ESSAYS IN POLITICAL ECONOMY

BY

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DISSERTATION

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Abstract

In this dissertation I consider the application of an economic framework to situations outside of traditional economics. I look at two areas: the first is the behavior of politicians in dynamic elections and the second is addiction in online video games. I show how the power of economic incentives shapes phenomena that are outside the realm of traditional economics.

Chapter 1 analyzes the incentives of politicians when their behavior in office affects the severity of issues upon which their re-election may depend. Elections are often about which candidate can best deal with the most pressing national issues. The severity of issues, however, is endogenous and depends on the actions taken by earlier politicians. If a politician has a comparative advantage in dealing with a particular issue, then this endogeneity creates an agency problem. An incumbent has an incentive to manipulate the severity of issues to create a more favorable environment for re-election. To analyze this incentive problem I develop a dynamic elections model in which candidates from differentiated parties have a comparative advantage in dealing with a specific issue. The incumbent chooses how much to invest in each issue, which endogenously determines the severity and relative importance of these issues to the voters in the future. I show that when politicians care about re-election they invest inefficiently. An incumbent invests less in the issue in which he has an advantage and more in the issue of his opponent. Because of this behavior, issues improve slower over time, are more severe in the stationary state, and parties remain in control of an office longer than is socially optimal.

Chapter 2 uses a unique panel dataset to analyze rational addiction in an online video game. As playing video games becomes more common among both children and adults, the extent to which video games can and should be considered addictive has become controversial. To look at this question from an economics perspective, I develop a model of rational addiction for video games. Using a unique and very large panel dataset collected from the online video game *Team Fortress 2* I show evidence of rational addiction: past and future consumption of this game play a significant role in determining how much an individual plays today. My data are rich enough that by estimating the model separately for each individual, I am able to identify and characterize potential addicts in a way that is consistent with rational addiction. The individuals identified as addicts in this way are very different than from using simple metrics, for instance by only looking at how much an individual plays, to define addiction. Estimating the model separately for each individual also provides evidence of significant heterogeneity among individuals, and suggests an individual-specific approach to analyzing addiction. Finally, I modify the model to allow for learning or endogenous skill development and provide evidence of a *skill-playtime feedback loop*: by playing today an individual improves his skill which reinforces his decision to play in the future and feeds back into addiction.

To my parents, with whom my education began.

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CHAPTER 1

DYNAMIC ELECTIONS WITH INVESTMENT IN ENDOGENOUS ISSUES

1.1 Introduction

Suppose voters elect a politician primarily because he is skilled at dealing with a particularly severe national issue. Once in office, if he focuses policy on this issue, then its condition should improve over time. In the next election this issue will be relatively less severe or important to voters, and the politician may have reduced his own electoral advantage. If a politician cares about re-election, then this endogeneity gives him an incentive to strategically manipulate the importance of issues. To analyze this incentive problem I emphasize three features of national issues in elections: (1) The severity of national issues is endogenous and depends on the policy of earlier politicians, (2) The candidate who can best deal with issues that are severe has an advantage with voters, and (3) Parties and their candidates are differentiated and have a comparative advantage in dealing with certain issues. The combination of these features creates an agency problem and an incentive for politicians to manipulate the severity of national issues. This structure results in strategic behavior that is important because it affects how politicians invest in issues that are important to voters, while also changing the dynamics of office-control and the severity of issues in the long-term. In this chapter I focus on analyzing this incentive problem.

When the severity of issues is endogenous, resolving the issues you were elected to resolve may reduce your chance of being re-elected. As an example, consider when George H.W. Bush ran for re-election. In early 1992 he was considered unbeatable due to his foreign policy successes. He presided over the end of the Cold War, formed an international coalition that was generally successful in Iraq, and at one point had approval ratings as high as 90% (Agiesta, 2007). Despite these accomplishments, he lost the election by a significant margin to Bill Clinton. Bush's loss was primarily attributed to the national focus on the economy rather than foreign policy in the election.[1] The phrase "It's the economy, stupid!" was popularized by Clinton's campaign. One reason foreign policy was relatively less important was that it had already been dealt with effectively by the president.

If improving an issue that a politician is skilled at means voters may no longer need him, then he has an incentive not to improve this issue as effectively as he could. In 2008 Newt Gingrich said that one of the great tragedies of the George W. Bush administration was that they were too successful at preventing foreign attacks. He even suggested say that the administration ought to have allowed an attack to get through once in awhile to remind us of the danger.[2] Though this sentiment is an extreme view, Gingrich's statement illustrates the incentive problem I focus on. When a politician "owns" an issue, or has an advantage dealing with a particular issue, then this endogeneity creates an agency problem and an incumbent has an incentive to manipulate the severity of issues and increase his chance of re-election.

To analyze this incentive problem I develop a dynamic elections model in which the relative importance of issues to a representative voter depends on the past policies of politicians. I assume there are two stochastic national issues, such as the economy and national security, and the role of a politician is to invest in these issues. Both issues naturally worsen over time, but greater investment reduces their severity. Politicians come from two differentiated parties, each of which has a comparative advantage in the social cost of investing in an issue. Throughout this paper I will use the United States as an example of this political structure.

[1] For further analysis see Apple (1992)

[2] The full quote is as follows: "This is, by the way, one of the great tragedies of the Bush administration. The more successful they've been at intercepting and stopping bad guys, the less proof there is that we're in danger. And therefore, the better they've done at making sure there isn't an attack, the easier it is to say, well, there was never going to be an attack anyway. And its almost like they should every once in a while have allowed an attack to get through just to remind us." - Newt Gingrich at a book talk in Huntington, NY, April 2008.

Based on Gallup Polls data from 2002 to 2009 shown in figure 1.1, I assume Republicans are better at investing in national security while Democrats are better at investing in the economy.

I first solve a two-period version of my model analytically to capture the intuition behind behavior. As a benchmark I look at the socially optimal investment, which is the optimal investment when politicians care about the utility of the voter and are not concerned with re-election. Even when the incentives of politicians are in line with those of the voter, there are some interesting results. For example, when an issue worsens it is socially optimal for the elected politician, regardless of his party, to invest more in both issues. This result means that when a Democrat is in office and the economy worsens he should invest more in not just the economy but also national security.

While the socially optimal investment provides a benchmark, I am primarily interested in behavior when politicians care about re-election. In this case an incumbent has an incentive to manipulate the severity of issues to improve his chance of re-election. He does this by

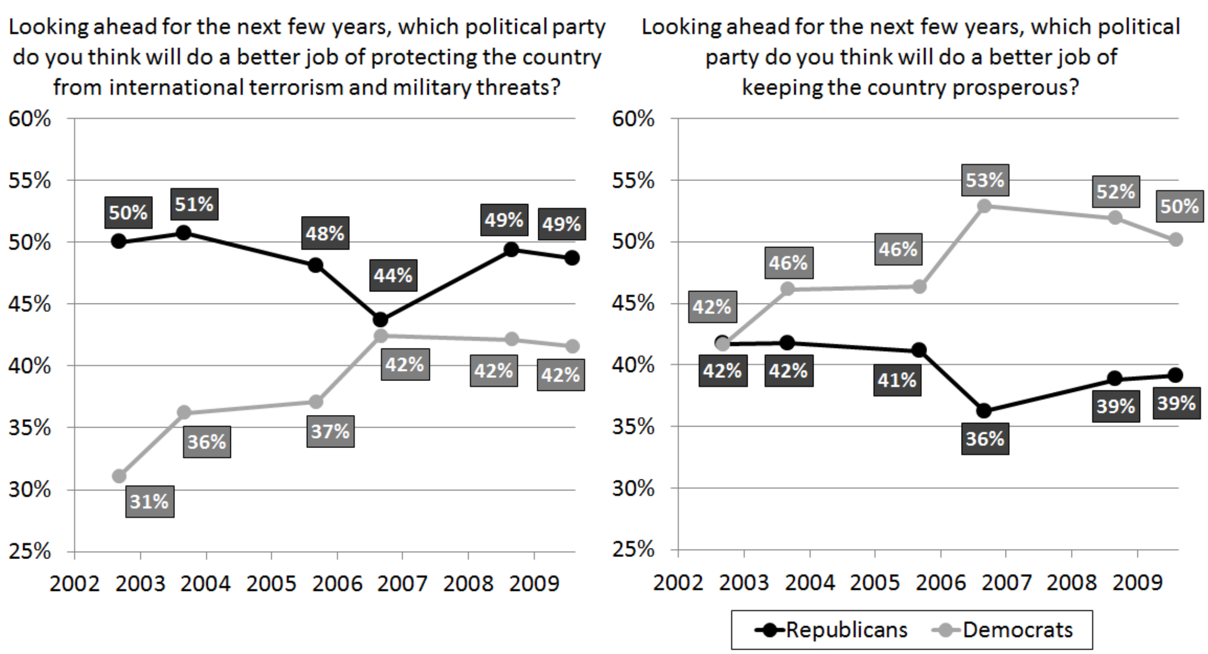


Figure 1.1: Polling Data from the Gallup Governance Series from 2002 to 2009 showing which party voters believe will do a better job handling two important national issues. Answers of "no difference," "don't know," and "refused" account for the remaining percent.

reducing investment in the issue in which he has an advantage and investing more in the other issue. This strategic behavior slows the improvement of the issue in which he specializes to allow for his comparative advantage to persist longer. At the same time, by investing more in the issue of his opponent he prevents that issue from becoming too severe and the opposing party from gaining an advantage. The incentive for manipulation is the strongest whenever an election outcome is expected to be close. When one party has an overwhelming advantage the cost of manipulating issues may outweigh the benefits. On the other hand, if an election is expected to be very close the incumbent has a strong incentive to behave strategically.

While the two-period model captures the intuition behind behavior and allows for analytical results I also extend the model to a finite, but arbitrarily long time horizon. I solve the longer time horizon numerically and show similar strategic behavior. Along the expected path the issue in which the current elected politician specializes improves slower than socially optimal, and control of office changes less frequently than it should. In the stationary state one or both issues may be more severe than socially optimal. My model also provides an endogenous explanation of the "incumbency effect" or the observation that incumbents tend to have an advantage over challengers in an election.

This chapter lies at the intersection of two main areas in the literature. The first is the topic of issue ownership and party differentiation. Petrocik (1996) presents a 1980 case study in the United States that shows there is significant difference in how voters perceive the competence of Democrats and Republicans in handling certain issues. Egan (2008) provides evidence that these perceived differences are greater than can be explained purely by policy preferences. Differentiated parties may lead to strategic behavior and Krasa and Polborn (2010) present a model in which candidates from differentiated parties compete for election and the elected politician chooses the amount of resources to allocate to each of two public goods. In equilibrium the winning candidate allocates less resources than preferred by the majority to the issue in which he has a comparative advantage. While this is similar to my result, the fundamental reasoning is different. In their model, candidates can commit to platforms, while in mine, the incumbent chooses policy to influence future elections.

The second relevant area in the literature is strategic decision making by politicians in a dynamic environment. Dellis (2009) analyzes a model of issue salience with heterogenous voters in which the elected politician is able to influence the importance of issues to the voter. A politician may have an incentive to make salient an issue on which he has an electoral advantage or reduce the salience of an issue on which he is electorally weak to increase the chance of re-election. While the results are similar Dellis focuses on a citizen-candidates in a deterministic environment. In contrast, I focus on the endogenous relationship between the choices of politicians and the severity of issues. Instead of looking at the evolution of issues, an alternative would be to consider the evolution of reputation. In a related paper Martinez (2009) analyzes a dynamic model where politicians develop reputations based on performance. The reputation of a politician conveys to the voter information about their underlying ability. Over time the voter can learns the ability of a politician and the the politician may have an incentive to manipulate this learning process before an election to improve his chance of re-election. Another related paper is Hess and Orphanides (1995) who present a model in which the elected politician provides a public good and faces the choice to go to war or not. When an incumbent becomes less likely to be re-elected on other issues, he may strategically begin an avoidable conflict in order to reveal his ability at war and improve his chance of re-election.

Finally there is some empirical support for the strategic manipulation of the severity of issues predicted by my model. Berrebi and Klor (2006) provides evidence of terrorist factions in Israel that choose to suppress or allow terrorist attacks in order to influence the political party likely to win election. Wilkinson (2006) shows examples of local government in India allowing or suppressing unrest before an election depending on the likely consequences of the unrest on the vote outcome.

The main contribution of this chapter to the literature on dynamic elections is a model in which the relative importance of issues to voters evolves endogenously and stochastically over time based on the past actions by politicians. I characterize the equilibrium, socially optimal investment and investment when politicians care about re-election analytically in a two-period model, as well as numerically for an arbitrarily long time horizon. My model predicts novel behavior on the part of politicians. When politicians care about re-election they invest less than socially optimal in the issue in which they specialize and more than socially optimal in the issue in which their opponent specializes.

The rest of this paper is organized as follows. Section 1.2 presents a two-period version of the model which allows me to focus on the intuition behind behavior. Section 1.3 solves this model and analytically characterizes the socially optimal behavior of politicians and the behavior of politicians who care about re-election. In this section I also discuss some of the special cases that can arise. Finally, I extend and numerically solve the model for a finite, but arbitrarily long time horizon. Section 1.4 provides some concluding comments.

1.2 Model

1.2.1 Setup

There is a two period sequence of repeated elections between candidates from two differentiated parties $\theta \in \{L, R\}$, a left-party and a right-party. In both periods (elections) each party provides a candidate for office and a representative voter chooses (elects) one of these candidates to hold office. The primary concern of the representative voter is the severity of two independent issues. Let $\omega_1, \omega_2 \in \mathbb{R}_+$ represent the severity of each issue at the beginning of the period before a politician is elected and is able to improve the issue. Larger values of ω_i represent increased severity (i.e. worsening) of the issue at the start of the period. For ease of discussion, I interpret the first issue ω_1 as "economic stability" and the second issue ω_2 as "national security." For example, if $\omega_1 > \omega_2$ then economic stability is more severe (i.e. worse) than national security at the start of a period.

The role of an elected politician is to choose social investment in both issues, $x_1, x_2 \in \mathbb{R}$. Higher values of x_i represent greater social investment and reduce the severity of issue i. If the severity of an issue at the start of the period is ω_i and the elected politician invests x_i then the severity of the issue after investment will be s_i, where:

$$s_i = \omega_i - x_i \tag{1.1}$$

If the elected politician chooses $x_1 > x_2$ then he invests more in the economy than national security, which improves the economy more than national security.[3]

While investment reduces the severity of an issue, it also has a social cost for the representative voter. The social cost of investment in an issue x_i is given by a party specific cost function $C_{i\theta}(x_i)$, which is increasing, convex, and twice continuously differentiable in $|x_i|$.[4] The heterogeneity between parties or their comparative advantage (issue ownership) comes from differences in this social cost function. Each party is specialized in one of the issues and their politicians have a lower marginal social cost of investing in that issue. If party θ has a comparative advantage in issue i then $\frac{\partial C_{i\theta}}{\partial x_i} < \frac{\partial C_{i,-\theta}}{\partial x_i}$. I assume that the L-party has a comparative advantage in the first issue while the R-party has a comparative advantage in the second issue. Finally, I assume symmetry or that $C_{1L}(x) = C_{2R}(x)$ and $C_{2L}(x) = C_{1R}(x)$.

To aid discussion I say that the economy (issue 1) is the advantaged issue for the L-party and the disadvantaged issue for the R-party. Likewise, national security (issue 2) is the disadvantaged issue for the L-party and the advantaged issue for the R-party. Note that all heterogeneity (comparative advantage) between parties is due to differences in the cost of investment in issues. An equivalent formulation would be differences in the efficacy of

[3]Making investment affect the severity of an issue additively is particularly convenient to discuss and interpret the behavior of each politician type. In principle investment could affect issue severity by $s_i = F(\omega_i, x_i)$ provided $\frac{\partial F}{\partial \omega_i} > 0$ and $\frac{\partial F}{\partial x_i} < 0$ and the results would be similar.

[4]Increasing and convex in the absolute value of x_i is important because a politician may choose negative investment which should still have a direct social cost.

investment, so that the same investment by the L-party on issue 1 reduces it more than the equivalent investment by the R-party.

In each period the two parties are also characterized by a random but persistent net-valence term v, which is endowed to politicians from both parties. This net-valence term represents the relative attractiveness to the representative voter of one candidate compared to the other in ways not captured by the model, such as appearance, likability and positions or abilities on other issues outside the model. Positive net-valence ($v > 0$) will be interpreted as higher exogenous attractiveness for the politician from the R-party, while negative net-valence ($v < 0$) will be interpreted as higher exogenous attractiveness for the L-party. To ease discussion of changes in net-valence, I define the net-valence advantage of party θ as v_θ where $v_R = v = -v_L$.

There is a single representative voter who is far-sighted. In each period this voter elects a politician to maximize the sum of her discounted lifetime expected utility. The voter cares about the severity of each issue after the politician invests in them, the social cost of this investment and the net-valence of the politician. The voter's one-period utility when a politician from the party θ with a net-valence v is elected and invests x_1, x_2 to improves the severity of the issues to $s_1 = \omega_1 - x_1$ and $s_2 = \omega_2 - x_2$ is:

$$u_\theta(s_1, s_2, x_1, x_2, v) \equiv -\rho(s_1) - \rho(s_2) - C_{1\theta}(x_1) - C_{2\theta}(x_2) + \frac{v_\theta}{2}. \tag{1.2}$$

The first two terms represent the dis-utility the voter receives from the severity of issues after the politician has invested in each issue. $\rho(s_i) \geq 0$ is increasing, convex, and twice continuously differentiable for positive values of s_i and constant for negative values of s_i. The third and fourth term are the social cost of investment and the last term represents the net-valence advantage of the elected candidate where $v_R = v = -v_L$.[5]

The politicians are also far-sighted. An elected politician cares about both the discounted

[5]This term is divided by 2 to normalize net-valence into utility terms such that the difference in utility from two candidates who adopt the same policies, excluding the social cost, will be exactly the net-valence.

lifetime expected utility of the voter and the value from holding office in each period. I define $\psi \geq 0$ as the perquisites of office, or the incremental benefit a politician from either party receives from being elected.[6] The representative voter and politicians share the same discount factor β.

