Warranty cost analysis for non-repairable services products

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Many services products are installed in a complex system that is operated only when the entire system is completed. The time from their installation to commissioning, called a dormant state in this paper, may take several years for systems such as complete buildings or aircraft. Warranties for the products may cover the time starting from their installation to a certain time. Warranty cost on replacements for such products is different from normal products without any dormant state. This paper analyses the replacement cost for non-repairable services products from a manufacturer perspective. We consider four warranty policies which include two types of warranty terms (i.e., non-renewing or renewing) and two types of replacements (i.e., with preventive replacement or replacement only upon failures). Relationships between the failure patterns at the dormant state and at the operating state are also discussed. Numerical examples and sensitivity analysis are presented to demonstrate the applicability of the methodology described in the paper.

Keywords: Warranty cost, Preventive replacement, Corrective maintenance, Non-renewing warranty policy, Renewing warranty policy, Dormant state.

1 Introduction

Warranty is a duty attached to a product and requires manufacturers to offer pre-specified compensation to buyers when the product fails to perform its designed functions under normal usage within the warranty period. Nowadays, product warranty becomes increasingly important in consumer and commercial transactions, and is widely used.

Selecting a suitable warranty policy is an optimization process in which both costs and profits should be considered from a manufacturer's perspective. On the one hand, providing

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warranty leads to additional costs on maintaining the products within the warranty period. On the other hand, warranties also serve as indicators to inform customers of the product quality and reliability that could give the company a competitive advantage.

Many systems including building services systems are installed in a building to support the functions required. Most building products are protected under a kind of warranty or insurance policy starting from their installation time. These products, for example, heating, ventilation, air conditioning (HVAC) systems and so forth, may be installed while a building is constructed. They are usually used until the building is completed and commissioned. The time from the installation to the commissioning, called dormant state in what follows, may take several years. It is not a short time period. In addition to construction, buildings can also be left in a dormant state if the owner can not find someone to occupy the building. For example, a construction company has such a building where the services have been installed and commissioned, but during the handover period, the client refused to accept the curtain walling and this has had to be replaced. Throughout this period the systems have not been in use.

Unlike other products that are usually put into use after they are sold, the services products have the following characteristics. At the dormant state,

- the products can age and deteriorate, and they can therefore fail to function when they are put into use at the commissioning time, and

- no inspection, or maintenance is conducted.

Hence, the dormant state should be taken into consideration when replacement costs within the warranty period are analyzed to avoid unrealistic results.

Warranty analysis for products with varying usage intensity is studied by researchers. When products are used intermittently, Murthy (1992) estimates the expected warranty cost/item sold for the case where the product usage is intermittent over the warranty period and the failure of the product is dependent on product usage. However, in the Murthy's model, the failure rate of the product at the dormant state is constant. This can not be applied in the building services product case because the failure rate of the products can be decreasing within its dormant period. Kim et al. (2001) studies the expected warranty cost for products sold with free replacement warranty with varying usage intensity. He does not consider the situation that products may age and deteriorate when the products are not used. Wu and Clements-Croome (2007) studies burn-in policies for systems with dormant states, and Wu and Li (2007) consider warranty policies for repairable systems. So far,

little research has been found on warranty cost analysis for non-repairable products with a dormant state.

There are mainly two basic warranty policies, a renewing and a non-renewing warranty policy (Blischke, Wallace, and Murthy, 1994). Under a renewing warranty, a product which fails within its warranty period is replaced by a new one, and the warranty is renewed at no charge to the buyer or at a partial cost to the buyer. Under a non-renewing warranty, the manufacturer offers a satisfactory service only within the original warranty period, and a failed product is replaced or repaired by the manufacturer at no cost to the buyer or at a pre-specified cost to the buyer within the original warranty period, and the original warranty is not renewable. A warranty policy may also be a mixture of the above two basic ones. The buyer incurs the full replacement cost on failures of the product after the original warranty period has expired.

There is considerable research on warranty analysis recently. For example, warranty analysis for repairable systems (Jhang, 2005 and 2005), and nonrepairable systems (Yeh et al., 2005); maintenance policy optimisation under warranty contracts (Pascual and Ortega, 2006, and Chien, 2005); and warranty analysis for products with a typical lifetime distribution. For more comprehensive information and understanding of different types of warranty policies, the reader is referred to the work of Blischke et al. (1994), Murthy et al. (2004), and Thomas and Rao (1999) for the taxonomy of warranty and the relevant research.

The objective of this paper is to help manufacturers on choosing the most cost-effective way when developing their warranty polices for products with a dormant state. It analyses replacement cost for four warranty policies.

