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AN OPTIMISATION APPROACH TO THE LOCATION AND SIZING OF MULTIPLE LEAKS IN A DUCT

D Sharp   DEME, Faculty of Technology, Open University, Walton Hall, Milton Keynes
V Chilekwa   DEME, Faculty of Technology, Open University, Walton Hall, Milton Keynes

1   INTRODUCTION

The detection of leaks in ducts is of crucial importance in many industrial situations. The leaks could be cracks or holes in the wall of the duct, or some other unexpected porosity. One of the standard methods of testing the integrity of a duct involves sealing the end of the duct, increasing the pressure within to a certain level and confirming that this level remains constant. However, in industry, many systems are inaccessible and impossible to seal, so in these cases a non-invasive technique is necessary. Various non-invasive methods have been developed for monitoring and measuring leaks but at present there is no universal method, each technique being suitable only for a specific leak type and rate[1].

In a paper published in 1997, Sharp and Campbell[2] describe acoustical procedures for locating and sizing a single leak in a duct. Using the technique of acoustic pulse reflectometry, the duct is probed with a sound pulse to enable the input impulse response of the duct to be measured. Suitable analysis of this impulse response then yields the position of the leak. Next, the acoustic impedance of the duct is calculated from the impulse response and, together with the leak position, is fed into a theoretical model which solves an inverse problem and provides the hole radius. However, while these methods enabled single leaks to be analysed successfully, extending this analytical approach to the study of multiple leaks proved impossible.

In this paper, a methodology is presented which utilises the technique of numerical optimisation, together with theoretical input impedance models, to detect, locate and size multiple leaks in a straight cylindrical tube. First of all, the input impedance of the duct under investigation is measured. This input impedance is then treated as the target value for the optimisation routine. Next, a numerical model of a duct of arbitrary length containing several leaks is derived. The theoretical input impedance of this duct model is calculated and used as the start value for the optimisation routine. The optimisation routine then proceeds to adjust the length of the modelled duct, together with the positions and sizes of the leaks, recalculating the theoretical impedance each time until it matches the target value. At this stage, the length, position and size of leaks in the numerical duct model should match those of the duct under investigation. It should be noted that the

initial values of the length, number of holes and hole sizes in the numerical duct model are not critical to the success of the optimisation.

In the next section, the numerical model of the duct is described and a theoretical expression for its input impedance is derived. Following this, in Section 3, the basic principles behind numerical optimisation are introduced. Finally, in Section 4, the results of applying this optimisation approach to leak detection to a duct that contains two leaks are presented and discussed.

2   NUMERICAL MODEL OF A DUCT CONTAINING LEAKS

2.1   Input impedance

The input impedance, z, of a duct provides useful quantitative information about the duct's resonance properties. It is defined as being the ratio of the acoustic pressure p to the air volume flow rate U at the entrance of the duct:

$$z = \frac{p}{U} \tag{1}$$

As the pressure and volume velocity are both complex quantities, the input impedance also comprises a real and imaginary part.

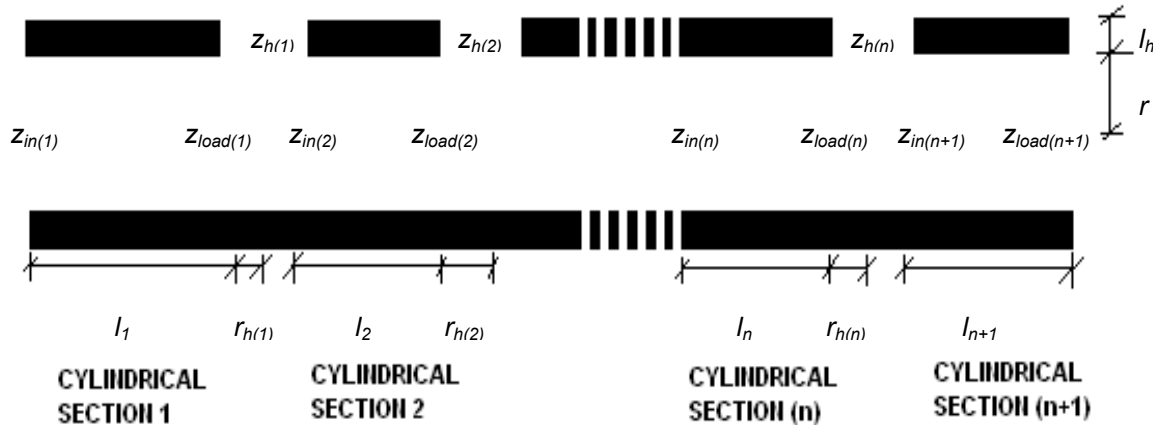## 2.2   Input impedance of a duct with multiple leaks

