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Contrasting energy scales of reentrant integer quantum Hall states

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We report drastically different onset temperatures of the reentrant integer quantum Hall states in the second and third Landau level. This finding is in quantitative disagreement with the Hartree-Fock theory of the bubble phases which is thought to describe these reentrant states. Our results indicate that the number of electrons per bubble in either the second or the third Landau level is likely different than predicted.

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In systems of charged particles strong Coulomb interactions stabilize a periodic ground state called the Wigner solid (WS).[1,2] The WS has been observed in two-dimensional electron gases (2DEGs) floating atop superfluids,[3] in 2DEGs confined to GaAs/AlGaAs heterostructures,[4] and electron bilayers.[5] There is a resurgence of interest in the WS stimulated by recent work on electrons confined to less than two dimensions,[6] 2DEGs in complex oxide heterostructures,[7] and in graphene.[8] The WS was also realized in ion clouds[9] and, most recently, in cold atomic gases with dipolar interactions[10] and it plays a role in charged colloidal suspensions[11] and neutron stars.[12]

Long range interactions may also stabilize periodic ground states which are more intricate than the WS.[13,14] One such many-body ground state is the electronic bubble phase which was predicted to form in the 2DEG subjected to a perpendicular magnetic field B.[13–20] Electrons in this system move on circular Landau orbitals, their energy is quantized to equidistant Landau levels (LLs), and their ground states are labeled by the LL filling factor ν at which they form. According to theory, the guiding centers of the Landau orbitals cluster into so-called electron bubbles and, furthermore, the bubbles order into an isotropic lattice. Such a bubble phase can therefore be thought of as a WS with an internal degree of freedom, i.e., with several electrons per unit cell.[13]

The experimentally measured reentrant integer quantum Hall states (RIQHSs) have been identified with the bubble phases.[21–30] Indeed, dc[21–27] and microwave transport features[28–30] of the RIQHSs are, generally speaking, consistent with the bubble interpretation. However, for the RIQHSs the number of electrons per bubble remains unknown to date. With the lack of any direct measurements on the structure of the bubbles one has to turn to the theory. In the second Landau level (SLL) both two- and one-electron bubble phases are predicted to form[19] while in the third Landau level (TLL) only two-electron bubble phases are expected.[15,17–20] These theories, however, have their limitations. The Hartree-Fock approach, the only one used for bubble phases of both the TLL[15,19,20] and the SLL,[19] is exact only in the limit of large LL occupation,[14,16] and may therefore not capture all aspects of bubbles at the lowest LL occupation, i.e., those in the second and third LLs. In addition, the presence of competing nearby fractional quantum Hall states in the SLL[25,26] is likely to enhance fluctuations and may therefore influence electron ordering. Finally, none of the theoretical techniques include LL mixing, an electron-electron interaction effect known to strongly affect the energy gaps of fractional quantum Hall ground states in the SLL.[31]

In this Rapid Communication we report sharp peaks in the temperature dependent longitudinal resistance of the RIQHSs in the TLL, which are similar to those of the RIQHSs in the SLL. This shared property highlights the common origin of these RIQHSs. The sharp peaks allowed us to extract the onset temperatures of the RIQHSs in the TLL, which enabled a quantitative comparison of the RIQHSs forming in the TLL with those in the SLL as well as with the theoretically predicted bubble phases. Our measurements of the onset temperatures are at odds with the cohesive energy calculations obtained within the Hartree-Fock approximation and indicate that the assignment of the RIQHSs to the various bubble phases is likely different than predicted.

We measured a high quality 2DEG confined to a 30 nm wide GaAs/AlGaAs quantum well with a density $n = 2.8 \times 10^{11}$ cm^{-2} and mobility 15×10^6 cm^2/V s grown at Purdue. The low frequency magnetotransport measurements were performed at dilution refrigerator temperatures while our sample was immersed into a liquid He-3 bath.[25,32] The He-3 bath facilitates cooling of the sample[25] and it enables B-field independent temperature measurements by the use of a quartz tuning fork viscometer.[32] Due to its large heat capacity, He-3 also serves as a thermal ballast which stabilizes the sample temperature.