This paper is organised as follows. Section 2 presents the assumptions about the product on its usage, failure rates at different states, replacement strategy, and warranty policies. Section 3 introduces four warranty polices. These policies can be non-renewed or renewed when preventive replacement is implemented or only corrective replacement is carried out. Replacement cost is obtained for the four warranty policies. In Section 4, three scenarios considering the application of the warranty policies are presented. Section 5 gives numerical examples when the lifetime distribution of the product is a 2-parameter Weibull distribution. It obtains replacement cost of the four warranty policies. Conclusions are presented in Section 6.

2 Warranty Models and Policies

2.1 *Model description*

We consider services products that are non-repairable. If such a product fails within the warranty period, it is replaced by the manufacturer. The following assumptions are made.

1. The product is dormant from its installation time 0 to commissioning time t_0, and operated within time interval $[t_0, t_0 + w]$. No replacement is implemented within time interval $(0, t_0)$. If the product fails to operate at time t_0, it is replaced with an new identical one immediately. Replacement time is assumed negligible.

2. The product is protected under either a renewing or a non-renewing free replacement warranty policy within $[t_0, t_0 + w]$. That is, in both the renewing case and the non-renewing case, the buyer is charged no cost on replacement within warranty period w.

3. The product has a lower failure rate within $(0, t_0)$, and a higher failure rate within $[t_0, t_0 + w]$. The failure rate is $r_1(t)$ if the product is at the dormant state, and $r_2(t)$ if the product is operated. $r_1(t)$ and $r_2(t)$ are non-decreasing functions. If a new product is operated from time 0, its failure rate is $r_0(t)$. Denote $F_k(t) = 1 - \bar{F}_k(t) = 1 - \exp\{-\int_0^t r_k(x)dx\}$, where $k = 0, 1$, and $F_2(t) = 1 - \bar{F}_2(t) = 1 - \exp\{-\int_0^t r_2(x)dx\}$. A possible scenario is shown in Figure 1.

4. Unit cost of replacement on failures is c_f, and unit cost of preventive replacement is c_r.

2.2 *Four warranty policies*

In what follows, preventive replacements mean that replacements are implemented either upon failures or on reaching a specified age τ_0, whichever occurs first. Consider the following four warranty policies.

Policy 1. The product is sold under non-renewing warranty, and replacements are executed upon failures within the warranty period.

Policy 2. The product is sold under non-renewing warranty, and preventive replacements are executed within the warranty period.

Policy 3. The product is sold under renewing warranty, and no preventive replacement is executed within the warranty period.

Policy 4.   The product is sold under renewing warranty, and preventive replacements are executed within the warranty period. If the product is preventively replaced by a new one on reaching age τ_0, the warranty is not renewed. If the product is replaced upon a failure, the warranty is renewed.

Policy 2 and Policy 4 may be considered when the failure rate of the product or business losses due to product failure is high, They are not often used because preventive replacements within warranty period are not often implemented. However, these two policies can potentially useful for the manufacturers as the warranty cost may be cheaper than those from other two policies. Replacement cost analysis for all of the above four polices are therefore considered in the paper.

There are three possible cases about the occurrence of the first failure of the product.

Case A.   The product is found failed to operate at time t_0, and a new identical product replaces it. The failure rate of the new product will be $r_0(t)$, where t starts from time 0 (see Figure 2).

Case B.   The product can operate at time t_0. The first failure of the product occurs within (t_0, w), and a new identical product replaces it. The failure rate of the products will be $r_2(t)$ between time t_0 to the occurrence of the first failure. The new product has failure rate $r_0(t)$ (see Figure 3).

Case C.   The first failure of the product occurs after the warranty term is expired.

Obviously, Case C does not incur any replacement cost to the manufacturer, so we will not discuss it below.

3   Warranty cost for different policies

In this section, the warranty cost functions are derived for the four warranty models. An example is shown later to illustrate the use of them.

3.1   *Main results*

For the replacement cost, we can obtain the following four theorems.

Theorem 1 *(Policy 1) A product is sold under Policy 1. The expected replacement cost for the product within warranty period w is*

$$C_1(w) = (M_{1A}(w) + M_{1B}(w))c_f \tag{1}$$

where $M_{1A}(w) = F_1(t_0)(1+M_1(w))$, $M_{1B}(w) = \bar{F}_1(t_0)\int_0^w(1+M_1(w-y))dF_2(y)$, *and* $M_1(t) = \sum_{k=1}^\infty F_0^{(k)}(t)$.

