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Influence of Boundary Conditions on the Prediction Accuracy of a Biot-based Poroelastic Model for Melamine Foam

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http://docs.lib.purdue.edu/herrick/66

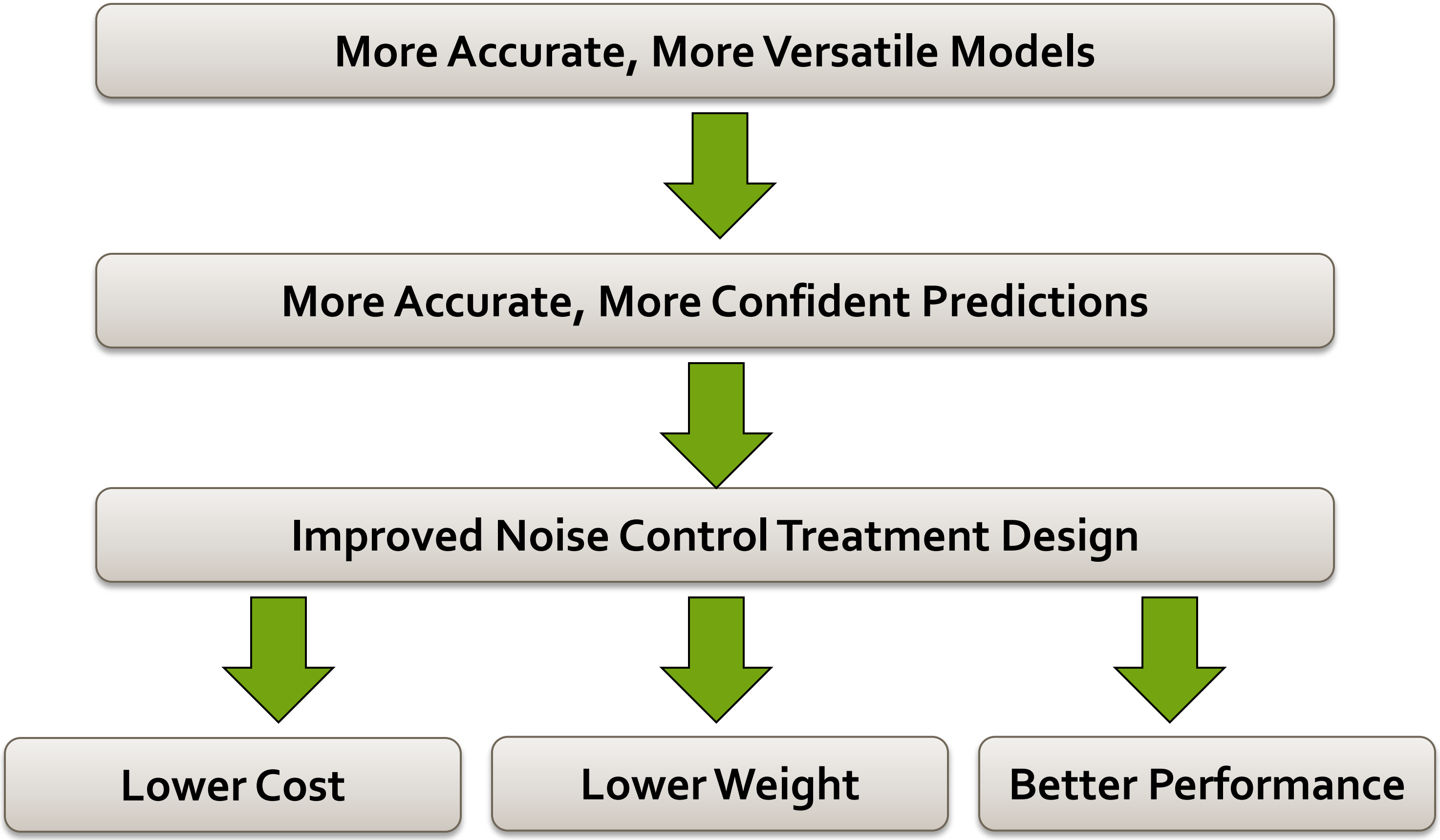
Influence of Boundary Conditions on the Prediction Accuracy of a Biot-based Poroelastic Model for Melamine Foam

Research Assistant: Ryan Schultz

Advisor: Dr. J. S. Bolton

Contents

But Why?



More Accurate, More Versatile Models

More Accurate, More Confident Predictions

Improved Noise Control Treatment Design

- Lower Cost
- Lower Weight
- Better Performance

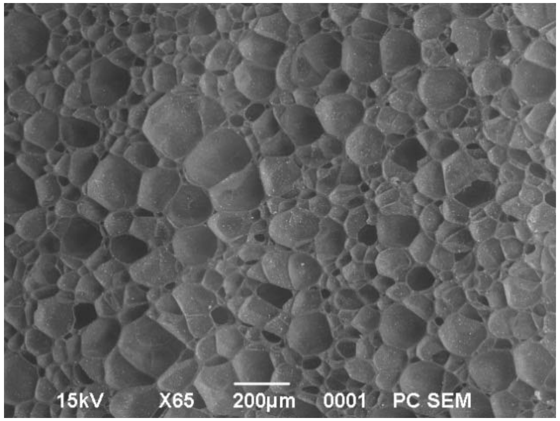
Poroelastic Materials

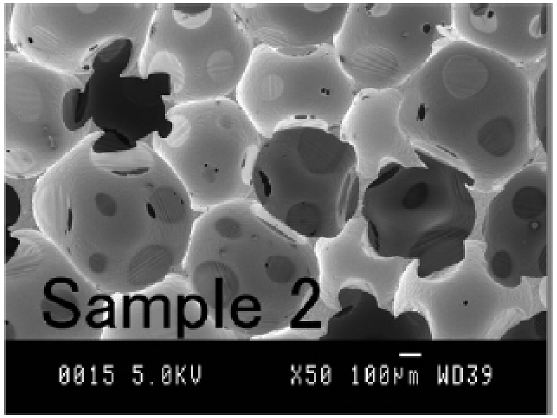
◆ **Poroelastic Materials:**

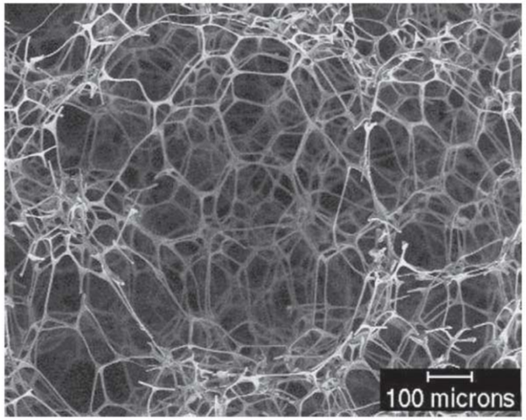
- 2 phases: solid frame saturated with fluid
- Pore cells: closed, open, partially reticulated