At the end of each period, the severity of each issue worsens through a stochastic shock. The severity of issue i evolves according to

$$\omega_i' = \max\{s_i, 0\} + \eta_i, \tag{1.3}$$

where $s_i = \omega_i - x_i$ is the severity of the issue after the elected politician invests in this issue, and $\eta_i \sim U[0, K]$ represents a stochastic worsening of the issue.[7] For the sake of completeness the transition is written to prevent negative values however in equilibrium the politician has no incentive to improve the severity of the issue beyond zero and we always have $s_i \geq 0$. Because of this, it will be safe to write $\omega_i' = s_i + \eta_i$.

The net-valence persists, but decays stochastically over time. I assume that the net-valence evolves according to

$$v' = \delta v + \varepsilon \tag{1.4}$$

where $0 \leq \delta < 1$ and $\varepsilon \sim U[-\bar{v}, \bar{v}]$.

I focus on a subgame perfect equilibrium. Due to the sequential nature of the model I can solve for equilibrium recursively. My main interest is in the optimal strategies (investment) of each politician type in the first period.

[6] Note that ψ can be interpreted as either the utility-normalized value of re-election to a politician or as the weight a politician places on the voter utility versus the value of re-election. As $\psi \to \infty$, the model converges to one in which the politicians are purely office motivated and only consider voter utility in how it affects their re-election chances.

[7] Making the stochastic shock follow a uniform positive distribution is particularly convenient because it makes it easier to discuss and interpret the behavior of each politician type. In principle the transition could follow some other distribution $\omega_i' \sim F_i(s_i)$ provided $\partial F_i / \partial s_i > 0$ and the results would be similar.

1.2.2 Second period

In the second period ($t = 2$) there is no future and an elected politician invests to maximize voter's single-period utility. The value function in this period for a politician from party θ who is elected when issue severity is ω_1, ω_2, has the net-valence v, and chooses investment x_1, x_2 is:

$$W_\theta^{(2)}(\omega_1, \omega_2, x_1, x_2, v) = -\rho(\omega_1 - x_1) - \rho(\omega_2 - x_2) - C_{1\theta}(x_1) - C_{2\theta}(x_2) + \frac{v_\theta}{2}. \tag{1.5}$$

The assumptions on utility and cost functions above guarantee that a unique solution exists. Let $x_{i\theta}^{(2)}(\omega_1, \omega_2, v)$ for $i = 1, 2$ be the optimal investment that solves the problem of a politician from party θ in the second period.

Next consider the problem faced by the representative voter in the second and final period. Given that the voter is able to correctly forecast the strategies of the politicians from each party, $x_{i\theta}^{(2)}$,the value function of the representative voter in the second period who elects a politician from party θ is:

$$V_\theta^{(2)}\left(\omega_1, \omega_2, x_{1\theta}^{(2)}, x_{2\theta}^{(2)}, v\right) = -\rho(\omega_1 - x_{1\theta}^{(2)}) - \rho(\omega_2 - x_{2\theta}^{(2)}) - C_{1\theta}(x_{1\theta}^{(2)}) - C_{2\theta}(x_{2\theta}^{(2)}) + \frac{v_\theta}{2} \tag{1.6}$$

The representative voter therefore faces the discrete choice problem:

$$\hat{V}^{(2)}(\omega_1, \omega_2, v) = \max_{\theta \in \{L,R\}} V_\theta^{(2)}\left(\omega_1, \omega_2, x_{1\theta}^{(2)}, x_{2\theta}^{(2)}, v\right) \tag{1.7}$$

Because the value from electing each politician type exhibits a single crossing property in net-valence v a unique solution exists and takes the form of a cut-off rule in net-valence. Let $\theta^{(2)}(\omega_1, \omega_2, v)$ be the optimal solution to the problem of the representative voter in the second period based on this cut-off rule.

1.2.3 First period

In the first period ($t = 1$) there exists a second period in which an election will occur so the elected politician cares about both the discounted lifetime expected utility of the voter and the value of holding office in the next period. Each politician is able to correctly forecast the election strategy of the voter, $\theta^{(2)}(\omega_1, \omega_2, v)$, the optimal investment of each politician type in the next period, $x_{i\theta}^{(2)}(\omega_1, \omega_2, v)$, and can determine how his investment choice will affect the probability of being re-elected. The value function in the first period for a politician from party θ who is elected when issue severity is ω_1, ω_2, has the net-valence v, and chooses investment x_1, x_2 is:

$$\begin{aligned} W^{(1)}(\omega_1, \omega_2, x_1, x_2, v) = u_\theta(\omega_1 - x_1, \omega_2 - x_2, x_1, x_2, v_\theta) + \beta E\left[\hat{V}^{(2)}(\omega_1', \omega_2', v')\right] \\ +\beta\psi E\left[Pr\left(\theta^{(2)}(\omega_1', \omega_2', v') = \theta\right)\right]. \end{aligned} \tag{1.8}$$

The first two terms are the lifetime discounted utility for the voter and the last term is the value of holding office, ψ, discounted and times the expected probability of being re-elected in the next period. Both expectations are conditioned on the net-valence v, the severity of issues after investment s_i, and the correctly predicted optimal strategies of the voter and politicians in the next period. The severity of issues after investment depends on the initial issue severity and investment in each issue. The problem of a politician in the first period is, therefore:

$$\max_{x_1, x_2} \; W_\theta^{(1)}(\omega_1, \omega_2, x_1, x_2, v). \tag{1.9}$$

In the event of a tie between two policies, I assume the politician chooses the policy that gives higher utility to the voter.[8] I show in appendix A.1 that a unique solution exists. Let $x_{i\theta}^{(1)}(\omega_1, \omega_2, v)$ for $i = 1, 2$ be the optimal investment that solves the problem of a politician from party θ in the first period.

[8] A tie-breaking rule is only necessary for uniqueness if the politician's problem is not single peaked. For a discussion of this case see section 1.3.4 on possible discontinuities in investment.

The optimal investment of each politician type in the first period captures the strategic behavior of politicians and is the main focus of my results. In the first period a politician faces a trade-off between investment in the best interest of the voter and investment to increase his chance re-election. The results in section 1.3 focus on characterizing this optimal investment.

To complete the model consider the problem of the voter in the first period. The representative voter is able to correctly forecast the optimal strategies of each politician type in both periods and her own strategy in the next period. The value function of the representative voter in the first period who elects a politician from party θ is:

$$V_\theta^{(1)}(\omega_1, \omega_2, x_{1\theta}^{(1)}, x_{(2\theta)}^{(1)}, v) = u_\theta(\omega_1 - x_{1\theta}^{(1)}, \omega_2 - x_{2\theta}^{(1)}, x_{1\theta}^{(1)}, x_{2\theta}^{(1)}, v_\theta) + \beta E\left[\hat{V}^{(2)}(\omega_1', \omega_2', v')\right] \quad (1.10)$$

I show in appendix A.1 that a unique solution again exists and takes the form of a cut-off rule in net-valence. Let $\theta^{(1)}(\omega_1, \omega_2, v)$ be the optimal solution to the problem of the representative voter in the first period that corresponds to this cut-off rule.

1.3 Results

The main focus of the results is characterizing the optimal investment of politicians in the first period. To do this I begin by characterizing the optimal election strategy of the voter in the second period, which will be important for the problem of the politician in the first period. Section 1.3.1 characterizes the socially optimal investment, or the investment when politicians care only about the voter's utility and not re-election. The main result follows in section 1.3.2, where I characterize the optimal investment when politicians are office-motivated and care about both the voter's utility and re-election. Sections 1.3.3 and 1.3.4 consider some special cases, for instance when a politician chooses negative investment in (i.e. sabotages) an issue, and when optimal investment may be discontinuous. Finally, section 1.3.5 considers extending the model for a finite but arbitrarily long time horizon that allows us to look at the long-run dynamics of office control and the stationary state. While the results in this section are for general utility and cost functions that satisfy the assumptions of the model above, the figures and numerical examples used to aid discussion are based on the functional forms and parameter values in appendix A.3.

We are primarily interested in the investment choice of an elected politician in the first period. Since this behavior depends on the election strategy of the voter in the second period, we begin by characterizing the optimal election strategy of the voter. After the second period there is no future, so the optimal investment of each politician and thus the optimal election strategy of the voter does not depend on the value ψ that a politician places on holding office.

PROPOSITION 1 *The optimal election strategy of the representative voter in the second period takes the form of a cut-off rule in net-valence $v^*(\omega_1, \omega_2)$, which is continuous in ω_1, ω_2. Such that:*

$$\theta^{(2)}(\omega_1, \omega_2, v) = \begin{cases} L & \text{If } v \leq v^*(\omega_1, \omega_2) \\ R & \text{otherwise.} \end{cases}$$

Furthermore, the net-valence cutoff rule is such that if i *is the advantaged issue and* -i *is the disadvantaged issue of a politician from party θ, and v_θ is the net-valence advantage then:*

$$\frac{\partial Pr(\theta^{(2)} = \theta)}{\partial s_i} \geq 0 \qquad \frac{\partial Pr(\theta^{(2)} = \theta)}{\partial s_{-i}} \leq 0 \qquad \frac{\partial Pr(\theta^{(2)} = \theta)}{\partial v_\theta} \geq 0$$

Where s_i, s_j is the severity of each issue after investment and the inequalities are strict when $0 < Pr(y' = \theta) < 1$ ***Proof.*** *See appendix A.1* ■

According to proposition 1, the optimal election rule in the second period for the voter behaves the way we intuitively expect it to behave. When the severity of an issue is worse, the politician from the party that specializes in this issue is more likely, and his opponent less likely, to be elected in the next period. This rules out, for example, a situation in which the economy worsens and ceteris paribus the R-party becomes more likely to be elected. In addition, if a politician becomes more attractive through net-valence advantage, then he is more likely to be elected in the next period

1.3.1 Socially optimal investment

To characterize the optimal investment it will be useful to compare and contrast two cases: First, the socially optimal investment, and second, the optimal investment when politicians are office-motivated. The socially optimal investment will provide a baseline or efficient level of investment with which we can compare the behavior of office-motivated politicians. The socially optimal investment can be defined according to:

DEFINITION 1 *Let the socially optimal investment be the optimal investment of politicians when* $\psi = 0$.

The socially optimal investment is the optimal investment when there is no value of re-election for a politician. In this case an elected politician behaves like a social planners and chooses investment to maximize the discounted lifetime expected utility of the voter. This is also equivalent to the representative voter herself being able to directly choose investment while still constrained to pick a politician from one of the parties to implement the investment. The following proposition characterizes the monotonicity of the socially optimal investment as a function of the severity of each issue for an elected politicians in the first period.

PROPOSITION 2 *The socially optimal investment in each issue in the first period is increasing in the severity of both issues. Or for* $i, j = 1, 2$ *and* $\theta = L, R$ *we have:*

$$\frac{\partial x_{i\theta}^{(1)}}{\partial \omega_j} \geq 0.$$

Proof. See appendix A.1. ■

According to proposition 2, the worsening of either issue leads to an increase in the socially optimal investment in both issues for both parties. For example, if the economy becomes more severe, it is socially optimal for the elected politician (regardless of party) to invest more in the economy and more in national security. To see why this is true, it helps to identify the ways a change in the severity of an issue can affect the benefit and cost of investment to the voter.

A change in the severity of an issue has three effects on the utility of the voter. The first effect, or the direct effect, is on the one-period utility of the voter. The second effect, or indirect effect, is on the expected severity and distribution of the issue in the next period. The third effect, or the office-control effect, is how this change in distribution over future issues affects the party likely to be elected in the future. When the severity of an issue changes we can identify the direction and size of each of these effects to determine the net effect on investment. To begin, consider an increase in the severity of an issue and how this affects the optimal investment in this same issue.

The first effect, or direct effect, from an increase in the severity of an issue is an increase in the marginal value of investment in this issue through one-period utility. When an issue becomes more severe, the politician has an incentive to invest more in this issue because the voter receives dis-utility from its severity. Or in other words, since the value of investment depends on the severity of an issue, an increase in severity increases the marginal value of investment.

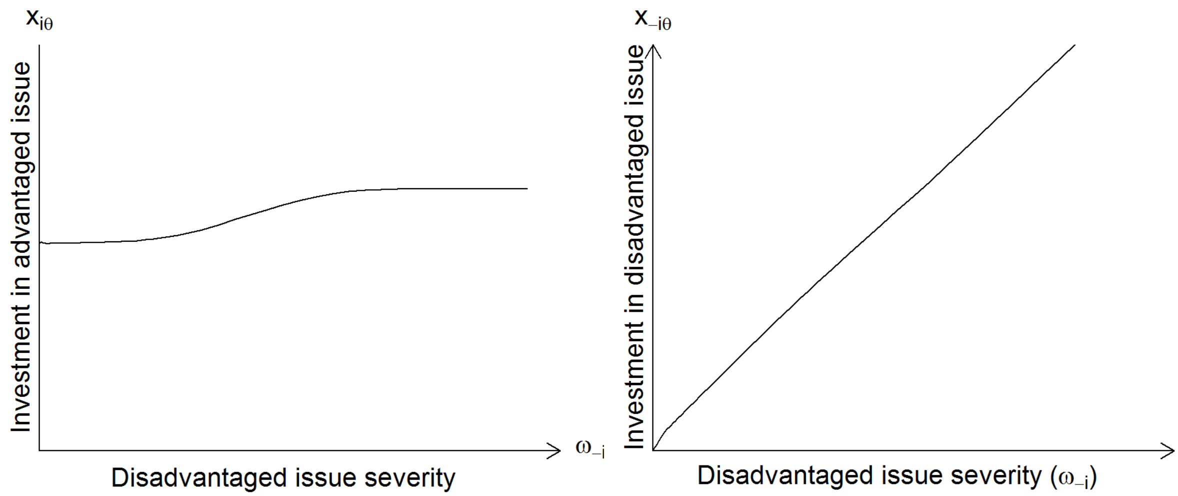
The second, or indirect effect, is an increase in the marginal value of investment through continuation utility. Since there is persistence in the severity of an issue, when an issue becomes more severe today, the expected severity in the next period is also worse. Since continuation utility is decreasing in the severity of issues and investment can reduce this severity, an increase in severity of an issue will increase the marginal benefit of investment and the optimal investment in this issue.

Finally the third, or office-control effect, is a decrease in the marginal benefit of investment in this issue. When an issue becomes more severe it becomes more likely that in the next period the politician who specializes in this issue will be elected. Since the future politician can invest in this issue at a low cost this makes investment today less valuable or decreases the marginal benefit of investment and optimal investment in this issue.

The direct and indirect effect on investment from an increase in the severity of an issue is positive while the office-control effect is negative. In appendix A.1 I show that the first two effects dominate the third and the overall effect is positive or optimal investment in an issue is increasing in the severity of that issue. The second frame of figure 1.2 shows how the socially optimal level of investment in a disadvantaged issue depends on the severity of that issue.

Next, consider the same three effects on investment in the other issue. In this case only the office-control effect matters. Because the severity of one issue enters one-period utility independently from investment in the other issue, an increase in the severity of one issue has no direct effect on the value of investment in the other issue (the direct effect is zero). Likewise, since the severity of one issue today does not directly affect the severity of the other issue in the future, there is no indirect effect. There is, however, an office-control effect. An increase in the severity of an issue increases the chance of electing a politician in the next period who specializes in this issue, and this increases the benefit of investing in the other issue today. The direct and indirect effect of an increase in the severity of an issue on investment in the other issue is zero, while the office-control effect is positive. The first frame of figure 1.2 shows the socially optimal level of investment in the advantaged issue as a function of the disadvantaged issue severity. Thus optimal investment is increasing in the severity of both issues.

The following proposition characterizes the monotonicity of the socially optimal investment as a function of the net-valence advantage of a candidate for an elected politicians in the first period.



(a) (b)

Figure 1.2: Frames (a) and (b) show how the socially optimal investment in each issue is increasing in the severity of the disadvantaged issue. By symmetry, investment in each issue as a function of the advantaged issue will also be increasing and will look similar, only reversed.

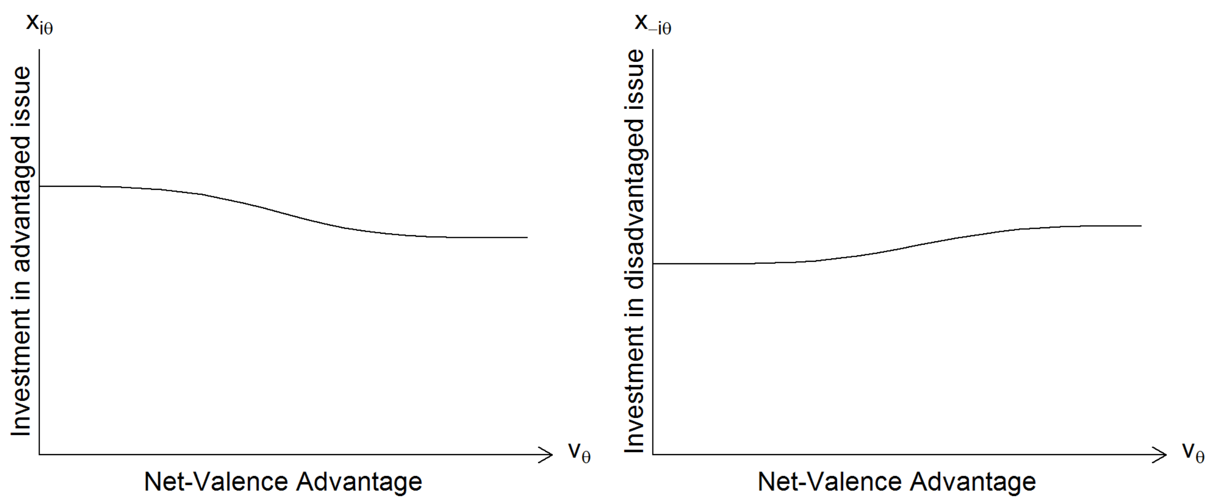
PROPOSITION 3 *When the net-valence advantage of a politician increases, it is socially optimal for the elected politician to invest less in the issue in which he has an advantage, and invest more in the other issue. If* i *is the advantaged issue for a politician from party* θ *and* -i *is the disadvantaged issue, then:*

$$\frac{\partial x_{i\theta}^{(1)}}{\partial v_\theta} \leq 0 \qquad\qquad \frac{\partial x_{-i\theta}^{(1)}}{\partial v_\theta} \geq 0.$$

Proof. See appendix A.1. ■

According to proposition 3, when a politician becomes more attractive due to a change in net-valence advantage, the socially optimal investment in the issue in which he specializes decreases and investment in the other issue increases regardless of who is in office. For example, if a Democrat candidate becomes exogenously more attractive, then it is socially optimal for the elected candidate (regardless of party) to invest less in the economy and more in national security. This may at first seem counter-intuitive, but suppose there is an increase in the net-valence advantage of the L-party and consider the three effects from before.