Proof.   From renewal theory (Ross, 1986), the number of replacements, $N(t)$, within time interval $(0,t)$ is given by a renewal process with the time between adjacent renewals distributed according to $F_0(t)$. The probability of k renewals in $[0,t)$ is given by

$$\Pr\{N(t) = k\} = F_0^{(k)}(t) - F_0^{(k+1)}(t) \tag{2}$$

where $F_0^{(k)}(t)$ is the k-fold convolution of $F_0(t)$ with itself. The expected number of renewals $M_1(t)$, equals the expected number of replacements, is given by

$$M_1(t) = \sum_{k=1}^\infty F_0^{(k)}(t) \tag{3}$$

$M_1(t)$ satisfies the renewal integral equation

$$M_1(t) = F_0(t) + \int_0^t M_1(t-x)dF_0(x) \tag{4}$$

The probability of the event that the product is found failed at time t_0 is $F_1(t_0)$. Hence, for Case A, the expected number of replacements, $M_{1A}(w)$, is given by

$$M_{1A}(w) = F_1(t_0)(1 + M_1(w)) \tag{5}$$

For Case B, if the first failure occurs within $(t_0, t_0 + y)$ with $y < w$, the expected number of replacements within time interval $(t_0 + y, w)$ is $M_1(w - y)$. From time t_0 to the occurrence of the first failure of the product, the product has failure rate $r_2(t)$ and life time distribution $F_2(t)$. Then, the expected total number of replacements within $[t_0, t_0 + w]$ for Case B is given by

$$M_{1B}(w) = \bar{F}_1(t_0)\int_0^w(1 + M_1(w-y))dF_2(y) \tag{6}$$

Therefore, the expected number of failures is $F_1(t_0)M_{1A}(w) + \bar{F}_1(t_0)M_{1B}(w)$. As the cost per replacement is c_f, the expected replacement cost within the warranty period is given by

$$C_1(w) = (M_{1A}(w) + M_{1B}(w))c_f \tag{7}$$

This ends the proof.

A warranty policy in the construction industry may cover as long as 5 years or more. During such a long term, an optimal preventive replacement policy can be implemented by the manufacturer. The most common and popular replacement policy might be the age-dependent preventive replacement policy. Under this policy, the product is always replaced

by a new identical one upon a failure or on reaching age τ_0, whichever occurs first, where τ_0 is a constant (Barlow and Hunter, 1960). If a preventive replacement policy is implemented within the warranty time period $[t_0, t_0 + w]$, then the replacement he warranty period for Policy 2 is as follows.

Theorem 2 *(Policy 2) A product is sold under Policy 2. The expected replacement cost within warranty period w is*

$$C_2(w, \tau_0) = M_{2A}(w, \tau_0) + M_{2B}(w, \tau_0) \tag{8}$$

where $M_{2A}(w, \tau_0) = F_1(t_0)C_R w$, $M_{2B}(w, \tau_0) = \bar{F}_1(t_0)(F_2(\tau_0)(c_f + C_R w) - C_R \int_0^{\tau_0} t dF_2(t) + \bar{F}_2(\tau_0)(c_r + C_R(w - \tau_0)))$, and $C_R = \frac{F_0(\tau_0)c_f + \bar{F}_0(\tau_0)c_r}{\int_0^{\tau_0} \bar{F}_0(t)dt}$.

Proof. The product is either replaced if it fails before reaching age τ_0, or preventively replaced by a new identical one if it survives longer than time τ_0. Denote the lifetime of a new product by X. The lifetime of a new product to a replacement is given by

$$\min(X, \tau_0) = \begin{cases} X & \text{if } X < \tau_0 \\ \tau_0 & \text{if } X \geq \tau_0 \end{cases} \tag{9}$$

The mean time between replacements is obtained by

$$E[\min(X, \tau_0)] = \int_0^\infty \Pr\{\min(X, \tau_0) > x\}dx = \int_0^{\tau_0} \bar{F}_0(x)dx$$

The expected cost between replacements is given by

$$\Pr\{X \leq \tau_0\}c_f + \Pr\{X > \tau_0\}c_r = F_0(\tau_0)c_f + \bar{F}_0(\tau_0)c_r$$

Hence, the long-run average cost, C_R, is obtained by

$$C_R = \frac{F_0(\tau_0)c_f + \bar{F}_0(\tau_0)c_r}{\int_0^{\tau_0} \bar{F}_0(t)dt} \tag{10}$$

In Case A, the product fails to operate at time t_0, and has a time length w in which replacements may needed. Therefore, the cost on possible replacements in Case A is $F_1(t_0)C_R w$.