In Fig. 1 we show the longitudinal magnetoresistance R_{xx} and the Hall resistance R_{xy} plotted against B and filling factor ν in the SLL and TLL. Here $\nu = nh/eB$, where h is Planck's constant and e is the elementary charge. It is important to appreciate that a completely filled orbital Landau level is spin split into two distinct energy levels and, hence, its filling factor is $\nu = 2$. Therefore the lowest Landau level corresponds to filling factors $\nu < 2$, the SLL corresponds to $2 < \nu < 4$, while the TLL to $4 < \nu < 6$. Data in the TLL is collected at 77 mK while those in the SLL at 6.9 mK.

The well known integer quantum Hall states are seen in Fig. 1 as plateaus in R_{xy} quantized to h/ie^2, with $i = 2$, 3, 4, 5, and 6. Each of these plateaus straddle the corresponding integer filling factor $\nu = i$. As B is varied, R_{xy} deviates from these plateaus. There are, however, other regions for which R_{xy} returns to an integer quantization but, in contrast to the plateaus

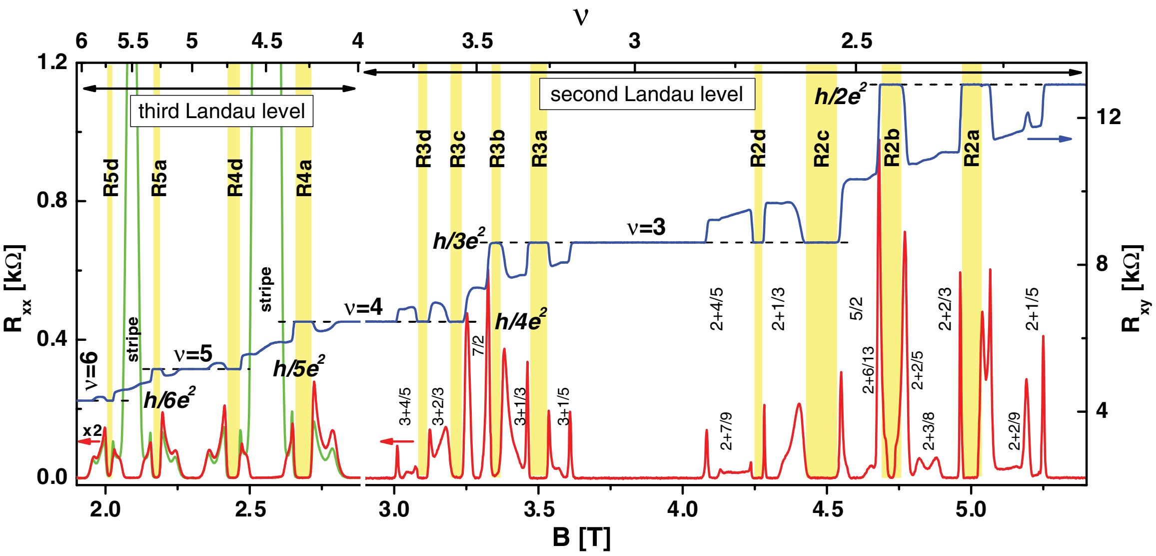

FIG. 1. (Color online) The magnetoresistance in the second ($2 < \nu < 4$) and third ($4 < \nu < 6$) Landau levels. Both the longitudinal (R_{xx}) and the Hall (R_{xy}) resistances are measured at 77 mK in the TLL and at 6.9 mK in the SLL. RIQHSs are marked by shaded stripes and FQHSs by their filling factors. In the TLL the two R_{xx} traces shown are measured along mutually perpendicular directions and, for clarity, are magnified by a factor of 2.