◆ **Melamine Foam**

- Open Cell
- Good acoustic, fire, thermal properties







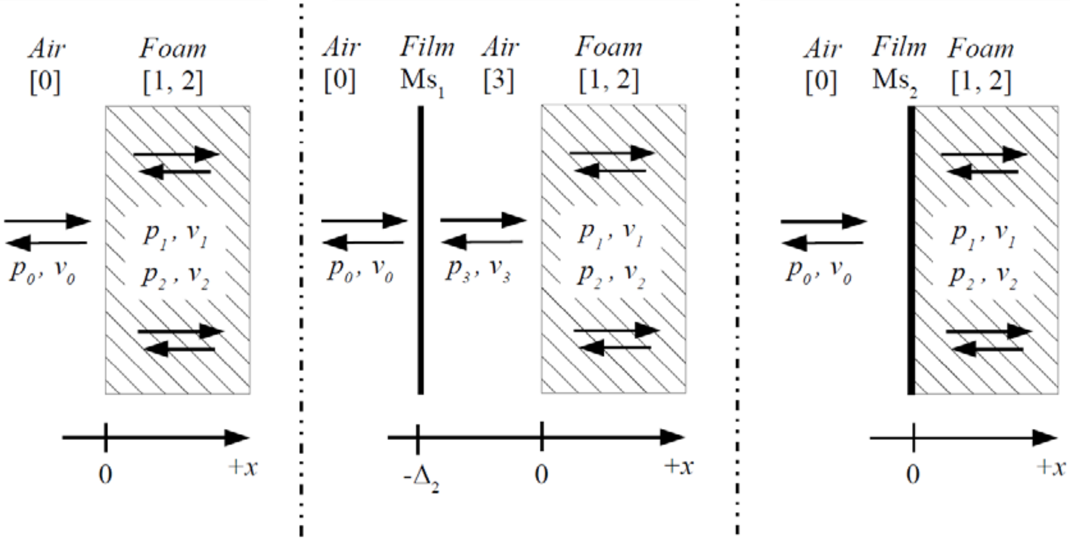
Boundary Conditions

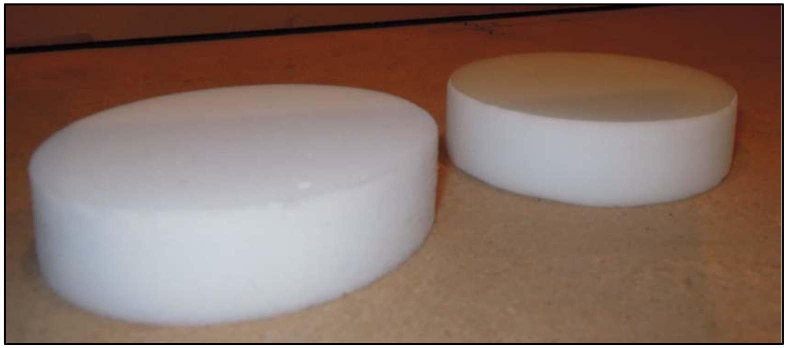
Facing Conditions

No Facing

Loose Facing

Glued Facing

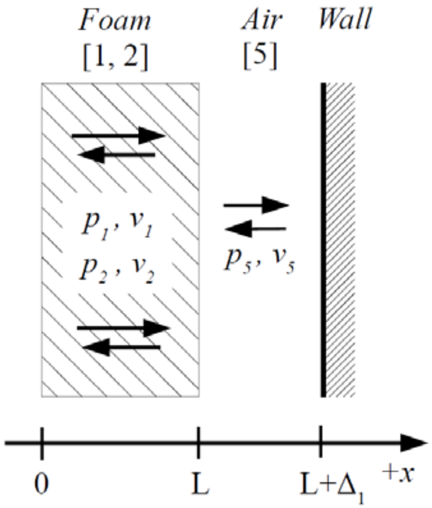




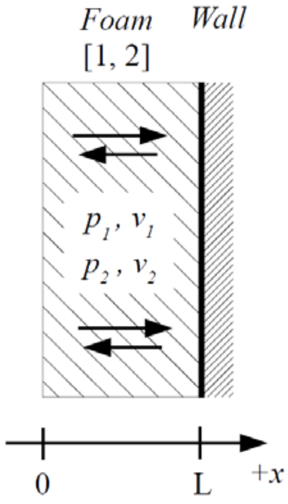
Boundary Conditions

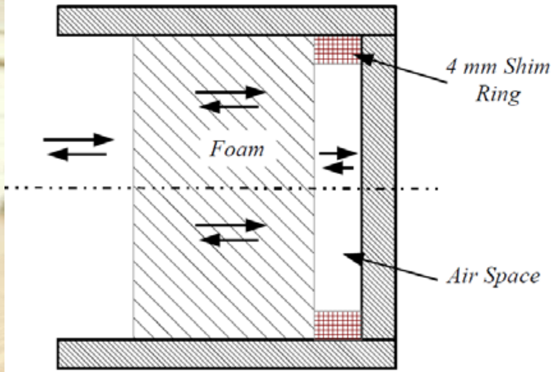
Mounting Conditions

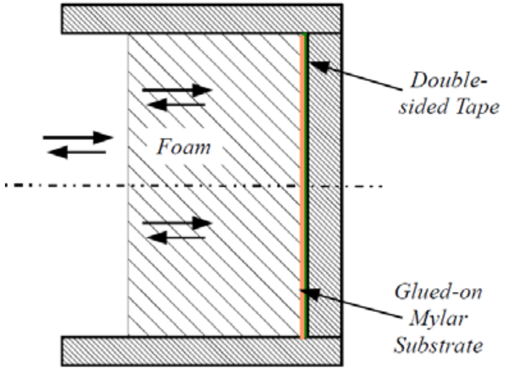
Gap



Fixed

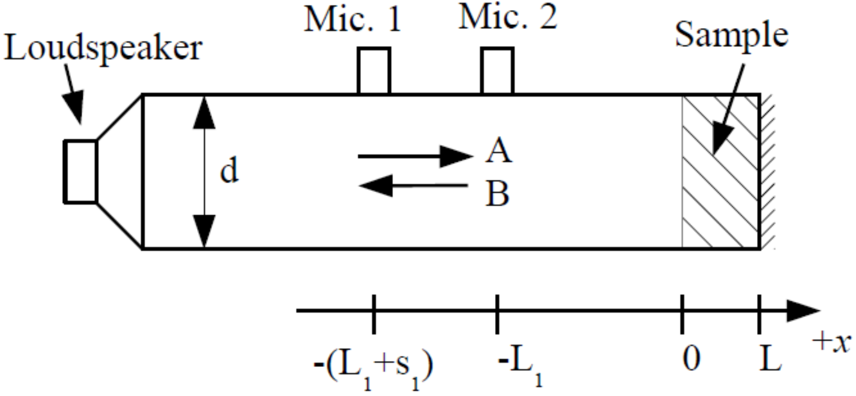




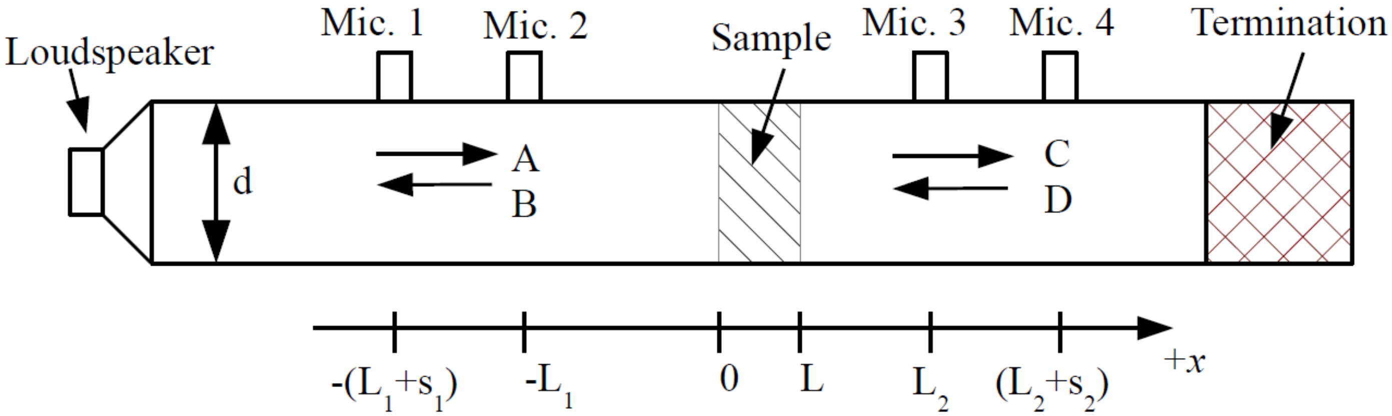


Measurement: Apparatus

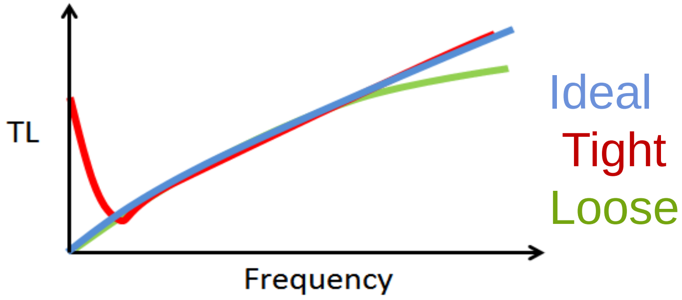
- **2-Mic Absorption**

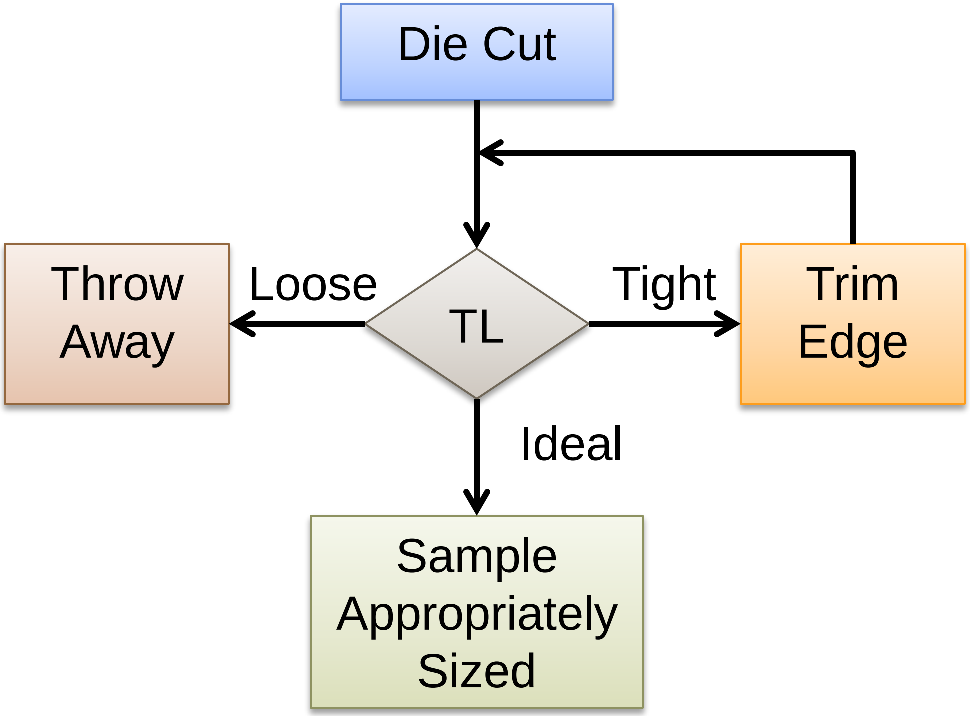


- **4-Mic Transmission Loss**



Measurement: Sample Fit




Die Cut

Throw Away

Loose

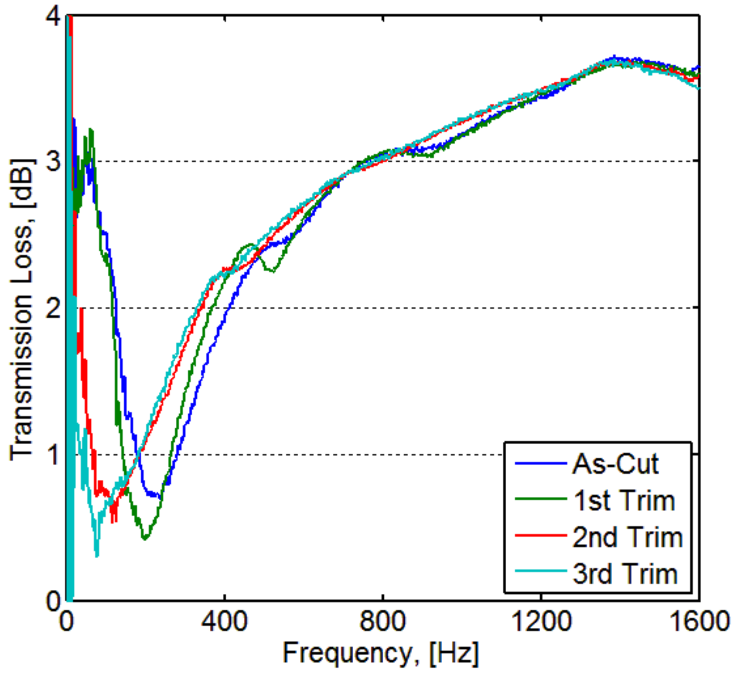
TL

Tight

Trim Edge

Ideal

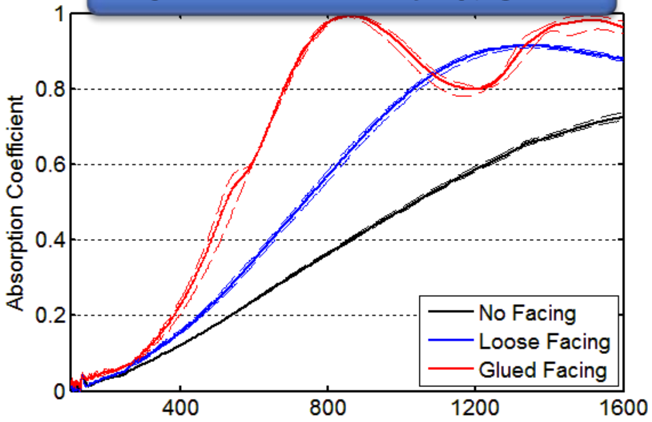
Sample Appropriately Sized



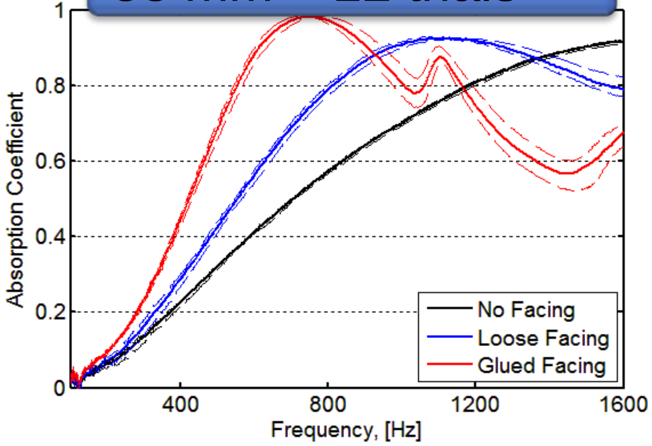
Absorption Measurement Results

Gap Mounting

25 mm – 12 trials

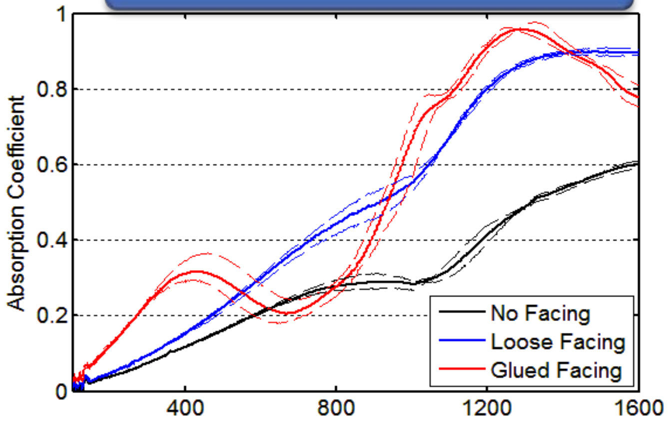


38 mm – 12 trials

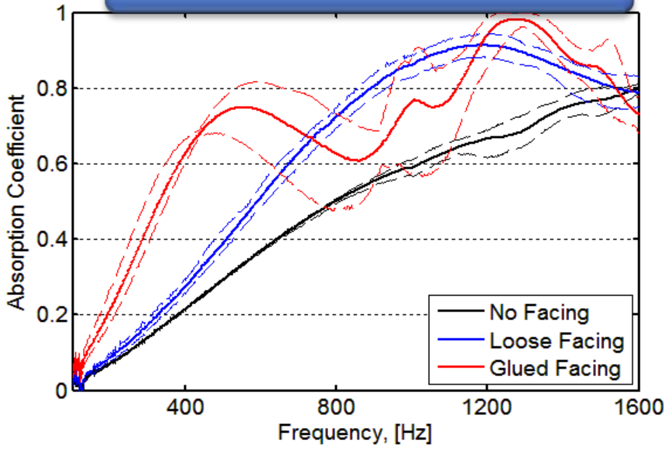


Fixed Mounting

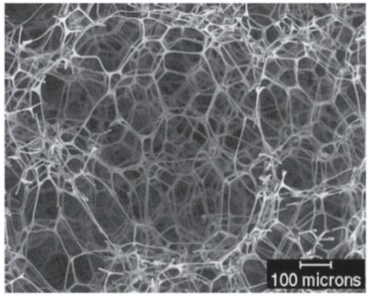
25 mm – 12 trials



38 mm – 12 trials

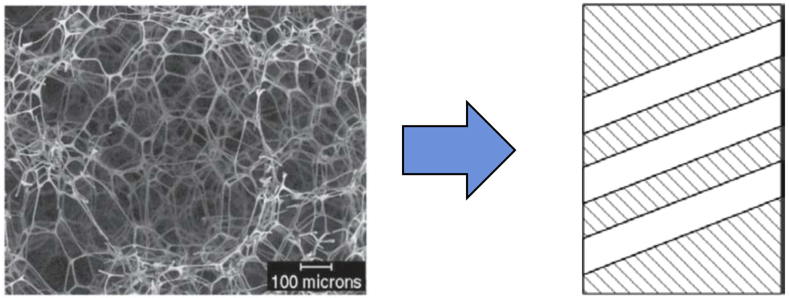


The Biot Theory: Terminology



Symbol	Parameter	Unit
ϕ	Porosity	-
σ	Flow Resistivity	$\mathrm{Ns/m^4}$
α_∞	Tortuosity	-
N	Shear Modulus	kPa
ν	Poisson's Ratio	-
η	Loss Factor	-
Λ	Viscous Characteristic Length	$\mu\mathrm{m}$
Λ'	Thermal Characteristic Length	$\mu\mathrm{m}$
ρ_1	Bulk Density	$\mathrm{kg/m^3}$

The Biot Theory



- **Stress-Strain Relations**

$$\sigma_x = Pe_x + Q\epsilon_x$$

$$s = R\epsilon_x + Qe_x$$

- **Dynamic Relations**

