers are free to choose whether to look for unskilled jobs. Skilled workers who work for unskilled firms can search for a skilled job while employed, i.e., there is on the job search. Since all jobs are homogenous within a sector, workers do not get better off by searching for a job within the same sector. Thus, I assume only skilled workers who are employees of unskilled jobs search on the job.

### 2.3.1 Matching Markets

The number of matches between firms and workers is endogenous in the model and it is determined in matching markets. Firms and workers that meet start producing in the subsequent period.

Matching markets are differentiated by the skill requirement of firms. Workers can direct their search by the skill type of the firm. Let us assume that these markets operate sequentially and the skilled job market operates first. After the skilled job market closes, the unskilled job market opens. Skilled unemployed workers who could not find a skilled job, as well as unskilled unemployed workers, can

participate in unskilled job search.

The heterogenous matching literature does not have a unanimous way of modeling matching. For instance, Albrecht and Vroman (2002) assume a global matching market which implies that neither firms nor workers can direct their search by skill. I choose to have sequential markets because it is a more realistic way of modeling job competition, since skilled workers primarily look for skilled jobs. Moreover, undirected search by skill artificially amplifies interaction. That is because with a global matching market, skilled firms can meet with unskilled workers, whom they will not hire, which results in lower incentives for skilled firms to open vacancies.

The meetings are governed by a matching function  $M(\tilde{u}_j, v_j(z))$  in each job market, where  $v_j$  is the ratio of vacancies of skill type  $j$  to the labor force of type  $j$ , and  $\tilde{u}_j$  is the ratio of the total number of job searchers in market  $j$  to the labor force of skill type  $j$ .

There are only skilled firms and workers in the skilled job market. Market tightness in the skilled market is the ratio of vacancies to unemployed and on the job searchers. The probability that a skilled firm meets a skilled worker is  $M(\tilde{u}_s, v_s(z))/v_s(z)$ , and the probability that a skilled worker meets a skilled firm is  $M(\tilde{u}_s, v_s(z))/\tilde{u}_s$ . Note that  $q_{us}(z) = 0$ , since unskilled workers do not participate in skilled job search. Let  $e_{su}$  be the fraction of skilled labor force that works at

unskilled jobs. Then,

$$\begin{aligned}
\tilde{u}_s &= u_s + e_{su} \\
\theta_s(z) &= \frac{v_s(z)}{\tilde{u}_s} \\
q_{ss}(z) &= \frac{M(\tilde{u}_s, v_s(z))}{v_s(z)} = q(\theta_s(z)) \\
q_{us}(z) &= 0 \\
f_{ss}(z) &= \frac{M(\tilde{u}_s, v_s(z))}{\tilde{u}_s} = f(\theta_s(z)).
\end{aligned} \tag{2.2}$$

Note that the probability that an unemployed skilled worker finds a skilled job,  $f_{ss}$ , is not affected by job competition directly. This is because, due to sequential job markets assumption, the possibility of search for an unskilled job does not affect the number of unemployed skilled workers who search for a skilled job, per se. The only other channel through which the skilled job finding probability will be affected is through the number of vacancies. However, job competition does not affect vacancy creation through the vacancy filling rates (or market tightness), per se. Firms are aware that skilled workers who find an unskilled job will be back in the market. Thus, job competition does not affect the firms' probability of finding a worker. As will be discussed later, the effect of job competition on skilled vacancy creation occurs through wages.

There can be skilled unemployed workers, in addition to unskilled job searchers, in the unskilled job market. Let  $\lambda(z)$  be the probability that skilled workers participate in unskilled job search. Note that those skilled unemployed who cannot find a skilled job,  $(1 - f_{ss}(z))pu_s$ , may participate in unskilled job search. Moreover, let  $\psi^s(z)$  be the probability that unskilled firms can meet with a skilled worker.

$$\begin{aligned}
\tilde{u}_u(z) &= (1-p)u_u + \lambda(z)(1-f_{ss}(z))pu_s \\
\theta_u(z) &= \frac{(1-p)v_u(z)}{\tilde{u}_u(z)} \\
q_{uu}(z) &= \frac{M(\tilde{u}_u(z), v_u(z))}{v_u(z)}(1-\psi^s(z)) = q(\theta_u(z))(1-\psi^s(z)) \\
q_{su}(z) &= \frac{M(\tilde{u}_u(z), v_u(z))}{v_u(z)}\psi^s(z) = q(\theta_u(z))\psi^s(z) \\
f_{uu}(z) &= \frac{M(\tilde{u}_u(z), v_u(z))}{\tilde{u}_u(z)} = f(\theta_u(z)) \\
f_{us}(z) &= 0 \\
f_{su}(z) &= \lambda(z)(1-f_{ss}(z))\frac{M(\tilde{u}_u(z), v_u(z))}{\tilde{u}_u(z)} = \lambda(z)(1-f_{ss})f(\theta_u(z))
\end{aligned} \tag{2.3}$$

Job competition affects the probability that a skilled unemployed worker finds a job since it gives the workers the opportunity to have an unskilled job.

The market tightness in the unskilled job market, which determines the job finding probability of unskilled workers, is directly affected by job competition. If skilled workers were not to participate in unskilled job search, there would be more open positions per unskilled worker. Thus, they would find jobs more easily.

However, job competition gives unskilled firms incentives to open vacancies, since a higher number of job searchers lets vacancies be filled more quickly. Although one would expect the latter effect to be weaker than the former, theoretically job competition need not hurt unskilled workers.

### 2.3.2 Timing

- A period starts with worker-firm pairs in a meeting, unemployed workers, and vacancies.

- Shocks are realized.
- Worker-firm pairs decide whether to produce or search.
- Firm-worker pairs that stay in the match start producing
- Skilled job market opens. Unemployed skilled workers and skilled workers who hold unskilled jobs search for a skilled job.
- Skilled job market closes. Skilled workers decide whether to participate in unskilled job search.
- Unskilled job market opens.
- New meetings form.
- Some fraction of jobs are destroyed.

### 2.3.3 Firms' Bellman Equations

Let  $V_j(z)$  denote the discounted present value of a vacancy to a firm with skill type  $j \in \{s, u\}$ . The vacant firm needs to incur the vacancy cost  $c_j$ . The firm meets with a worker with some endogenous probability and decides whether to produce with that worker or continue to search in the following period. The firm with skill type  $j$  matches with a worker with skill type  $i$  with probability  $q_{ij}(z)$ . The discounted present value of a firm that produces is  $J_{.j}(z)$ .<sup>4</sup>

Recall that the vacancy filling probability is a function of market tightness. Firms need to know the market tightness in order solve their optimization problem.

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<sup>4</sup>First notation of the subscript denotes the worker's skill type.

Hence, it is part of the state of the economy for firms. For notational ease, I do not explicitly write the market tightness as an argument.

$$V_u(z) = -c_u + q_{su}(z)\beta E_{z'|z} J_{su}(z') + q_{uu}(z)\beta E_{z'|z} J_{uu}(z') + (1 - q_{su}(z) - q_{uu}(z))\beta E_{z'|z} V_u(z') \quad (2.4)$$

$$V_s(z) = -c_s + q_{ss}(z)\beta E_{z'|z} J_{ss}(z') + (1 - q_{ss}(z))\beta E_{z'|z} V_s(z') \quad (2.5)$$

Observe that skilled firms can only meet with skilled workers. Moreover, skilled firms do not differentiate whether the worker they hire is coming from employment or unemployment. I assume skilled employees of unskilled jobs quit their jobs at the end of the period if they find a skilled job.<sup>5</sup> Thus, in the following period, they negotiate with a skilled firm as having the same outside option as unemployed skilled workers. If firms were to differentiate between employment histories, skilled workers who transit from employment would get higher first period wages, since they would have had higher outside options (if their jobs did not get destroyed by exogenous shocks). In this case, skilled firms' incentives to open vacancies would be negatively affected, since their expected profits from a job would go down. However, given the relatively small increase in outside options of employed skilled job searchers, their small numbers in comparison to unemployed job searchers, and the fact that outside employment option holds only for the first period will make this effect small.

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<sup>5</sup>Observe that the model does not differentiate between quits or fires.

A skilled firm in a match with a skilled worker, has the following optimization problem:

$$J_{ss}(z) = \max \left\{ V_s(z), \quad y_s(z) - w_{ss}(z) + \beta \delta_s E_{z'|z} V_s(z') + \beta (1 - \delta_s) E_{z'|z} J_{ss}(z') \right\} \quad (2.6)$$

The firm will choose the better option of either continuing to search or hiring the skilled worker. If it searches, the firm gets the present discounted value of having a vacancy,  $V_s(z)$ . If the firm hires the worker, then they produce in that period, the firm gets the output net of the worker's wage, and with some exogenous probability continues to the following period.