An increase in net-valence advantage of the L-party has no direct or indirect effect on investment since net-valence enters utility additively, and does not directly affect the distribution of issue severity. There is, however, an office-control effect. An increase in net-valence advantage makes the politician from the L-party exogenously more attractive, and his election in the next period becomes more likely. Since it is more likely that a politician from the L-party will be in office in the next period, investment in national security will be costly next period and there is more value in investing today. Likewise, investment in the economy is more likely to be cheap in the next period, so there is less value in investing today. Figure 1.3 shows the socially optimal investment in each issue as a function of net-valence advantage.



(a) (b)

Figure 1.3: Frames (a) and (b) show how the socially optimal investment in each issue depends on the net-valence advantage. When a politician becomes more attractive for some exogenous reason then it is socially optimal for the elected politician to invest less in the issue in which he has an advantage and more in the issue in which he has a disadvantage.

1.3.2 Investment under Office-motivated Politicians

In this section I characterize the optimal investment of politicians when they care about re-election (i.e. are office-motivated), and then compare this characterization to the socially optimal investment. To aid discussion I use the following terminology:

DEFINITION 2 *Let $x_{i\theta}^{SP}(\omega_1, \omega_2, v)$ and $x_{i\theta}^{\psi}(\omega_1, \omega_2, v)$ for $i = 1, 2$ and $\theta = L, R$ be the optimal investment in each issue of an elected politician from party θ in the first period for (i) social planner politicians when $\psi = 0$ and (ii) office-motivated politicians when $\psi > 0$ respectively.*

The following theorem is the main result in this section and characterizes optimal investment of a office-motivated politician compared to the socially optimal investment.

THEOREM 1 *An office-motivated politician invests less than socially optimal in the issue in which he specializes and more than socially optimal in the other issue. If* i *is the advantaged issue for a politician from party θ and* -i *is the disadvantaged issue then:*

$$x_{i\theta}^{\psi} \leq x_{i\theta}^{SP}, \qquad\qquad x_{-i\theta}^{\psi} \geq x_{-i\theta}^{SP}.$$

If when evaluated at the socially optimal investment the marginal effect of investment on re-election is non-zero, then the inequalities will be strict. That is if:

$$\frac{\partial}{\partial x_i} Pr\left(y' = \theta | \dots\right)\Big|_{x_{1\theta}^{SP}, x_{2\theta}^{SP}}, \neq 0$$

then the inequalities for investment in issue i will be strict. The larger the absolute value of the above term the greater the inequalities.

Proof. See appendix A.1. ■

Accoding to theorem 1, when a politician cares about re-election he deviates from the socially optimal by investing less in the issue in which he specializes and more in the other issue. Office-motivation adds an additional incentive to reduce investment in his advantaged issue and increase investment in his disadvantaged issue to slow the improvement of the advantaged issue, and increase his chance of re-election in the future.

The first row of figure 1.4 shows how the optimal investment in each issue of an office-motivated politician depends upon the severity of the disadvantaged issue. An office-motivated politician invests less than socially optimal in the advantaged issue and more than socially optimal in the disadvantaged issue. The second row of figure 1.4 shows how the optimal policy for an elected politician is affected by his net-valence advantage. Again the politician invests less in the issue in which he specializes and more in the other issue. The third row of figure 1.4 shows how this strategic behavior increases the probability of being re-elected.

When politicians are office-motivated, investment will no longer always be monotone in issue severity. When an election outcome is expected to be close, an increase in the severity of the issue the incumbent is bad at might cause him to reduce his investment in the issue he is good at. As an example, a politician who specializes in national security may invest less in national security and more in the economy when the economy worsens. While this may not seem surprising, in this case the under-investment is being driven not by the allocation of scarce resources to the more severe issue but rather by the desire to avoid a situation in which the opposing party becomes too attractive to the voter. Perhaps more surprising, in some situations the investment of an office-motivated politician in an issue may be decreasing in its own severity. For example, a politician who has an advantage in the economy may invest less in the economy when the economy worsens. This behavior is likely when the value politicians place on re-election is high or there is a discontinuity in investment, which is discussed in section 1.3.3.

How far the elected politician deviates from socially optimal depends on the relative costs and benefits of deviating. In states in which there is greater uncertainty about who will win the election in the next period (the probability of re-election is not one or zero), the benefit

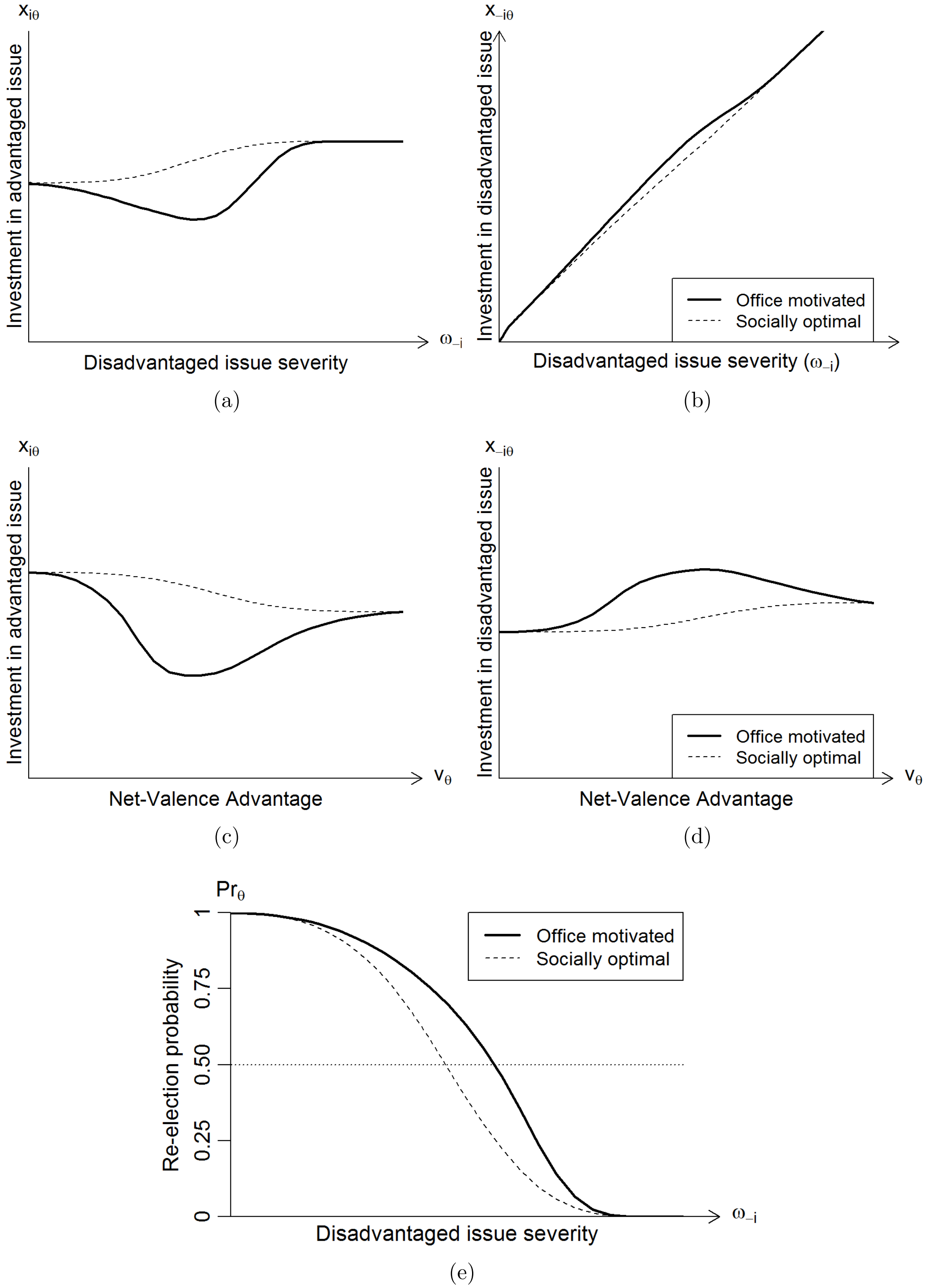


Figure 1.4: Frames (a) and (c) show how an office-motivated politician invests less than socially optimal in the issue in which he has a advantage. Frames (b) and (d) show how he invests more than socially optimal in the disadvantaged issue. Frame (e) shows shows how this strategic behavior increases the chance of re-election.

to the incumbent of deviating is higher. When one or both issues are severe, the cost of deviating is higher. The next proposition characterizes the difference between the socially optimal investment and investment when politicians are office-motivated.

PROPOSITION 4 *The difference between the socially optimal investment and the investment of an office motivated politician in the first period, or* $\left|x_{i\theta}^{\psi} - x_{i\theta}^{SP}\right|$*, is a single-peaked increasing then decreasing function in issues severity and net-valence advantage.*

According to proposition 4, as the severity of an issue increases the elected politician begins to deviate from the socially optimal investment in order to increase the chance of his re-election. As the the issue continues to worsen, eventually the politician reaches a maximum deviation, after which point his investment converges back to the socially optimal. In each frame of figure 1.4, the point at which the largest deviation from the social optimal occurs is where there is an equal probability of either party winning the election in the next period.

A natural question that might arise at this point in the chapter is: If the voter knows that an L-politician will invest less than socially optimal in the economy when elected, why would the voter continue to elect an L-politician when the economy is bad? The short answer is that the L-politician is still better than the alternative, or the lesser of two evils. To illustrate why, consider the following numerical example that uses the functional forms and parameter values in appendix A.3.

EXAMPLE 1

Suppose the severity of issues is $(\omega_1, \omega_2) = (20, 10)$ *in the first period and neither party has a net-valence advantage,* $v = 0$*. In this case the economy is significantly more severe than national security. The socially optimal investment for the politician from each party is:*

$$\left(x_{1\theta}^{SP}, x_{2\theta}^{SP}\right) = (17.63, 6.17) \qquad \textit{and} \qquad \left(x_{1R}^{SP}, x_{2R}^{SP}\right) = (11.76, 9.33).$$

The voter would prefer to elect the politician from the L-party who has a comparative advantage and will invest more in the economy. Unfortunately, if politicians are office-motivated then the voter has to choose between politicians who once elected will invest either:

$$\left(x_{1L}^{\psi}, x_{2L}^{\psi}\right) = (16.02, 7.23) \qquad \textit{or} \qquad \left(x_{1R}^{\psi}, x_{2R}^{\psi}\right) = (12.35, 8.69).$$

While an office-motivated L-politician invests less than socially optimal in the economy (but still more than the R-politician), and more than socially optimal in national security, it is still better to elect the L-politician than the R-politician. By manipulating the severity of the economy the L-politician is able to increase the chance of re-election from 38.7% *to* 59.5%*.*

While it is possible to create extreme examples, in general, as long as the differences in the marginal social cost of investment (the size of the comparative advantage) are large, then an office-motivated politician will always invest more in the issue in which they specialize than the politician from the other party. In most cases, an office-motivated politician from the R-party is still more desirable when national security is poor than the alternative. Or put another way, an office motivated politician from the L-party is still more desirable when the economy is poor than the alternative.

In the next two sections I discuss some results that may arise in the model in certain situations. The first is sabotage, or negative investment in an issue. The second is discontinuity in the optimal investment.

1.3.3 Special case: sabotage

One feature that can occur is negative investment or sabotage. Negative investment occurs when an elected politician deliberately disinvests (chooses negative investment or sabotages) in an issue to give rise to a state in the next period that is more desirable for future election. The following proposition states this more formally.

PROPOSITION 5 *There always exists some state in which it is optimal for a office-motivated politician to invest negatively in (i.e. sabotage) the issue in which he specializes. That is if a politician from party θ has an advantage in issue* i *then there exists some (ω_1, ω_2, v) such that:*

$$x_{i\theta}^{\psi}(\omega_1, \omega_2, v) < 0.$$

Proof. *Consider the case where $(\omega_1, \omega_2, v) = (0, 0, 0)$ and a politician from the L-party is elected. The socially optimal investment in the first issue is $x_{1L}^{SP*} = 0$. At this socially optimal level of investment a marginal decrease in investment in the first issue will increase the expected probability of re-election. By theorem 1 it must be $x_{1L}^{\psi*} < x_{1L}^{SP*} = 0$ or that an office-motivated politician chooses negative investment.* ■

Proposition 5 states that there are always some states in which it is optimal for an elected politician to invest negatively in (i.e. sabotage) the issue in which he specialize in order to increase his chance of re-election. To help understand the intuition behind sabotage, it is useful to consider a numerical example using the functional forms and parameter values from appendix A.3.

EXAMPLE 2

Suppose the severity of issues is $(\omega_1, \omega_2) = (0.5, 0)$ in the first period and neither party has a net-valence advantage, $v = 0$. The economy is more severe than national security, but neither is very severe. The socially optimal investment is the same for an elected politician from either party, $\left(x_{1\theta}^{SP*}, x_{2\theta}^{SP*}\right) = (0.5, 0)$. The voter will elect the politician from the L-party since the cost of this investment will be less. Unfortunately, if politicians are office-motivated then the voter has to choose between politicians from each party who once elected will invest either:

$$\left(x_{1L}^{SP*}, x_{2L}^{SP*}\right) = (-0.32, 0) \qquad or \qquad \left(x_{1R}^{SP*}, x_{2R}^{SP*}\right) = (0.5, -0.73)$$

Both politicians will choose negative investment in, or sabotage, the issue in which they have an advantage. Despite this sabotage it is still optimal for the voter to elect the politician from the L-party because his deviation is less costly to the voter. By behaving strategically the L-politician is able to increase the chance of re-election from 50% *to* 55.7%*.*

While sabotage always exists for some states, it should not occur frequently. Sabotage occurs only when the severity of both issues is relatively low (the cost of deviating is low) and issue severity and net-valence are such that the election in the next period is expected to be close (the value from deviating is high).

1.3.4 Special case: investment discontinuity

Despite the standard assumptions on utility, cost, and distribution functions, the problem of the politician is not generally globally concave. If the problem of the politician is not concave then there may be more than one peak and optimal investment may be discontinuous. Any discontinuity in investment does not effect the results above for the two-period model, but may pose a problem if we wish to extend the model beyond two periods. For a longer time horizon, the conditions under which optimal investment is discontinuous may continue to spawn a discontinuity in each period that we extend the horizon.

To understand why investment may be discontinuous, consider how non-concavity might enter the problem of the politician. The problem of an elected politician in the first period, or equation (1.8) from above, is:

$$\begin{aligned} W^{(1)}(\omega_1, \omega_2, x_1, x_2, v) = u(\omega_1 - x_1, \omega_2 - x_2, x_1, x_2, v_\theta) + \beta E\left[\hat{V}^{(2)}(\omega_1', \omega_2', v')\right] \\ +\beta\psi E\left[Pr\left(\theta^{(2)}(\omega_1', \omega_2', v') = \theta\right)\right]. \end{aligned} \tag{1.8}$$

The first two terms, which represent the lifetime discounted utility of the voter are strictly concave in investment by assumption. The final term, the expected value from re-election, cannot be strictly concave in investment simultaneously for both politician types.[9] As a result, in order for the politician's problem to be strictly concave, any convexity from the re-election probability must be dominated by the concavity of the voter's value function.

Since the non-concave term in equation (1.8) is weighted by the value of re-election, as you increase ψ the degree of non-concavity increases until the problem of the politician is no longer single-peaked. Once this happens, there will be two peaks to the politician's problem: one peak corresponds to a higher voter utility and lower re-election probability while the other peak to lower voter utility and higher re-election probability. In equilibrium the optimal investment of the elected politician will become discontinuous as his policy jumps from one peak to the other.

Figure 1.5 shows what happens to the investment function as ψ increases above $\overline{\psi}$ in a numeric example. In the figure, the lines corresponding to $\psi = 0$ and $\psi = 200$ are identical to those in the first row of figure 1.4. For larger values of ψ as the severity of the disadvantaged issue increases the cost of strategic behavior increases and eventually the optimal investment discontinuously jumps from the peak with lower voter utility and higher re-election probability to the peak with higher voter utility and lower re-election probability.

While discontinuous investment is interesting in the two-period model, it only arises in situations where ψ is large. As ψ increases politicians place less weight on the utility of the voter and more on the perquisites of office; in this case a model of pure office-motivation (i.e. one in which politicians do not care about the utility of the voter) is more appropriate to investigate investment decisions. In the next section where I extend the model for a finite but arbitrarily long time horizon, I focus on the case where optimal investment is continuous.

A sufficient condition for optimal investment to be continuous is for the problem of the politician to be globally concave. Since the non-concave term in equation (1.8) is also

[9] If $Pr(y' = L|\cdots)$ was strictly concave in x_{1L} and x_{2L}, then it would follow that the complement probability $Pr(y' = R|\cdots)$ could not be concave in x_{1R} and x_{2R}.