In Case B, the first failure occurs after t_0. The lifetime distribution between time t_0 and the occurrence of the first failure is $F_2(t)$. Hence, the expected replacement cost before the occurrence of the first failure is given by

$$\int_0^{\tau_0} (c_f + (w - t)C_R)dF_2(t)$$
$$= \bar{F}_1(t_0)(F_2(\tau_0)(c_f + C_R w) - C_R \int_0^{\tau_0} t dF_2(t) \tag{11}$$

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The expected replacement cost from the occurrence of the first failure to the end of the warranty period $t_0 + w$ is $\bar{F}_2(\tau_0)(c_r + C_R(w - \tau_0)))$. By adding the two items, we have the replacement cost for Case B:

$$M_{2B}(w, \tau_0) = \bar{F}_1(t_0)(F_2(\tau_0)(c_f + C_R w) - C_R \int_0^{\tau_0} t dF_2(t) + \bar{F}_2(\tau_0)(c_r + C_R(w - \tau_0)))$$

Therefore, the expected cost on replacements is $M_{2A}(w, \tau_0) + M_{2B}(w, \tau_0)$, and this completes the proof of Theorem 2. When the warranty is renewing, we can obtain the following results.

Theorem 3 (Policy 3) *A product is sold under Policy 3. The expected replacement cost within warranty time w is given by*

$$C_3(w) = (M_{3A}(w) + M_{3B}(w))c_f \tag{12}$$

where $M_{3A}(w) = \frac{1}{\bar{F}_0(w)}$, and $M_{3B}(w) = \frac{F_2(w)}{\bar{F}_0(w)}$.

Proof. Under a renewing warranty policy, a failed product is replaced and the warranty term is renewed. Therefore, for Case A, the expected number of replacements is given by

$$
\begin{aligned}
M_{3A}(w) &= F_1(t_0)(1 + \sum_{j=1}^{\infty} F_0^j(w)) \\
&= \frac{F_1(t_0)}{\bar{F}_0(w)} \tag{13}
\end{aligned}
$$

For Case B, from time t_0 to the occurrence of the first failure of the product, the product has failure rate $r_2(t)$ and life time distribution $F_2(t)$. If the first failure occurs within $(t_0, t_0 + w)$, the manufacturer needs $F_2(w)$ times replacements. After the first replacement, the expected number of replacements is $F_2(w) \sum_{j=1}^{\infty} F_0^j(w)$. The expected number of replacement in Case B is given by

$$
\begin{aligned}
M_{3B}(w) &= \bar{F}_1(t_0)F_2(w)\left(1 + \sum_{j=1}^{\infty} F_0^j(w)\right) \\
&= \frac{\bar{F}_1(t_0)F_2(w)}{\bar{F}_0(w)} \tag{14}
\end{aligned}
$$

The expected total number of replacements is $M_{3A}(w) + M_{3B}(w)$. Considering the replacement cost, and multiplying the expected total number of replacement by c_f, we obtain Theorem 3. For Policy 4, we have the following results.

Theorem 4 (Policy 4) *A product is sold under Policy 4. The expected replacement cost within warranty time w is given by*

$$C_4(w, \tau_0) = F_1(t_0)(c_f + M_{4A}(w, \tau_0)) + \bar{F}_1(t_0)M_{4B}(w, \tau_0) \tag{15}$$

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where

$$M_{4A}(w, \tau_0) = \frac{F_0(\tau_0)c_f + \bar{F}_0(\tau_0)(1 - \bar{F}_0^N(\tau_0))c_r}{(1 - F_0(w - N\tau_0))\bar{F}_0^N(\tau_0)F_0(\tau_0)} - c_f \tag{16}$$

with $N\tau_0 \leq w < (N+1)\tau_0$.

If $N \geq 2$,

$$\begin{aligned}
M_{4B}(w, \tau_0) &= F_2(\tau_0)(c_f + M_{4A}(w, \tau_0)) + \bar{F}_2(\tau_0)c_r \\
&+ \{F_0(\tau_0)(c_f + M_{4A}(w, \tau_0)) + \bar{F}_0(\tau_0)c_r\} \sum_{j=0}^{N-2} (\bar{F}_2(\tau_0)\bar{F}_0^j(\tau_0)) \\
&+ (c_f + M_{4A}(w, \tau_0))\bar{F}_2(\tau_0)\bar{F}_0^{N-1}(\tau_0)F_0(w - N\tau_0)
\end{aligned} \tag{17}$$

If $N = 1$,

$$\begin{aligned}
M_{4B}(w, \tau_0) &= F_2(\tau_0)(c_f + M_{4A}(w, \tau_0, 1)) + \bar{F}_2(\tau_0)c_r \\
&+ (c_f + M_{4A}(w, \tau_0, 1))\bar{F}_2(\tau_0)F_0(w - \tau_0)
\end{aligned} \tag{18}$$

Proof. For a new product with a life time distribution $F_0(t)$, the expected total cost for replacements, $M_{4A}(w, \tau_0)$, can be obtained as follows.