An unskilled firm in a meeting with a worker has an optimization problem similar to the skilled firm:

$$J_{uu}(z) = \max \left\{ V_u(z), \quad y_u(z) - w_{uu}(z) + \beta \delta_u E_{z'|z} V_u(z') + \beta (1 - \delta_u) E_{z'|z} J_{uu}(z') \right\} \quad (2.7)$$

$$J_{su}(z) = \max \left\{ V_u(z), \quad y_u(z) - w_{su}(z) + f_{ss}(z) \beta E_{z'|z} V_u(z') + \beta (1 - f_{ss}(z)) \delta_u E_{z'|z} V_u(z') \right. \\ \left. + (1 - f_{ss}(z)) (1 - \delta_u) \beta E_{z'|z} J_{su}(z') \right\} \quad (2.8)$$

If an unskilled firm considers hiring a skilled worker, the firm takes into account that the skilled worker searches for a skilled job, thus she may leave the firm in the following period. On the job search will reduce the value of hiring a skilled worker as opposed to an unskilled worker. This will be reflected on wages. One would expect skilled workers to earn less than unskilled for an unskilled job, since skilled workers need to compensate the firm for higher job termination probability. On the other hand, skilled workers have higher outside options than unskilled workers.

Thus, unskilled firms may need to offer skilled workers higher wages for skilled workers to accept the job offer.

Note that all firms within a skill group are identical, i.e., they all receive the same output realization. To avoid endogenous separation (which would mean no employment in that period), one needs to put some restrictions on output values. Assume that  $b_s < \min\{y_s(1), y_u(1)\}$  and  $b_u < y_u(1)$ . This assumption guarantees that even in the worst possible output realization, firms and workers will not endogenously terminate a job.<sup>6</sup>

### 2.3.4 Workers' Bellman Equations

Let  $U_i(z)$  denote the discounted present value of unemployment to a worker with skill type  $i \in \{s, u\}$ . The unemployed worker receives some income  $b_i$  during the current period and looks for a job. She meets with a skilled and unskilled firm with some endogenous probabilities, or she does not meet with any firm and remains unemployed.

$$U_s(z) = b_s + f_{ss}(z)\beta E_{z'|z} W_{ss}(z') + f_{su}(z)\beta E_{z'|z} W_{su}(z') + (1 - f_{ss}(z) - f_{su}(z))\beta E_{z'|z} U_s(z') \quad (2.9)$$

$$U_u(z) = b_u + f_{us}(z)\beta E_{z'|z} W_{us}(z') + f_{uu}(z)\beta E_{z'|z} W_{uu}(z') + (1 - f_{us}(z) - f_{uu}(z))\beta E_{z'|z} U_u(z') \quad (2.10)$$

An unskilled worker in a meeting with an unskilled firm needs to decide whether she wants to continue to search or form an employment relationship with

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<sup>6</sup>This restriction on output is sufficient, but not necessary. It is possible to avoid endogenous separation with lower values of outputs.



this firm. If she stays unemployed she receives  $U_u(z)$ . If she chooses the job, she receives a wage for the current period and with some exogenous probability keeps her job in the following period.

$$W_{uu}(z) = \max\left\{U_u(z), \quad w_{uu}(z) + \beta\delta_u E_{z'|z} U_u(z') + \beta(1 - \delta_u) E_{z'|z} W_{uu}(z')\right\} \quad (2.11)$$

The unskilled worker will not be hired by a skilled firm. Thus, if she meets with a skilled firm, she receives  $U_u(z)$ .

$$W_{us}(z) = U_u(z) \quad (2.12)$$

A skilled worker has essentially the same optimization problem. If the skilled worker is in a meeting with an unskilled firm, she takes into account her option of on the job search when considering the employment relationship with this firm.

$$W_{su}(z) = \max\left\{U_s(z), \quad w_{su}(z) + f_{ss}(z)\beta E_{z'|z} W_{ss}(z') + \beta(1 - f_{ss}(z))\delta_u E_{z'|z} U_s(z')\right. \\ \left. + (1 - f_{ss}(z))(1 - \delta_u)\beta E_{z'|z} W_{su}(z')\right\} \quad (2.13)$$

$$W_{ss}(z) = \max\left\{U_s(z), \quad w_{ss}(z) + \delta_s\beta E_{z'|z} U_s(z')\right. \\ \left. + (1 - \delta_s)\beta E_{z'|z} W_{ss}(z')\right\} \quad (2.14)$$

### 2.3.5 Wage Determination

The wage between a firm and a worker is determined through Nash bargaining. Worker's bargaining power is  $\mu_{ij}$ . The worker and the firm bargain over their share of the match surplus, net of their outside options. A firm's outside option is always being vacant. A worker's outside option is always being unemployed. Note that workers and firms bargain at the beginning of the period, before job markets

open. Thus, a skilled worker, regardless of the skill type of the job she is matched with, has unemployment as her outside option. The bargaining problem between a firm with skill requirement  $j$  and a worker with skill type  $i$  can be formalized as follows:

$$w_{ij}(z) = \operatorname{argmax} [J_{ij}(z) - V_j(z)]^{(1-\mu_{ij})} [W_{ij}(z) - U_i(z)]^{\mu_{ij}}, \quad (2.15)$$

## 2.4 Equilibrium

For given values of  $y_i(z)$  and  $\Pi_i$ , and initial unemployment rates, the equilibrium of the model is the set of functions  $\{J_{ij}(z), W_{ij}(z), w_{ij}(z)\}_{i,j \in \{s,u\}}$ ,  $V_s(z), V_u(z), U_s(z), U_u(z), \psi^s(z), v_s(z), v_u(z), u_s(z), u_u(z)\}_{\forall z}$  such that

- the value of vacancies in both sectors is zero (free entry condition):

$$V_s(z) = 0 \quad (2.16)$$

$$V_u(z) = 0 \quad (2.17)$$

- value functions satisfy equations from (2.5) to 2.14.
- wage rates satisfy equation 3.7.
- The probability that an unskilled firm meets a skilled worker is

$$\psi^s(z) = \frac{\lambda(z)(1 - f_{ss}(z))pu_s}{(1 - p)u_u + \lambda(z)(1 - f_{ss}(z))pu_s}. \quad (2.18)$$

- $\lambda(z) = 1$  if and only if  $E_{z'|z} W_{su}(z') > U_s(z)$ .
- Flows across employment states are

$$u'_u(z) = (1 - f_{uu}(z))u_u + \delta_u(1 - u_u) \quad (2.19)$$

$$u'_s(z) = (1 - f_{ss}(z) - f_{su}(z))u_s + \delta_u e_{su} + \delta_s(1 - u_s - e_{su}(z)) \quad (2.20)$$

$$e'_{su}(z) = f_{su}(z)u_s + (1 - \delta_u)e_{su}(1 - u_s - e_{su}). \quad (2.21)$$

### 2.4.1 Implications of job competition

Job competition will have effects on the following labor market outcomes. First of all, job competition affects the job finding probability of skilled workers. Thus it is expected to reduce their unemployment rate. Moreover, since they compete with unskilled workers, unskilled unemployment rate is expected to go up.

Job competition also has effects on the wages of skilled workers. The option value of getting unskilled jobs (which should be paying higher wages than unemployment incomes, otherwise skilled workers would have not participated) strengthens the bargaining position of skilled workers. Consequently, it is expected to increase the wages they get at skilled jobs.

Note that these are primary effects. Firms would respond to these effects in equilibrium. For instance, if there are skilled job searchers in unskilled market, unskilled firms will have higher expected returns, since they will be more likely to fill their positions. Thus, the number of unskilled vacancies would go up. As a result, It is not clear whether job competition would increase the unskilled unemployment rate.

Moreover, higher outside options of skilled workers would reduce skilled firms' incentives to open vacancies since their expected returns would go down. Thus, job competition would reduce the number of available skilled vacancies, which

would reduce skilled workers' wages. Hence, general equilibrium effects makes effects of job competition on wages and unemployment ambiguous.

## 2.5 Calibration and Computation

The equilibrium of this economy is characterized by the decision rules for vacancy creation and the wage determination rules for given values of unemployment rates and the aggregate state of the economy.

To describe the equilibrium, it is helpful to work with the match surplus  $S_{ij}(z)$ , which is defined as follows:

$$S_{ij}(z) = W_{ij}(z) - U_i(z) + J_{ij}(z) - V_j(z). \quad (2.22)$$

Moreover, Nash bargaining implies the following relationship between shares workers and firms get from the surplus.

$$W_{ij}(z) - U_i(z) = \mu_{ij} S_{ij}(z) \quad (2.23)$$

Using bellman equations and identity 2.23, we get

$$\begin{aligned} S_{us}(z) &= 0 \\ S_{uu}(z) &= \max \left\{ 0, \quad y_u(z) + (1 - \delta_s)\beta E_{z'|z} S_{uu}(z') + \beta E_{z'|z} U_u(z') - U_u(z) \right\} \\ S_{su}(z) &= \max \left\{ 0, \quad y_u(z) + (1 - f_{ss}(z))(1 - \delta_u)\beta E_{z'|z} S_{su}(z') - U_s(z) + \beta E_{z'|z} U_s(z') \right. \\ &\quad \left. + f_{ss}(z)\beta \mu_{ss} E_{z'|z} S_{ss}(z') \right\} \\ S_{ss}(z) &= \max \left\{ 0, \quad y_s(z) + (1 - \delta_s)\beta E_{z'|z} S_{ss}(z') + \beta E_{z'|z} U_s(z') - U_s(z) \right\}. \end{aligned} \quad (2.24)$$

Similarly, unemployment value functions are

$$\begin{aligned}
U_u(z) &= b_u + f_{uu}(z)\beta\mu_{uu}E_{z'|z}S_{uu}(z') + \beta E_{z'|z}U_u(z') \\
U_s(z) &= b_s + f_{ss}(z)\beta\mu_{ss}E_{z'|z}S_{ss}(z') + f_{su}(z)\mu_{su}\beta E_{z'|z}S_{su}(z') + \beta E_{z'|z}U_s(z').
\end{aligned}
\tag{2.25}$$

Vacancy value equations become

$$\begin{aligned}
c_u &= q_{uu}(z)\beta(1 - \mu_{uu})E_{z'|z}S_{uu}(z') + q_{su}(z)\beta(1 - \mu_{su})E_{z'|z}S_{su}(z') \\
c_s &= q_{ss}(z)\beta(1 - \mu_{ss})E_{z'|z}S_{ss}(z').
\end{aligned}
\tag{2.26}$$

### 2.5.1 Computing equilibrium

In order to solve for equations (3.12), we need skilled and unskilled market tightnesses  $\theta_s$  and  $\theta_u$ , respectively. Observe that these values also depend on unemployment rates of the period.