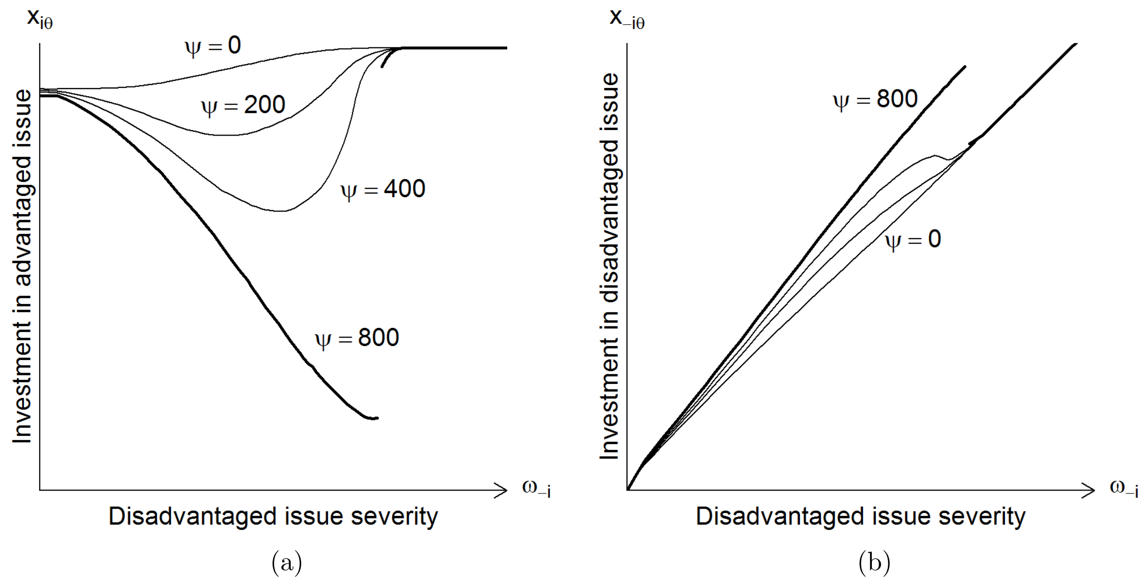


Figure 1.5: Frames (a) and (b) show how the socially optimal investment in each issue can be discontinuous for an office-motivated politician in the disadvantaged issue severity if ψ is sufficiently large. By symmetry, investment in each issue as a function of the advantaged issue will also have a discontinuity and look similar, only reversed.

weighted by the discount factor β, one way to guarantee global concavity of the politician's problem is to place conditions on β and ψ. The following proposition says that it is always possible to choose a combination of β and ψ so that the problem of the politician is globally concave, which is a sufficient condition to guarantee that optimal investment is continuous in the two-period model:

PROPOSITION 6 *Given any initial values for ω_1, ω_2, v and some $\psi = \overline{\psi}$, there exists some $\overline{\beta} > 0$ such that for all $\beta \leq \overline{\beta}$ and $\psi \leq \overline{\psi}$ the decision problem of the politician from each party is globally concave and optimal investment will be continuous.*

Proof. See appendix A.1 ■

Another way to guarantee global concavity without placing a condition on the discount factor would be if there was sufficient smoothing from uncertainty. By increasing the width of the distribution over the states in the next period by making K and/or $\overline{v}$ sufficiently large would also guarantee global concavity. For example, as $\overline{v} \to \infty$ the expected probability of re-election goes to 0.5 regardless of investment and if investment has no effect on re-election there is no incentive to manipulate the issues.

1.3.5 Extending the Horizon

The two-period model allows an intuitive characterization of the optimal investment decision of politicians. To understand how these results apply to long-term dynamics, such as stationary state and equilibrium path, we need to extend the model for a longer time horizon. Because of the capital-investment nature of the model, it extends naturally for a longer horizon. Due to the strategic complexity of an arbitrarily long time horizon I do make one simplifying assumption before solving the model numerically.

To extend the model I modify the incentive structure of politicians by replacing office-motivation with a special form of myopia, that of successor-motivation. If politicians are successor-motivated then they have preferences over the party of the politician that is elected in the immediately following term. A politician cares about the re-election of a politician from his party rather than the re-election of himself specifically. Successor-motivation gives an elected politician the same incentives as if he was in first term of office with a two-term limit. Successor-motivation greatly simplifies the equilibrium while retaining similar incentives to office-motivation. Note that in the two-period model office-motivation and successor-motivation are equivalent.

One motivation for this incentive structure would be if an elected politician is rewarded by his party, perhaps with a high-ranking position within the party or a lucrative speaking contract, for using his power while in office to increase the chance of the party maintaining control of the office. Another motivation would be if there is significant uncertainty about the identity of the candidate who will be selected by the party in the next period to run for office. These motivations are the most reasonable for public positions in which the term limit is either relatively short (such as president) or there is frequent change in the identity of elected officials.

For a finite but arbitrarily long time horizon T, the value function and problem for each politician type and the representative in the final period is the same as equations 1.5 through 1.7 only with the index (2) being replaced with (T). Likewise for an arbitrary period $t < T$ the value function and problem for each politician type and the representative voter is the same as equations 1.8 through 1.10 only with the index (1) replaced by the index (t) and the index (2) being replaced with the index $(t+1)$.

As with the two-period mode, proposition 6 continues to hold for a longer horizon. It is always possible to choose β and ψ sufficiently small for the politician's problem to be globally concave. Because of the sequential nature, the model can be solved recursively. Appendix A.2 provides details on the extension of the two-period model.

The results from the two-period model continue to hold numerically for a longer time horizon. The socially optimal investment and optimal investment for an office-motivated politician in the first period of an arbitrarily long time horizon continue to have the same properties as the two period-model, characterized by propositions 2 through 5 and Theorem 1. Optimal investment for a social planner as well as office-motivated politician in this longer time horizon are similar to those of the two-period model shown in Figure 1.4. The elected politician continues to under-invest the issue in which he has an advantage and over-invest the other issue. The advantage of the longer time horizon is that we can look at long-run dynamics.

Figure 1.6 shows the expected path and stationary states for both the socially optimal investment and investment by office-motivated politicians beginning in an initial state in which national security is slightly more severe than the economy, and neither party has a net-valence advantage.[10] Along the expected path net-valence is zero and the voter elects the politician who has an advantage in the issue that is more severe. In states above the the 45 degree line, the economy is more severe than national security and the voter elects the

[10]By expected path I mean the path if the realization of each random variable occurs at its expected value.

L-politician. In states below the 45 degree line, national security is relatively more severe and the voter elects the R-politician. The dashed line in figure 1.6 shows the socially optimal expected path.

Along the socially optimal expected path the voter first elects a politician from the R-party who has an advantage in national security, which is more severe. The R-politician then invests more in national security and less in the economy. In the next period since national security has improved more than the economy and the economy is now relatively more severe the voter elects an L-politician. Continuing along the expected path office control changes between the parties in each period as the voter alternates politicians in order to use the comparative advantage of each party to ratchet down first one issue then the other. In the steady state the voter alternates in expectation between the two parties to prevent either issue from becoming too severe.

When politicians care about the election of their successor they behave strategically and this affects both the expected stationary state and the expected path. If the issues are not too severe (i.e. the cost of deviating is not too high) then the politician is able to increase the chance of a successor from his party being elected in the next period by investing less in the issue in which his party has an advantage. In figure 1.6 the solid line shows the expected path when politicians are successor-motivated. Because a politician cares about re-election of his successor, he invests strategically to increase this chance. Both in the stationary state and along the expected path a successor from the same party is more likely to be elected than the opposing party.

When politicians are successor-motivated there will be two steady states, each of which corresponds to the case where in expectation one party controls the office in every period. In each steady state the severity of the issue in which the elected politician has an advantage will be worse and the severity of the disadvantaged issue will be better than when that politician is in office in socially optimal steady state. It is important to note that figure 1.6 shows the path and stationary state under expectation. While successor-motivated politicians maintain

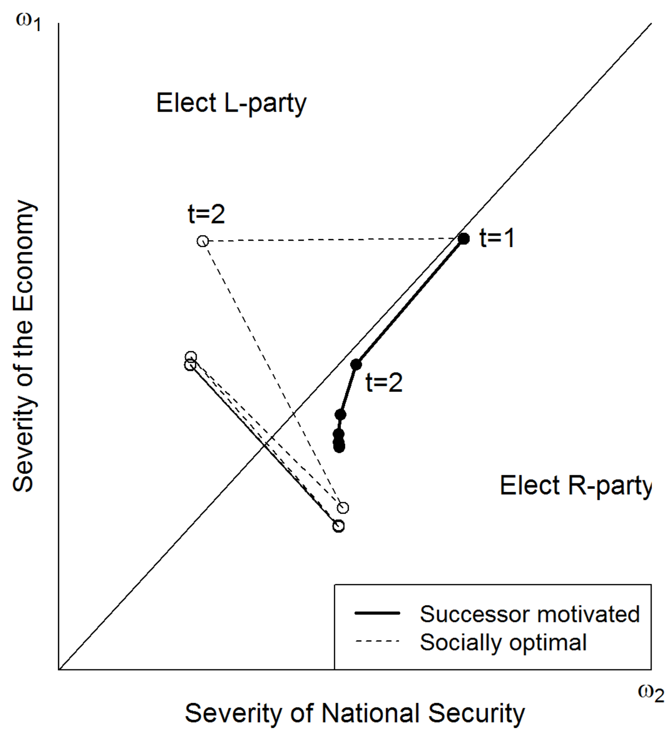


Figure 1.6: The expected path and stationary state for a particular example. The socially optimal is for office control to switch between parties in each period. When politicians are successor-motivated in each steady state one party is able to maintain office control indefinitely under expectation and at least one issue is more severe than socially optimal.

office control indefinitely under expectation, random shocks will switch between the steady states and which party is actually in office and maintains control.

This example illustrates how when politicians are successor-motivated it may be possible for the elected politician to maintain control of office indefinitely (under expectation). The social cost of this behavior is one or both issues are more severe in the long-run than socially optimal.

1.4 Conclusion

The policy choices of a politician in office affect not just the current issues, but also issues in the future upon which his re-election may depend. If a politician cares about re-election, then his incentives will not be the same as the voter's, and there is an agency problem. To investigate this incentive structure I presented a dynamic election model based on a capital-investment structure that endogenizes the relationship between the investment choices of a politician and the severity of issues over which future elections are determined.

My model is based on three assumptions: (1) The severity of national issues is endogenous and depends on the policy of earlier politicians, (2) The candidate who can best deal with issues that are severe has an advantage with voters, and (3) Parties and their candidates are differentiated and have a comparative advantage in dealing with certain issues. Under these assumptions, when politicians care about re-election, I show that they have an incentive to behave strategically and manipulate the severity of national issues to their advantage.

When a politician cares about re-election he has an incentive to under-invest in the issue in which he specializes and over-invest in the other issue. This strategic behavior slows the improvement of the issue in which he has an advantage, while also preventing the issue in which his opponent has an advantage from getting too severe. By keeping the issue in which he specializes relevant for the voter, the politician is able to create an environment in which his comparative advantage is more valuable. By behaving strategically the politician makes it possible for him to say to the voter: "I know this issue is a major concern for you and I have been working to improve it. I'm better than my opponent on dealing with this issue, and if you reelect me, I'll be able to continue improving it." What's omitted from this argument is that while the the politician is indeed improving the issue, he is deliberately improving it slower than socially optimal, or slower than the voter would like, in order to be able to continue making exactly this argument for as long as possible.

When the election in the following period is more competitive, the marginal effect of investment on re-election is higher and the politician has a stronger incentive to behave strategically. If one party has a decisive advantage, or the issues are very severe then the elected politician returns to the socially optimal investment. The circumstances under which politicians behave strategically can be viewed in either two ways. On one hand, when one or both issues are very severe politicians are less likely to behave strategically, which is precisely when socially optimal investment matters the most to the voter. On the other hand, strategic behavior is the strongest when elections are expected to be close, which is common in many elections.

Because the model is based on a capital investment structure it can easily be extended for a longer time horizon or to incorporate other changes. If we extend the time horizon, then along the socially optimal path office control is likely to switch in each period. The elected politician uses his comparative advantage to improve the more severe issue before passing control to the other party who can invest more efficiently in the other issue. When politicians care about the re-election of their party, this path breaks down and politicians invest less than socially optimal in the issue in which they specialize in order to slow its improvement and maintain control of the office longer. In the example of figure 1.6 the outcome may be extreme in the sense that in expectation one party may always have control of the office until a sufficiently large shock switches control of the office in expectation entirely to the other party.

Endogenous issues generate very specific strategic behavior by elected politicians. This behavior has significant short-term and long-term effects on the severity of issues and office control that have not previously been considered. When the policy of a politician influences his chance of re-election by changing the relative severity of issues, he invests inefficiently. In particular, an elected politician invests less than socially optimal in the issue in which he specializes and more than socially optimal in the other issue. By focusing on the the "wrong" issue a politician is able to slow the improvement of the issue in which he specializes to maintain his comparative advantage longer. In the long-run, this leads to more severe national issues and office control changes less frequently than is socially optimal.

CHAPTER 2

RATIONAL ADDICTION AND VIDEO GAMES

2.1 Introduction

In 2007 the American Medical Association was called upon to consider "video game addiction" as a condition for formal diagnosis in the 2012 Diagnostic and Statistical Manual of Mental Disorders (DSM-V). Their followup report acknowledged possible connections between video games and addictive behavior and concluded that "more research is needed in this area."[1] This focus on potential addiction in video games reflects both the rise in use of video games and controversy over their effects. A 2008 Survey by the Pew Institute found that 97% of teens play video games (Lenhart et al., 2008). The Entertainment Software Association (ESA) reports that 72% of American households play video games, 82% percent of gamers are 18 years of age or older, and the average game player is 37 years old (ESA 2011). The effects of video games are controversial. Some studies find that video games lead to greater aggression and violence in the real world, while others find a positive correlation between playing video games and creativity, productivity and increased mental health.[2] Ultimately, as with any consumption good, too much can be harmful and in several cases playing video games has lead to death. In 2010 a South Korean couple was arrested for neglecting and starving their baby while playing an online video game.[3] As video games

[1]For background see Council on Science and Public Health (2007), APA News Release 2007 and Elliot (2007).

[2]See Jackson et al. (2012), Whitlock et al. (2012), Gentile (2011), and McGonigal (2011) for examples of benefits from video games.

[3]For more on this story, in which, ironically, they were caring for a virtual child online, see *South Korea child 'starves as parents raise virtual baby'* BBC News (2010). For other examples of deaths related to video games see *Chinese online gamer dies after three-day session* BBC News (2011*a*) and *South Korean dies after*

become a more ubiquitous part of everyday life it is important to understand their effects, and if and when they should be classified as addictive.

In economics addiction is frequently studied in the context of a rational addiction model, originally formalized by Becker and Murphy (1988). In a model of rational addiction, the value of consuming an addictive good depends on past consumption. The more an individual consumed in the past, the greater value from consuming today. Individuals are rational and forward looking; they make consumption choices based on the knowledge of how their decision today will affect the value of consumption in the future. The degree to which a good is considered to be addictive depends on the extent that past and future consumption influence current consumption. For a more detailed explanation of how addiction has been approached with economics in the past see Becker (1996).

I analyze addiction in video games using a rational addiction model in a manner similar to how Becker et al. (1994) study cigarettes. A video game differs from traditional consumption goods(e.g. cigarettes), in significant ways. First, unlike purchasing a pack of cigarettes, time spent playing a video game does not have a direct monetary cost. Once an individual owns a game, the relevant cost of playing for an hour is the value of the next best alternative that hour could have been spent on (such as watching television). Second, the value an individual receives from playing a video game depends on his performance in that game. If he does poorly at the game he may enjoy it less. At the same time, the performance of an individual is not fixed but may change over time. While my main focus is addiction, I also consider some aspects of learning and investigate how learning may feedback into the decision to play more.

I apply this model of rational addiction to unique panel data collected from an online video game called *Team Fortress 2 (TF2)* and provide evidence of rational addiction. Both past and future consumption of this video game are significant in determining how much an individual plays today. This result suggests the game is addictive in the sense that the value

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games session BBC News (2011*b*)

of playing today depends on the amount an individual played in the past. The size of this effect is similar in magnitude to what Becker et al. (1994) find for cigarettes.

My dataset is rich enough in the time dimension that I can go beyond analyzing addiction at the game level and estimate the rational addiction model at an individual level. This estimation allows me to identify and characterize individual addicts in a way that is consistent with rational addiction. The set of individuals I identify in this way is different from those identified as addicts using a simple metric, such as the average amount of time spent playing. Using a simple cut-off, such as playing two hours or more per day, to define addiction does not take into consideration differences in the marginal value of time between individuals. An individual who plays a lot but also has a lot of free time for entertainment may not necessarily be addicted. Applying the rational addiction at the individual level provides a method to identify addicts that allows for differences in the marginal value of time between individuals.

Finally, I consider a version of the model that allows for learning, or endogenous skill formation. When skill is endogenous, I provide evidence of a *skill-playtime feedback loop*: By playing today an individual improves his future skill which in turn reinforces his decision to play in the future. Learning affects the value of playing in the future, and this can contribute to rational addiction.

My results provide evidence in support of rational addiction in video games. Because I estimate the model for a single game, these results should be interpreted as a lower bound on possible addiction. Many individuals likely play multiple games and I do not observe substitution between games that contributes to addiction. Even so, I can provide strong evidence of rational addiction in video games. At the same time, these results show that addiction may depend on the individual as much as on the nature of the game.

2.1.1 Related literature

The psychology literature on the effects of video games is fairly recent and most articles fall in one of two groups based on their conclusions. One group finds a significant link between violent and aggressive video games and violence and aggression in the real world. For a background see Anderson et al. (2004) and Anderson and Bushman (2001). For example, Fischer et al. (2007) provides evidence of a link between playing virtual driving video games and aggressive driving behavior. A second strand of the literature finds beneficial effects of playing video games. Jackson et al. (2012) provides evidence that playing video games is strongly linked to creativity. Video games have also been shown to have strong educational benefits (Griffiths, 2002) as well as improve cognitive ability in older adults (Whitlock et al., 2012). While these results may appear diametrically opposed, Gentile (2011) argues that video games affect children and adolescents in multiple dimensions and we need a more nuanced approach rather than simply labeling video games "good" or "bad."

In economics, addiction was first approached as non-rational habit formation. The theory behind the economics of habit formation was initially investigated by Pollak (1970) and Pollak (1976). More recently the standard approach to modeling addiction is with the theory of rational addiction proposed by Becker and Murphy (1988). While rational addiction is controversial,[4] it remains popular primarily for two reasons: First, it is the most successful method at this time for explaining addiction that is consistent with rational forward-looking agents. Second, the model provides empirical implications that can be easily tested. The rational addiction model has been applied to a variety of goods including cigarettes in Becker et al. (1994) and Chaloupka (1991), cocaine in Grossman et al. (1999), and gambling in Mobilia (1993).

[4]See Elster (1999), Elster (1997), and Rogeberg (2004) for critiques of rational addiction.

2.2 Model

To analyze addiction in video games I use a model of rational addiction adapted for video games. In a rational addiction model, forward-looking agents have utility that depends on current and past consumption. The degree to which past and future consumption influence current consumption is a measure of the addictiveness of the good.