Figure 1. Schematic diagram of a cylindrical duct containing multiple leaks

Figure 1 shows a schematic diagram of a straight tube containing n leaks (and comprising $n+1$ cylindrical sections). The input impedance of the whole duct, $z_{in(1)}$, is made up of contributions from the impedances of the $n+1$ cylindrical sections and the impedances of the n holes in the duct walls[2,3].

To calculate $z_{in(1)}$, the load impedance, $z_{load(n+1)}$, of the final cylindrical section must first be determined. As this is simply the radiation impedance at the end of the duct, then $z_{load(n+1)} = z_{rad}$ where[4]:

$$z_{rad} = 0.25 \frac{\rho \omega}{\underline{k} \pi r^2} \underline{k}^2 r^2 + j0.6 \frac{\rho \omega}{\underline{k} \pi r^2} \underline{k} r \tag{2}$$

and ω is the angular frequency, ρ is the density of air, $\underline{k}$ is the complex propagation constant and r is the radius of the duct. The next step involves calculating the input impedance, $z_{in(n+1)}$, of the $(n+1)^{th}$ cylindrical section and the impedance, $z_{h(n)}$, of the nth hole as follows:

$$z_{in(n+1)} = \frac{\rho\omega}{\underline{k}\pi r^2} \left(\frac{\frac{z_{load(n+1)}\underline{k}\pi r^2}{\rho\omega} + j\tan \underline{k}l_{n+1}}{1 + j\frac{z_{load(n+1)}\underline{k}\pi r^2}{\rho\omega}\tan \underline{k}l_{n+1}} \right) \tag{3}$$

$$z_{h(n)} = \frac{\rho\omega\underline{k}}{4\pi} + j\frac{\rho\omega(l_h + Er_{h(n)})}{\pi r_{h(n)}^2} \tag{4}$$

where l_{n+1} is the length of the $(n+1)^{th}$ cylindrical section, l_h is the thickness of the duct wall, $r_{h(n)}$ is the radius of the nth hole and $E = 1.595 - 0.58(r_h/r)^2$ is the sum of the inner and outer end corrections for a hole set flush with the cylinder wall. As $z_{in(n+1)}$ and $z_{h(n)}$ form a parallel acoustic circuit, $z_{load(n)}$ can be calculated as follows:

$$z_{load(n)} = \frac{z_{h(n)}z_{in(n+1)}}{z_{in(n+1)} + z_{h(n)}} \tag{5}$$

In the same manner as was seen in equation 3, the input impedance of the nth cylindrical section is then calculated from the load impedance $z_{load(n)}$ where:

$$z_{in(n)} = \frac{\rho\omega}{\underline{k}\pi r^2} \left(\frac{\frac{z_{load(n)}\underline{k}\pi r^2}{\rho\omega} + j\tan \underline{k}l_n}{1 + j\frac{z_{load(n)}\underline{k}\pi r^2}{\rho\omega}\tan \underline{k}l_n} \right) \tag{6}$$

and l_n is the length of the nth cylindrical section. The procedure for calculating the load impedance, hole impedance and input impedance is repeated for each subsequent section until the first cylindrical section is reached and $z_{in(1)}$ is found.