I use the following algorithm to solve for equilibrium:

1. Let  $Z = \{z_1, z_2, \dots, z_{n^2}\}$  be the index for the aggregate state of the economy such that

$$Z = \left\{ \begin{pmatrix} y_u(1) \\ y_s(1) \end{pmatrix}, \begin{pmatrix} y_u(1) \\ y_s(2) \end{pmatrix}, \dots, \begin{pmatrix} y_u(1) \\ y_s(n) \end{pmatrix}, \begin{pmatrix} y_u(2) \\ y_s(1) \end{pmatrix}, \dots, \begin{pmatrix} y_u(n) \\ y_s(n) \end{pmatrix} \right\}$$

Let  $Z$  be the size  $N_z$  vector of possible states of the economy, which is governed by joint probabilities of  $y_s$  and  $y_u$  Markov processes.

2. Generate a grid for each of skilled and unskilled unemployment rates, and for cross skill employment,  $U^s$ ,  $U^u$ ,  $E^{su}$ , respectively. Let  $N$  be the size of each of these grids.

3. Set upper and lower bound for  $\theta_u$  and  $\theta_s$ . Boundaries are  $N_z \times N \times N \times N$  dimensional quadruples.
4. Guess a  $\theta_u$  value within these boundaries.  $\theta_u(i, j, k, m)$  is the guess for market tightness when the economy has  $Z(i)$  shocks, skilled unemployment rate is  $U^s(j)$ , unskilled unemployment rate is  $U^u(k)$ , and employment of skilled workers in the unskilled sector is  $E^{su}(m)$ .
5. Guess a  $\theta^s$  value within boundaries.
6. Compute job finding and vacancy filling probabilities using guessed values in equations (2.2) and (2.3).
7. Compute surplus values and unemployment values using equations 3.10 and 3.11, via value function iteration. Notice that one needs to interpolate for surplus values of the subsequent period to calculate expected surplus values.
8. Compute values of vacancies using equations (3.12).
9. Update guess of  $\theta_s$ .
  - If  $V_s > 0$ ; then increase  $\theta_s$ .
  - If  $V_s < 0$ ; then decrease  $\theta_s$ .
10. Repeat steps starting from step 5 until convergence
11. Update guess of  $\theta_u$ .
  - If  $V_u > 0$ ; then increase  $\theta_u$ .

- If  $V_u < 0$ ; then decrease  $\theta_u$ .

12. Repeat steps starting from step 4 until convergence

### 2.5.1.1 Calibration

In order to solve the system of equations, parameter values should be assigned. Parameters of the model are:  $\{p, c_s, c_u \bar{y}_s, \bar{y}_u \delta_s, \delta_u \mu_s, \mu_u, \mu_{su} b_s, b_u, \alpha, \beta\}$ , and parameters that govern shocks.

The model period is one month, so  $\beta$  is set to 0.996 to match annual interest rate of four percent. The skilled fraction of the labor force  $p$  is set to 0.27, its average value from 1996 to 2006. The average value of output produced by unskilled labor is normalized to 1. Unemployment income is assumed to be the same across skill groups and is set to 0.54. This number is the average replacement ratio for the U.S.. The matching function is of the form

$$M(\tilde{u}, v) = \frac{\tilde{u}v}{(\tilde{u}^\rho + v^\rho)^{1/\rho}}$$

The matching parameter is set to 1.1, which gives the equilibrium matching elasticity of unemployment if the market tightness is the data average. All workers are assumed to have the same bargaining power, which is 0.36. This number is generally assumed in the literature (Shimer (2005)).

Shock processes are taken from Hornstein et al (2005). Hornstein et al report that quarterly labor productivity follows an AR1 process with auto-correlation coefficient of 0.89 and standard deviation of 0.02.<sup>7</sup> I assume that the baseline AR1

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<sup>7</sup>Measured as output per worker in the non-farm sector.

process for both skilled and unskilled output is the same except for the means. The intent behind this assumption is to generate the interactions only through the matching markets.

I normalize the average value of unskilled output to 1 and set the average value of skilled output to 1.79. I use the Tauchen approximation to AR1 processes to get Markov values and transition probabilities.

$$y_u = \begin{bmatrix} 0.956 \\ 1 \\ 1.044 \end{bmatrix}, \quad \Pi_u = \begin{bmatrix} 0.804 & 0.195 & 0.001 \\ 0.136 & 0.727 & 0.136 \\ 0.001 & 0.195 & 0.804 \end{bmatrix}$$

$$y_s = \begin{bmatrix} 1.746 \\ 1.79 \\ 1.834 \end{bmatrix}, \quad \Pi_s = \begin{bmatrix} 0.804 & 0.195 & 0.001 \\ 0.136 & 0.727 & 0.136 \\ 0.001 & 0.195 & 0.804 \end{bmatrix}$$

The remaining parameters are the costs of posting vacancies ( $c_u, c_s$ ). They are set so that the average job finding probabilities of both skill groups are close to the data.

## 2.6 Results

To see the effects of job competition on the labor market outcomes of skill groups, I simulate an economy of 10,000 agents for 1400 periods. I use the last 1100 periods to run the same probit regression as discussed in previous sections on simulated data.

Table 2.2 shows the coefficients on skilled and unskilled unemployment rates for probit regressions. Although these coefficients are not the marginal effects, they have the same sign. The first column of Table 2.2 has the results for the employment



probability of an unskilled worker. An increase in the unskilled unemployment rate reduces the employment probability. However, a change in skilled unemployment rate does not have a significant effect on this probability. The third column of Table 2.2 has probit regression results for the employment probability of a skilled worker. Skilled unemployment rate affects the probability of being employed for a skilled worker. However, unskilled unemployment rate is not effective.

These probit regressions suggest that the interaction between labor markets of skill groups through job competition is not significant enough to influence a specific skill group's employment probability. Note that for skilled vacancies changes in the unskilled sector is not important per se because, due to on the job search, changes in the unskilled job market does not change the number of skilled job searchers per se. As discussed earlier, the only other effect is through changes in skilled workers' outside options, hence through wages. The probit results suggest that these secondary effects are not strong enough.

The reason that changes in the skilled unemployment rate does not affect the employment probability of an unskilled worker is less obvious. One possible explanation is that skilled workers who search for unskilled jobs are a relatively small fraction of the total unskilled job searchers. Thus, fluctuations in the number of skilled workers in the unskilled job market may have small effects on the total number of unskilled vacancies created.

To see the effects of the existence of job competition on labor market outcomes, I do the following counterfactual exercise. Using the same values of the parameters as in the economy with the interaction, I calculate the equilibrium of

an economy without job competition. Thus, the only difference between these two economies is the presence of the participation of skilled workers in unskilled job search, or lack there of. I simulate these two economies for the same shock realizations. Results are reported on Table 2.3.

The first two columns of Table 2.3 show means and standard deviations of some variables of the simulated economy with interaction. Numbers in parentheses are the standard errors. Observe that skilled workers that hold unskilled jobs are a small fraction of the skilled labor force. The skilled unemployment rate is lower compared to the unskilled unemployment rate. Note that, on average, skilled workers are paid slightly higher wages at unskilled jobs than the unskilled workers.

The simulated data moments of the economy without job competition are displayed in last two columns of the Table 2.3. Note that, the existence of job competition widens the unemployment rate gap in favor of the skilled workers.

The number of skilled workers who are employed in skilled jobs is almost the same in both economies. Note that the sum of the number of skilled unemployed workers and the number of skilled workers employed at unskilled jobs in the economy with interaction is almost the same as the number of unemployed skilled workers in the economy with no interaction. Interestingly, the unskilled unemployment rate decreases in the presence of job competition.

The average value of wages is not affected significantly by the existence of job competition. This is because in both of these equilibria, the number of vacancies adjusts so that workers' outside options do not change significantly enough to generate wage differences across these economies.

## 2.7 Concluding Remarks

What are the implications of the interaction between heterogeneous groups through matching markets (job competition)? This paper aims to analyze this question. Data suggest that a skilled (unskilled) worker's probability of being employed is affected by unskilled (skilled) unemployment. Job competition is in line with this fact.