A replacement for a new product that fails within time interval $(t_0, t_0 + \tau_0)$ costs c_f, and cost for further replacement is $M_{4A}(w, \tau_0)$. The expected total cost for replacements on failure is given by $\{F_0(\tau_0)(c_f + M_{4A}(w, \tau_0))\} \sum_{j=0}^{N-1} \bar{F}_0^j(\tau_0)$.

If the product is still operating at time $t_0 + \tau_0$, it is preventively replaced by a new one. This preventive replacement costs c_r. The expected total costs for preventive replacements is $c_r \bar{F}_0(\tau_0) \sum_{j=0}^{N-1} \bar{F}_0^j(\tau_0)$.

Within the time interval $(t_0 + N\tau_0, t_0 + w)$, the cost of the replacement is $(c_f + M_{4A}(w, \tau_0))\bar{F}_0^N(\tau_0)F_0(w - \tau_0)$. Therefore, the expected total cost on replacements in Case A is given by

$$\begin{aligned}
M_{4A}(w, \tau_0) &= \{F_0(\tau_0)(c_f + M_{4A}(w, \tau_0)) + \bar{F}_0(\tau_0)c_r\} \sum_{j=0}^{N-1} \bar{F}_0^j(\tau_0) \\
&+ (c_f + M_{4A}(w, \tau_0))\bar{F}_0^N(\tau_0)F_0(w - N\tau_0)
\end{aligned} \tag{19}$$

From the above equation, M_{4A} is obtained by

$$\begin{aligned}
M_{4A}(w, \tau_0) &= \frac{(F_0(\tau_0)c_f + \bar{F}_0(\tau_0)c_r) \sum_{j=0}^{N-1} \bar{F}_0^j(\tau_0) + \bar{F}_0^N(\tau_0)F_0(w - N\tau_0)c_f}{1 - F_0(\tau_0) \sum_{j=0}^{N-1} \bar{F}_0^j(\tau_0) - \bar{F}_0^N(\tau_0)F_0(w - N\tau_0)} \\
&= \frac{(F_0(\tau_0)c_f + \bar{F}_0(\tau_0)c_r)(1 - \bar{F}_0^N(\tau_0)) + \bar{F}_0^N(\tau_0)F_0(\tau_0)F_0(w - N\tau_0)c_f}{(1 - F_0(w - N\tau_0))\bar{F}_0^N(\tau_0)F_0(\tau_0)} \\
&= \frac{F_0(\tau_0)c_f + \bar{F}_0(\tau_0)(1 - \bar{F}_0^N(\tau_0))c_r}{(1 - F_0(w - N\tau_0))\bar{F}_0^N(\tau_0)F_0(\tau_0)} - c_f
\end{aligned} \tag{20}$$

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In Case B, the lifetime distribution after time t_0 is $F_2(t)$. If the first failure occurs within time interval $(t_0, t_0 + \tau_0)$, the expected replacement cost before the occurrence of the first failure is $F_2(\tau_0)(c_f + M_{4A}(w, \tau_0, 1)) + \bar{F}_2(\tau_0)c_r$. The replacement cost in the rest time can be analyzed as in Case A. The expected total cost for the warranty policy can be obtained.

3.2 *Discussion*

For various scenarios regarding failure rates and replacement costs, an optimal warranty policy can be selected from the above four policies. The following three optimal scenarios can be considered.

Optimization Scenario 1. Suppose no preventive replacement is performed. If both the non-renewing warranty policy and the renewing warranty policy can be considered, a more cost-effective warranty policy can be selected by minimising Equations (1) and (12) from Policy 1 and Policy 3.

Optimization Scenario 2. If preventive replacement policies are implemented within the warranty period, time interval τ_0 can be optimized from Equations (8) and (15) based on Policy 2 and Policy 4.

Optimization Scenario 3. If preventive replacements, renewing warranty policies, and non-renewing warranty policies are allowed, costs from Equations (1), (8), (12) and (15) can be optimized from all the four policies.

Note. *Although Section 3 considers the replacement cost for a product having a dormant state, the results in the section can be applied for a product without any dormant state. This can simply be obtained by the following two ways.*

- *Set $t_0 = 0$, which indicates there is no dormant state in the product's lifetime, or*

- *set $r_1(t) = r_0(t)$, which means that the dormant state can be ignored because the product at the dormant state has the same failure rate as at the operating state.*

Therefore, a product without any dormant state in its lifetime can be considered as a special case.