3   NUMERICAL OPTIMISATION

Numerical optimisation is a technique for finding the minimum or maximum of a function. In most cases it is desirable to find the minimum of the function, which will coincide with the most efficient solution of that function[5]. Numerical optimisation is used in a whole range of fields including economics, weather forecasting, chemistry, biology, physics and signal processing.

Numerical optimisation is usually used when an analytical solution to a function is non-existent or when the variables affecting the solution being sought are more than two[6]. The technique can be used for both linear and nonlinear functions.

3.1   Rosenbrock algorithm

The Rosenbrock algorithm is a 0th order form of numerical optimisation which uses steps of varying lengths and alternating search directions to minimise a function[7]. The function variables are treated as forming a base of vectors in an N-dimensional coordinate system, where N is the number of

function variables. The first search is conducted along a line parallel to the x_1-axis, the second search along a line parallel to the x_2-axis, the third search along a line parallel to the x_3-axis until the search along the x_N-axis is made and one search cycle is completed.

Initially, the length of the step in the search direction of a given vector is set to be of arbitrary size. If this succeeds in yielding a lower value of the function, the step length is multiplied by a positive number greater than one. However, in the case of a failure, the step length is multiplied by a negative number between 0 and 1. Each step made in the direction of the vectors is called a trial.

The alternating searches are repeated from x_1 to x_N until at least one trial has been successful in each direction, and one has failed. A new set of directions for the vectors is then calculated. The method used to calculate the new search directions ξ_1 to ξ_N (orthogonal to the original search directions) is based on the Gram-Schmidt procedure. The algorithm is illustrated as follows:

$$B_1 = A_1 ,$$

$$\xi_1 = \frac{B_1}{|B_1|} ,$$

$$B_2 = A_2 - A_2 \cdot \xi_1 \, \xi_1 ,$$

$$\xi_2 = \frac{B_2}{|B_2|} ,$$

and so on, until

$$B_N = A_N - \sum_{j=1}^{N-1} A_N \cdot \xi_j \xi_j ,$$

$$\xi_N = \frac{B_N}{|B_N|}$$

where A_1 is the vector joining the initial and final points obtained by use of the vectors $x_1, x_2 \ldots x_N$, A_2 is the sum of advances made in directions other than the first (i.e. the vector joining the points obtained by use of the vectors $x_2, x_3 \ldots x_N$), and so on.

The set of trials made with one set of directions, and the subsequent change of these directions, is known as an iteration stage. Once new search directions have been calculated at the end of an iteration stage, the whole process is repeated over and over again until eventually the minimum of the function is found.

It should be noted that, during each iteration stage conducted in a particular search direction, it is necessary to ensure that the minimum value of the function found is a global minimum rather than a local minimum. This is achieved by using large initial step lengths to leave local minima and go on with the search for a function value representing the global minimum.

4   APPLICATION TO INVESTIGATION OF MULTIPLE LEAKS

To apply the Rosenbrock algorithm to the problem of investigating multiple leaks in a duct, parameters of a numerical model duct with three leaks were used to create a 7-dimensional coordinate system of base vectors comprising l_1 = 0.15m, l_2 = 0.2m, l_3 = 0.2m, l_4 = 0.1m, r_1 = 0.6mm, r_2 = 0.6mm and r_3 = 0.6mm. Using this model any number of leaks up to a maximum of three can be detected, located and sized. By increasing the number of parameters in the model,

unknown ducts containing greater numbers of holes can be measured at the cost of computational speed.

To initiate the optimisation routine, the input impedance of the model duct is calculated. This forms the start value for the optimisation routine. The difference between the input impedance of the duct under investigation (target value) and that of the numerical model (start value) is used to test the progress of the optimisation and is called the test value. During the process of optimisation, the parameter values of the numerical model are updated after each successful trial and the test value recalculated. If the calculated test value is lower than the previous value, the updated parameters of the numerical model are taken as the new optimised values and stored until another reduction in the test value is achieved. The optimisation process is stopped when the test value is minimised to a near zero figure and the optimised value makes a close approximation to the target value to within a specified percentage error.