I use a heterogeneous agents model where the only interaction between different groups comes from job competition. I run a probit regression on simulated data and find that the skilled (unskilled) unemployment rate does not significantly affect the probability of an unskilled (skilled) worker being employed. I also simulate the economy in the presence and the absence of job competition. I find that although the presence of job competition increases unemployment inequality, it reduces both skilled and unskilled unemployment rates.

Future research should first look at other ways of empirically testing the relevance of job competition. It is also important to figure out what channels are more important in generating the interaction we see in the data between labor market outcomes of different skill groups.

Table 2.1: Parameter Values

Parameter	Value	Source
$p$	0.27	Skilled fraction of Labor Force, average <sup>+</sup>
$\beta$	0.996	interest rate
$\alpha$	1.1	match elasticity, Pissarides et al (2000)
$b$	0.54	replacement ration
$\mu_u, \mu_s$	0.36	Shimer (2005)
$\delta_u$	0.017	Separation rate of unskilled labor, average <sup>+</sup>
$\delta_s$	0.006	Separation rate of skilled labor, average
$\bar{y}_u$	1	Normalization
$\bar{y}_s$	1.79	Acemoglu et al (2002?)

<sup>+</sup> Average over 1996-2006

Table 2.2: Results: Probit Estimation

	$\mathbf{P}(e_u = 1)$	$\mathbf{P}(e_s = 1)$
$unrt_{sk}$	-0.001 (0.27)	-19.664 (.93)
$unrt_{un}$	-9.34 (0.254)	-.027 (0.89)

Table 2.3: Results: Model Moments

	Interaction		No Interaction	
	mean	std dvn.	mean	std dvn.
$u_s$	0.012	(0.000)	0.020	(0.000)
$u_u$	0.05	(0.002)	0.048	(0.002)
$e_{su}$	0.01	(0.000)	-	-
$f_{ss}$	0.276	(0.004)	0.288	(0.006)
$f_{su}$	0.235	(0.009)	-	-
$f_{uu}$	0.325	(0.012)	0.336	(0.012)
$Y_u$	0.695	(0.024)	0.693	(0.024)
$Y_s$	0.474	(0.009)	0.474	(0.009)
$w_s$	1.65	(0.02)	1.653	(0.030)
$w_{su}$	0.983	(0.025)	-	-
$w_u$	0.955	(0.025)	0.956	(0.026)

## Chapter 3

# On the Unemployment Response to a Skill Biased Technological Change

### 3.1 Introduction

The higher unemployment rate of less skilled workers and the increasing skill premium are the stylized facts in labor markets over the last decades. Technological change that has favored skilled workers, called skill biased technological change (SBTC), is one possible explanation for these facts. This paper analyzes the importance of skill biased vs. neutral shocks in generating labor market facts we see in the data.

Nickell and Bell (AER, 1996) analyze unemployment rates and wages for men. They document a rise in both skilled and unskilled unemployment rates as well as a rise in relative wages of skilled workers from 1970s to 1980s in the US (and some European countries).<sup>1</sup> They argue that demand shift by itself cannot account for the rise in overall unemployment.

There is a large body of literature that documents a skill biased technological change (See the review of Katz and Autor (1999) and the references contained therein). There are also a number of studies that analyze the effect of skill biased shifts in labor demand on aggregate unemployment. For example, Nickell and Bell

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<sup>1</sup>Also see Katz and Murphy (1992), OECD (1994), and Nickell and Bell (1995)

(1995) suggest that a SBTC can account for 10 to 30 percent of the rise in overall unemployment from the 1970s to the 1980s in OECD countries where unemployment rose significantly. Blanchard and Katz (1997) and Mortensen and Pissarides (1999) emphasize the importance of labor market institutions in determining the effects of SBTC on unemployment. Albrecht and Vroman (2002) develop a heterogenous agents framework and show that SBTC can increase the unskilled unemployment rate. Krusell, Ohanian, and Rios Rull (2005) look at how SBTC can generate wage dispersion. They show that capital skill complementarity can account for most of the variation in skill premia. Stadler and Wapler (2004) endogenize skill biased technological change and show that it reduces skilled unemployment rate and increases wage disparity. Cuadras-Morato and Mateos-Planas (2004) show that a SBTC and increased “employment friction” can account for almost all the variation in unskilled unemployment, the skill premium, and the increase in skilled labor force. Cuadras-Morato and Mateos-Planas (2004) also show that SBTC accounts for almost half of the increase in the skilled unemployment rate.

This paper analyzes how unemployment rates of different skill groups respond to skill biased and neutral shocks, and how these shocks interact with each other. Although SBTC is known to be important, its interaction with skill neutral shocks is not well-studied. Exploring the effects of SBTC in the presence of skill neutral shocks will contribute to our understanding of overall labor markets and their responses to shocks.

I use a heterogenous labor matching model. There are fixed numbers of skilled and unskilled workers with an exogenous skill distribution. There are two

intermediate goods that are produced by workers of a particular skill type. Skilled and unskilled intermediate goods are produced in final good production. The economy is hit by two shocks. The first is a skill neutral shock that affects the overall productiveness of each intermediate good. The other is a skill biased shock that increases relative productivity of the skilled intermediate good.

The results suggest that the economy responds similar to the skill biased vs. skill neutral shock. The main difference is in the co-movement of skilled and unskilled unemployment rates and the volatility of their ratios. When the economy is hit only by skill neutral shocks (with the realization of the same skill biased shock every period), then skilled and unskilled unemployment rates move in the same direction. When the economy is hit by only skill biased shocks, then unemployment rates move in opposite directions. The correlation is stronger, resulting in less volatility in unemployment ratios. If there are both skill biased and skill neutral shocks in the economy, then unemployment rates move in the same direction, with less correlation than the case with only skill neutral shocks. In this case, the volatility of unemployment ratios increases.

The remainder of this paper is organized as follows. The next section describes the model. Computing the equilibrium of the model and calibration of parameter values are discussed in section 2. The following section documents results, and section 4 concludes.

### 3.2 Model

The economy is inhabited by workers and entrepreneurs. Workers can be employed or unemployed. If employed, they inelastically supply their labor and consume wages. If unemployed, they receive unemployment income and search for a job. There are two types of workers: skilled and unskilled. Skilled workers work in firms that produce high-tech intermediate goods (skilled firms). Unskilled workers only work for firms that produce low-tech intermediate good (unskilled firms). Each firm has only one job. There is also a final good sector. Firms in the final good sector buy intermediate goods and produce the final good. They do not hire labor. All these markets are competitive.

Entrepreneurs can stay idle (earn nothing) or start a firm. They can choose to produce an intermediate good or a final good. If they choose to produce an intermediate good, they choose the type of the good, they incur a vacancy cost, and search for workers. If they decide to have a final good producing firm, they buy the intermediate goods, produce the final good, and sell it. They do not hire extra labor.

The final good sector is subject to two shocks. The first is “neutral” shock, denoted by  $A$ , which affects productiveness of both skilled and unskilled inputs. The other shock,  $\varphi$ , directly affects the productivity of the skilled intermediate good. These shocks follow a Markov process with  $A \in \{A_1, A_2, \dots, A_n\}$  and  $\varphi \in \{\varphi_1, \varphi_2, \dots, \varphi_m\}$ . Let  $\Pi_A$  and  $\Pi_\varphi$  be the transition matrices where  $\Pi_A(i, j)$  ( $\Pi_\varphi(i, j)$ ) is the probability that the subsequent period’s shock will be  $A_j$  ( $\varphi_j$ ) if the current period’s shocks is  $A_i$  ( $\varphi_i$ ).



Let  $z = \{A, \varphi\}$  be the aggregate state of the economy. Let  $Y_s(z)$  and  $Y_u(z)$  be the amounts of skilled and unskilled intermediate goods used in production of the final good,  $Y(z)$ . Then, the final good production function is

$$Y(z) = A[\gamma(\varphi Y_s(z))^\rho + (1 - \gamma)Y_u(z)^\rho]^{\frac{1}{\rho}}$$

where  $\gamma$  is the share parameter. Let the final good price be the numeraire, and  $P_s(z)$  and  $P_u(z)$  denote the prices of skilled and unskilled intermediate goods, respectively. Observe that skill biased shock will also affect the unskilled sector through the price  $P_u(z)$ .

### 3.2.1 Intermediate Goods Sectors

#### 3.2.1.1 Workers' Bellman Equations

Skilled and unskilled workers have similar optimization problems to solve. A worker, skilled or unskilled, can be in one of two states: employed or unemployed. If unemployed, the worker receives income  $b_i$ , where  $i \in \{s, u\}$ . Moreover, she searches for a job. Let  $U_i(z)$  denote the value of unemployment for that worker when the aggregate state of the economy is  $z$ .

$$U_i(z) = b_i + \beta\{f_i(z)E_{z'|z}W_i(z') + (1 - f_i(z))E_{z'|z}U_i(z')\} \quad (3.1)$$

where  $\beta$  is the discount rate,  $W_i(z)$  is the value of being in a match with a firm, and  $f_i(z)$  is the probability that the worker will find a job. Although this probability is exogenous to the worker, it is endogenous to the model.