$$\frac{\partial \sigma_x}{\partial x} = \rho_1 \frac{\partial^2 u_x}{\partial t^2} + \rho_a \frac{\partial^2}{\partial t^2}(u_x - U_x) + b\frac{\partial}{\partial t}(u_x - U_x)$$

$$\frac{\partial s}{\partial x} = \rho_a \frac{\partial^2}{\partial t^2}(U_x - u_x) + b\frac{\partial}{\partial t}(U_x - u_x)$$

$$Qe_x + R\epsilon_x = -\omega^2(\rho_{12}u_x + \rho_{22}U_x)$$

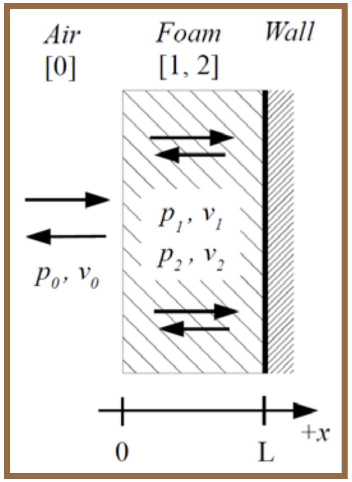
$$Pe_x + Q\epsilon_x = -\omega^2(\rho_{11}u_x + \rho_{12}U_x)$$

$$\begin{tikzcd}
 & \nabla^4 e_x + A_1\nabla^2 e_x + A_2 e_x = 0 \arrow[dr] & \\
e^{-j\bar{k}\bar{r}}e^{j\omega t} \arrow[ur] & & k_{1,2}^2 = \frac{1}{2}\left(A_1 \pm \sqrt{A_1^2 - 4A_2}\right)
\end{tikzcd}$$

Solution of Field Variables: Example

Front Condition: No Facing

Mounting Condition: Fixed



Solution for Field Variables:

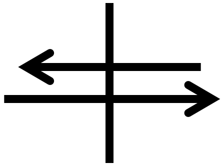
- Insert expressions for field variables into constraint relations for Front & Rear surfaces and Mounting condition.
- Number of equations = Number of unknown field variables + Reflection coefficient
- Solve the linear system

$$\begin{aligned}(1-\phi)p_{\text{ext}} &= p_1\\ \phi p_{\text{ext}} &= p_2\\ v_{\text{ext}} &= (1-\phi)v_1+\phi v_2\\ v_1 &= 0\\ v_2 &= 0\end{aligned}$$

$$\begin{bmatrix} -(1-\phi) & -f_1 & f_1 & -f_2 & f_2\\ -\phi & -g_1 & g_1 & -g_2 & g_2\\ 1/Z_c & d_1 & d_1 & d_2 & d_2\\ 0 & W_1 & W_1^{-1} & W_2 & W_2^{-1}\\ 0 & b_1W_1 & b_1W_1^{-1} & b_2W_2 & b_2W_2^{-1}\end{bmatrix}\begin{Bmatrix} R\\ C_1\\ C_2\\ C_3\\ C_4\end{Bmatrix} = \begin{Bmatrix} (1-\phi)\\ \phi\\ 1/Z_c\\ 0\\ 0\end{Bmatrix}$$

Intensity in the Porous Material

For Plane Harmonic Waves:



Acoustic Pressure

$$p = Ae^{-jk_0x} + Be^{jk_0x}$$

Particle Velocity

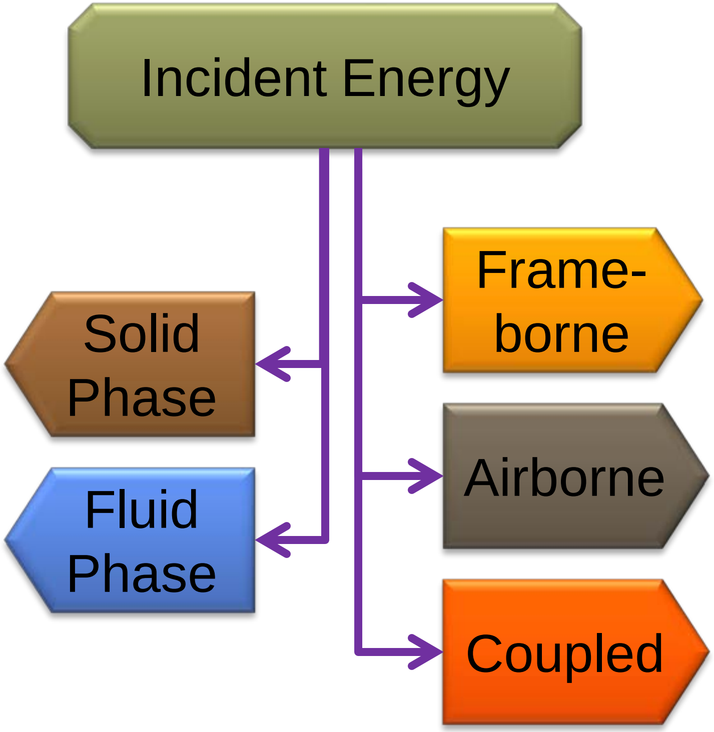
$$v = \frac{1}{\rho c}\left(Ae^{-jk_0x} - Be^{jk_0x}\right)$$