of the integer quantum Hall states, these plateaus develop at ranges of ν which do not contain any integer values. These features define the RIQHSs.[21–24] As an example, the RIQHS labeled $R2c$ in Fig. 1 has $R_{xy} = h/3e^2$ and it stretches between $2.54 < \nu < 2.60$, a region which does not contain any integers. Quantization of R_{xy} is accompanied by a vanishing R_{xx}. Altogether, in the SLL there are eight RIQHS labeled $R2a$, $R2b$, $R2c$, $R2d$, $R3a$, $R3b$, $R3c$, and $R3d$,[23] while in the TLL there are only four such states labeled $R4a$, $R4d$, $R5a$, and $R5d$.[21,22] The RIQHSs are clearly marked and shaded in Fig. 1.

In Fig. 1 we also identify anisotropic ground states called stripe phases[13–15] in the vicinity of $\nu = 9/2$ and $11/2$,[21,22] a very strong fractional quantum Hall state (FQHS) at $\nu = 5/2$ (Ref. 33) with a gap of 0.50 K, a well quantized $\nu = 2 + 2/5$ FQHS, and we discern developing FQHSs at $\nu = 2 + 6/13$, $2 + 2/9$, $2 + 7/9$, and $2 + 3/8$.[25,26] We also observe a split-off RIQHS at B fields exceeding that of the $R2a$ state, which was discovered in Ref. 25 and studied in detail in Ref. 27. In addition to these known aspects, we observe a feature in the Hall resistance at $B = 5.196$ T or $\nu = 2.214$. This feature is a clear deviation from the classical Hall line and it may signal the development of another RIQHS.

A notable difference between the RIQHSs is that there are twice as many of them in the SLL than in the TLL. Despite this disparity in their numbers, the RIQHSs in the SLL and high LLs share common features in the quantized reentrant transport[21–24] and microwave response.[28–30] In the following we establish two additional common transport signatures of the RIQHSs in the SLL and TLL: spikes flanking the vanishing regions of the R_{xx} vs B curves and a peak in the temperature dependent R_{xx}. These findings further strengthen the argument that the RIQHSs of different LLs have similar origins.

One similarity between the RIQHSs in the SLL and TLL we find is the presence of two sharp spikes in the flanks of the vanishing region of the R_{xx} vs B curves, i.e., the edges of the shaded areas of Fig. 1. Such spikes are known to be present in the flanks of the RIQHSs in the SLL,[25–27] and now we observe them in the TLL as well. With the exception of the data in Ref. 34, earlier R_{xx} vs B curves showed a single broad peak in the region separating the RIQHS from the nearby integer plateau; the width at half height of this peak near the RIQHSs $R4a$ was measured to be about 0.05 T. In contrast, our data in Fig. 1 at the corresponding fields, i.e., in the range of 2.7–2.85 T, has a more complex structure which exhibits a sharp spike at 2.72 T of width 0.016 T. We think that the richer structure in R_{xx} and the presence of the sharp spikes are due to an improved sample uniformity.

Contrary to a previous report,[35] in our sample there are no magnetoresistance features which may be associated with a FQHS in the TLL. We find that the $\nu = 4 + 1/5$ and $4 + 4/5$ filling factors, as seen in Figs. 1 and 2, are part of the complex behavior of R_{xx} described above. Local minima do develop, but they are not located at $\nu = 4 + 1/5$ or $4 + 4/5$ and, furthermore, they are not accompanied by a quantized Hall plateau in R_{xy} (not shown) in the 6.9–300 mK temperature range. Thus, in our sample there is no evidence for the formation of any FQHS in the TLL.

We find that the temperature evolution of R_{xx} of the RIQHS in the TLL and that of RIQHSs in the SLL[27] share the following common features: At the lowest temperatures there are two well separated spikes of finite resistance flanking the vanishing R_{xx}; with increasing T these two spikes merge into a single peak, and this peak disappears into a smooth background with a further increase in T. Such a temperature dependence for the $R4a$ state of the TLL is shown in Fig. 2. We define the center of a RIQHS as the location ν_c at which the extent of the vanishing R_{xx} plateau is nearly zero. For example, the curve at 128 mK of Fig. 2 exhibits a $R4a$ state of nearly zero width at $\nu_c = 4.287$. The partial filling factor ν_c^* is the decimal part of ν_c, and values for the various RIQHSs are summarized in Table I.