When the worker is in a match with a firm, she decides whether to attach to the firm or detach and continue searching. Employed workers receive a wage and

with an exogenous positive probability  $1 - s_i$  worker's employment survives into the subsequent period. The equation below formalizes the value of being in a match with a firm for a worker.

$$W_i(z) = \max\{U_i(z), w_i(z) + \beta\{(1 - s_i)E_{z'|z}W_i(z') + \beta s_i E_{z'|z}U_i(z')\}\} \quad (3.2)$$

### 3.2.1.2 Firms' Bellman Equations

This section describes the problem faced by intermediate good producers. There are skilled and unskilled firms which can produce  $y_s$  and  $y_u$  amounts of skilled and unskilled goods, respectively. I assume that skilled jobs are more productive, i.e.  $y_s > y_u$ . Observe that the amount of output a firm can produce is independent of the state of the economy. However, firms sell their products to final good producers at some endogenously determined market price, which is affected by shocks.

A job can be either vacant or filled. A vacant job incurs a cost,  $c_j$ , where  $j \in \{s, u\}$ . It will be filled with some positive probability,  $q_j(z)$ . A filled job produces  $y_j$ , and sells the output at market price  $P_j(z)$ . Let  $V_j(z)$  denote the value of a vacant job. It can be formally defined as:

$$V_j(z) = -c_j + \beta\{q_j(z)E_{z'|z}J_j(z') + (1 - q_j(z))E_{z'|z}V_j(z')\} \quad (3.3)$$

where  $J_j(z)$  is the value of being in a match with a worker.

The firm in a match with a worker decides between continuing to search and hiring the worker. If the job is filled, it produces output  $y_j$ , sells it at price  $P_j(z)$ , and pays the worker wage  $w_j(z)$ . The job will be destroyed in the subsequent period

with exogenous probability  $s_j$ .

$$J_j(z) = \max\{V_j(z), P_j(z)y_j(z) - w_j(z) + \beta\{(1 - s_j)E_{z'|z}J_j(z') + \beta s_j E_{z'|z}V_j(z')\}\} \quad (3.4)$$

### 3.2.1.3 Matching and Wage Determination

There are skill specific matching markets. The matching process in each skill market is presented via a matching function. Arguments of a matching function are vacancies,  $v_j(z)$ , and unemployment levels,  $u_j$  of skill type  $j$ . Note that the number of vacancies opened in a given period is a function of the current aggregate state. However, the current unemployment rate is already determined, as it is a state vector.<sup>2</sup> The matching function is homogenous of degree one, concave, and increasing in both arguments. If the number of vacancies seeking workers is  $v_j(z)$  and  $M(u_j, v_j(z))$  of them actually are filled, then the probability that a vacancy will be filled is

$$q_j(z) = M(u_j, v_j(z))/v_j(z) \quad (3.5)$$

The probability that a worker searching for a job will find one is

$$f_j(z) = M(u_j, v_j(z))/u_j \quad (3.6)$$

Because the matching function is homogeneous of degree one, these probabilities are functions of vacancy unemployment ratios. Let  $\theta_j(z) = v_j(z)/u_j$ , which is known as the market tightness. As the market tightness increases, it becomes easier for an unemployed worker to match with a vacancy. Consequently, an increase

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<sup>2</sup>I do not include that in the vector  $z$  for notational convenience.

in the number of unskilled (skilled) unemployed will decrease the market tightness, and will make it harder for unskilled (skilled) workers to find a job.

Describing the wage determination process will conclude this section. I assume that workers (firms) get a constant  $\mu_j$  ( $1 - \mu_j$ ) fraction of match surplus, net of their outside options. A worker's outside option is staying unemployed, whereas a firm's outside option is staying vacant. Let  $S_j$  denote the match surplus if a firm with skill type  $j$  hires a worker with the same skill type.

$$\begin{aligned} S_u(z) &= W_u(z) - U_u(z) + J_u(z) - V_u(z) \\ S_s(z) &= W_s(z) - U_s(z) + J_s(z) - V_s(z) \end{aligned}$$

The wage determination rule implies

$$W_i(z) - U_i(z) = \mu S_i(z) \quad J_i(z) - V_i(z) = (1 - \mu) S_i(z)$$

Substituting values of  $W_i(z)$ ,  $J_i(z)$ ,  $U_i(z)$ , and  $V_i(z)$  from equations above into  $W_i(z) - U_i(z) = \mu S_i(z)$  (or into  $J_i(z) - V_i(z) = (1 - \mu) S_i(z)$ ) gives the following wage equation:

$$w_i(z) = \mu P_j(z) y_j(z) + (1 - \mu) U_i(z) - (1 - \mu) \beta E_{z'|z} U_i(z') + \mu V_j(z) - \mu \beta E_{z'|z} V_j(z') \quad (3.7)$$

This wage determination rule makes workers and firms unanimously determine whether parties should form an employment relationship or continue searching.

### 3.2.2 Equilibrium

The stationary equilibrium of the model is the set of functions  $\{J_s(z), J_u(z), V_s(z), V_u(z), W_s(z), W_u(z), U_s(z), U_u(z), w_s(z), w_u(z), u_s, u_u, v_s(z), v_u(z)\}$ , and initial condition of unemployment, such that

- the value functions satisfy the Bellman equations described above.
- the value of a vacant job is zero (free entry condition);  $V_s(z) = 0$  and  $V_u(z) = 0$ .
- wage rates satisfy equation (3.7)
- Prices of final goods are governed by the following functions:

$$P_s(z) = A[\gamma(\varphi Y_s(z))^\rho + (1 - \gamma)Y_u(z)^\rho]^{\frac{1}{\rho}-1} \gamma \varphi^\rho Y_s(z)^{\rho-1}$$

$$P_u(z) = A[\gamma(\varphi Y_s(z))^\rho + (1 - \gamma)Y_u(z)^\rho]^{\frac{1}{\rho}-1} (1 - \gamma)Y_u(z)^{\rho-1}$$

- Intermediate good markets should clear, i.e.

$$Y_s(z) = p(1 - u_s)y_s \quad Y_u(z) = (1 - p)(1 - u_u)y_u$$

- Flows across employment states are

$$u'_u(z) = (1 - f_u(z))u_u + \delta_u(1 - u_u) \quad (3.8)$$

$$u'_s(z) = (1 - f_s(z))u_s + \delta_s(1 - u_s) \quad (3.9)$$

In this economy, prices of both intermediate goods go up with a positive shock, whether it is skill biased or neutral. Since the employment levels are determined at the beginning of the period, realized shocks in the current period do not

have any effect on the current period employment, resulting in no effect on the current period intermediate outputs. However, these shocks affect the current period prices, thus the current wages. Current period shocks affect future employment, since they carry information about the future shocks. Firms decide whether to open vacancies or not by gathering the information about the future from the current shocks. If firms observe positive shocks in the current state, they know that a positive shock realization in the next period is more (or less) likely, thus they would have incentives to open (or not) vacancies in the current period, reducing (or increasing) the unemployment in the subsequent period.

### 3.3 Computation and Calibration

The equilibrium of this economy is characterized by decision rules for vacancy creation and wage determination rules, which are functions of the current period unemployment rates and aggregate state of the economy.

To describe an equilibrium, it is helpful to work with match surplus  $S_{ij}(z)$ , which is defined above.

Using the Bellman equations and the definition of the match surplus, we get

$$\begin{aligned} S_{uu}(z) &= \max \left\{ 0, \quad P_u(z)y_u + (1 - \delta_s)\beta E_{z'|z} S_{uu}(z') + \beta E_{z'|z} U_u(z') - U_u(z) \right\} \\ S_{ss}(z) &= \max \left\{ 0, \quad P_s(z)y_s + (1 - \delta_s)\beta E_{z'|z} S_{ss}(z') + \beta E_{z'|z} U_s(z') - U_s(z) \right\} \end{aligned} \quad (3.10)$$

Similarly, unemployment value functions are

$$\begin{aligned} U_u(z) &= b_u + f_u(z)\beta\mu_u E_{z'|z} S_{uu}(z') + \beta E_{z'|z} U_u(z') \\ U_s(z) &= b_s + f_s(z)\beta\mu_s E_{z'|z} S_{ss}(z') + \beta U_s(z') \end{aligned} \quad (3.11)$$

Vacancy value equations become

$$\begin{aligned} c_u &= q_u(z)\beta(1 - \mu_u)E_{z'|z}S_{uu}(z') \\ c_s &= q_s(z)\beta(1 - \mu_s)E_{z'|z}S_{ss}(z') \end{aligned} \tag{3.12}$$

### 3.3.1 Computing equilibrium

The equilibrium of the model can be characterized by the laws of motion for unemployment rates and market tightness for the labor markets that clear the market. Observe that market tightness for a particular labor market (or the number of vacancies created) is a function of the unemployment rates in both markets (since these determine the amount of each intermediate good supplied, thus the relative prices) and the shocks to the economy.