The failure rate of a product at a dormant state can be associated with its operating failure rate, the variation between the operating and dormant environments and so on. Understanding the failure rates at the dormant state is important for replacement policy development and warranty cost analysis. For most of building services products, for example, some environmental parameters such as temperature and humidity are the similar in both

the operating and the dormant environment. Failures affected by such parameters within the operating time may also occur at the dormant state.

However, the relationship between the failure rates within the dormant state and the operating state is rarely discussed in the reliability literature. We assume this relationship as follows.

The failure rate at the dormant state is assumed to be the product of a failure rate at the operating state and a constant μ (where $0 < \mu < 1$). That is

$$r_1(t) = \lambda r_0(\mu t) \tag{21}$$

with $t \in (0, t_0)$ and $0 < \lambda < 1$. That follows

$$F_1(t) = 1 - (\bar{F}_0(\mu t))^{\frac{\lambda}{\mu}} \tag{22}$$

The residual lifetime distribution of a product that survives after dormant time t_0, or the lifetime distribution within time interval $(t_0, +\infty)$, can be obtained based on the lifetime distribution at the dormant state, $F_1(t)$. However, within time interval $(0, t_0)$, the product is at the dormant state whereas within time interval $(t_0, +\infty)$ it is at the operating state. Hence, the lifetime distribution within $(t_0, +\infty)$ is different from the residual lifetime distribution that is derived from the distribution within time interval $(0, t_0)$. In this paper, we assume the lifetime distribution of a product that survives after dormant time t_0 is

$$F_2(t) = F_0(\mu t_0 + t), t \geq 0 \tag{23}$$

4   Numerical examples

Weibull and related models are commonly used in reliability engineering and they have attracted a lot of attentions recently (see e.g. Xie et.al.(2004), Lai et. al. (2003)).

Consider a case where the life distribution of the product under consideration is a 2-parameter Weibull distribution at the operating state. The failure rate is assumed to be

$$r_0(t) = \frac{\beta}{\alpha}(\frac{t}{\alpha})^{\beta-1} \tag{24}$$

According to Assumption 3, the following distributions can be given

$$F_0(t) = 1 - \exp\{-(\frac{t}{\alpha})^{\beta}\}, \tag{25}$$

$$F_1(t) = 1 - \exp\{-\frac{\lambda}{\mu}(\frac{\mu t}{\alpha})^{\beta}\}, \tag{26}$$

and

$$F_2(t) = 1 - \exp\{-(\frac{\mu t_0 + t}{\alpha})^\beta\} \tag{27}$$

In obtaining costs from Policy 1 and Policy 2, we need to approximate the renewal functions. Given a renewal function

$$m(t) = F(t) + \int_0^t m(t - x) dF(x) \tag{28}$$

Suppose it is desired to approximate $m(t)$ with $0 \le t \le T^*$. Partition $[0, T^*]$ into N subintervals: $0 = T_0 < T_1 < \ldots < T_L = T^*$. The following recursive approximations to $m(T_i)$ in Xie (1989) can be used:

$$m(T_i) = \frac{F(T_i) + S_i - F(T_i - T_{i-\frac{1}{2}})m(T_{i-1})}{1 - F(T_i - T_{i-\frac{1}{2}})}, 1 \le i \le N \tag{29}$$

with $m(T_0) = 0$, where $S_1 = 0$,

$$S_i = \sum_{j=1}^{i-1} F(T_i - T_{j-\frac{1}{2}})(m(T_j) - m(T_{j-1})), 2 \le i \le N \tag{30}$$

and

$$T_{i-\frac{1}{2}} = \frac{1}{2}(T_{i-1} + T_i)$$

Some examples on studies on renewal equations and solutions can be found in Ran et. al. (2006), Tortorella (2005).

Set the parameters used in the paper as shown in Table 1 for Optimization Scenario 1, and Table 3 for both Optimization Scenario 2 and Optimization Scenario 3.

Table 2 and Figure 4 show replacement cost over warranty periods of Policy 1 and Policy 3. When warranty periods are lower than 2 (or 24 months), the two policies have the same replacement costs. When warranty periods are larger than 2, replacement cost incurred in Policy 2 changes faster than that from Policy 1.

Table 4 and Figure 5 show replacement cost over replacement interval of Policy 2 and Policy 4. The optimal replacement interval is 21 month for Policy 2 whereas it is 13 month for Policy 4.