Intensity*

$$I = \frac{1}{2}\mathrm{Re}\{pv^*\}$$

*time-averaged rate of energy transmission through a unit area

Intensity



Incident Energy

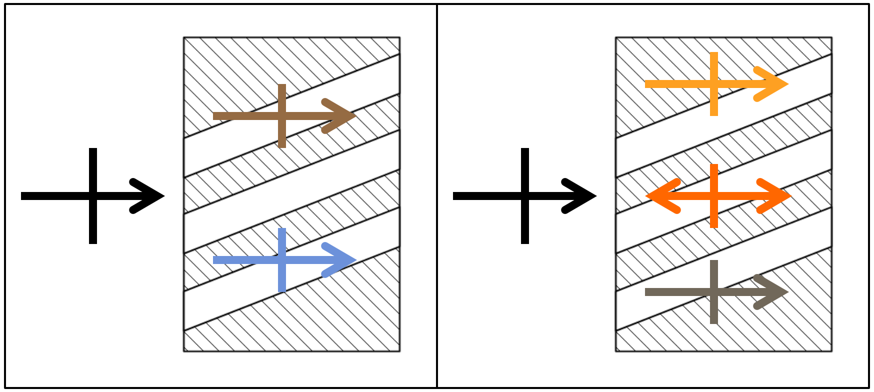
Solid Phase

Fluid Phase

Frame-borne

Airborne

Coupled



$$I_{foam} = I_{solid} + I_{fluid}$$

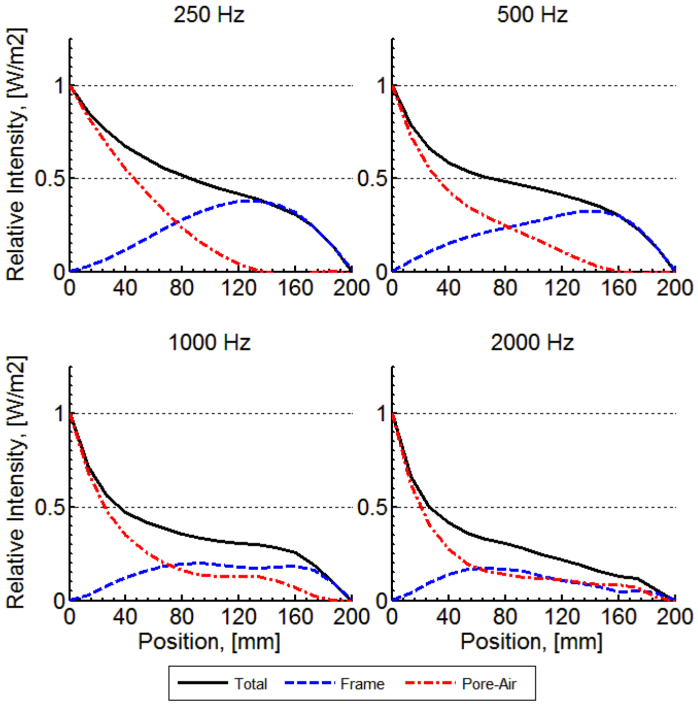
$$I_{foam} = I_{k_1} + I_{k_2} + I_{coupled}$$

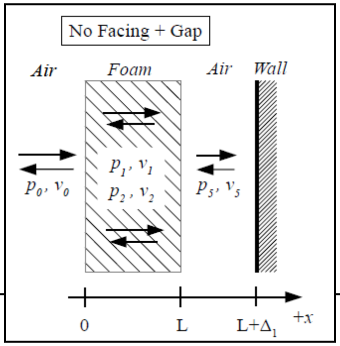
$$I_{plane\ wave} = \frac{1}{2} Re\{pv^*\}$$

$$I_{foam} = \frac{1}{2} Re\{p_1 {v_1}^*\} + \frac{1}{2} Re\{p_2 {v_2}^*\}$$

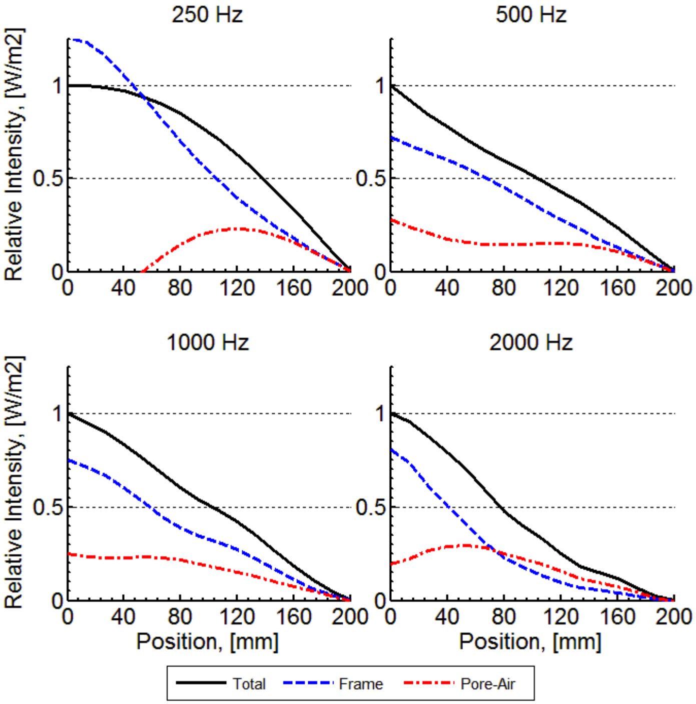
Intensity: By Phases

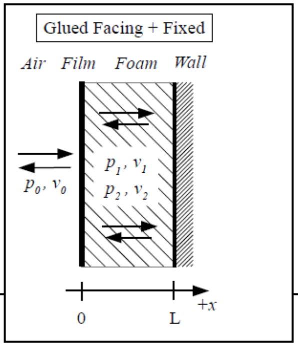
No Facing + Gap





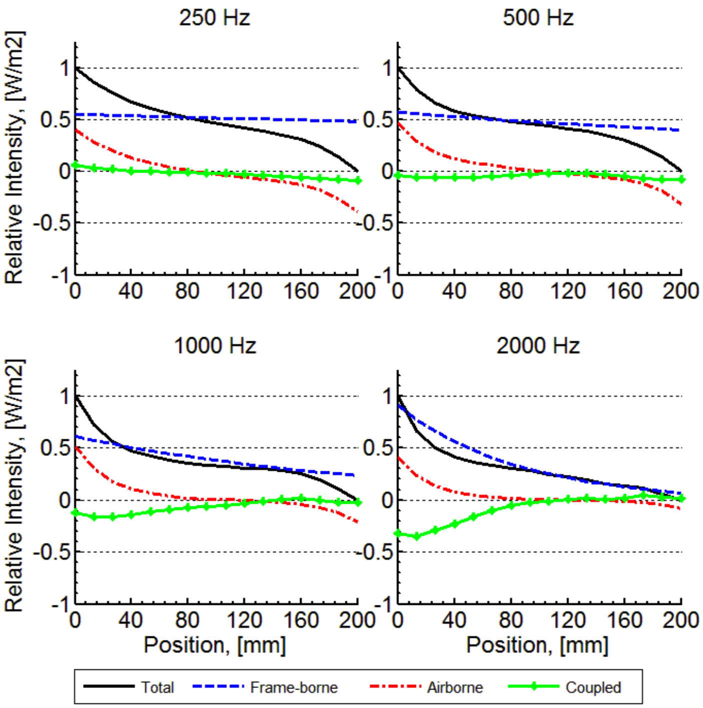
Glued Facing + Fixed

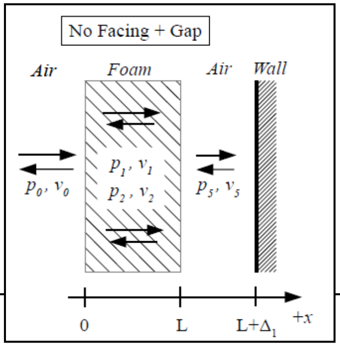




Intensity: By Wave Type

No Facing + Gap





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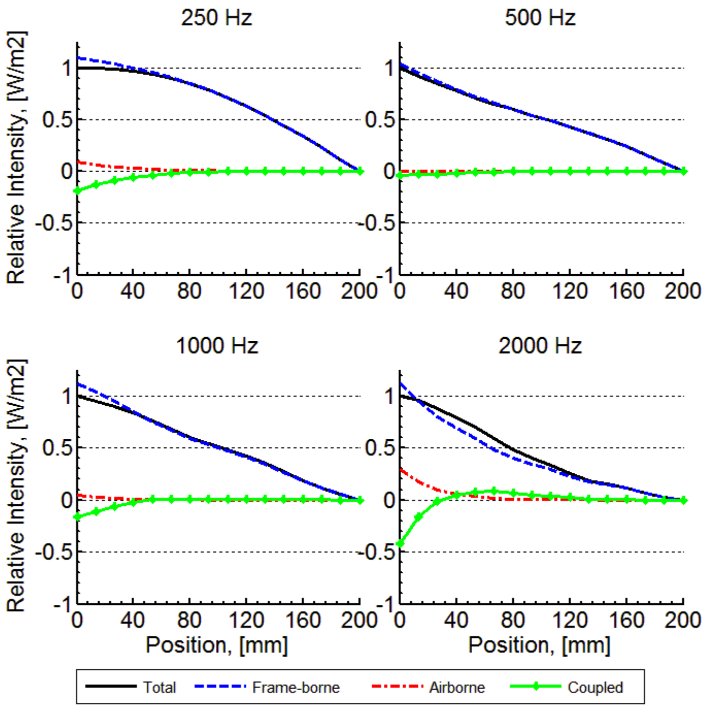
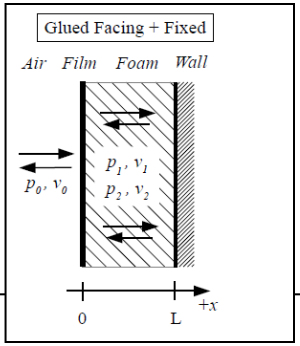
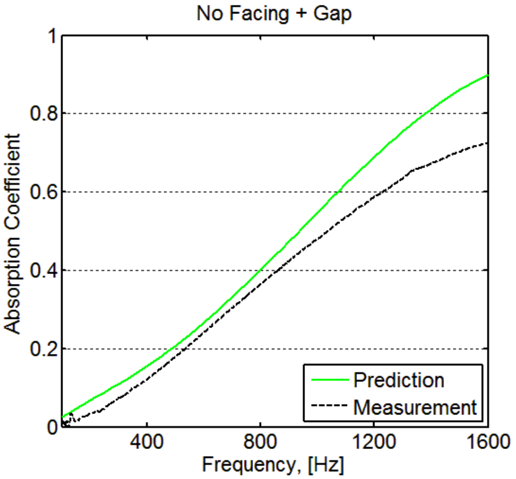
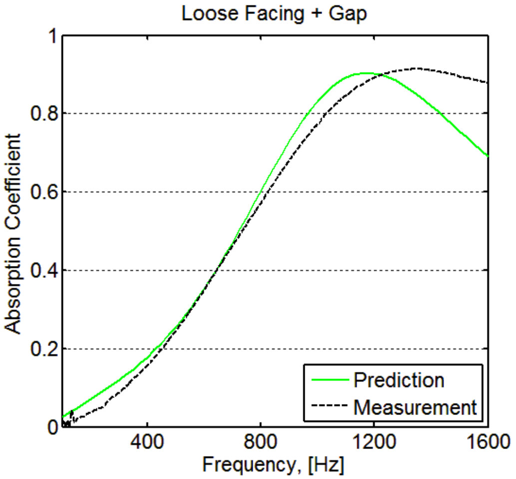
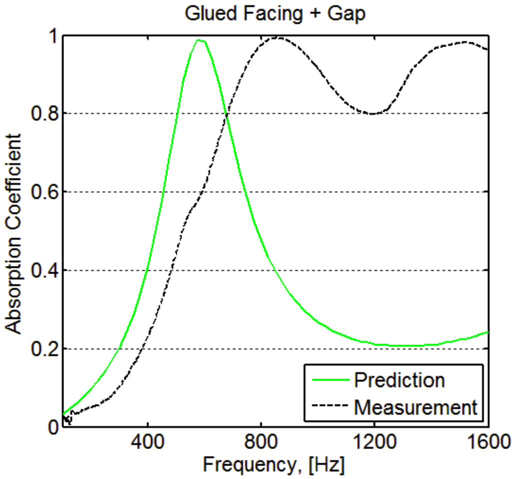