I use the following algorithm to solve for equilibrium:

1. Recall that both shocks follow Markov processes. Let  $Z = \{1, 2, \dots, n^2\}$  be the index for the aggregate state of the economy such that

$$Z = \left\{ \begin{pmatrix} A(1) \\ \psi(1) \end{pmatrix}, \begin{pmatrix} A(1) \\ \psi(2) \end{pmatrix}, \dots, \begin{pmatrix} A(1) \\ \psi(n) \end{pmatrix}, \begin{pmatrix} A(2) \\ \psi(1) \end{pmatrix}, \dots, \begin{pmatrix} A(n) \\ \psi(n) \end{pmatrix} \right\}$$

Let size of  $Z$  be  $N_z$ . State of the economy is governed by by joint probabilities of the Markov processes  $A$  and  $\psi$ .

2. Generate a grid for skilled and unskilled unemployment rates,  $U_s$ ,  $U_u$ , respectively.. Let  $N_s = N_u$  be the size of this grid.
3. Generate upper and lower bounds for  $\theta_u$  and  $\theta_s$ . Boundaries are  $N_z \times N \times N$  dimensional triples.

4. Guess a  $\theta_u$  triple.  $\theta_u(i, j, k)$  is the guess for market tightness when the economy has  $Z(i)$  shocks, skilled unemployment rate is  $Us(j)$ , and unskilled unemployment rate is  $Uu(k)$ .
5. Guess a  $\theta_s$  triple.
6. Compute job finding and vacancy filling probabilities using the guessed values in equations (3.5) and (3.6).
7. Compute surplus values and unemployment values using equations 3.10 and 3.11, via value function iteration.
8. Compute values of vacancies using equations (3.12).
9. Update guess of  $\theta_s$  using vacancy value for skilled firms  $V_s$ .
  - If  $V_s > 0$ ; then increase  $\theta_s$ .
  - If  $V_s < 0$ ; then decrease  $\theta_s$ .
10. Repeat steps starting from step 5 until convergence
11. Update guess of  $\theta_u$  using vacancy value for unskilled firms  $V_u$ .
  - If  $V_u > 0$ ; then increase  $\theta_u$ .
  - If  $V_u < 0$ ; then decrease  $\theta_u$ .
12. Repeat steps starting from step 4 until convergence.



### 3.3.2 A Numerical Exercise

In order to solve the system of equations, parameter values should be assigned. Parameters of the model are:  $\{p, c_s, c_u, y_s, y_u, \delta_s, \delta_u, \mu_s, \mu_u, \mu_{su}, b_s, b_u, \alpha, \beta\}$ , and parameters that govern the shocks.

The model period is one month, so  $\beta$  is set to 0.996 to match the annual interest rate of four percent. The skilled fraction of the labor force  $p$  is set to 0.27, its average value from 1996 to 2006. The average value of the output produced by an unskilled firm is normalized to 1. Unemployment income is assumed to be the same across skill groups and is set to 0.54. This number is the average replacement ratio for the US. The parameter of the matching function is  $\alpha = 1.1$ . The matching function has the form

$$M(u, v) = \frac{uv}{(u^\alpha + v^\alpha)^{1/\alpha}}$$

All workers are assumed to have the same bargaining power, which is 0.36. This number is generally assumed in the literature (Shimer, 2005).

Acemoglu (2003) argues that the elasticity of substitution between skilled and unskilled labor should be more than 1 (citing Freeman, 1986). Acemoglu sets this parameter to 1.4. Acemoglu and Zilibotti (2001) assume the elasticity to be 2. Krusell, Ohanian, Rios-Rull, and Violante (2000) calibrate the elasticity to 1.67. I set  $\rho = 1/3$  so that the elasticity of substitution between skilled and unskilled labor is 1.5.<sup>3</sup>

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<sup>3</sup>Recall that the elasticity of substitution is  $\frac{1}{1-\rho}$ .

The remaining parameters are the costs of posting vacancies ( $c_u$ ,  $c_s$ ). They are set so that the average job finding probabilities of skill groups are close to the data.

Table 3.1: Parameters

<b>Parameter</b>	<b>Value</b>
$\beta$ Discount Rate	0.996
p High-Skill fraction LF	0.27
$s_s$ Job separation rate, skilled	0.006
$s_u$ Job separation rate, unskilled	0.017
$\alpha$ Matching parameter	1.1
$\mu_s$ Bargaining parameter, skilled	0.36
$\mu_u$ Bargaining parameter, unskilled	0.36
$b_s$ Unemployment benefit, skilled	0.54
$b_u$ Unemployment benefit, unskilled	0.54
$y_u$ Output; unskilled jobs	1
$y_s$ Output; skilled jobs	1.79
$c_s$ Vacancy cost, skilled	3.94
$c_u$ Vacancy cost, unskilled	1.8
$\gamma$ parameter in agg. production fct	0.4

I assume the processes that govern both skill biased and skill neutral shocks are the same. This way the only difference between responses to shocks will come from them being skill biased vs. neutral. I derive values and transition probabilities of shocks from an underlying AR1 process with standard deviation 0.3 and the auto-regression coefficient of 0.8. The mean values of these processes are 1.5.<sup>4</sup> With these values, the dispersion between different values of a shock is the same in terms of percentages. Values and transition probabilities of shocks are:

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<sup>4</sup>I use Tauchen approximation to approximate implied Markov processes, and I assume the boundary value is 1.

$$A = \begin{bmatrix} 1 \\ 1.5 \\ 2 \end{bmatrix}, \quad \Pi_A = \begin{bmatrix} 0.6915 & 0.2934 & 0.0151 \\ 0.2023 & 0.5953 & 0.2023 \\ 0.0151 & 0.2934 & 0.6915 \end{bmatrix}$$

$$\varphi = \begin{bmatrix} 1 \\ 1.5 \\ 2 \end{bmatrix}, \quad \Pi_\varphi = \begin{bmatrix} 0.6915 & 0.2934 & 0.0151 \\ 0.2023 & 0.5953 & 0.2023 \\ 0.0151 & 0.2934 & 0.6915 \end{bmatrix}$$

### 3.3.3 Results

For given values of parameters, I simulate the time series data for the economy for the following cases: First, to see the effects of skill neutral shocks on labor market outcomes, I do the following exercise. I simulate the economy 5000 times. In each simulation, the first 60 periods the economy receives the following shocks:  $A = 1.5$ ,  $\varphi = 1.5$ . At 61st period, the shock values are:  $A = 2$ ,  $\varphi = 1.5$ . The economy receives high skill neutral shock. Then, for the following 250 periods, value of skill biased shocks are 1.5 for each period, while skill neutral shocks are random realizations that evolve according to the transition matrix for the skill neutral shock. Then, I take the averages for each period across simulations to neutralize the effects of particular shock realizations.

Second, I analyze the effects of skill biased shocks on labor market outcomes. I simulate the economy 5000 times in this exercise as well. In each simulation, for the first 60 periods the economy receives the following shocks:  $A = 1.5$ ,  $\varphi = 1.5$ . At the 61st period, the shock values are:  $A = 1.5$ ,  $\varphi = 2$ . The economy receives high skill biased shock. Then, for the following 250 periods, value of skill neutral shocks are 1.5 for each period, while skill biased shocks are random realizations that evolve

according to the transition matrix for these shocks.

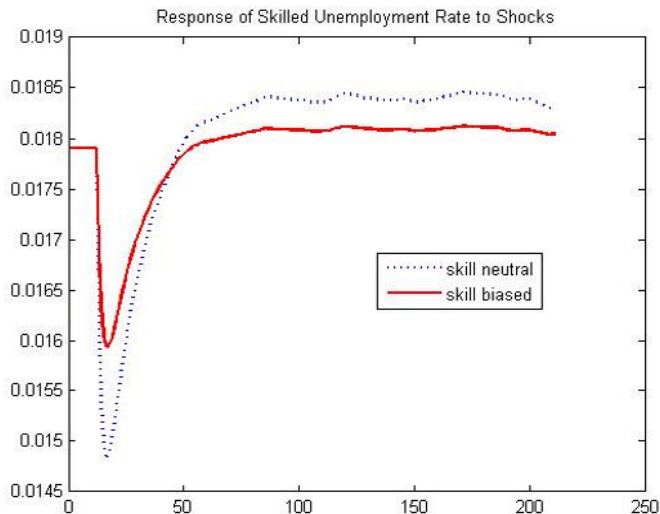


Figure 3.1: Response of Skilled Unemployment to Shocks

Figure 3.1 shows how the skilled unemployment rate responds to skill biased vs. skill neutral shocks. Since shocks are persistent, after observing a high shock in the current period, more vacancies are created, resulting in a lower unemployment in the subsequent period. Observe that, vacancy creation is higher for a skill neutral shock.