In the case of a traditional good like cigarettes, an individual has an income from which he can purchase the good. Once the good is consumed it must be repurchased in order to be consumed again. Its demand will, therefore, depend upon an individual's income, the current price of the good, and the current prices of other goods. In the case of a video game, however, once an individual has payed the fixed cost to acquire the game there is no direct monetary cost associated with playing the game.[5] Instead, the cost of playing the game will take the form of an opportunity cost, i.e. the value of what an individual could have been doing during this time. Because of this difference from a traditional consumption good with a market price, I adapt the Becker and Murphy (1988) model of rational addiction so that an individual has a fixed endowment of entertainment or leisure time that can be allocated between playing a video game and another entertainment activity. The value derived from playing a video game depends on the expected performance of an individual in the game. In addition, the value of the other entertainment options may change over time.

At the beginning of each period t an individual chooses how to allocate a fixed time endowment between two activities. An individual spends time playing a video game, y_t, and time consuming another outside entertainment option, z_t. Instantaneous utility in period t is given by the concave utility function:

$$U(y_t, y_{t-1}, p_t, z_t, e_t), \tag{2.1}$$

[5] Note that this is not true for games that continue to have associated monthly fees such as many massively multiplayer online role playing games (MMORPGs) that are traditionally considered to have strong addictive features.

where p_t represents the performance of the individual in period t in the video game and e_t is an unobservable shock to the marginal value of the outside option, $e_t \sim F_e(0, \sigma_e^2)$. The performance of an individual in period t is a random variable and is unknown to the individual at the start of the period when he chooses to allocate his time for playing, y_t. However, p_t is based on the skill of the individual (which is known to the individual but not the econometrician) as well as a random shock:

$$p_t = s + \eta_t, \tag{2.2}$$

where s is the skill of an individual and $\eta_t \sim F_\eta(\eta_{t-1}, \sigma_\eta^2)$ is a Markov process shock to performance. While an individual's performance depends on his underlying skill, uncertainty arises because performance is a relative measure and depends on the skill of his teammates and opponents. For example, an individual may realize higher performance when playing against low skill opponents than against high skill opponents. The shocks to performance follow a Markov process to allow for persistence in who you play with or against, which is not observed.[6]

A rational forward-looking consumer, therefore, solves the problem:

$$\max_{\{y_t, z_t\}_{t=1}^{\infty}} E_{p_t}\left[\sum_{t=1}^{\infty} \beta^{t-1} U(y_t, y_{t-1}, s_t, z_t, e_t)\right] \tag{2.3}$$

$$s.t. \quad (i) \quad y_t + z_t = H$$

$$(ii) \quad p_t = s + \eta_t,$$

where H is the time available in each period to allocate between playing the video game and the other entertainment option. The expectation is taken with respect to p_t given the past history up to point t. Eliminating z_t through the budget constraint, the first order condition

[6] In section 2.4.4 I consider the case where the skill of an individual evolves endogenously over time based on past playtime.

is:

$$E_{p_t}\left[U_1(y_t, y_{t-1}, p_t, H - y_t, e_t) - U_4(y_t, y_{t-1}, p_t, H - y_t, e_t) \right.$$
$$\left. + \beta U_2(y_{t+1}, y_t, p_t, H - y_{t+1}, e_{t+1})\right] = 0 \tag{2.4}$$

Consider a utility function that is quadratic in y_t, y_{t-1}, z_t and e_t. Suppose performance p_t interacts only with playtime y_t but may do so in a non-linear way.[7] Because the shocks to performance are Markov, we have $E_{p_t}[p_t|p_{t-1}] = p_{t-1}$, or that the expected value of performance today is exactly the observed value of performance yesterday. This structure allows us to use performance in the previous period, p_{t-1}, as the expected value of performance today. We can then write the first order condition as the following demand equation:

$$y_t = \theta y_{t-1} + \beta\theta y_{t+1} + \theta_1 p_{t-1} + \theta_2 (p_{t-1})^2 + \theta_3 e_t + \theta_4 e_{t+1} + K, \tag{2.5}$$

where θ and each θ_i are constants. This is the main equation used for estimation with forward-looking rational individuals. See appendix B.1.1 for more details on the derivation of equation 2.5.

The model predicts that an individual's demand for playing this video game is a function of playtime yesterday, playtime tomorrow, performance yesterday, and unobservable shocks to the value of the outside option. One implication is that the coefficient on y_{t-1} and y_{t+1} both share the same θ which represents the marginal effect of future and past playtime on playtime today. This θ is a parameter of interest and represents the degree of addiction in a rational addiction model. A significant and positive θ suggests that past and future consumption play an important role in determining consumption today. Larger values of θ suggest stronger rational addiction.

[7] That is, performance affects the marginal utility of playtime (in possibly a non-linear way) but does not directly affect the marginal utility of the outside option. For a more complete explanation of functional forms and derivation of the estimation equation see appendix B.1.1.

2.2.1 Estimation strategy

An implication of equation 2.5 is that an OLS estimate attempting to explain playtime would be biased. There is positive autocorrelation in the shocks to the marginal value of the outside option e_t. Even if θ and α were actually zero, an OLS regression would suggest they were positive. In addition, even if the term e_{t+1} was not present in equation 2.5, estimates would still be biased due to its dynamic nature.

To deal with this endogeneity and autocorrelation I use the playtime of friends as an instrument for the playtime of an individual. Becker et al. (1994) uses the price of cigarettes as an instrument for consumption of cigarettes. The analogue in my model would be to use lagged performance as an instrument for playtime. However, because of the persistence of shocks in performance this is not a suitable instrument. A shock to performance affects all future performance and playtime. In addition to observing individual characteristics I also observe the identity of an individual's friends and can determine their characteristics. If an individual chooses friends who are similar to himself in terms of playing time, then the average playtime and performance of friends should be correlated with an individual's own playtime. In addition, an individual's choice of how much to play should not affect the average skill and playtime of his friends. This makes the average playtime and skill of friends valid instruments for an individual's own playtime. For more on the the validity of these variables as instruments see appendix B.1.3. Instrumental variable estimation is done using the panel data IV methods of Balestra and Varadharajan-Krishnakumar (1987).

Finally, the constant K includes the individual specific quantity of time available for entertainment H. Including individual fixed effects will allow H to differ for each individual.

2.2.2 Myopic model

The rational addiction model can also easily accommodate individuals who are not forward-looking, but who exhibit non-rational or myopic addiction in their playing. This would be consistent with earlier models of habit formation such as in Pollak (1970) and Pollak (1976). In the case of myopic addiction, past playtime affects the marginal utility from playing today, but individuals are myopic and do not consider how their decision today may affect their future utility. The first order condition equation is the same as equation 2.4 except that all forward-looking elements are zero. In this case we have:

$$y_t = \theta y_{t-1} + \theta_1 p_{t-1} + \theta_2 (p_{t-1})^2 + \theta_3 e_t + K \tag{2.6}$$

This is the main equation used for estimation of myopic addiction. As with forward-looking individuals, the main parameter of interest is θ. A significant and positive θ suggests that past consumption plays an important role in consumption today. Larger values of θ suggest stronger habit formation and addiction in the myopic sense.

2.3 Data

The majority of empirical research on addiction in both psychology and economics faces severe data limitations. Research in the field of Psychology tends to use either survey or experimental data. In both cases the sample size is usually small, there are potential selection issues and estimation is based on a single or small number of cross-sections. With survey data, individuals may not have incentives to report truthfully. Economics literature also relies on both survey and experimental data, as well as aggregate data at the national, state, or county level. While aggregate data allows multiple cross-sections over time (usually by month or year), because of its aggregate nature, it is impossible to determine what happens at the individual level.

Using aggregate data to estimate the rational addiction model leads to misleading results. For a detailed explanation of potential problems using aggregate data to estimate rational addiction see Auld and Grootendorst (2004). As an example why aggregate data can be misleading, imagine an example where every 18 year old consumes cigarettes for one year as soon as they are legally able to do so and never smokes again after that year. In this case, if there is any data aggregation, even at the city level, an estimation would conclude that cigarettes are highly addictive, which in this extreme example they are not. The persistence of consumption in aggregate data, which may not come from the same individuals, leads to a predisposition to identify rational addiction. While there are some exceptions that take advantage of micro-data to estimate rational addiction, such as Chaloupka (1991) and Labeaga (1999), even in these cases either the individual or time dimension is usually small.

My data does not have such weaknesses. I use a very large unbalanced panel dataset collected from publicly available information for the online video game Team Fortress 2 (TF2), developed by Valve Corporation.[8] TF2 is a first-person-shooter game in which individual players play as one of nine different classes (such as soldier, medic, spy, et cetera) to work together as a team and compete against an opposing team of other players to complete an objective. For more information about the game see appendix B.2.

TF2 was originally released in October 2007 as part of a package of five games called the Orange Box which sold over 3 million copies within the first year (Remo, 2008). Since its release TF2 has been primarily distributed through Valve Corporation's online digital distribution and multiplayer system called Steam. Despite being almost five years old, today TF2 remains the fifth most played game on the Steam system which distributes over 1,504 games.[9] Associated with the Steam system is an online community which publicly reports extensive gameplay statistics for individuals by game. The Steam community also allows individuals to form bi-lateral friendship links, which are also reported. Through friendship links individuals are notified when a friend plays, they can easily communicate with friends,

[8]This particular game was chosen due to the high quality and availability of data. There is no reason for this particular game to be any more "addictive" than any other video game. TF2 is likely to be less addictive than other games, particularly massively multiplayer online role-playing games (MMORPGs) which have progressive elements that are often cited as encouraging addiction.

[9]See `http://www.steampowered.com` and `http://www.valvesoftware.com/` for additional information about the Steam system. The popularity of TF2 determined from `http://store.steampowered.com/stats/` and number of games available on steam determined from `http://store.steampowered.com/search/` on 4/25/2012.

and they can join and play with a friend with a single click. My dataset consists of individual and friend statistics collected from this online Steam community.

I collected weekly cross-sections of all individual members in the "Team Fortress 2 Official Game Group" on Steam over a period of approximately 18 months from February 1st, 2010 until June 22nd 2011. By extrapolation, the "Offical Group" represents approximately 70-80% of all active TF2 players.[10] Using these data, I construct an unbalanced panel dataset of 160,810 individuals who are active in at least one cross-section. An individual is considered active in a particular cross-section if they played at least 30 minutes[11] in that week. The average user is active for 14 weeks with 2.8 gaps in their playtime.[12] One feature of TF2 is individuals play as one of nine distinct "classes." These classes are divided into three distinct roles: offense, defense and support. Each class has particular strengths and weaknesses designed to complement their teammates in different ways. At any time during a game individuals are free to change which class they are playing. The data I collect is reported at the class-level which I then aggregate by class for each individual.

For each individual in each week I observe the amount of time they play (hours per week) and the average number of points they earn per hour (if active), which is a measure of performance. For each individual I observe the set of their friends, which allows me to determine the characteristics of friends. The final result is a panel dataset of 160,810 individuals with 66 cross-sections and with a total of 2,247,239 individual-time observations. Table 2.1 presents a list of the important variables with descriptions and summary statistics for active individuals. The summary statistics are first averaged across cross-sections by individual and represent the mean and median individual.

While the average total playtime of 4.73 hours per week is not particularly high (approximately 40 minutes per day), the distribution is heavily skewed to the right. For example,

[10] Extrapolated using server statistics in November 2010.

[11] 30 minutes was the average length of one round on a non-arena (i.e. a cooperative) style map in November of 2010.

[12] A gap is defined as a week in which an individual does not play consecutively, excluding when they initially enter or exit the panel.

Table 2.1: Definition, mean, median, and standard deviations of variables

Variable	Mean	Median	SD	Definition
y_t	4.73	3.71	3.86	Total amount of time an individual plays in week t measured in hours.
p_t	93.8	87.4	55.8	Performance of an individual in week t measured by the average number of accumulated points per hour of playtime in the previous week.
F_t	4.31	1.67	6.88	Number of friends an individual has in week t.
y_t^F	7.10	6.85	3.75	Average playtime of an individual's friends in week t.
s_t^F	95.4	94.2	34.9	Average performance of an individual's friends in week t.
m_t	0.338	0.299	0.149	An index that measures the degree an individual specializes among classes. See appendix B.3.1 for details.

in an average cross-section 90% of individuals play 14 hours or less and these individuals average 4.57 hours per week. The remaining 10% that play more than 14 hours per week average 21 hours per week. Put another way, in an average week 50% of active individuals contribute 85% of all the time spent playing.

2.4 Results

In this section I present my main empirical results. I start by estimating the myopic model and then consider the rational model with forward-looking individuals. Next, I estimate the rational addiction model at the individual level to identify and characterize what it means to be an addict. Finally, I consider a modification to the model to allow for learning, or endogenous skill formation.

2.4.1 Myopic model

Before estimating the model of rational addiction with forward-looking individuals I estimate the myopic model from section 2.2.2. Table 2.2 presents the estimation results of the myopic model. Each column of this table is estimated using a panel regression with fixed effects. The fixed effects capture individual differences in the endowment of time. The first four columns are estimated with instrumental variables to correct for the endogeneity of playtime. Columns (i)a and (i)b use the same period average playtime of friends as an instrument for an individual's playtime, while columns (ii)a and (ii)b add lagged average performance of friends as an instrument. The final column (iv) is estimated without instruments. Each of the instrumental variable regressions include the χ^2 test statistic from a Hausman test to check if the non-IV estimate is consistent. Since this hypothesis is rejected in each case, I focus on the instrumental variable estimates in the first four columns. Also included in the regression are individual fixed effects. Columns (i)a and (ii)a estimate equation 2.6 with individual fixed effects and some date fixed effects[13] as explanatory variables. Columns (i)b and (ii)b add as additional explanatory variables non-linear specialization and number of friends.

The main estimates of interest are θ, the coefficient on lagged playtime y_{t-1}, as well as θ_1 and θ_2, the coefficient on lagged performance p_{t-1} and lagged performance squared, p_{t-1}^2. θ represents the persistence of playtime, or the degree to which past playtime affects current playtime. In each specification θ is positive and significant, ranging from 0.441 to 0.47 depending on instruments and specification. This coefficient represents the degree of habit formation or addiction when utility today depends on past playtime. For example, a value of $\theta = 0.4$ would imply that every hour that an individual played in the previous week increases the amount he plays this week by 24 minutes. These estimates are slightly smaller but similar in magnitude to those found in Becker et al. (1994) where they found

[13]The date fixed effects included are for the month of July (summer holiday), the last two weeks in December (winter holiday), and a number of weeks in which a major patch or update to the game was added.

Table 2.2: Estimates of Myopic Models of Addiction, Dependent Variable y_t

	Fixed effects with IV				Non-IV
	(i)a	(i)b	(ii)a	(ii)b	(iii)
y_{t-1}	0.47*** (84.2)	0.443*** (77.3)	0.47*** (73.7)	0.441*** (66.8)	0.34*** (452)
p_{t-1}	0.00266*** (12.3)	0.00216*** (9.22)	0.0035*** (13.1)	0.00283*** (9.74)	-0.000427** (-2.49)
p^2_{t-1}	-0.000003*** (-9.66)	-0.000003*** (-8)	-0.000004*** (-9.96)	-0.000004*** (-8.08)	1e-07 (0.415)
m_{t-1}	-	1.21*** (8.5)	-	1.18*** (7.1)	-0.076 (-0.745)
m^2_{t-1}	-	-0.987*** (-8)	-	-0.967*** (-6.67)	-0.277*** (-2.76)
F_{t-1}	-	-0.0877*** (-84.1)	-	-0.0847*** (-73.4)	-0.0936*** (-97.6)
summer holiday	3.36*** (122)	3.26*** (119)	3.47*** (109)	3.36*** (106)	3.13*** (131)
winter holiday	1.3*** (34.8)	1.22*** (32.7)	1.46*** (35)	1.36*** (32.8)	1.04*** (31.8)
major patch	2.22*** (109)	2.24*** (110)	2.4*** (98.7)	2.41*** (100)	2.11*** (118)
R^2	0.147	0.153	0.146	0.152	0.147
χ^2	1326	809	1402	997	-
n	108276	108276	87415	87415	130902
T	64	64	63	63	64
N	1358745	1358745	1056907	1056907	1640065

Significance levels: * = 10%, ** = 5%, *** = 1%, t-statistics in parentheses.

estimates of θ for cigarettes ranging from 0.478 to 0.602 in a myopic model of addiction using instruments.

Greater performance in the previous period also contributes to greater playtime this period. The estimates of the coefficients on lagged performance, θ_1 and θ_2, represents the effect of performance last period on playtime this period. In each of the instrumental variable estimations θ_1 and θ_2 are significant. θ_1 is positive and captures the first order effect of an increase in performance on playtime. θ_2 is negative and captures the diminishing marginal returns of performance on playtime. Evaluated at the median performance, a one standard deviation increase in skill increases the playtime of an individual by between 5 and 9 minutes per week.

The estimates of the remaining coefficients are also useful in explaining playtime. The estimates of the coefficients on specialization, m_{t-1} and m_{t-1}^2, are significant. The values of the specialization variable m_t range from complete diversification at $m_t = 1/9$ to complete specialization at $m_t = 1$. Going from complete diversification to a value of $m_t = 0.61$ increases an individual's playtime by 15 minutes per week, while continuing on to complete specialization reduces his playtime from this peak by 9 minutes. Each friend decreases an individual's playtime by 5 minutes per week. Finally, the date fixed effects are all positive. A summer or winter holiday increases the amount an individual plays by around 3.3 hours and 1.3 hours per week respectively, while the release of a major patch increases playtime by 2.3 hours per week.

2.4.2 Rational model

Estimating the myopic model provides evidence of habit formation or addiction in a myopic sense. However, the real focus is on rational addiction with forward looking individuals. To determine if there is evidence of rational addiction, I estimate equation 2.5 for the significance of the coefficient on leading playtime y_{t+1}. A criterion for rejecting a model of rational addiction is if the coefficient on leading playtime is not significant. Table 2.3 presents the estimation results of the rational model and shows that this coefficient is significant.