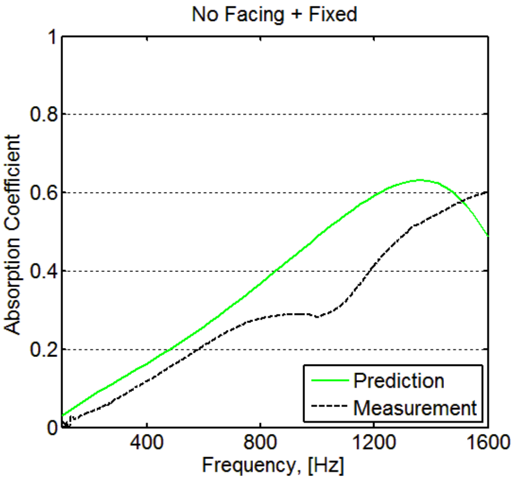


Predictions vs. Measurements


No Facing + Gap


Loose Facing + Gap


Glued Facing + Gap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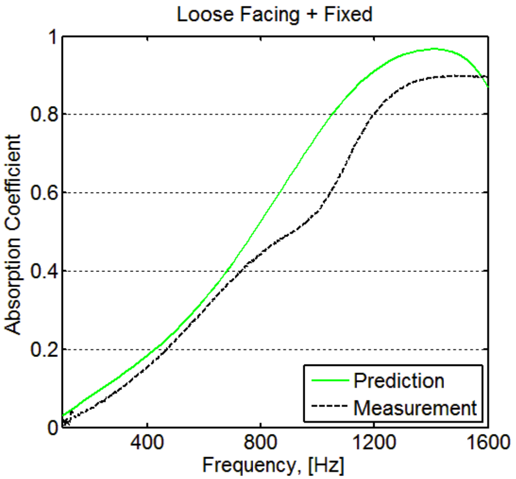
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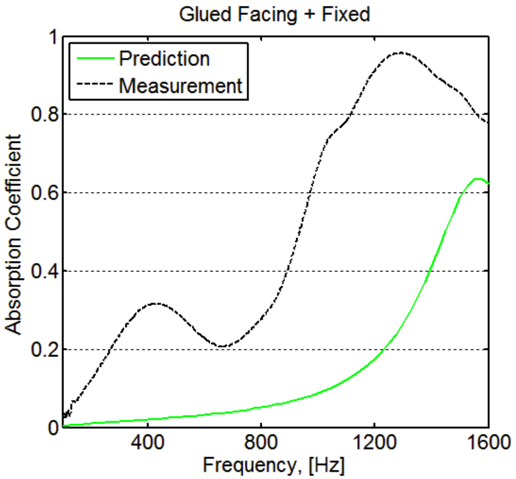
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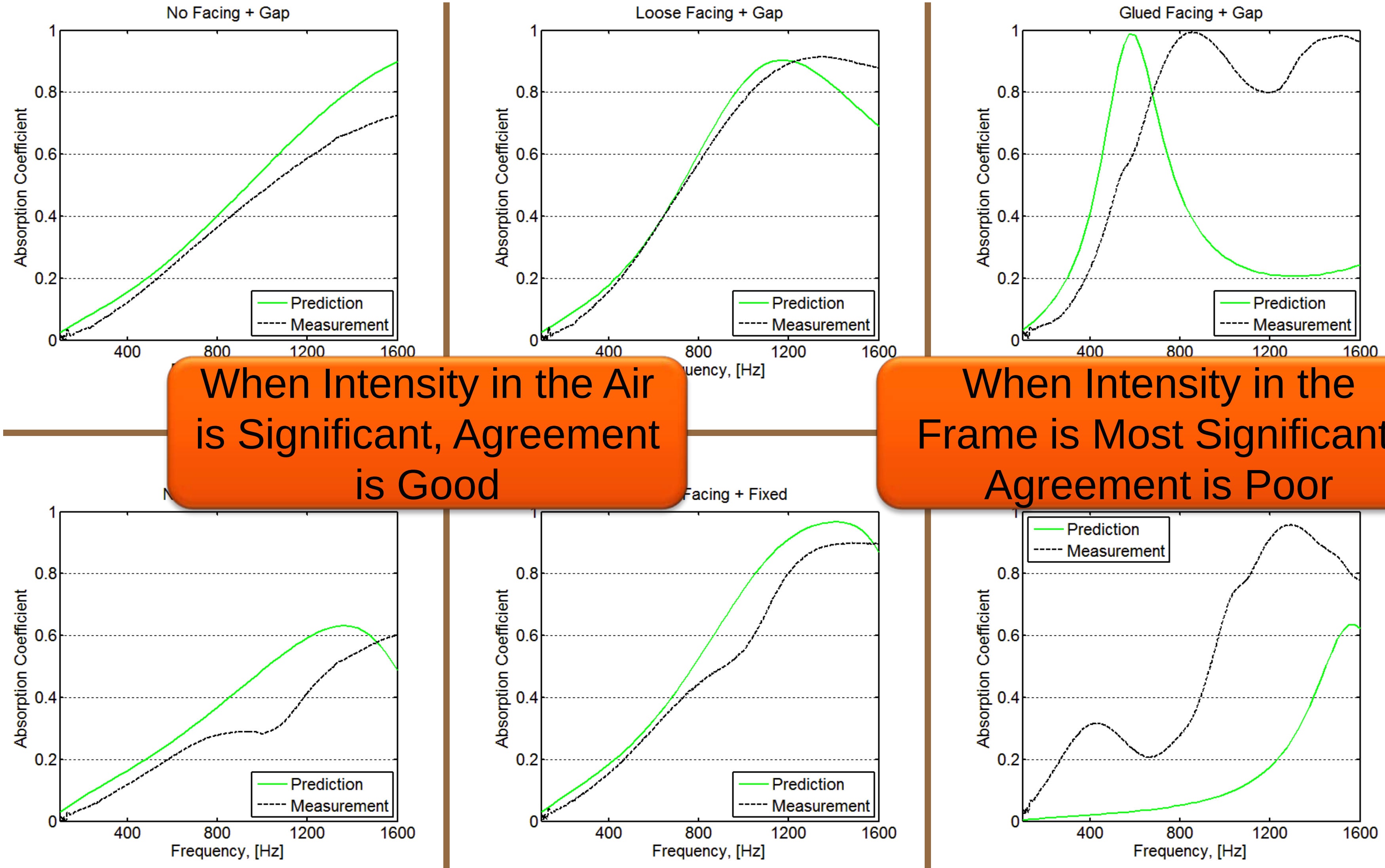
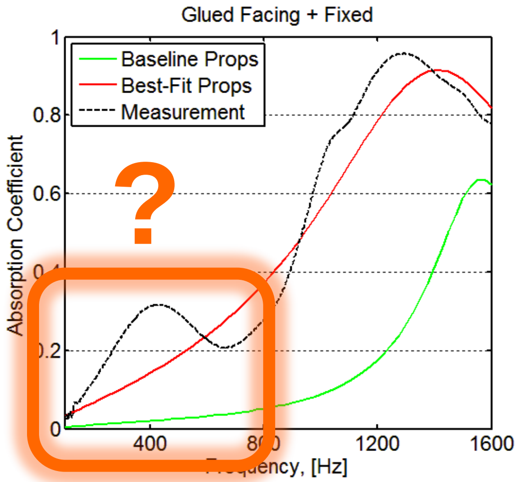
Predictions vs. Measurements

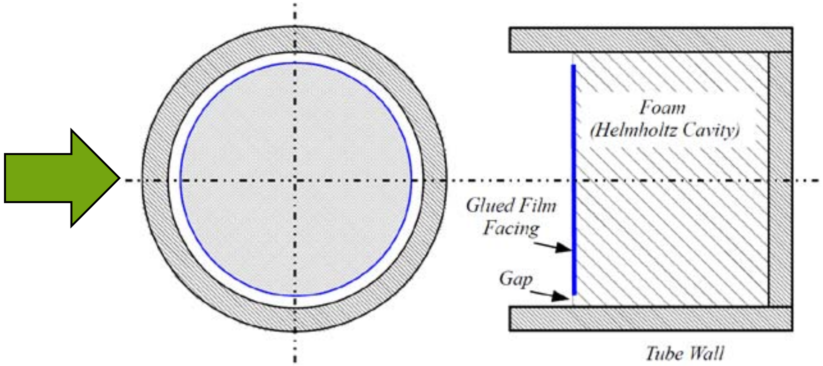


When Intensity in the Air is Significant, Agreement is Good

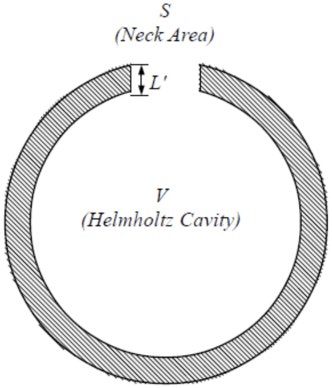
When Intensity in the Frame is Most Significant, Agreement is Poor

Helmholtz Resonator Effect





Mechanical Impedance	$z_m = R_r + j(\omega m - s/\omega)$
Mass	$m = \rho_0 S L'$
Stiffness	$s = \rho_0 c_0^2 S^2 / V$
Total Acoustic Impedance	$z = 1/(1/z_H + 1/z_f)$