Table 5 shows the change of replacement cost over warranty periods and replacement intervals. For example, when the warranty period is 1 year, Policy 1 is the optimal policy because it occurs the minimum replacement cost (79.27) among the four policies, and Policy 3 may be a potential because it occurs a little more than Policy 1 (80.18 in Policy 3). When the warranty policy is set to be 2.5 years, Policy 4 is the optimal one as it only incurs 294.94 unit cost if preventive replacement is implemented per year. When the warranty policy is set to be 5 years, Policy 2 is the optimal one as it only occurs 616.54 unit cost if preventive replacement is implemented per year.

5 Conclusions

A building services product protected by warranty is commonly at a dormant state from its installation to commissioning, and not put into use until the commissioning phase has been completed. However, the product may fail because of aging, deterioration or other causes arising within this dormant period. This makes the products different from normal products in terms of both usage intensity and failure rates.

Within the warranty period, replacements can be correctively or preventively implemented upon failures or on reaching a specific age. This paper analyzes replacement cost for four warranty policies that consider non-renewing and renewing free replacement warranty policies, provided that the preventive maintenance may be implemented or only replacement upon failure is carried out.

Replacement cost for the four policies are obtained. The four policies can be applied in different scenarios regarding the failure pattern and replacement cost of building services products. The results in the paper can also be applied to products having no dormant state in their lifetime. A typical case about the relationship between the failure rates at the dormant state and at the operating state is discussed. Numerical examples show each of the four warranty policies can be optimal over warranty periods and replacement intervals.

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t_0	λ	μ	α	β	c_f	c_r
3	0.2	0.2	6.0	5.0	100	50

Table 1: Parameters for Optimization Scenario 1.

w	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
$C_1(w)$	0.022	0.14	0.52	1.52	3.62	7.48	13.85	23.29	35.91	50.95
$C_3(w)$	0.022	0.14	0.53	1.52	3.66	7.72	14.81	26.54	45.43	75.85

Table 2: Replacement cost of Optimization Scenario 1.

t_0	w	λ	μ	α	β	c_f	c_r
3	3	0.2	0.2	2.0	4.0	100	50

Table 3: Parameters for Optimization Scenarios 2 and 3.

τ_0	1/12	3/12	5/12	7/12	9/12	11/12	13/12	15/12	17/12
C_2	2968.29	602.40	378.18	279.73	215.34	189.88	176.05	163.74	161.86
C_4	1802.25	605.06	360.35	268.95	232.09	196.61	164.66	189.31	234.39
τ_0	19/12	21/12	23/12	25/12	27/12	29/12	31/12	33/12	35/12
C_2	161.78	159.28	160.61	162.00	162.57	164.41	165.90	166.77	167.39
C_4	219.31	241.07	289.04	378.87	549.13	894.76	1668.18	3617.07	9260.59

Table 4: Replacement cost of Optimization Scenario 2.

	τ_0	w								
		1	1.5	2	2.5	3	3.5	4	4.5	5
$C_1(w)$		79.27	215.5	436.4	697.27	922.99	1084.94	1225.99	1392.99	1590.03
$C_2(w,\tau_0)$	0.5	217.31	318.03	418.74	519.46	620.17	720.88	821.6	922.31	1023.02
	1		226.81	282.49	338.16	393.84	449.51	505.19	560.87	616.54
	1.5			343.16	395.37	447.57	499.77	551.98	604.18	656.39
	2				541.24	609.27	677.31	745.34	813.37	881.41
	2.5					782.79	880.31	977.83	1075.35	1172.87
	3						1050.95	1183.74	1316.54	1449.33
	3.5							1345.91	1508.49	1671.08
	4								1623.01	1802.18
	4.5									1847.44
$C_3(w)$		80.18	228.55	526.43	1102.6	2377.05	6182.07	23488.64	157947.78	2244064
$C_4(w,\tau_0)$	0.5	220.34	321.27	422.28	523.36	624.52	725.76	827.08	928.47	1029.95
	1		181.03	293.99	294.94	409.23	410.26	525.89	527.02	644.01
	1.5			329.45	343.02	503.65	504.69	520.29	690.05	691.23
	2				627.45	642.81	711.46	1014.31	1015.67	1035.55
	2.5					1203.93	1223.97	1313.52	1578.17	2368.35
	3						2479.22	2512.06	2658.81	3092.53
	3.5							6287.07	6362.52	6699.77
	4								23606.92	23882.83
	4.5									158169.8