As in the estimation of the myopic model, each column of table 2.3 is estimated using a panel regression with fixed effects. The fixed effects capture individual differences in the endowment of time. The first four columns are estimated with instrumental variables to correct for the endogeneity of playtime. Columns (i)a and (i)b use the same period average playtime of friends as an instrument for an individual's playtime, while columns (ii)a and (ii)b add lagged average performance of friends as an instrument. The final column (iv) is estimated without instruments. Each of the instrumental variable regressions include the χ^2

test statistic from a Hausman test to check if the non-IV estimate is consistent. Since this hypothesis is rejected in each case, I focus on the instrumental variable estimates in the first four columns. Columns (i)a and (ii)a estimate equation 2.5 with individual fixed effects and some date fixed effects as explanatory variables. Columns (i)b and (ii)b add as additional explanatory variables non-linear specialization and number of friends.

Table 2.3: Estimates of Rational Models of Addiction, Dependent Variable y_t

	Fixed effects with IV				Non-IV
	(i)a	(i)b	(ii)a	(ii)b	(iii)
y_{t-1}	0.361*** (53.2)	0.351*** (51.3)	0.359*** (48.1)	0.347*** (46)	0.278*** (335)
y_{t+1}	0.371*** (50.2)	0.359*** (47.1)	0.364*** (46)	0.353*** (43.3)	0.289*** (339)
p_{t-1}	0.00131*** (5.03)	0.00118*** (4.2)	0.00183*** (5.94)	0.00158*** (4.75)	-0.00105*** (-5.3)
p^2_{t-1}	-0.000002*** (-3.77)	-0.000001*** (-3.46)	-0.000002*** (-4.34)	-0.000002*** (-3.76)	0.000001*** (3.05)
m_{t-1}	-	1.17*** (7.18)	-	1.1*** (5.98)	0.0682 (0.591)
m^2_{t-1}	-	-0.913*** (-6.46)	-	-0.878*** (-5.47)	-0.332*** (-2.9)
F_{t-1}	-	-0.042*** (-29.2)	-	-0.0416*** (-27.2)	-0.0551*** (-53.2)
summer holiday	2.32*** (65.2)	2.3*** (65.2)	2.43*** (61.1)	2.41*** (61)	2.36*** (90.5)
winter holiday	1.53*** (37.9)	1.49*** (36.8)	1.67*** (38)	1.62*** (36.7)	1.28*** (36.2)
major patch	2.23*** (99.7)	2.25*** (100)	2.37*** (92.4)	2.39*** (93.2)	2.15*** (110)
R^2	0.222	0.224	0.219	0.221	0.216
χ^2	974	687	1040	780	-
n	86535	86535	72148	72148	106883
T	63	63	62	62	63
N	1049626	1049626	850345	850345	1288930

Significance levels: * = 10%, ** = 5%, *** = 1%, t-statistics in parentheses.

In each specification the coefficient on leading playtime y_{t+1} is significant and positive which provides evidence of rational addiction. The estimates of θ, the coefficient on lagged playtime y_{t-1}, range from 0.347 to 0.361 in the specifications using instruments. These estimates are slightly smaller but similar in magnitude to those of Becker et al. (1994) for cigarettes where they find estimates ranging from 0.373 to 0.481 in their specifications using instruments. The estimates for the coefficient on y_{t+1} are all significant and range from 0.353 to 0.371 in the specifications using instruments. This is significantly higher than the estimates in Becker et al. (1994) which range from 0.135 to 0.236. One reason the estimates of the coefficient on y_{t+1} are much closer to the estimates of the coefficients on y_{t-1} is that my data are weekly and reflect weekly, rather than annual, discount rates. For example, a weekly discount rate of $\beta = 0.99$ is equal to an annual discount rate of $0.99^{52} \approx 0.59$. While this explains why the estimates may be closer, it does not explain why the coefficient on y_{t+1} appear to be consistently larger than the coefficient on y_{t-1}. One possible explanation is learning. If by playing an individual improves his future performance, then there is an additional benefit to playing today that will be realized in the future. In section 2.4.4 I investigate effects of learning and show that that this may provide an explanation for these results.

Greater performance in the previous period also contributes to greater playtime this period. The estimates of the coefficients on lagged performance, θ_1 and θ_2, represents the effect of performance last period on playtime this period. In each of the instrumental variable estimations θ_1 and θ_2 are significant. θ_1 is positive and captures the first order effect of an increase in performance on playtime. θ_2 is negative captures the diminishing marginal returns of performance on playtime. Evaluated at the median performance, a one standard deviation increase in skill increases the playtime of an individual by between 3 and 5 minutes per week.

The estimates of the remaining coefficients are also useful in explaining playtime. The estimates of the coefficients on specialization, m_{t-1} and m_{t-1}^2, are significant. The values of the specialization variable m_t range from complete diversification at $m_t = 1/9$ to complete specialization at $m_t = 1$. Going from complete diversification to a value of $m_t = 0.64$ increases an individual's playtime by 15 minutes per week, while continuing on to complete specialization reduces his playtime from this peak by 7 minutes. Each friend decreases an individual's playtime by 2.5 minutes per week. Finally, the date fixed effects are all positive. A summer or winter holiday increases the amount an individual plays by around 2.35 hours and 1.6 hours per week respectively while the release of a major patch increases playtime by 2.3 hours per week.

Since the coefficient on leading playtime, y_{t+1} is positive and significant this provides evidence in support of rational addiction in this video game. The estimates of θ are large, similar in magnitude to the estimates in Becker et al. (1994) for cigarettes, suggesting a strong addictive component to the game.

2.4.3 Identifying addicts

While the results above provide evidence of rational addiction in this game, a natural question that follows is how can we use this to identify and characterize addicts? While the game exhibits evidence of rational addiction, this does not mean that all individuals who play the game are addicted. The rational addiction model coupled with individual data over time suggests a natural way to identify individual addicts.

I apply the rational addiction model at the individual level and estimate equation 2.5 separately for each individual. This allows me to identify an individual-specific measure of the degree of addiction, θ_i. Estimating individual specific θ_i in this way does not introduce bias since all parameters are individual-specific (i.e. there are no common parameters), but the estimates may be imprecise. By estimating equation 2.5 separately for each individual using as instruments the playtime and skill of friends, I am able to identify the individuals with significant and positive θ_i, hereafter called "addicts".

This approach is related to Fernandez-Val and Lee (2010) in which they develop a GMM method to estimate panel data with nonaddative unobserved heterogeneity. They then apply this method to the original Becker et al. (1994) model and data to show there is significant State-specific heterogeneity in the price effect. In their case some parameters are not individual-specific, since the addictiveness of cigarettes should not depend on the State. Because of this, a separate OLS regression for each state would lead to severely biased estimates due to an incidental parameter problem. To address this problem, they develop a method to correct the bias in mean an variance. In my case, however, I believe that all parameters may be individual-specific which allows me to treat each individual as a separate regression.

Since the dataset is an unbalanced panel, some individuals have a small number of observations, due either to the individual leaving the dataset early or entering late. Among the 42,103 individuals with sufficient data to make estimation possible[14] I identify 5,626 or 13.4% as addicts with significant and positive θ. This addiction rate is significantly larger than found in Gentile (2009) who using a national survey data show that 8% of youths aged 8 to 18 had symptoms of video game addiction. Among the these addicts there are 677 (or 7.3% of addicts) who also have a positive and significant estimate for the coefficient on y_{t+1}, which is evidence of rational forward-looking addiction.

Table 2.4 presents the mean and median values for variables and estimates of coefficients when individuals are divided into subgroups of non-addicts, myopic addicts and rational addicts. These three subgroups are surprisingly similar in characteristics. Myopic addicts play slightly more (about 20%) than non-addicts and have slightly higher performance.[15] Both myopic and rational addicts tend to have more friends than non-addicts, and these friends tend to play more. Myopic addicts also tend to have friends that have higher performance than non-addicts.

[14]Limiting the estimation to only individuals with a large number of observations may lead to a selection problem since addiction should be correlated with remaining in the dataset longer. Most of the individuals dropped however are dropped based on when they entered the dataset. By design each observation for estimation requires a lagged and leading observation. The minimum requirements for estimating an individual is therefore seven three-consecutive period observations in the data. Those that cannot be estimated are primarily those that entered the dataset late or have gaps in their consecutive playtime. I also show later on that there is no evidence of selection problems.

[15]In the case of drug and alcohol abuse, typically one of the features of addiction is less social participation. In this case, since an online video game involves social interactions addiction leads to greater rather than less social interaction. While the quality of interactions may not be the same, this is possibly a positive side effect of online video game addiction.

Table 2.4: Differences between addicts and non-addicts

Variable or Estimate	Non-addicts Mean	Non-addicts Median	Myopic Addicts Mean	Myopic Addicts Median	Rational Addicts Mean	Rational Addicts Median
y_t	7.11	6.21	8.57	7.73	7.9	7.24
p_t	96.71	95.38	100.21	99.85	96.10	94.43
F_t	8.91	5.84	12.55	9.48	11.16	8.18
y_t^F	7.8	7.64	8.25	8.15	8.06	7.87
p_t^F	96.76	96.99	98.69	99.53	96.03	97.25
$\hat{\theta}_i$	-	-	0.93	0.88	0.842	0.65
$\hat{\beta\theta}_i$	-	-	-	-	0.845	0.651
N	17.84	14	27.6	25	22.70	18

The similarity between these groups (in particular in terms of playtime) suggests that using a simple metric to determine addiction, such as a cut-off rule in playtime of 14 hours or more per week, may lead to very different set of addicts than predicted by the rational addiction model. A cut-off rule identifies addicts based on the the absolute amount of time spent playing rather than the relative amount to available hours. Estimating rational addiction at the individual level captures persistence in playing as well as allowing for differences in the amount of time available for entertainment across individuals.

Among myopic and rational addicts the mean estimates for θ_i is 0.93 and 0.842 respectively. This estimate is much higher than in the panel data specifications and suggests very strong addiction. A value of $\theta_i = 0.8$ implies that an individual plays an extra 48 minutes this week for each hour he played in the previous week. Among rational addicts the mean estimate for $\hat{\beta\theta}$ is 0.845, or that future playtime is an important factor in determining current playtime.

In order to verify that dropping individuals with too few observations for estimation does not cause a selection problem, I check the addiction rate among individuals with an arbitrary but specific number of observations (rather than those with more than a certain number). Among individuals who have exactly 20 observations the addiction rate is 14% and among these addicts 9% show evidence of rational forward-looking addiction. This addiction rate is similar to the rate using all individuals for which estimation is possible, and this suggests that dropping individuals with too few observations to estimate does not necessarily lead to over-selection of addicts.

Until there is a formal definition and classification of addiction in video games it is not possible to conclude whether estimating the rational addiction model at the individual level is better than a simple metric for identifying addicts. However, the rational addiction model has the advantage of allowing for different marginal value of entertainment time by individuals, even when data on this is not available. This is important if addiction and the associated negative effects depend on the value of what an individual is giving up in order to play as well as the quantity of time spent playing. For example, if an individual has more free time for entertainment than another, the negative effects of both playing the same number of hours may be less for this individual. While playing more should be associated with greater addiction for an individual, comparing the amount of time two individual spend playing may not always correctly identify the addict.

Estimating the rational addiction model at the individual level also provides evidence that using micro data rather than aggregate data helps prevent spurious results. One of the main criticisms of Becker et al. (1994) is that aggregate data tends generate spurious evidence in favor of the rational addiction model. To understand why this is the case see Auld and Grootendorst (2004) as well as my simple example in section 2.3. Auld and Grootendorst (2004) also state that this particular problem of spurious results may not apply to micro data. These individual estimation results support their statement. If the results in section 2.4 were spurious, then it should follow that the addiction rate when estimating the model separately for each individual should be very high. Since the addiction rate is low, only 13.4%, and not an artifact of selection problems, this supports the argument that micro significantly reduces the risk of spurious results in the rational addiction model.

Finally, the results from the individual estimations suggest that there is very strong heterogeneity in the data. Estimating the model separately for each individual suggests that there is only a small proportion of individuals who are addicted. This heterogeneity may mean that it is not a characteristic of the game which leads to addiction, but rather some characteristic of an individual that determines addiction. This suggests an approach for looking at addiction that is individual-based: what is it about an individual that causes him to become addicted to different goods? Rather than a good-focused approach: what is it about this product that makes it addictive?

2.4.4 Learning

The demand equation 2.5, which follows from the rational addiction model, predicts that the estimate of the coefficient on y_{t+1} should represent $\beta\theta$ while the estimate of the coefficient on y_t should represent θ. Since $\beta \leq 1$, it follows that $\beta\theta \leq \theta$. Since the discount rate β is a weekly discount factor and may be very small (for example $\beta = 0.999$), the estimates of these two coefficients may not be significantly different from each other. For the purpose of rejecting the myopic model in favor of the rational model, the significance rather than the value of the coefficient on y_{t+1} is important. However, in a number of the instrumental variable specifications in table 2.3, the coefficient on y_{t+1} is statistically larger than the coefficient on y_{t-1}. A significantly larger coefficient on y_{t+1} is not consistent with the model. In this section I will consider learning, which provides one possible explanation for this inconsistency.

One significant difference between playing video games and consuming a good, such as cigarettes, is that video games may involve significant learning-by-doing. When an individual first begins to play a video game they may not be familiar with some aspects of the game, such as the game mechanics or layout of the maps. Playing the game over time may lead to higher level of skill, which then is reflected in performance. In the context of human capital development, learning means that investment by playing today increases the stock level of skill, which then affects future decisions. In this case the choice of action today affects all future decisions and introduces significant dynamic complications. One attractive features of the rational addiction model is that even though the assumption of additive separable utility in time is relaxed, the model can be reduced to a simple demand equation that depends only on your consumption in the previous and next future period. No dynamic programming is required to solve the rational addition model. Adding learning, however, changes this. If skill is a stock variable that persists each period, improvements to this stock level affect decisions not just tomorrow but in all future periods.

I introduce a modified version of the model from section 2.2 that allows an individual's skill, s_t, to evolve endogenously over time based on the past playtime decisions of that individual without resorting to dynamic programming techniques to solve the model. In contrast to the earlier model, I assume that the marginal utility of playing in period t depends on an individual's performance at the start of period, p_t, rather than the performance realized during period. At the start of period t, performance p_t is realized based on the underlying skill of an individual at the start of the period, s_t, and a random shock:

$$p_t = s_t + \eta_t, \tag{2.7}$$

where $\eta_t \sim F_\eta(0, \sigma_\eta^2)$. Performance p_t is observed by individuals and the econometrician and known when an individual chooses y_t. Skill s_t is known by the individual, but not the econometrician. At the end of each period (after utility for that period is realized) skill s_t transitions based on the amount an individual played during that period and some noise:

$$s_{t+1} = \delta s_t + b_1 y_t + \mu_t, \tag{2.8}$$

where $\mu_t \sim F_\mu(0, \sigma_\mu^2)$. While skill is not observable by the econometrician, observing only performance and playtime in period t is sufficient to determine the expected value of performance for the next period. Equations 2.7 and 2.8 imply that performance p_t transitions according to:

$$p_{t+1} = \delta p_t + b_1 y_t + \eta_{t+1} - \delta\eta_t + \mu_t, \tag{2.9}$$

so that in period t:

$$E\left[p_{t+1}|p_t, y_t\right] = \delta p_t + b_1 y_t. \tag{2.10}$$

The individual faces the same problem as before, expect now p_t is known in each period and the expectation is taken with respect to performance in future periods:

$$\begin{aligned} &\max_{\{y_t,z_t\}_{t=1}^{\infty}} E\left[\sum_{t=1}^{\infty} \beta^{t-1} U(y_t, y_{t-1}, p_t, z_t, e_t)\right] \qquad (2.11)\\ &\text{s.t.} \quad (i)\ p_t = s_t + \eta_t \\ &\qquad\quad (ii)\ s_{t+1} = \delta s_t + b_1 y_t + \mu_t \\ &\qquad\quad (iii)\ z_t + y_t = H. \end{aligned}$$

This is a relatively standard dynamic programming problem. However, using dynamic programming techniques is in a sense inconsistent with the model of rational addiction. One of the main features of the rational addiction model is that while an individual is full rational, his decision only depends on the previous and next future period and not all future periods. In the spirit of this structure, rather than resorting to dynamic programming, I approximate the problem to maintain a similar structure with the rational addiction model. For the full details see appendix B.1.2. The solution to the approximated problem can be written as the linear demand function:

$$y_t = \theta y_{t-1} + \beta(\theta + \alpha) y_{t+1} + \theta_1 p_{t-1} + \theta_2 e_t + \theta_3 e_{t+1} + K, \tag{2.12}$$

Where the additional term α represents the value of learning.

Equation 2.12 is a first order approximation of the true solution to the problem in 2.11 and there are two ways to interpret this demand function. The first is that it represents demand when individuals are forward-looking but only consider how playtime affects their skill in the immediately following period (rather than all future periods). This semi-myopic approach eliminates the longer-term effects of learning and collapses the problem back to the same horizon as the rational addiction model. Ultimately however, this interpretation is inconsistent with rational behavior. A second interpretation is that equation 2.11 represents a numerical approximation of the true demand function which should depend on all future values of playtime, but is not feasible to estimate.

If α is positive then an individual considers how his playing today affects his skill tomorrow which in turn affects the utility from playing tomorrow. Larger values of α imply stronger learning effects. With this specification it is not possible to directly estimate α, however with the estimate of θ it is possible to calculate an implied estimate of α for different values of β. Using the results in table 2.3 of section 2.4 we can calculate implied estimates for α.[16] Table 2.5 presents the implied estimates of α for a given discount rate β.

Table 2.5: Implied estimates of parameter α for different discount rates β

β	(i)a	(i)b	(ii)a	(ii)b	(iii)
0.999	0.011*	0.009	0.005	0.0066	0.011***
0.995	0.012**	0.01*	0.0065	0.008	0.012***
0.99	0.014**	0.012*	0.0083	0.0098	0.013***
0.98	0.018***	0.016**	0.012*	0.013*	0.016***
0.95	0.03***	0.028***	0.024***	0.025***	0.025***
0.9	0.051***	0.049***	0.045***	0.045***	0.042***

Significance levels: * = 10%, ** = 5%, *** = 1%

While the significance and magnitude of the implied α is directly tied to the assumed value of β, the imputed estimate for α is significant and positive in many of the specifications. α can be interpreted as the degree to which learning affects an individual's decision to play today. For example, an individual with a weekly discount rate of $\beta = 0.98$ (which corresponds to an annual discount rate of 0.35) and a value of $\alpha = 0.017$ will play an additional minute

[16]The change in notation for performance means that p_t in problem 2.11 is actually p_{t-1} in the model in section 2.2. This means that we can directly apply the results in table 2.3. While these results also include the term p_{t-1}^2, re-estimating without this term does not significantly change the coefficients of interest on y_{t-1} and y_{t+1}.

this week for each hour he anticipates playing next week in order to improve his skill and increase the value of playing next week.