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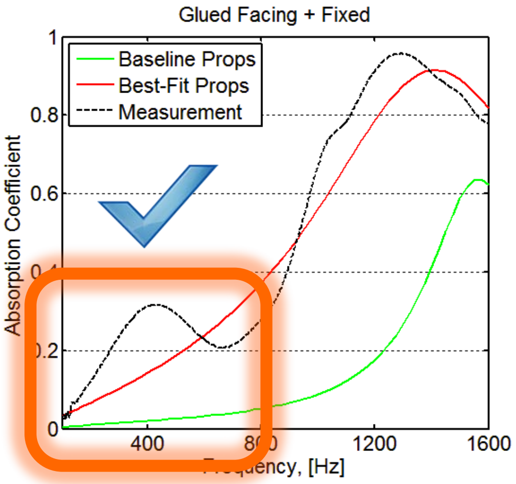
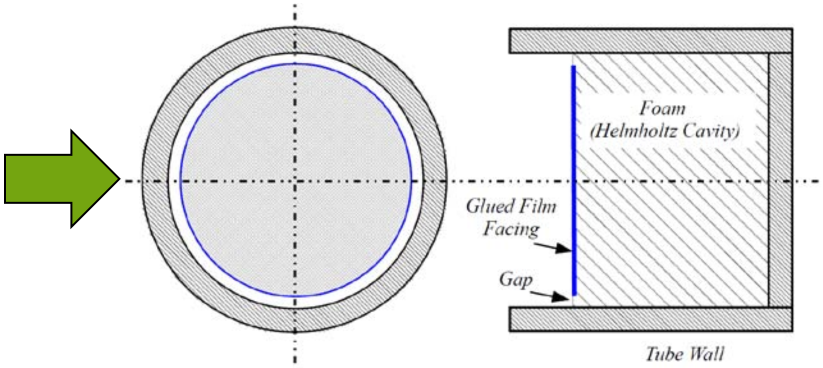
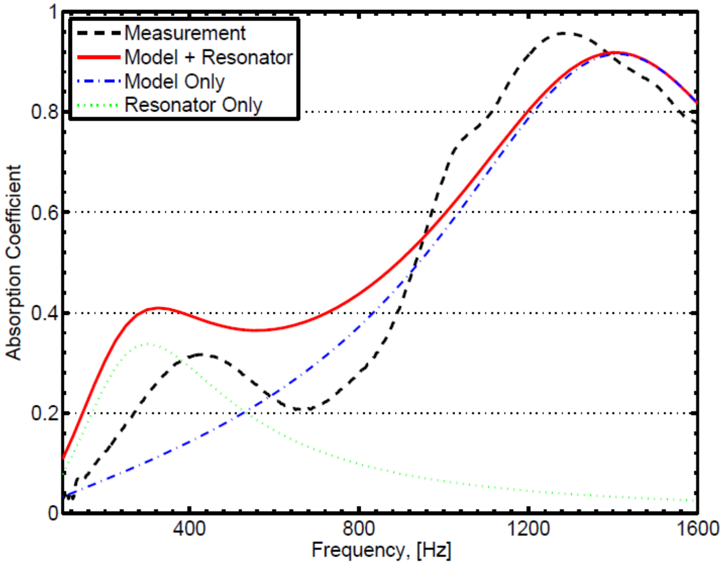




Combined Foam + Helmholtz Resonator System is Similar to Measured System



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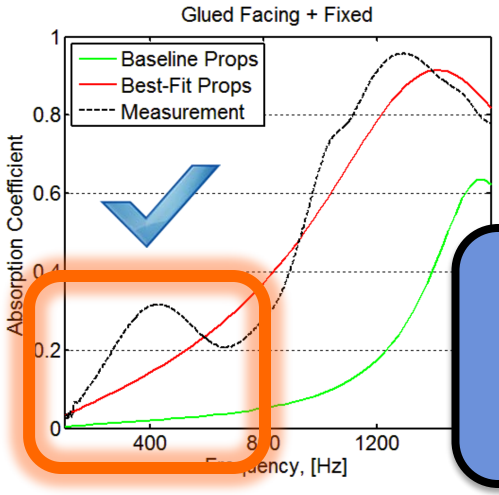
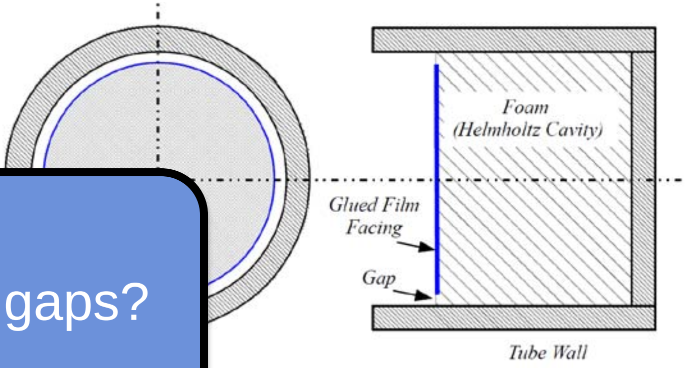
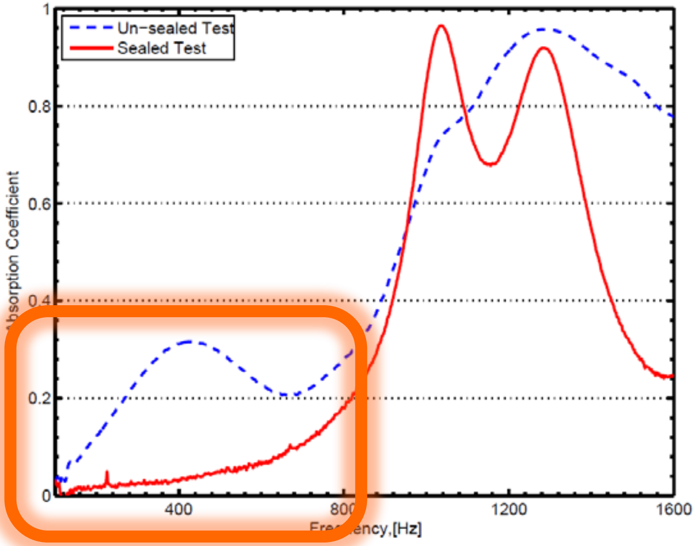




But is it really due to edge gaps?

Measured Glued Facing + Fixed with Edge Sealed



Summary

- **Measured acoustical properties of melamine foam using two- and four-microphone standing wave tube techniques**
- **Developed a 1-dimensional formulation of the Biot theory for wave propagation in poroelastic materials**
- **Explored the intensity distribution among the two phases & two wave types**
  - Intensity distribution is dependent on the imposed boundary conditions
  - Frame-borne wave type decays much more slowly than airborne wave type
  - Exchange between wave types is more significant at higher frequencies
- **Compared model predictions with measurements**
- **Determined that model-measurement agreement appears to be dependent on the boundary conditions applied to the foam sample**
  - Agreement appears to deteriorate when Frame plays a larger role

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Thank you!

Questions?

Measurement: Sample Fit

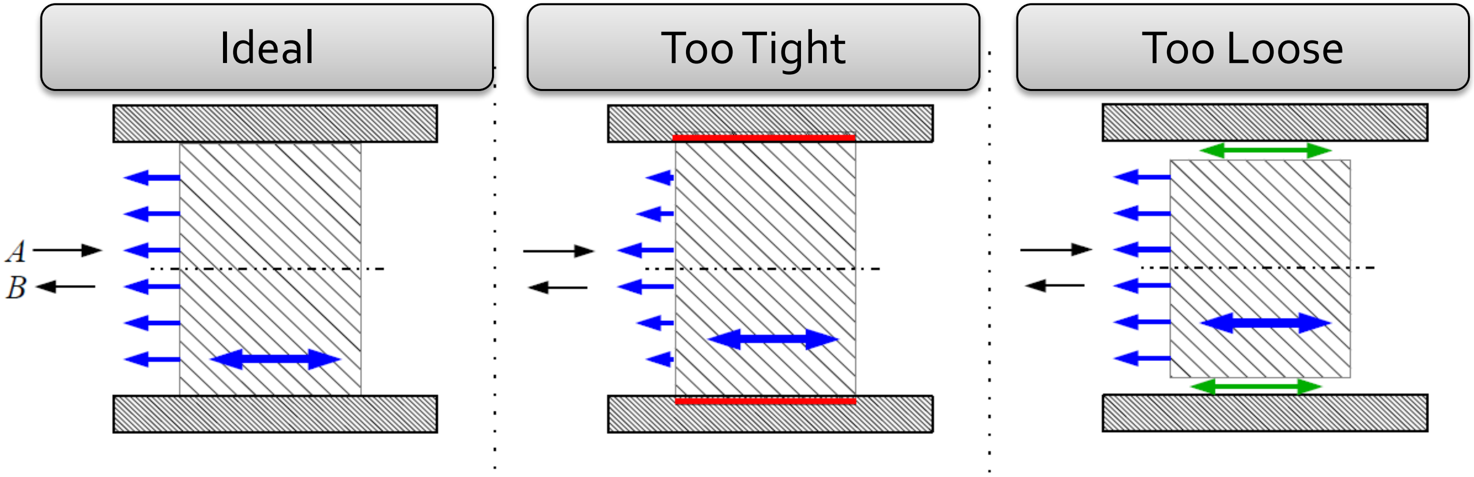
Ideally:

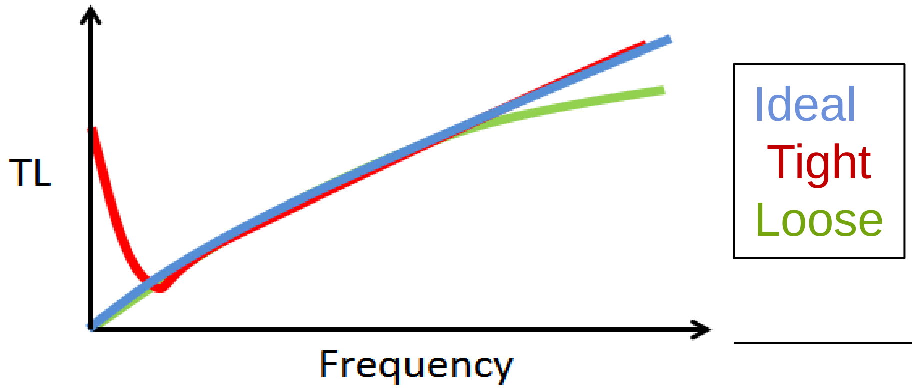
Sample
=
Layer of infinite lateral extent

Reality:

Sample either over- or under-sized

- Constrained
- Leaky





Field Variables

Frame

Particle Velocity

$$v_1 = j\omega\left(C_1 e^{jk_1 x} + C_2 e^{-jk_1 x} + C_3 e^{jk_2 x} + C_4 e^{-jk_2 x}\right)$$

Acoustic Pressure

$$p_1 = -f_1 C_1 e^{jk_1 x} + f_1 C_2 e^{-jk_1 x} - f_2 C_3 e^{jk_2 x} + f_2 C_4 e^{-jk_2 x}$$