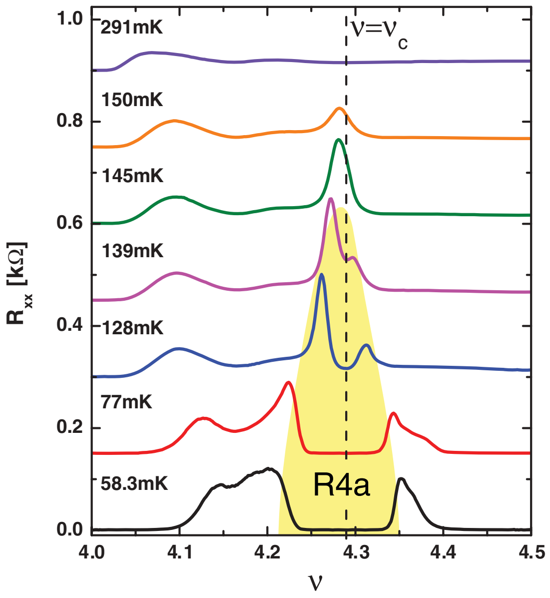

FIG. 2. (Color online) The evolution with temperature of the *R4a* RIQHS of the third Landau level. For clarity traces are shifted by 150 Ω relative to another and the reentrant region is shaded.

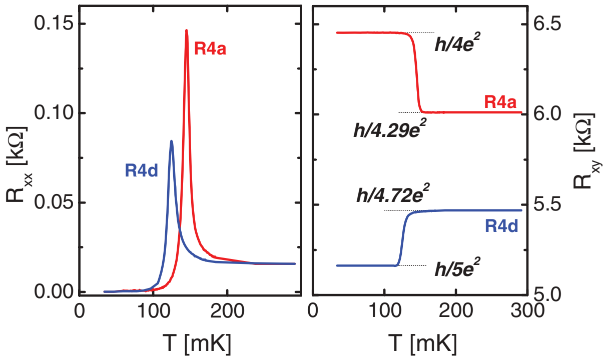

FIG. 3. (Color online) The magnetoresistance R_{xx} and the Hall resistance R_{xy} of two RIQHSs in the third Landau level measured at $\nu = 4.29$ and $\nu = 4.72$.

A second shared feature of the RIQHSs in the TLL and in the SLL[27] is the similar R_{xx} and R_{xy} vs T curves measured at a fixed ν. In Fig. 3 we show such curves for the *R4a* and *R4d* states of the TLL in close vicinity to their respective central filling factors. As the temperature is increased, the Hall resistance undergoes an extremely abrupt change from the nearest integer quantized value to the classical Hall value $B/ne = h/\nu e^2$. Simultaneously, with the sharp change in R_{xy} the longitudinal resistance R_{xx} for the *R4a* state exhibits a sharp peak of width at half height of only 10 mK. We have recently reported similar dependences of both R_{xy} and R_{xx} of the RIQHSs in the SLL of a higher density sample and have interpreted the peak temperature as the onset temperature T_c of the RIQHSs.[27] We thus find that a peak in the R_{xx} vs T curves accompanied by a sharp transition of R_{xy} from the classical Hall to a quantized value is not specific to the SLL, but is also a property of the RIQHSs forming in the TLL.