2.5 Conclusion

In this chapter I analyze and provide evidence in support of addiction in video games. I adapt the Becker and Murphy (1988) model of rational addiction for video games and estimate this model using unique panel data I collected from the online video game Team Fortress 2.

The empirical results provide evidence in support of rational addiction in video games. Past and future consumption of this game play a significant role in determining the amount an individual plays today. The degree to which past consumption affects current consumption is similar in magnitude to what Becker et al. (1994) find for cigarettes.

By using data collected at the individual rather than aggregate level, I am able to avoid some empirical problems with estimating the rational addiction model. Auld and Grootendorst (2004) show that when aggregate data with strong auto-correlation is used to estimate rational addiction it leads to spurious results in favor of addiction. The data are also rich enough to allow me to estimate the model separately for each individual. When I do this, I find an addiction rate of approximately 13.4%, and of these addicts 7.3% show evidence of rational addiction. In this case both myopic and rational addicts show much stronger addiction than when estimated as a whole population. Estimating the rational addiction model separately for individuals allows me to identify addicts in a way that is consistent with the rational addiction model.

The results of the individual estimations suggests that the addiction trends identified in the full data may be driven primarily by a small number of addicted individuals rather than a particular feature of the game. At the same time, these estimates of addiction for this particular video game represent a lower bound on the addictiveness of video games in general.

Finally, I present a simplified model that allows for learning and endogenous skill. Using this model I provide evidence of a skill-playtime feedback look: By playing today an individual improves his future skill which in turn reinforces his decision to play in the future. Learning affects the value of playing in the future, and this can contribute to rational addiction. This model provides the first step to looking at learning in more depth.

APPENDIX A

APPENDIX FOR CHAPTER 1

A.1 Characterization of equilibrium and optimal policies

A.1.1 Second period

In the final period the value function for each politician type is globally concave in x_1, x_2 and there exists a unique optimal investment $x_{i\theta}^{(2)}(\omega_1, \omega_2, v)$ for $i = 1, 2$ and $\theta \in \{L, R\}$ that is continuous in all parameters. The solution is given by the first order condition is:

$$-\frac{d\rho}{ds_i}\frac{\partial s_i}{\partial x_i} = \frac{\partial C_{i\theta}}{\partial x_i},$$

where $\frac{\partial s_i}{\partial x_i} = -1$, or it is optimal to invest until the marginal benefit (marginal reduction in the severity of an issue) is equal to the marginal social cost of investment. Since the marginal social cost, $\frac{\partial C_{i\theta}}{\partial x_i}$, is lower for the issue in which the elected politician has a comparative advantage it follows that $x_{1L}^{(2)} > x_{1R}^{(2)}$ and $x_{2R}^{(2)} > x_{2L}^{(2)}$, or that an elected politician invests more in the issue in which they specialize than the other party would.

From the implicit function theorem:

$$\frac{\partial x_{1\theta}^{(2)}}{\partial \omega_1} = \left(\frac{\partial^2 \rho}{\partial^2 s_2} + \frac{\partial^2 C_{i\theta}}{\partial^2 x_2}\right)\left(\frac{\partial^2 \rho}{\partial^2 s_1}\right) \Big/ \left|H_\theta^{(2)}\right| > 0,$$

where $\left|H_\theta^{(2)}\right|$ is the determinant of the Hessian matrix for politician type θ in the second period which is positive:

$$\left|H_\theta^{(2)}\right| = \left(\frac{\partial^2 \rho}{\partial^2 s_1} + \frac{\partial^2 C_{1\theta}}{\partial^2 x_1}\right)\left(\frac{\partial^2 \rho}{\partial^2 s_2} + \frac{\partial^2 C_{2\theta}}{\partial^2 x_2}\right) > 0.$$

Furthermore by simplifying we have:

$$\frac{\partial x_{1\theta}^{(2)}}{\partial \omega_1} = \frac{\left(\frac{\partial^2 \rho}{\partial^2 s_1}\right)}{\left(\frac{\partial^2 \rho}{\partial^2 s_1} + \frac{\partial^2 C_{1\theta}}{\partial^2 x_1}\right)} < 1,$$

or that $0 < \frac{\partial x_{1\theta}^{(2)}}{\partial \omega_1} < 1$. The optimal policy is always increasing in severity of the underlying issue but does not increase faster than the issue itself. Doing the same for investment in the second issue, we can fully characterize optimal investment and for any i, $j \neq i$ and θ:

$$0 < \frac{\partial x_{i\theta}^{(2)}}{\partial \omega_i} < 1, \qquad \frac{\partial x_{i\theta}^{(2)}}{\partial \omega_j} = 0, \qquad \frac{\partial x_{i\theta}^{(2)}}{\partial v} = 0.$$

Next, consider the value function of the voter in the second period for each politician type evaluated at the optimal investment. Define:

$$\hat{V}_\theta^{(2)}(\omega_1, \omega_2, v) = V_\theta^{(2)}\left(\omega_1, \omega_2, x_{1\theta}^{(2)*}(\omega_1, \omega_2, v), x_{2\theta}^{(2)*}(\omega_1, \omega_2, v), v\right).$$

Since there is no future, the value function of the voter from electing politician from party θ is equivalent to that of the politician:

$$V_\theta^{(2)}\left(\omega_1, \omega_2, x_1, x_2, v\right) \equiv W_\theta^{(2)}\left(\omega_1, \omega_2, x_1, x_2, v\right).$$

In particular, at the optimum we have:

$$\frac{\partial W_\theta^{(2)}}{\partial x_i} = 0 = \frac{\partial V_\theta^{(2)}}{\partial x_i},$$

which implies that

$$\frac{\partial \hat{V}_\theta^{(2)}}{\partial \omega_i} = -\frac{\partial \rho}{\partial s_i}\frac{\partial s_i}{\partial \omega_i} < 0,$$

since $\frac{\partial s_i}{\partial \omega_i} = 1$. An increase in the severity of an issue decreases the value to the voter from electing each politician type. In addition since the elected politician invests more in the issue in which he specializes we can order the monotonicity when evaluated at the optimal policy of the politician by:

$$\frac{\partial \hat{V}_R^{(2)}}{\partial \omega_1} < \frac{\partial \hat{V}_L^{(2)}}{\partial \omega_1} < 0, \qquad\qquad \frac{\partial \hat{V}_L^{(2)}}{\partial \omega_2} < \frac{\partial \hat{V}_R^{(2)}}{\partial \omega_2} < 0.$$

It follows from the concavity of the politicians problem that $\hat{V}_\theta^{(2)}(\omega_1, \omega_2, v)$ is globally concave in ω_1, ω_2.

Next, to find the optimal policy for the voter notice that since net-valence enters additively:

$$\frac{\partial \hat{V}_L^{(2)}}{\partial v} = -\frac{1}{2} < 0 < \frac{1}{2} = \frac{\partial \hat{V}_R^{(2)}}{\partial v}.$$

For any fixed ω_1, ω_2 as you increase the net-valence the value of electing a politician from the R-party increases while the value from electing a politician from the L-party decreases. Therefore, the unique optimal strategy for voter takes the form of a continuous cut-off rule $v^{(2)*}(\omega_1, \omega_2)$ in net-valence such that:

$$\theta^{(2)}(\omega_1, \omega_2, v) = \begin{cases} L & \text{if } v \leq v^{(2)*}(\omega_1, \omega_2) \\ R & \text{otherwise.} \end{cases}$$

To characterize the cut-off rule, notice that by design at the cut-off:

$$\hat{V}_L^{(2)}\left(\omega_1, \omega_2, v^{(2)*}(\omega_1, \omega_2)\right) \equiv \hat{V}_R^{(2)}\left(\omega_1, \omega_2, v^{(2)*}(\omega_1, \omega_2)\right).$$

Differentiating we have

$$\frac{\partial \hat{V}_L^{(2)}}{\partial \omega_i} - \frac{\partial \hat{V}_R^{(2)}}{\partial \omega_i} = \left(\frac{\partial v^{(2)*}}{\partial \omega_i}\right)\left(\frac{\partial \hat{V}_R^{(2)}}{\partial v} - \frac{\partial \hat{V}_L^{(2)}}{\partial v}\right).$$

Since $\frac{\partial \hat{V}_R^{(2)}}{\partial v} - \frac{\partial \hat{V}_L^{(2)}}{\partial v} = \frac{\partial u_R}{\partial v} - \frac{\partial u_L}{\partial v} = 1$, the change in the optimal cut-off rule when the severity of an issue changes is exactly the difference in slope of the value function for each politician type at the point where they cross:

$$\frac{\partial v^{(2)*}}{\partial \omega_i} = \left(\frac{\partial \hat{V}_L^{(2)}}{\partial \omega_i} - \frac{\partial \hat{V}_R^{(2)}}{\partial \omega_i}\right)_{v=v^{(2)*}(\omega_1,\omega_2)}.$$

Using the ordering on the monotonicity of the value functions from above we have:

$$\frac{\partial v^{(2)*}}{\partial \omega_1} > 0, \qquad \frac{\partial v^{(2)*}}{\partial \omega_2} < 0.$$

When the severity of an issue in which a politician specializes increases, the voter is willing to elect that politician with lower net-valence advantage than before.

Before looking at the problem of a politician in the first period, consider how the election strategy of the voter affects the expected probability of a politician being elected in the final period. Define the expected probability of a politician from party θ being elected in the next period when the severity of each issue after investment is $s_i = \omega_i - x_i$ as:

$$G_\theta(s_1, s_2, v) = E\left[Pr(\theta^{(2)}(\omega_1', \omega_2', v') = \theta),\right]$$

where the expecation is conditioned on the severity of issues after investment, s_1, s_2, the net-valence v as well as the correctly predicted optimal strategies of the voter in the next period. This can also be written as:

$$G_\theta(s_1, s_2, v) = \begin{cases} E\left[F_v\left(v^{(2)*}(\omega_1', \omega_2')|v\right)|s_1, s_2\right] & \text{if } \theta = L \\ 1 - E\left[F_v\left(v^{(2)*}(\omega_1', \omega_2')|v\right)|s_1, s_2\right] & \text{if } \theta = R, \end{cases}$$

where F_v is the CDF of v' or $F_v(x|v) = Pr(x \leq v')$. For now focus on the probability of a politician from the L-party being elected in the next period. We can expand the expectation:

$$G_L(\cdot) = \int_0^K \int_0^K F_v\left(v^{(2)*}(s_1 + \eta_1, s_2 + \eta + 2)|v\right) \frac{d\eta_1 d\eta_2}{K^2}$$

Taking the derivative with respect to s_1 we get:

$$\frac{\partial G_L}{\partial s_1} = \int_0^K \int_0^K \left(\frac{\partial v^{(2)*}}{\partial \omega_1'}\right) f_v\left(v^{(2)*}(s_1 + \eta_1, s_2 + \eta_2\right) \frac{d\eta_1 d\eta_2}{K^2}.$$

This term is positive since $v^{(2)*}(\omega_1', \omega_2')$ is increasing in ω_1'. If a politician from party θ has an advantage in issue i and a disadvantage in issue j then likewise we can characterize:

$$\frac{\partial G_\theta}{\partial s_i} \geq 0 \qquad\qquad \frac{\partial G_\theta}{\partial s_j} \leq 0$$

Or that a politician is more likely to be elected if either the issue he specializes in is more severe after investment. It also follows trivially:

$$\frac{\partial G_\theta}{\partial v_\theta} \geq 0,$$

or that when a politician becomes exogenously more attractive then everything else being equal he is more likely to be elected. While the second and cross derivative of $G_\theta(\cdot)$ will be important in determining the concavity of the problem of an office-motivated politician I will return to this later.

A.1.2 First period

In the first period ($t = 1$) the value function for an elected politician from party θ is:

$$W_\theta^{(1)}(\omega_1, \omega_2, x_1, x_2, v) = u_\theta(s_1, s_2, x_1, x_2, v) + \beta E\left[\hat{V}^{(2)}(\omega_1', \omega_2', v')\right] + \beta\psi G_\theta(s_1, s_2, v),$$

where $s_i = \omega_i - x_i$ and $G_\theta(s_1, s_2, v)$ is the expected probability of being elected in the next period defined as above:

$$G_\theta(s_1, s_2, v) = E\left[Pr(y'(\omega_1', \omega_2') = \theta)|s_1, s_2.\right]$$

The expectation can be expanded:

$$\begin{aligned}
W^{(1)}(\omega_1, \omega_2, x_1, x_2, v) =& u_\theta(\omega_1 - x_1, \omega_2 - x_2, x_1, x_2, v) \\
&+ \beta \int_0^K \int_0^K \left[\int_{-\infty}^{v^{(2)*}} \hat{V}_L^{(2)}(\omega_1', \omega_2', v') f_v(v'|v) dv' \right.\\
&+ \left. \int_{v^{(2)*}}^{\infty} \hat{V}_R^{(2)}(\omega_1', \omega_2', v') f_v(v'|v) dv'\right] \frac{d\eta_1 d\eta_2}{K^2} \\
&+ \beta\psi G_\theta(s_1, s_2, v),
\end{aligned}$$

where $\omega_i' = \omega_i - x_i + \eta_i$ and $v^{(2)*} = v^{(2)*}(\omega_1', \omega_2')$. Since at the optimal cut-off point the utility from the two politician types are exactly equivalent:

$$\hat{V}_L^{(2)}\left(\omega_1, \omega_2, v^{(2)*}(\omega_1, \omega_2)\right) \equiv \hat{V}_R^{(2)}\left(\omega_1, \omega_2, v^{(2)*}(\omega_1, \omega_2)\right),$$

the first order condition can be written as:

$$\begin{aligned}
&\frac{\partial \rho}{\partial s_i} - \frac{\partial C_{i\theta}}{\partial x_i} - \beta \int_0^K \int_0^K \left[\int_{-\infty}^{v^{(2)*}} \left(\frac{\partial \hat{V}_L^{(2)}}{\partial \omega_i'}\right) f_v(v'|v) dv' \right.\\
&\quad + \left. \int_{v^{(2)*}}^{\infty} \left(\frac{\partial \hat{V}_R^{(2)}}{\partial \omega_i'}\right) f_v(v'|v) dv'\right] \frac{d\eta_1 d\eta_2}{K^2} - \psi\beta \frac{\partial G_\theta}{\partial s_i} = 0,
\end{aligned} \tag{A.1}$$

where $s_i = \omega_i - x_i$, $\omega_i' = \omega_i - x_i + \eta_i$, and $v^{(2)*} = v^{(2)*}(\omega_1', \omega_2')$. When $\psi = 0$ the problem of politician is reduced to that of a social planner, or the politician simply maximizes the expected discounted utility of the voter.

In order for a unique solution to the first order conditions to exist, a sufficient condition is for the problem to be globally concave. To look at the concavity, first consider the cross-partial derivative:

$$\frac{\partial^2 W_\theta^{(1)}}{\partial x_1 \partial x_2} = \beta \int_0^K \int_0^K \left(\frac{\partial v^{(2)*}}{\partial \omega_1'}\right)\left(\frac{\partial v^{(2)*}}{\partial \omega_2'}\right) f_v(v^{(2)*}|v) \frac{d\eta_1 d\eta_2}{K^2} \leq 0. \tag{A.2}$$

Next, consider the second derivative of the problem with respect to x_i:

$$\begin{aligned}
\frac{\partial^2 W_\theta^{(1)}}{\partial^2 x_i} = & -\frac{\partial^2 \rho}{\partial^2 s_i} - \frac{\partial^2 C_{i\theta}}{\partial^2 x_i} + \beta \int_0^K \int_0^K \left\{ \left(\frac{\partial \hat{V}_L^{(2)}}{\partial \omega_i'} - \frac{\partial \hat{V}_R^{(2)}}{\partial \omega_i'}\right) f_v(v^{(2)*}|v) \left(\frac{\partial v^{(2)*}}{\partial \omega_i'}\right) \right. \\
& + \int_{-\infty}^{v^{(2)*}} \left(\frac{\partial^2 \hat{V}_L^{(2)}}{\partial^2 \omega_i'}\right) f_v(v')dv' \\
& \left. + \int_{v^{(2)*}}^{\infty} \left(\frac{\partial^2 \hat{V}_R^{(2)}}{\partial^2 \omega_i'}\right) f_v(v')dv' \right\} \frac{d\eta_1 d\eta_2}{K^2} + \psi\beta \frac{\partial^2 G_\theta^{(2)}}{\partial^2 s_i}.
\end{aligned}$$

Re-writing the second derivative we have:

$$\begin{aligned}
\frac{\partial^2 W_\theta^{(1)}}{\partial^2 x_i} = & -\left(\frac{\partial^2 \rho}{\partial^2 s_i} + \frac{\partial^2 C_{i\theta}}{\partial^2 x_i}\right) + \beta \int_0^K \int_0^K \left[\int_{-\infty}^{v^{(2)*}} \left(\frac{\partial^2 \hat{V}_L^{(2)}}{\partial^2 \omega_i'}\right) f_v(v'|v)dv' \right. \\
& \left. + \int_{v^{(2)*}}^{\infty} \left(\frac{\partial^2 \hat{V}_R^{(2)}}{\partial^2 \omega_i'}\right) f_v(v'|v)dv' \right] \frac{d\eta_1 d\eta_2}{K^2} \\
& + \beta \int_0^K \int_0^K \left(\frac{\partial v^{(2)*}}{\partial \omega_i'}\right)^2 f_v(v^{(2)*}|v) \frac{d\eta_1 d\eta_2}{K^2} + \beta\psi \frac{\partial^2 G_\theta^{(2)}}{\partial^2 s_i}.
\end{aligned} \tag{A.3}$$

Each term in the equation above is negative except for the last two terms which are non-negative. Therefore, the problem of the politician is not generally globally concave. Even when $\psi = 0$ and the problem of the politician is equivalent to that of a social planner the second to last term remains and introduces non-concavity. This non-concavity exists because the voter faces a discrete choice between two politicians. While the value function from electing each politician type may be globally concave the max of two concave functions is not concave.