4.1　Results

To demonstrate its effectiveness, the optimisation approach was applied to a test duct containing two leaks (with parameters $l_1 = 0.2m$, $l_2 = 0.3m$, $l_3 = 0.36m$, $r_1 = 2mm$ and $r_2 = 2mm$). The input impedance of the test duct was measured using the technique of acoustic pulse reflectometry and then used as the target value for the optimisation routine.
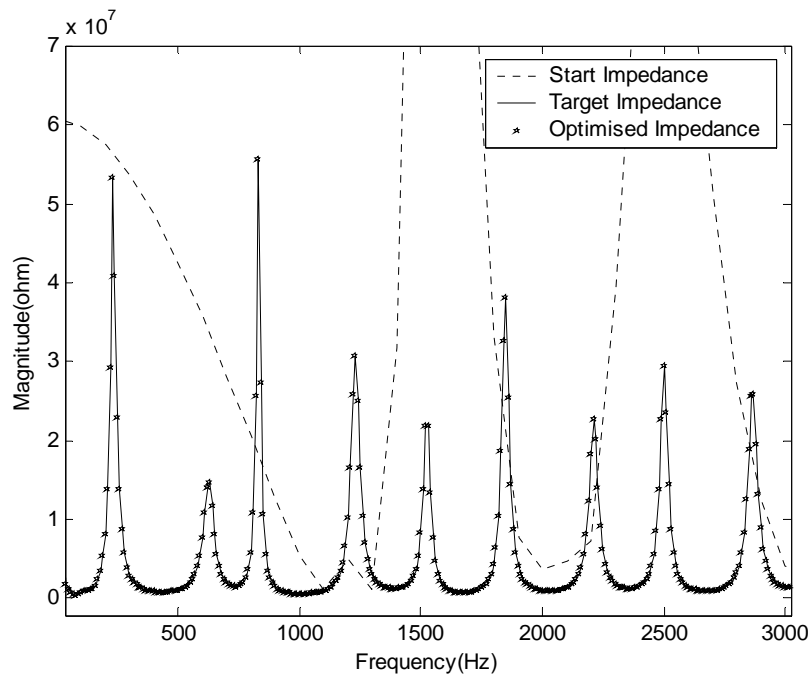

Figure 2. Plots of the 'target' impedance curve, 'start' impedance curve and the 'optimised' impedance curve.

Figure 2 compares the 'start' impedance curve, the 'target' impedance curve and the 'optimised' impedance curve. Table 1 shows the final parameters of the numerical model which result after the optimisation procedure. The parameters for the start and target impedances are also tabulated.

The prediction error, taken as the percentage difference between the optimised values and the target values, was used to analyse the performance of the optimisation routine in making the predictions about multiple leaks. Using this indicator, all the predictions were consistent to within a 10% error. The prediction accuracy can be improved by setting the number of iterations in the optimisation routine to a larger value at the cost of computation time.

	Start Value (SV)	Target Value (TV)	Optimised Value (OV)	$\dfrac{\lvert TV - OV \rvert}{TV} \times 100$
	(m)	(m)	(m)	(%)
l_1	0.15	0.2	0.219	9.5
l_2	0.2	0.3	0.309	3
l_3	0.2	0.36	0.365	1.4
l_4	0.1	0	0	0
r_1	0.0006	0.002	0.0019	5
r_2	0.0006	0.002	0.00185	7.5
r_3	0.0006	0	0	0

Table 1. Parameters for the start impedance, target impedance and the optimised impedance.

5   CONCLUSIONS

The preliminary results presented in this paper clearly indicate the potential of the numerical optimisation approach for predicting the number, location and sizes of multiple leaks in a duct. The predictions of hole size and position agree with the directly measured values to within a 10% error. By increasing the number of iterations in the optimisation routine, it should be possible to improve this level of accuracy.

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