In the following we compare the locations, i.e., the filling factors of the RIQHSs. Surprisingly, the filling factors of the RIQHSs in the TLL have yet to be measured with high precision.[21–24] Inspecting Table I we find that *R2a*, *R3a* from the SLL and *R4a*, *R5a* from the TLL develop at similar partial filling factors. Indeed, $\nu_c^*|_{R3a} = \nu_c^*|_{R4a} = \nu_c^*|_{R5a}$ within our measurement error of ±0.003. Furthermore, this common value is in close proximity to $\nu_c^*|_{R2a}$. Nonetheless, we measure a significant difference between the common value of $\nu_c^*|_{Ria}$, with $i = 3{,}4{,}5$ and $\nu_c^*|_{R2a}$. This is seen in Fig. 4 as an alignment of data points associated with *R3a*, *R4a*, and *R5a* onto a vertical dashed line and a slight horizontal departure of the point associated with *R2a* from this line. A similar alignment occurs for the particle-hole symmetric states *R2d*, *R3d*, *R4d*, and *R5d*. We summarize thus that RIQHSs *Ria* with $i = 2$, 3, 4, and 5 form at similar partial filling factors and yet theory favors different types of order for these states: one-electron bubbles or WS at *R2a* and *R3a* (Ref. 19) and two-electron bubbles for *R4a* and *R5a*.[15,17–20]

As a further test we examine the energy scales of the RIQHSs. The cohesive energy of the bubble phase E_{coh} is readily obtained from the Hartree-Fock theories.[13–15,19,20] It is customary to calculate the reduced cohesive energy $e_{\text{coh}} = E_{\text{coh}}/E_c$, where $E_c = e^2/4\pi\epsilon l_B$ is the Coulomb energy and $l_B = \sqrt{\hbar/eB}$ the magnetic length. Experimentally we measure the onset temperature T_c and we consider the reduced onset temperature $t_c = k_B T_c/E_c$. Figure 4 summarizes the t_c of the RIQHSs in the SLL and TLL as function of ν_c^*. We assume that, within the bubble interpretation, the onset temperature of a RIQHS is a measure of its cohesive energy.[2] We find that the reduced onset temperatures t_c of the RIQHSs in the SLL and TLL are more than two orders of magnitude smaller than the reduced cohesive energies $e_{\text{coh}} = E_{\text{coh}}/E_c$ of the associated bubble phases.[13–15,19,20] We think this difference is most likely due to disorder and Landau level mixing effects which are not included in the Hartee-Fock estimations.[13–15,19,20] Furthermore, similarly to a recent report,[27] in the SLL we find a good collapse of t_c's from different spin branches and a nonmonotonic dependence of t_c of ν_c^*. As shown in Fig. 4, t_c in the TLL is in the vicinity of 16×10^{-4}, but the collapse of values from the two different spin branches is not as good as for the RIQHSs in the SLL.

Our most interesting finding is the disproportionately large energy scale of the RIQHSs in the TLL as compared to those in the SLL. The most striking disagreement is between the

TABLE I. Central filling factors ν_c^* and onset temperatures T_c of the RIQHSs measured.

	R2a	*R2b*	*R2c*	*R2d*	*R3a*	*R3b*	*R3c*	*R3d*	*R4a*	*R4d*	*R5a*	*R5d*
ν_c^*	0.300	0.438	0.568	0.700	0.288	0.430	0.576	0.713	0.287	0.714	0.286	0.714
T_c (mK)	45.3	29.8	39.9	29.5	38.1	25.4	31.0	25.5	145	125	111	100