Table 5: Replacement cost of Optimization Scenario 3.
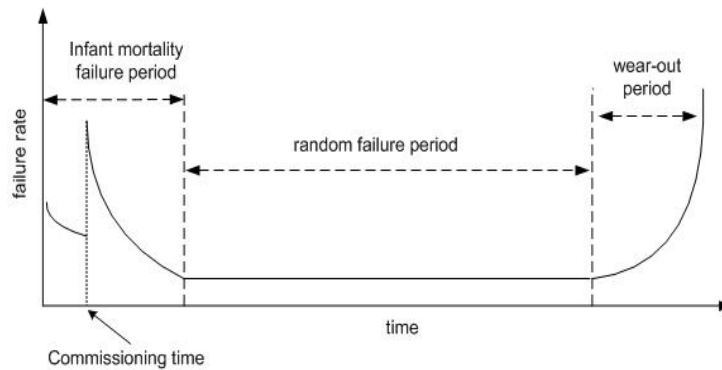

Figure 1: Failure patterns of an item with a dormant state
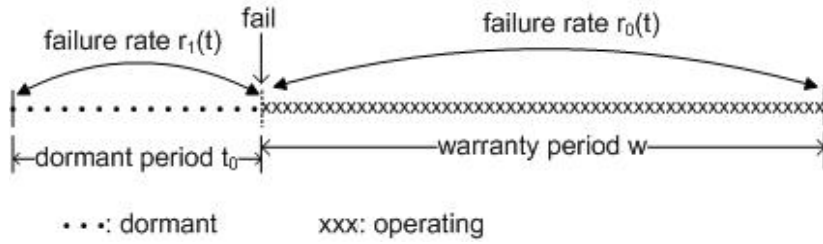
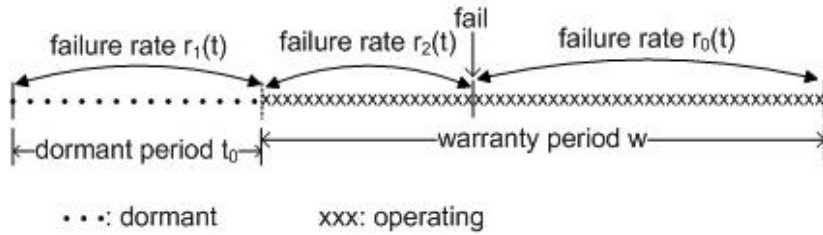Figure 2: Case A - the product is found failed to operate at time t_0

Figure 3: Case B - the product is found failed to operate after time t_0
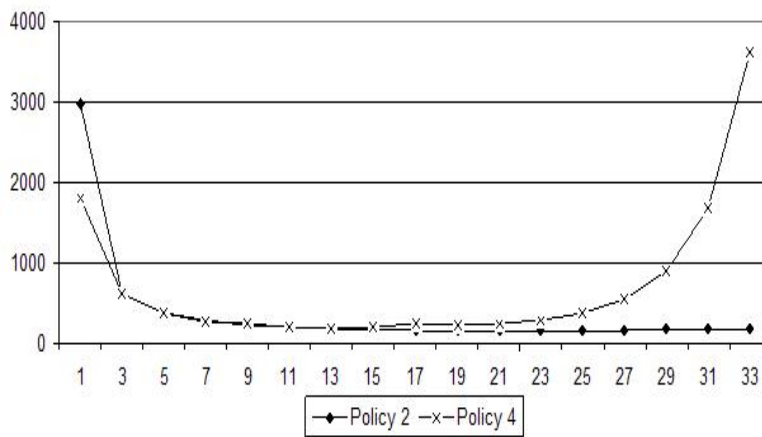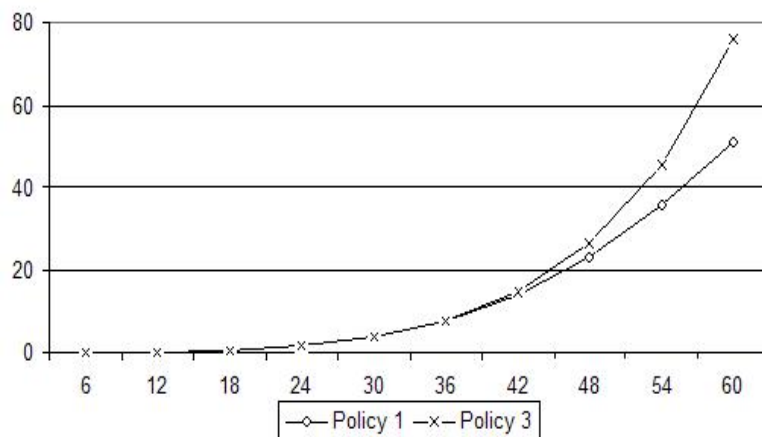

Figure 4: Replacement cost of Optimization Scenario 1.

Figure 5: Replacement cost of Optimization Scenario 2.