Air

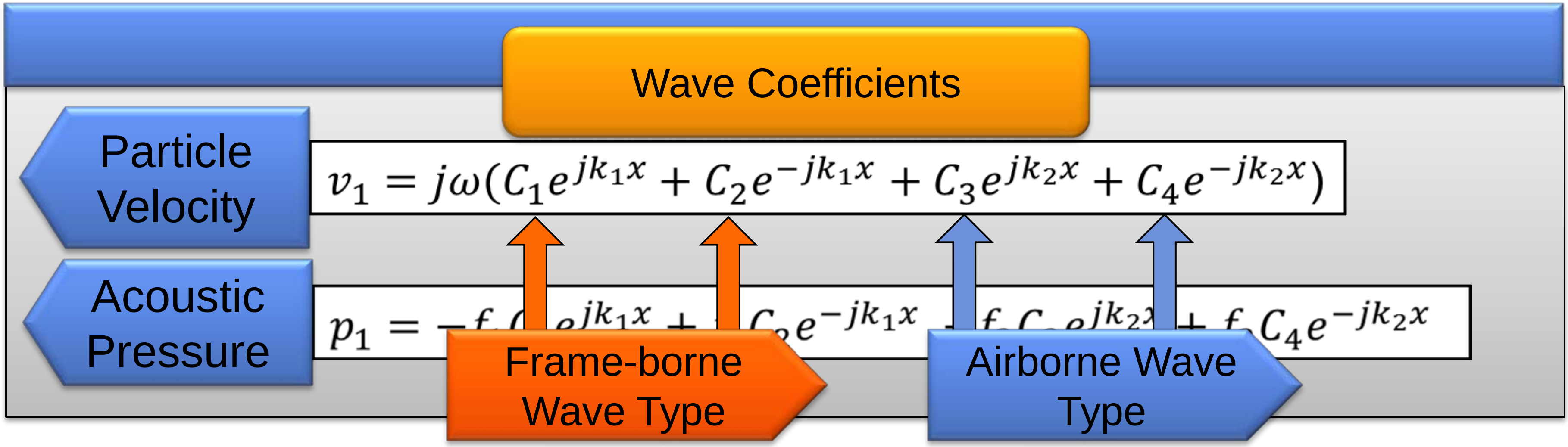
Particle Velocity

$$v_2 = j\omega\left(b_1 C_1 e^{jk_1 x} + b_1 C_2 e^{-jk_1 x} + b_2 C_3 e^{jk_2 x} + b_2 C_4 e^{-jk_2 x}\right)$$

Acoustic Pressure

$$p_2 = -g_1 C_1 e^{jk_1 x} + g_1 C_2 e^{-jk_1 x} - g_2 C_3 e^{jk_2 x} + g_2 C_4 e^{-jk_2 x}$$

Field Variables



Wave Coefficients

Particle Velocity

$$v_1 = j\omega(C_1e^{jk_1x} + C_2e^{-jk_1x} + C_3e^{jk_2x} + C_4e^{-jk_2x})$$

Acoustic Pressure

$$p_1 = -f\,C\;e^{jk_1x} + \;C_2e^{-jk_1x} - f\,C\;e^{jk_2x} + f\,C_4e^{-jk_2x}$$

Frame-borne Wave Type

Airborne Wave Type

Air

Particle Velocity

$$v_2 = j\omega(b_1C_1e^{jk_1x} + b_1C_2e^{-jk_1x} + b_2C_3e^{jk_2x} + b_2C_4e^{-jk_2x})$$

Acoustic Pressure

$$p_2 = -g_1C_1e^{jk_1x} + g_1C_2e^{-jk_1x} - g_2C_3e^{jk_2x} + g_2C_4e^{-jk_2x}$$

Intensity

$$I_{\text{foam}} = \frac{1}{2}\text{Re}\{p_1 v_1^*\} + \frac{1}{2}\text{Re}\{p_2 v_2^*\}$$

We have expressions for the acoustic velocity and pressure in each phase

Frame

Particle Velocity

$$v_1 = j\omega\left(C_1 e^{jk_1 x} + C_2 e^{-jk_1 x} + C_3 e^{jk_2 x} + C_4 e^{-jk_2 x}\right)$$

Acoustic Pressure

$$p_1 = -f_1 C_1 e^{jk_1 x} + f_1 C_2 e^{-jk_1 x} - f_2 C_3 e^{jk_2 x} + f_2 C_4 e^{-jk_2 x}$$

Air

Particle Velocity

$$v_2 = j\omega\left(b_1 C_1 e^{jk_1 x} + b_1 C_2 e^{-jk_1 x} + b_2 C_3 e^{jk_2 x} + b_2 C_4 e^{-jk_2 x}\right)$$

Acoustic Pressure

$$p_2 = -g_1 C_1 e^{jk_1 x} + g_1 C_2 e^{-jk_1 x} - g_2 C_3 e^{jk_2 x} + g_2 C_4 e^{-jk_2 x}$$

Intensity

$$I_{\text{foam}} = \frac{1}{2}\text{Re}\{p_1 v_1^*\} + \frac{1}{2}\text{Re}\{p_2 v_2^*\}$$

Apply the field variable equations, expand and separate terms by wave type

Frame-borne

Solid

$$I_{\text{frame } k_1} = \frac{1}{2}\text{Re}\{(j\omega f_1)[C_1C_1^* e^{-2\beta_1 x} + C_1C_2^* e^{j2\alpha_1 x} - C_2C_2^* e^{2\beta_1 x} - C_1^* C_2 e^{-j2\alpha_1 x}]\}$$

Fluid

$$I_{\text{pore } k_1} = \frac{1}{2}\text{Re}\{(j\omega g_1 b_1^*)[C_1C_1^* e^{-2\beta_1 x} + C_1C_2^* e^{j2\alpha_1 x} - C_2C_2^* e^{2\beta_1 x} - C_1^* C_2 e^{-j2\alpha_1 x}]\}$$

Airborne

Solid

$$I_{\text{frame } k_2} = \frac{1}{2}\text{Re}\{(j\omega f_2)[C_3C_3^* e^{-2\beta_2 x} + C_3C_4^* e^{j2\alpha_2 x} - C_4C_4^* e^{2\beta_2 x} - C_3^* C_4 e^{-j2\alpha_2 x}]\}$$

Fluid

$$I_{\text{pore } k_2} = \frac{1}{2}\text{Re}\{(j\omega g_2 b_2^*)[C_3C_3^* e^{-2\beta_2 x} + C_3C_4^* e^{j2\alpha_2 x} - C_4C_4^* e^{2\beta_2 x} - C_3^* C_4 e^{-j2\alpha_2 x}]\}$$

Coupled

Solid

$$I_{\text{frame coupled}} = \frac{1}{2}\text{Re}\{j\omega f_1 X_1 + j\omega f_2 X_2\}$$

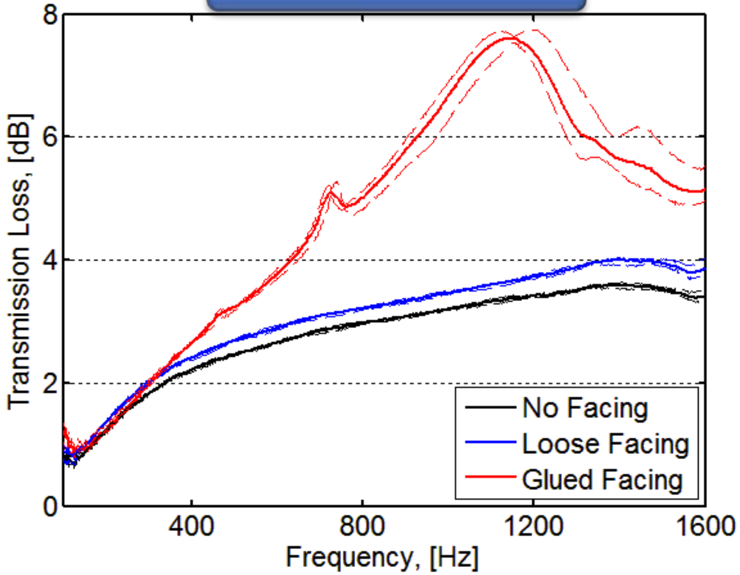
Fluid

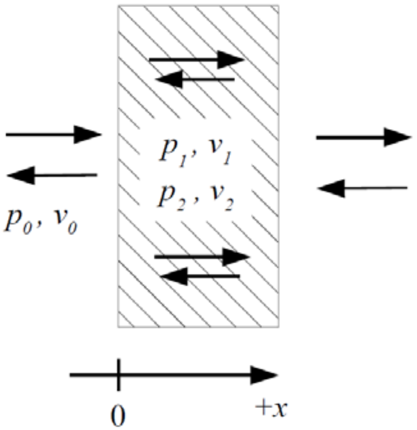
$$I_{\text{pore coupled}} = \frac{1}{2}\text{Re}\{j\omega g_1 b_2^* X_1 + j\omega g_2 b_1^* X_2\}$$

$$X_1 = C_1C_3^* e^{jx(k_1-k_2^*)} + C_1C_4^* e^{jx(k_1+k_2^*)} - C_2C_3^* e^{-jx(k_1+k_2^*)} - C_2C_4^* e^{-jx(k_1-k_2^*)}$$

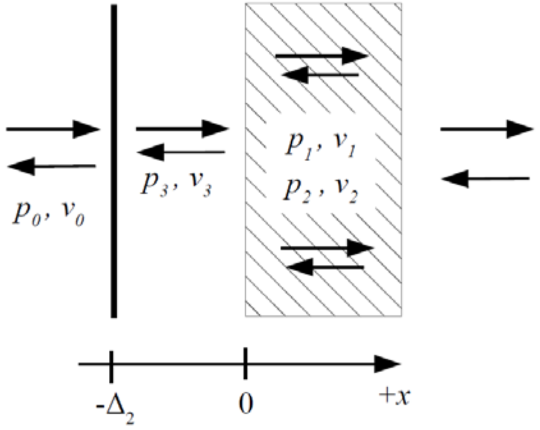
$$X_2 = C_1^*C_3 e^{jx(k_2-k_1^*)} + C_2^*C_3 e^{jx(k_2+k_1^*)} - C_1^*C_4 e^{-jx(k_2+k_1^*)} - C_2^*C_4 e^{-jx(k_2-k_1^*)}$$