Figure 3.2 shows response of unskilled unemployment rate to skill biased vs. skill neutral shocks. Like skilled firms, unskilled firms create more vacancies when they observe high levels of shocks. As a result, unemployment in the subsequent period is lower. A skill biased shock reduces the unemployment rate of unskilled workers as well since there is complementarity between unskilled and skilled intermediate goods. Unskilled unemployment is affected more from a skill neutral shock

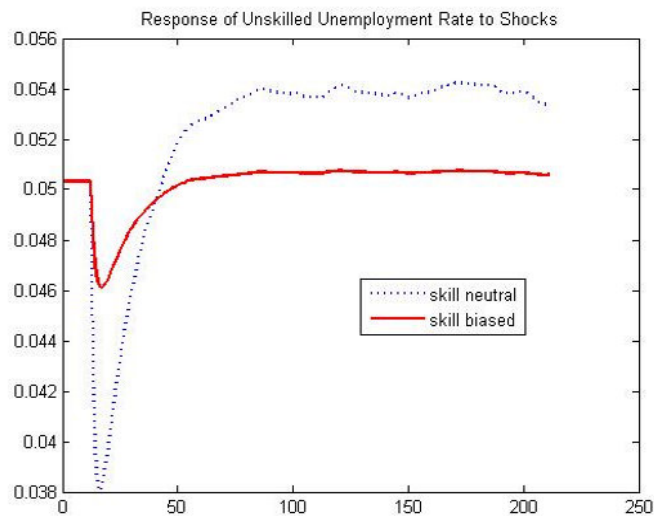


Figure 3.2: Response of Unskilled Unemployment to Shocks

compared to the effect of a skill biased shock.

As expected, a skill biased shock, increases the gap between the skilled and unskilled unemployment rates. Figure 3.3 displays that effect. This happens not because unskilled unemployment increases, but because the skilled unemployment rate decreases more. On the other hand, the unskilled unemployment rate declines relatively more than skilled unemployment rate as a result of a favorable skill neutral shock.

One parameter that is expected to affects responses of unemployment rates to shocks is the elasticity of substitution. To see how the economy behaves at different elasticity levels, I conduct the following exercise. I find the equilibrium for different  $\rho$  values (all other parameters are the same). Then I simulate these economies with

Table 3.2: Response of the Economy to Different Elasticities

	$\rho = 2/3$ ( $\epsilon = 3^{\dagger}$ )	$\rho = 1/3$ ( $\epsilon = 1.5$ )	$\rho = -1/3$ ( $\epsilon = 0.75$ )	$\rho = -1$ ( $\epsilon = 0.5$ )
$u_u$	0.053 (0.01)	0.053 (0.01)	0.053 (0.010)	0.054 (0.012)
$u_s$	0.018 (0.002)	0.018 (0.002)	0.018 (0.002)	0.019 (0.002)
$f_u$	0.313 (0.079)	0.314 (0.079)	0.314 (0.086)	0.311 (0.097)
$f_s$	0.329 (0.054)	0.326 (0.051)	0.321 (0.046)	0.316 (0.046)
$u_u/u_s$	2.913 (0.29)	2.874 (0.235)	2.833 (0.273)	2.839 (0.465)
$Y_u$	0.691 (0.007)	0.692 (0.007)	0.691 (0.008)	0.691 (0.009)
$Y_s$	0.475 (0.001)	0.474 (0.001)	0.474 (0.001)	0.474 (0.001)
$Y$	1.064 (0.299)	1.061 (0.298)	1.055 (0.297)	1.049 (0.296)
$P_u$	0.912 (0.234)	0.914 (0.242)	0.918 (0.266)	0.921 (0.301)
$P_s$	0.911 (0.302)	0.901 (0.273)	0.884 (0.234)	0.868 (0.228)
$wage_u$	0.873 (0.151)	0.875 (0.155)	0.878 (0.17)	0.878 (0.190)
$wage_s$	1.568 (0.346)	1.553 (0.315)	1.525 (0.274)	1.502 (0.268)

Implied elasticity of substitution  $\epsilon = \frac{1}{1-\rho}$   
 Values are mean and standard of time series

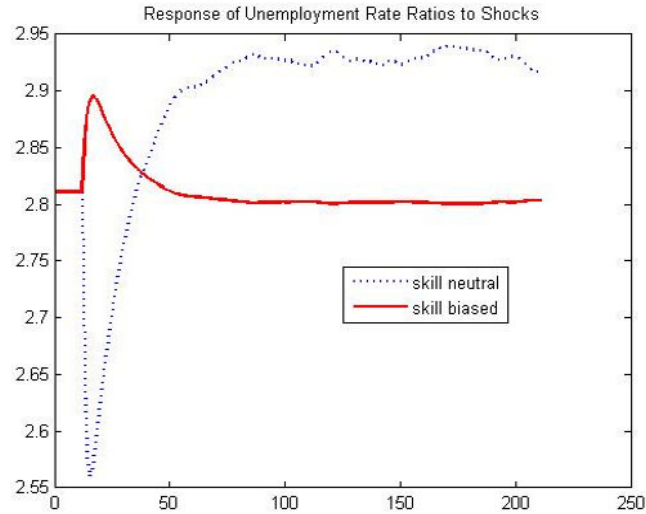


Figure 3.3: Response of Unemployment Ratios to Shocks

the same shock realizations for 1000 periods. As Table 3.2 shows, there is not much difference between these economies in mean values of key variables. There are some differences across economies in second moments.

As  $\rho$  goes down, i.e. elasticity of substitution goes down, we expect less co-movement between prices of intermediate goods, thus lower correlation between unemployment rates. However, for different  $\rho$  values, response of prices to different shock states also changes, both quantitatively and qualitatively. That generates different responses of vacancies to the same state for different  $\rho$  values.

### 3.4 Concluding Remarks

This chapter aims to analyze effects of skill biased and skill neutral shocks on labor market outcomes of different skill groups. Skill based shocks are discussed

as one of the sources behind higher unemployment and lower wages of unskilled workers.

I use an otherwise standard search model with a final good sector that uses skilled and unskilled intermediate goods in production. This sector is hit by two shocks: a skill biased shock that affects productivity of the skilled intermediate good and a skill neutral shock that affects overall productivity.

The results show that skilled unemployment is more responsive to skill biased shocks, compared to unskilled unemployment, than skill neutral shocks. The unskilled unemployment rate goes down as a result of a favorable skill biased shock, since goods these skill groups produce are complements. However, a high level skill biased shock increases the gap between unemployment rates of skill groups (although it reduces the levels), while a high level of skill neutral shock reduces that gap.



## Appendices

# Appendix A

## Chapter 1 Appendix

### A.1 Existence of the Equilibrium

It is easier to work with the match surplus to characterize the equilibrium in matching models. The match surplus is defined as the sum of the value of a match to the firm and to the worker, net of their outside options.

$$S(\pi, \lambda) = J(\pi, \lambda) - V(\lambda) + W(\pi, \lambda) - U(\lambda)$$

Substituting in values of  $J(\pi, \lambda)$  and  $W(\pi, \lambda)$  from the Bellman equations result in <sup>1</sup>

$$S(\pi, \lambda) = \max \left\{ E(y|\pi) + \beta(1 - \delta)\pi S(1, \lambda) - (1 - \beta)U(\lambda) - (1 - \beta)V(\lambda), 0 \right\}$$

Note that the match surplus is linear in  $\pi$  for  $\pi > \pi^*$ , thus we can write the surplus as  $S(\pi, \lambda) = (\pi - \pi^*)S'(\pi^*, \lambda)$  where  $S'(\pi^*, \lambda)$  is <sup>2</sup>

$$S'(\pi^*, \lambda) = \frac{y^g - y^b}{1 - \beta(1 - \delta)(1 - \pi^*)} \tag{A.1}$$

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<sup>1</sup>One can write wages as a function of match surplus as well

$$w(\pi, \lambda) = (1 - \beta)U(\lambda) + \mu S(\pi, \lambda) - \beta(1 - \delta)\mu\pi S(1, \lambda)$$

<sup>2</sup>I take the derivative of  $S(\pi, \lambda)$  with respect to  $\pi$ , substitute  $S(1, \lambda) = (1 - \pi^*)S'(\pi^*, \lambda)$  in, and rearrange the terms to get this expression.

The equilibrium can be characterized by three variables. These are reservation probability, market tightness, and the choice of the employee selection technology. For a given value of  $\lambda$ , the values of  $(\theta_\lambda, \pi_\lambda^*)$  are determined by the intersection of two curves: the optimal match formation curve (OMF) and the free entry curve (FE).