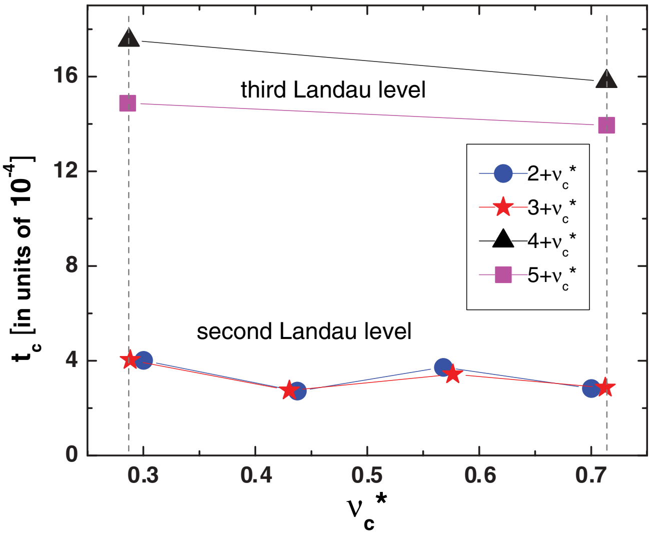

FIG. 4. (Color online) The reduced onset temperatures $t_c = k_B T_c / E_c$ of the RIQHSs in the SLL and TLL plotted as function of the partial filling factor ν_c^*. Lines are guides to the eye.

RIQHSs $R4a$ and $R2b$ believed to be two-electron bubbles. In Ref. 19 the cohesive energies are calculated for both RIQHSs and they are found to be similar, $e_{\rm coh}^{R4a}/e_{\rm coh}^{R2b} \approx 1.2$. In contrast to these predictions, we measure a large difference in the onset temperatures, $t_c^{R4a}/t_c^{R2b} = 6.4$. In Ref. 19 it is found that $e_{\rm coh}^{R4a}/e_{\rm coh}^{R2a} \approx 1$,[19] while we measure $t_c^{R4a}/t_c^{R2a} = 4.3$. In another work[20] $e_{\rm coh}^{R4a}$ is larger by a factor 2 as compared to that in Ref. 19. When considering $e_{\rm coh}^{R2b}$ from Ref. 19, the discrepancy between $e_{\rm coh}^{R4a}/e_{\rm coh}^{R2b}$ and t_c^{R4a}/t_c^{R2b} is reduced by the same factor of 2, but it still remains considerable. Taken together, we conclude that there are clear quantitative inconsistencies between the measured and calculated energy scales of the RIQHSs. We note that, within the SLL, the measured and theoretical energy scales of $R2a$ and $R2b$ states compare surprisingly well: $t_c^{R2a}/t_c^{R2b} = 1.5$ and $e_{\rm coh}^{R2a}/e_{\rm coh}^{R2b} \approx 1.2$.[19]

One scenario which could account for our onset temperature data is that, contrary to the theory,[19] all of the RIQHSs in the SLL are bubble phases of the same type and those in the TLL are bubbles of a different kind. We cannot, however, discard the possibility that the RIQHSs of the second and third LLs are the same type of bubble phases. The large difference in onsets could be caused by an effect dependent on LL occupancy. Because of the presence of one extra filled LL, screening of the disorder potential in the TLL is expected to be more effective than that in the SLL.[13,36] The substantially larger onsets of the RIQHSs in the TLL as compared to those in the SLL could thus be a consequence of a smoother effective disorder potential due to screening of one extra filled LL.

Finally we note that there are two recent reports of reentrant behavior in the lowest LL in 2DEGs forming in GaAs/AlGaAs hosts. One such observation is made in a heterostructure which has short range neutral scattering centers.[37] Another experiment was performed on wide quantum wells.[38] In both of these experiments[37,38] reentrance has been associated with the formation of electron solids similar to the WS since electron-electron interactions in the lowest LL are not expected to promote electronic bubble phases.[15] However, the relationship between these electron solids and those in higher LLs we have studied is not understood at this time.

To conclude, the reported common features in the transport of the RIQHSs both in the TLL and SLL, together with the reentrant behavior and radiofrequency response, support the idea that the RIQHSs belong to the same family of ground states, irrespective of the LL they form in. These features are qualitatively consistent with the bubble interpretation of these phases. We found, however, that the very different energy scales of the RIQHSs in different LLs are inconsistent with quantitative predictions of the theory of the bubbles. This disagreement is suggestive of an assignment of the RIQHSs to bubble phases different than that proposed by the theory. Our results call for further work in order to elucidate the nature of the RIQHSs.

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