TL Measurement Results

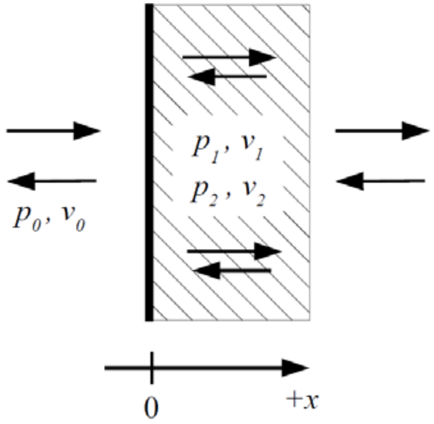

25 mm – 12 trials



F: No Facing
R: No Facing



F: Loose Facing
R: No Facing

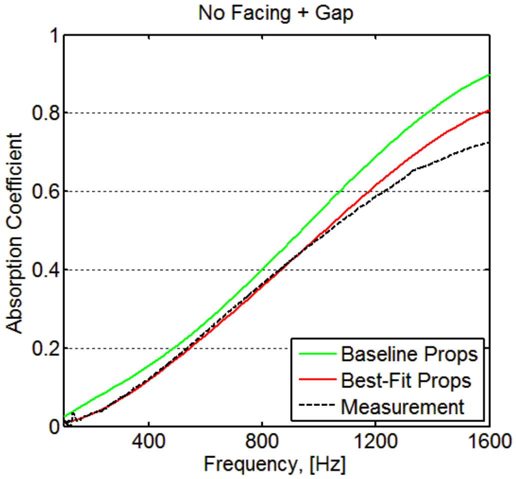


F: Glued Facing
R: No Facing

Parameter Estimation

Improve Prediction-Measurement Agreement

- Uncertain of parameter values:
  - Porosity
  - Flow Resistivity
  - Tortuosity
  - Characteristic Length
  - Shear Modulus
  - Loss Factor
- Allow these to take a range of values & predict absorption coefficients
- Find the parameter set that best predicts the measured absorption coefficient

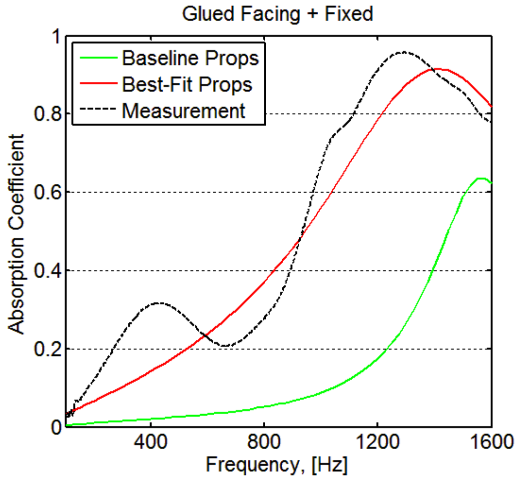


Flow Resistivity

Thermal C.L.

Shear Modulus

Loss Factor



Parameter Estimation

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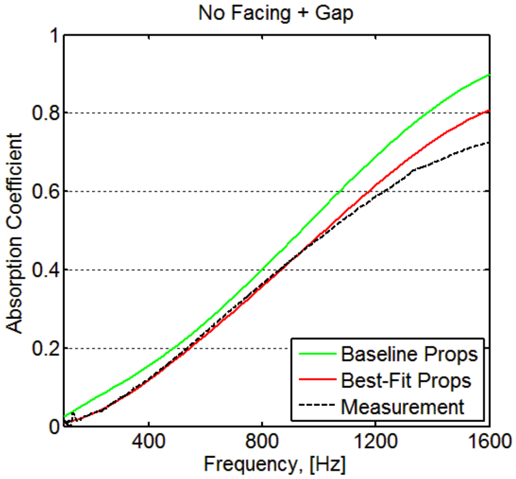


Flow Resistivity

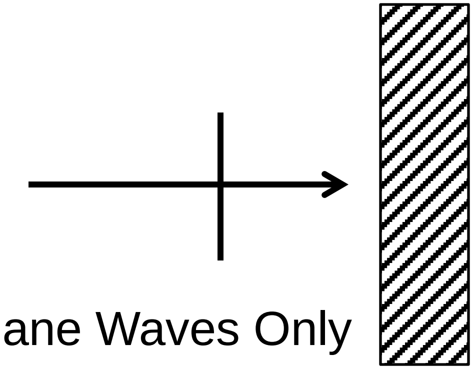
Thermal C.L.

Shear Modulus

Loss Factor

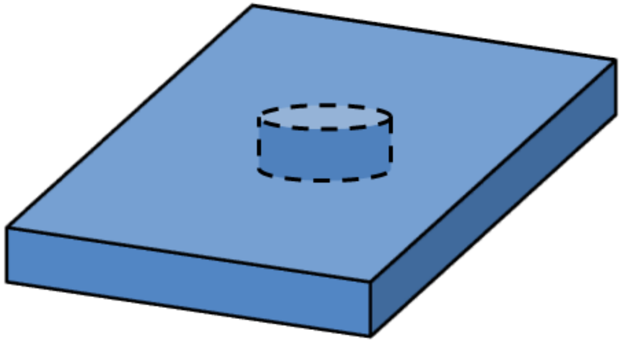
Measurement: Theory

◆Key Assumptions:



Plane Waves Only

Finite Sample = Layer of Infinite Lateral Extent



Sound Field:

$$P = Ae^{-jk_0x} + Be^{jk_0x}$$

Unknowns

$$P = e^{-jk_0x} + Re^{jk_0x}$$

$$v = \frac{1}{\rho c}\left(e^{-jk_0x} - Re^{jk_0x}\right)$$

Pressure at 2 Points:

$$P_1 = Ae^{jk_0(L_1+s_1)} + Be^{-jk_0(L_1+s_1)}$$

$$P_2 = Ae^{jk_0L_1} + Be^{-jk_0L_1}$$

Acoustical Properties:

$$R = B/A \qquad \alpha = 1 - |R|^2$$

Poroelastic Material Modeling

Zwikker & Kosten:

- Extension of Kirchhoff's theory for cylindrical pore sections
- Found there are two waves that propagate in an elastic porous material
- Elastic frame, normal incidence
- Rosin, Lauriks et al., Bolton

Biot:

- Generalized theory using 3-D continuum mechanics, allows for 3-D wave propagation by 2 dilatational and 1 shear wave
- Widely applied: soils, foams
- Described & modified by: Allard and Atalla, Bolton et al.

Modifications of Biot Theory:

- Modifications of Coupling Terms:
  - Bolton, Johnson et al., Pride et al., Wilson, Kino
  - Bolton, Champoux-Allard, Lafarge, Kino

Mass & Elastic Coefficients

Elastic Coefficients

$$N = G$$

$$P = A + 2G + K_f(1-\phi)^2/\phi$$

$$Q = (1-\phi)K_f$$

$$R = \phi K_f$$

Mass Coefficients

$$\rho_{11} = \rho_1 + \rho_a + b/j\omega$$

$$\rho_{12} = -\rho_a - b/j\omega$$

$$\rho_{22} = \phi\rho_0 + \rho_a + b/j\omega$$

$$\frac{\partial \sigma_x}{\partial x} = \rho_1 \frac{\partial^2 u_x}{\partial t^2} + \rho_a \frac{\partial^2}{\partial t^2}(u_x - U_x) + b\frac{\partial}{\partial t}(u_x - U_x)$$

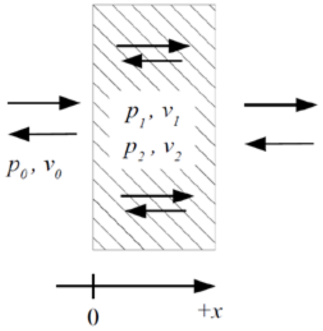
Inertial Coupling Term

$$\rho_a = \phi\rho_0(\alpha_\infty - 1)$$

Viscous Coupling Term

$$b = \sigma\phi^2 G(w)/j\omega$$

Boundary Condition Constraints

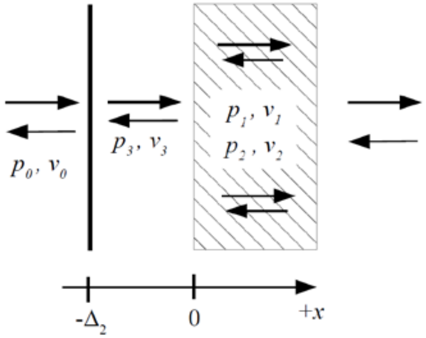


No Facing

$$v_{\text{ext}} = (1-\phi)v_1 + \phi v_2$$

$$(1-\phi)p_{\text{ext}} = p_1$$

$$\phi p_{\text{ext}} = p_2$$

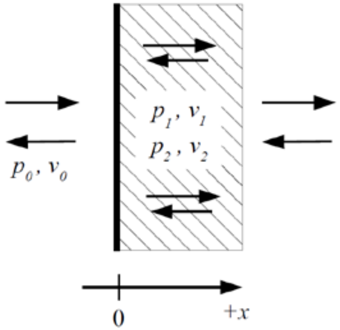


Loose Film Facing

$$v_{\text{side1}} = v_{\text{film}}$$

$$v_{\text{side2}} = v_{\text{film}}$$

$$p_{\text{side1}} - p_{\text{side2}} = M_{s1}\frac{d}{dt}v_{\text{film}}$$



Glued Film Facing

$$v_{\text{ext}} = v_{\text{film}}$$

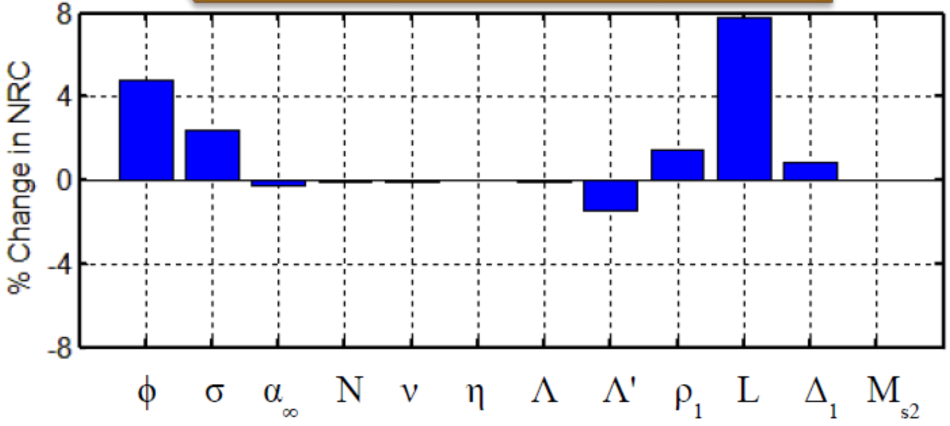
$$v_1 = v_{\text{film}}$$

$$v_2 = v_{\text{film}}$$

$$p_{\text{ext}} - p_1 - p_2 = M_{s2}\frac{d}{dt}v_{\text{film}}$$

Sensitivity to Model Inputs

No Facing + Gap Configuration



Key Findings:

- Sample Depth
- Porosity
- Flow Resistivity
- Thermal Characteristic Length
- Bulk Density

Key Findings:

- Facing Area Density
- Porosity
- Bulk Density
- Shear Modulus
- Poisson’s Ratio
- Loss Factor

Glued Facing + Fixed Configuration