The optimal match formation curve delivers the reservation probability for any given market tightness level. Recall that the  $\pi_\lambda^*$  leaves workers and firms indifferent between forming the production unit or staying unattached, i.e., it solves the equation  $S(\pi_\lambda^*) = 0$ . Using this condition, and substituting  $S'(\pi_\lambda^*, \lambda)$  into  $S(\pi_\lambda^*, \lambda) = 0$  equation yields

$$(1 - \beta)U(\lambda) = y^b + \pi_\lambda^* S'(\pi_\lambda^*, \lambda)$$

We can rewrite the equation above<sup>3</sup>

$$\theta_\lambda \mu \frac{c + \lambda \kappa}{(1 - \mu)} = (y^b - b) + \pi_\lambda^* S'(\pi_\lambda^*, \lambda) \quad (\text{A.2})$$

Note that  $\pi_\lambda^* S'(\pi_\lambda^*, \lambda)$  is increasing in  $\pi_\lambda^*$ . Thus, we have an upward sloping line in the  $(\pi_\lambda^*, \theta_\lambda)$  space. The intuition for the upwards sloping optimal match

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<sup>3</sup>Recall that in equilibrium

$$(1 - \beta)U(\lambda) = b + \beta f(\theta) \mu \left\{ \lambda \int_0^1 S(\pi, \lambda) d\Omega + (1 - \lambda) \int_0^1 S(\pi, \lambda) d\Gamma \right\}$$

$$(1 - \beta)V(\lambda) = -c - \lambda \kappa + \beta q(\theta)(1 - \mu) \left\{ \lambda \int_0^1 S(\pi, \lambda) d\Omega + (1 - \lambda) \int_0^1 S(\pi, \lambda) d\Gamma \right\}$$

Substituting for surplus values from  $V(\lambda)$  equation into  $U(\lambda)$  equations yields

$$(1 - \beta)U(\lambda) = b + \theta \mu \frac{c + \lambda \kappa}{1 - \mu}$$

formation curve is as follows: At any  $(\theta, \pi_\lambda^*)$  pair on the curve, we know that the match surplus is zero. As market tightness increases, so does the worker's outside option (firm's outside option is always zero in equilibrium). Since the worker's outside option is higher at the new market tightness, the match surplus at the new market tightness with the old reservation probability will be negative. Thus, the firm and the worker will decide to separate. They will be indifferent between being unattached and forming a production unit at a higher  $\pi_\lambda^*$  level, resulting in a positive slope.

The second curve that determines the equilibrium is the free entry (or vacancy creation) condition. We get this condition in the following way: The equilibrium value of the market tightness should be such that the vacancies earn zero profit in equilibrium, i.e.,  $\theta_\lambda$  solves  $V(\lambda) = 0$ ; thus

$$c + \lambda\kappa = \beta q(\theta)(1 - \mu) \left( \lambda \int_{\pi_\lambda^*}^1 S(\pi, \lambda) d\Omega + (1 - \lambda) \int_{\pi_\lambda^*}^1 S(\pi, \lambda) d\Gamma \right) \quad (\text{A.3})$$

Note that, since  $S(\pi, \lambda)$  is linear in  $\pi$ , we can write the expected surplus as follows:

$$\int_0^1 S(\pi, \lambda) d\cdot = S'(\pi_\lambda^*, \lambda) \int_{\pi_\lambda^*}^1 (\pi - \pi^*) d\cdot$$

Thus, the free entry condition can be written as

$$c + \lambda\kappa = \beta q(\theta)(1 - \mu) S'(\pi_\lambda^*, \lambda) \left( \lambda \int_{\pi_\lambda^*}^1 [\pi - \pi_\lambda^*] d\Omega + (1 - \lambda) \int_{\pi_\lambda^*}^1 [\pi - \pi_\lambda^*] d\Gamma \right) \quad (\text{A.4})$$

Since both  $S'(\pi_\lambda^*, \lambda)$  and  $\int_{\pi_\lambda^*}^1 [\pi - \pi_\lambda^*] d\cdot$  are decreasing in  $\pi_\lambda^*$ , this equation has a negative slope in  $(\pi_\lambda^*, \theta_\lambda)$  space. The intuition for the downward sloping free entry curve is as follows: As the  $\pi_\lambda^*$  increases, it gets harder for firms to find workers, thus

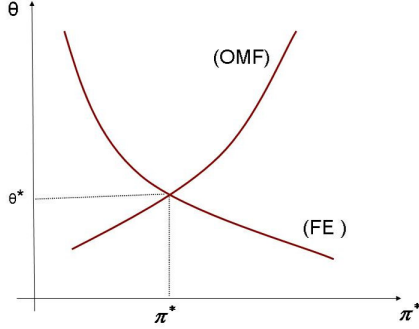


Figure A.1: Existence of Equilibrium

they need to incur vacancy costs longer (value of opening a vacancy decreases), thus the number of vacancies, and hence the market tightness, goes down.

These two curves are required to intersect for the existence of an equilibrium. To get the conditions for the existence of an equilibrium, I need a functional form for the firms' matching probability. I assume the matching function to be  $M = \frac{uv}{(u^\alpha + v^\alpha)^{\frac{1}{\alpha}}}$  to derive the condition below.

*Proposition 1.* If  $\frac{(c + \lambda\kappa)(1 - \beta(1 - \delta))}{\beta(y_g - y_b)\left(\lambda \int_0^1 \pi d\Omega + (1 - \lambda) \int_0^1 \pi d\Gamma\right)} < 1$ , then for any  $\lambda \in [0, 1]$  there is a pair  $(\pi_\lambda^*, \theta_\lambda)$  such that equations (6) and (8) are satisfied.

*Proof.* Since the OMF and the FE curves have monotonically increasing and decreasing slopes, respectively, it is sufficient to show that there exists  $\pi_1, \pi_2$  such that

$$\pi_1 < \pi_2, \quad \theta_{OMF}(\pi_1) < \theta_{FE}(\pi_1) \quad \text{and} \quad \theta_{OMF}(\pi_2) > \theta_{FE}(\pi_2)$$

Then, by intermediate value theorem, there exists  $\pi^* \in [\pi_1, \pi_2]$  such that  $\theta_{OMF}(\pi^*) = \theta_{FE}(\pi^*)$

As  $\pi_2$  be a number close to 1. As  $\pi$  goes to 1,  $\theta_{FE}(\pi)$  goes to zero. Moreover,  $\theta_{OMF}(1) = \frac{(1-\mu)}{\mu(c+\lambda\kappa)}(y^g - b) > 0$ . Thus for a value of  $\pi_2$  close to 1;  $\theta_{OMF}(\pi_2) > \theta_{FE}(\pi_2)$ .

Let  $\pi_1 = 0$ .

$$\theta_{OMF}(\pi_1) = \frac{(1-\mu)}{\mu(c+\lambda\kappa)}(y^b - b) \leq 0$$

Substituting  $q(\theta) = \frac{1}{(1+\theta^\alpha)(1/\alpha)}$  and rearranging terms yields

$$\theta_{FE}(\pi_1) = \left[ \frac{\beta(1-\mu)(y^g - y^b) \left( \lambda \int_0^1 \pi d\Omega + (1-\lambda) \int_0^1 \pi d\Gamma \right)^\alpha}{(c+\lambda\kappa)(1-\beta(1-\delta))} - 1 \right]^{1/\alpha}$$

If,  $\frac{\beta(1-\mu)(y^g - y^b) \left( \lambda \int_0^1 \pi d\Omega + (1-\lambda) \int_0^1 \pi d\Gamma \right)}{(c+\lambda\kappa)(1-\beta(1-\delta))} > 1$ , then  $\theta_{FE}(\pi_1) > 0$ . Consequently,  $\theta_{OMF}(\pi_1) < \theta_{FE}(\pi_1)$ .  $\square$

The inequality will not hold if the means of distributions are sufficiently close to zero, the output difference by the quality of a match is implausibly small, or the total cost of opening a vacancy is implausibly high<sup>4</sup>. The condition does not bind for any plausible parameter values.

For a given  $\lambda$ ,  $(\theta_\lambda, \pi_\lambda^*)$  pair is an equilibrium if firms do not deviate from

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<sup>4</sup>Examples of such implausible values would include the total vacancy cost to be 200 times higher than the low quality output, high quality output to be less than one per cent higher than the low quality output, or the mean of the distributions to be 0.01.

their selection technology choice, i.e, if the following condition is satisfied.

$$\frac{\int_{\pi_{\lambda}^*}^1 (\pi - \pi_{\lambda}^*) d\Omega}{\int_{\pi_{\lambda}^*}^1 (\pi - \pi_{\lambda}^*) d\Gamma} \begin{cases} \geq \frac{c+\kappa}{c} & \text{if } \lambda = 1 \\ = \frac{c+\kappa}{c} & \text{if } \lambda \in (0, 1) \\ \leq \frac{c+\kappa}{c} & \text{if } \lambda = 0 \end{cases} \quad (\text{A.5})$$

Observe that it is possible to have multiple equilibria. Recall that other firms' actions affect the firm via affecting the outside option of workers, thus affecting the wages the firm pays. Whether there are multiple equilibria depends on how market tightness reacts to changes in  $\lambda$  (which depends on the parameter values).

## Appendix B

### Chapter 2 Appendix

#### B.1 Probit Estimation

CMAAs used in probit estimation

1. New York–Northern New Jersey–Long Island, NY–NJ–CT–PA
2. Los Angeles–Riverside–Orange County, CA
3. Chicago–Gary–Kenosha, IL–IN–WI
4. Washington–Baltimore, DC–MD–VA–WV
5. San Francisco–Oakland–San Jose, CA
6. Philadelphia–Wilmington–Atlantic City, PA–NJ–DE–MD
7. Boston–Worcester–Lawrence, MA–NH–ME–CT
8. Detroit–Ann Arbor–Flint, MI
9. Dallas–Fort Worth, TX
10. Houston–Galveston–Brazoria, TX
11. Miami–Fort Lauderdale, FL
12. Seattle–Tacoma–Bremerton, WA



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## Vita

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This dissertation was typeset with L<sup>A</sup>T<sub>E</sub>X<sup>†</sup> by the author.

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<sup>†</sup>L<sup>A</sup>T<sub>E</sub>X is a document preparation system developed by Leslie Lamport as a special version of Donald Knuth's T<sub>E</sub>X Program.