While there are two possible sources of convexity, we want to focus on convexity that arises when politicians are office-motivated, rather than any convexity from the discrete choice nature of the voter's problem. I assume that the distribution over future states is sufficiently wide, or K and δ are sufficiently large, so that any convexity introduced by the discrete choice nature of the voter's problem is dominated by the concavity of the voter's one-period utility function. This is equivalent to assuming that when $\psi = 0$ the above term is always negative. Since $\frac{\partial v^{(2)*}}{\partial \omega_i'}$ is fully characterized by $\rho(s_i)$ and $C_i(x_i)$ for $i = 1, 2$ this should be seen as an assumption on the concavity of these functions and the degree of smoothing that enters from uncertainty. Formally, the assumption is that K and δ are sufficiently large that for $i = 1, 2$ and $\theta = L, R$ when $\psi = 0$ we have:

$$\frac{\partial^2 W_\theta^{(1)}}{\partial^2 x_i} < \frac{\partial^2 W_\theta^{(1)}}{\partial^2 x_i} + \frac{\partial^2 C_{i\theta}}{\partial^2 x_i} \leq \frac{\partial^2 W_\theta^{(1)}}{\partial x_1 \partial x_2} \leq 0. \tag{A.4}$$

This assumption can be thought of as an extension of the concavity assumption of the one-period utility function to the dynamic model in order to guarantee that the value function of the voter is globally concave at least when evaluated near the socially optimal. This allows us to focus on any convexity (and subsequent discontinuities) that may arise when politicians are office-motivated. If the assumption is violated, then it does not fundamentally change any of the results, but it may introduce some additional discontinuities in the optimal policies due to the discrete choice nature of the voter's problem. This complicates the results without adding useful economic interpretations.

A.1.3 Characterization of socially optimal investment

By the assumptions above, the problem of a politician who does not care about re-election (i.e. a social planner) is globally concave and there exists a unique socially optimal investment in each issue for each politician type. Because of the additive nature of the transition technology, a marginal increase in the severity of an issue has the same effect on the continuation value of the voter as a marginal decrease in the investment on that issue excluding any change in the social cost of investment. This means that for i and $j \neq i$ we have:

$$\frac{\partial^2 W_\theta^{(1)}}{\partial x_i \partial \omega_i} = -\frac{\partial^2 W_\theta^{(1)}}{\partial^2 x_i} - \frac{\partial^2 C_\theta}{\partial^2 x_i}, \qquad \frac{\partial^2 W_\theta^{(1)}}{\partial x_i \partial \omega_j} = -\frac{\partial^2 W_\theta^{(1)}}{\partial x_1 \partial x_2}.$$

Using the implicit function theorem and the above we have:

$$\frac{\partial x_{1\theta}^{(1)*}}{\partial \omega_1} = \frac{\left(\frac{\partial^2 W_\theta^{(1)}}{\partial^2 x_1} + \frac{\partial^2 C_\theta}{\partial^2 x_1}\right)\left(\frac{\partial^2 W_\theta^{(1)}}{\partial^2 x_2}\right) - \left(\frac{\partial^2 W_\theta^{(1)}}{\partial x_1 \partial x_2}\right)^2}{|H^{(1)}|} \geq 0.$$

The numerator is positive and the denominator is the determinant of the Hessian:

$$\left|H_\theta^{(1)}\right| = \left(\frac{\partial^2 W_\theta^{(1)}}{\partial^2 x_1}\right)\left(\frac{\partial^2 W_\theta^{(1)}}{\partial^2 x_2}\right) - \left(\frac{\partial^2 W_\theta^{(1)}}{\partial x_1 \partial x_2}\right)^2 > 0.$$

Likewise we have:

$$\frac{\partial x_{2\theta}^{(1)*}}{\partial \omega_1} = -\frac{\left(\frac{\partial^2 W_\theta^{(1)}}{\partial x_1 \partial x_2}\right)\left(\frac{\partial^2 C_{1\theta}}{\partial^2 x_1}\right)}{|H^{(1)}|} \geq 0.$$

By similar steps we can characterize $x_{2\theta}^{(1)*}$ and for $\theta = L, R$ and any $i, j = 1, 2$, we have:

$$\frac{\partial x_{i\theta}^{(1)}}{\partial \omega_j} \geq 0,$$

which proves proposition 2.

For the derivative with respect to net-valence we have:

$$\frac{\partial x_{1\theta}^{(1)*}}{\partial v} = -\frac{\left(\frac{\partial^2 W_\theta^{(1)}}{\partial^2 x_2}\right)\left(\frac{\partial^2 W_\theta^{(1)}}{\partial x_1 \partial v}\right) - \left(\frac{\partial^2 W_\theta^{(1)}}{\partial x_1 \partial x_2}\right)\left(\frac{\partial^2 W_\theta^{(1)}}{\partial x_2 \partial v}\right)}{|H^{(1)}|}.$$

Since we have $\frac{\partial^2 W_\theta^{(1)}}{\partial^2 x_i} < \frac{\partial^2 W_\theta^{(1)}}{\partial x_1 \partial x_2} \leq 0$, what remains is to determine the sign of $\frac{\partial^2 W_\theta}{\partial x_i \partial v}$.

Consider the first derivative of the politician's problem in equation A.1. Since $\psi = 0$ and the first two terms are independent of v the only term that depends on v in the first order condition is:

$$-\beta \int_0^K \int_0^K \left\{ \int_{-\infty}^{v^{(2)*}} \left(\frac{\partial \hat{V}_L^{(2)}}{\partial \omega_i'} \right) f_v(v'|v)dv' + \int_{v^{(2)*}}^{\infty} \left(\frac{\partial \hat{V}_R^{(2)}}{\partial \omega_i'} \right) f_v(v'|v)dv' \right\} \frac{d\eta_1 d\eta_2}{K^2}.$$

For a particular realization of η_1, η_2 we have:

$$-\beta \int_{-\infty}^{v^{(2)*}} \left(\frac{\partial \hat{V}_L^{(2)}}{\partial \omega_i'} \right) f_v(v'|v)dv' - \beta \int_{v^{(2)*}}^{\infty} \left(\frac{\partial \hat{V}_R^{(2)}}{\partial \omega_i'} \right) f_v(v'|v)dv'.$$

To differentiate this with respect to v it will be easier to first re-write the term as a finite integration. For every possible realization of ω_1', ω_2' the optimal net-valence cut-off is either within the range of possible net-valences in the next period or it is not. If for a particular realization of ω_1', ω_2' we have $v^{(2)*}(\omega_1', \omega_2') < \delta v - \overline{v}$, then the politician from the R-party will be elected with certainty and this term will reduce to

$$-\frac{\beta}{2\overline{v}} \int_{\delta v - \overline{v}}^{\delta v + \overline{v}} \left(\frac{\partial \hat{V}_R^{(2)}}{\partial \omega_i'} \right) dv' = -\frac{\partial \hat{V}_R^{(2)}}{\partial \omega_i'}$$

The derivative of this term with respect to v will be zero since $\frac{\partial^2 V_\theta^{(2)}}{\partial \omega_i \partial v} = 0$. Likewise if $v^{(2)*}(\omega_1', \omega_2') > \delta v + \overline{v}$, then the politician from the L-party will be elected and the derivative of this term with respect to v will be zero.

Therefore, the only case in which this term is non-zero is when for a particular realization of ω_1', ω_2' we have $\delta v - \overline{v} \leq v^{(2)*}(\omega_1', \omega_2') \leq \delta v + \overline{v}$. Define $\hat{\Omega}'$ as the set of all possible future issue realizations such that the net-valence cut-off is interior to the distribution of net-valence. That is, define $\hat{\Omega}' \equiv \left\{ \omega_1', \omega_2' \in \hat{\Omega}' | \forall \omega_1', \omega_2' \in \hat{\Omega}', \delta v - \overline{v} \leq v^{(2)*}\left(\omega_1', \omega_2'\right) \leq \delta v + \overline{v} \right\}$ and we can re-write the expectation as:

$$-\frac{\beta}{2\overline{v}}\int_{\delta v-\overline{v}}^{v^{(2)*}}\left(\frac{\partial \hat{V}_L^{(2)}}{\partial \omega_i'}\right)dv' - \frac{\beta}{2\overline{v}}\int_{v^{(2)*}}^{\delta v+\overline{v}}\left(\frac{\partial \hat{V}_R^{(2)}}{\partial \omega_i'}\right)dv'.$$

Taking the derivative with respect to v we have:

$$\frac{\beta\delta}{2\overline{v}}\left(\left.\frac{\partial \hat{V}_L^{(2)}}{\partial \omega_i'}\right|_{v'=\delta v-\overline{v}} - \left.\frac{\partial \hat{V}_R^{(2)}}{\partial \omega_i'}\right|_{v'=\delta v+\overline{v}}\right).$$

Since $\frac{\partial \hat{V}_\theta^{(2)}}{\partial \omega_i'}$ does not depend on v', it does not matter where v' is evaluated and putting this all together we have:

$$\frac{\partial^2 W_\theta^{(1)}}{\partial x_i \partial v} = \frac{\beta\delta}{2\overline{v}}\int_{\hat{\Omega}'}\left(\frac{\partial \hat{V}_L^{(2)}}{\partial \omega_i'} - \frac{\partial \hat{V}_R^{(2)}}{\partial \omega_i'}\right)d\hat{\Omega}'.$$

From above we have $\frac{\partial \hat{V}_R^{(2)}}{\partial \omega_1} < \frac{\partial \hat{V}_L^{(2)}}{\partial \omega_1} < 0$ and $\frac{\partial \hat{V}_L^{(2)}}{\partial \omega_2} < \frac{\partial \hat{V}_R^{(2)}}{\partial \omega_2} < 0$ which gives us:

$$\frac{\partial^2 W_\theta^{(1)}}{\partial x_1 \partial v} > 0 \qquad\qquad \frac{\partial^2 W_\theta^{(1)}}{\partial x_2 \partial v} < 0.$$

Returning to the implicit function theorem we now have:

$$\frac{\partial x_{1L}^{(1)*}}{\partial v} > 0 \qquad\qquad \frac{\partial x_{2L}^{(1)*}}{\partial v} < 0.$$

Which proves proposition 3.

A.1.4 Characterization of optimal investment with office motivation

When $\psi > 0$ the optimal investment of a politician does not coincide with the investment that maximizes the lifetime discounted utility of the voter and the the problem of the politician is not in generally globally concave. Recall that and first order conditions are for the problem of the elected politician for $i = 1, 2$ is:

$$\frac{\partial \rho}{\partial x_i} - \frac{\partial C_{i\theta}}{\partial x_i} - \beta \int_0^K \int_0^K \Bigg[\int_{-\infty}^{v^{(2)*}} \left(\frac{\partial V_L^{(2)}}{\partial \omega_i'} \right) f_v(v'|v) dv' + \int_{v^{(2)*}}^{\infty} \left(\frac{\partial V_R^{(2)}}{\partial \omega_i'} \right) f_v(v'|v) dv' + \psi \left(\frac{\partial G_\theta^{(2)}}{\partial \omega_i'} \right) \Bigg] \frac{d\eta_1 d\eta_2}{K^2} = 0.$$

Given a realization of ω_1, ω_2, v consider the first order condition evaluated at the socially optimal investment in both issues $x_{1\theta}^{SP*}, x_{2\theta}^{SP*}$ there are two possible cases. First, that at the socially optimal investment the marginal effect of investment on re-election is zero. That is:

$$\frac{\partial Pr\,(y' = \theta)}{\partial x_i} = -\int_0^K \int_0^K \left(\frac{\partial G_\theta^{(2)}}{\partial \omega_i'} \right) \frac{d\eta_1 d\eta_2}{K^2} = 0.$$

In this case the re-election term in the first order condition is zero and the first order condition for a office motivated politician is satisfied at the socially optimal investment.

The second possibility is that the re-election term in the first order condition is not zero. Then by proposition 1, the marginal effect of investment in the issue in which the politician specializes is negative and the marginal effect of investment on the other issue is positive. This indicates which direction the optimal investment for an office motivated politician is relative to that of a social planner. For an office-motivated politician there is an additional marginal cost to investing in the issue in which he specializes, and an additional marginal benefit to investment in the other issue. If the problem of the politician is globally concave, then it immediately follows that the optimal investment of an office-motivated politician in the issue in which the he specializes must be less than socially optimal while optimal investment in the other issue must be more than socially optimal.

If the problem of the politician is not necessarily globally concave then we need to characterize the non-concavity which enters from the probability of re-election. From before we have that if issue i is the issue in which a politician from party θ has an advantage and issue j is the issue in which he has a disadvantage, then the probability of re-election $G_\theta(s_1, s_2, v)$ is increasing in s_i and decreasing in s_j. In addition since the cut-off rule is continuous and the shocks to issue severity and net-valence are distributed uniformly we have that $G_\theta(s_1, s_2, v)$ is first convex and then concave in s_i and concave and then convex in s_j. This inflection point of $G_\theta(s_1, s_2, v)$ in s_1, s_2 will play an important role.

If ψ is sufficiently large, then the convexity from the probability of re-election may be large enough generate a second peak in the problem of the politician. In this case there will be two potential candidates for optimal investment, one which corresponds to a larger deviation from the socially optimal and higher re-election probability, and one which corresponds to a lower deviation and lower re-election probability. As the severity of the issue in which a politician specializes increases, the benefit of deviation decreases while the cost increases. At some point the optimal investment will jump from the higher re-election probability peak to the lower re-election probability peak and generate a discontinuity in investment. This jump is in the "right direction" in the sense that any discontinuous change in optimal investment goes in the same direction as the slope of the optimal investment function.

Finally, this allows us to characterize how the investment of an office-motivated politician differs from the socially optimal investment. Suppose a politician from party θ specializes in issue i. When ω_i is large (i.e. severe) this politician is re-elected with certainty and the optimal investment for an office-motivated politician coincides with that of the socially optimal: $x_{i\theta}^{\psi} = x_{i\theta}^{SP}$. As ω_i decreases (i.e. improves) because investing in issue i has an additional cost for the office-motivated politician we have $\partial x_{i\theta}^{\psi}/\partial\omega_i > \partial x_{i\theta}^{SP}/\partial\omega_i$ of the politician decreases his investment faster than socially optimal. At the inflection point in the re-election probability since $\partial^2 G_\theta/\partial^2 s_i = 0$ we have $\partial x_{i\theta}^{\psi}/\partial\omega_i = \partial x_{i\theta}^{SP}/\partial\omega_i$, or the slope of the investment for an office-motivated politician is the same as the slope of the socially optimal investment. After the inflection point, since larger and more costly deviations are required to influence the probability of re-election, the benefit of deviating from the socially optimal decreases and we have $\partial x_{i\theta}^{\psi}/\partial\omega_i < \partial x_{i\theta}^{SP}/\partial\omega_i$ or the investment returns to the socially optimal level. Therefore, the difference between the socially optimal and office-motivated investment is a first increasing then decreasing function which has a maximum (i.e. the largest deviation) at the inflection point in the re-election probability.

While the possible discontinuity of investment is interesting, we may want to adopt conditions that guarantee global concavity and a unique continuous optimal investment function for each politician type. To look at the concavity, we return to the cross derivative and second derivative of the problem of the voter with respect to x_i:

$$\frac{\partial^2 W_\theta^{(1)}}{\partial^2 x_i} = \frac{\partial^2 V_\theta^{(1)}}{\partial^2 x_i} + \psi\beta \frac{\partial^2 G_\theta}{\partial^2 x_i'} \qquad \frac{\partial^2 W_\theta^{(1)}}{\partial x_1 \partial x_2} = \frac{\partial^2 V_\theta^{(1)}}{\partial x_1 \partial x_2} + \psi\beta \frac{\partial^2 G_\theta}{\partial x_1 \partial x_2}.$$

Since by assumption $\frac{\partial^2 V_\theta^{(1)}}{\partial^2 x_i} < 0$ and $\frac{\partial^2 V_\theta^{(1)}}{\partial^2 x_i} = 0$, the global concavity of the politician's problem will depend on how much convexity enters from the probability of re-election and we need to characterize $\frac{\partial^2 G_\theta}{\partial^2 x_i'}$ and $\frac{\partial^2 G_\theta}{\partial x_1 \partial x_2}$. In particular, I show that any convexity can be bounded. First consider $\frac{\partial^2 G_L}{\partial x_1 \partial x_2}$:

$$\frac{\partial^2 G_L}{\partial x_1 \partial x_2} = \frac{\partial^2 G_L}{\partial \omega_1 \partial \omega_2} = \int_0^K \int_0^K \left(\frac{\partial^2 v^{(2)*}}{\partial \omega_1' \partial \omega_2'} \right) f_v \left(v^{(2)*}(\omega_1', \omega_2') | v \right) \frac{d\eta_1 d\eta_2}{K^2} = 0,$$

since from above $\frac{\partial^2 v^{(2)*}}{\partial \omega_1' \partial \omega_2'} = 0$ and $\frac{\partial^2 G_R}{\partial x_1 \partial x_2} = 0$. Next consider $\frac{\partial^2 G_L}{\partial^2 x_1'}$:

$$\frac{\partial^2 G_L}{\partial^2 x_1} = \frac{\partial^2 G_L}{\partial^2 \omega_1} = \int_0^K \int_0^K \left(\frac{\partial^2 v^{(2)*}}{\partial^2 \omega_1'} \right)^2 \frac{d\eta_1 d\eta_2}{K^2} = \int_0^K \int_0^K \left(\frac{\partial^2 \hat{V}_L^{(2)}}{\partial^2 \omega_1} - \frac{\partial^2 \hat{V}_R^{(2)}}{\partial^2 \omega_1} \right)^2 \frac{d\eta_1 d\eta_2}{K^2}.$$

Given an initial severity of issues ω_1, ω_2, net-valence v and $\psi = \hat{\psi}$ this term will be bounded and there exists some $\hat{\beta} > 0$ such that if $\beta \leq \hat{\beta}$ and $\psi \geq \hat{\psi}$, then the problem of each politician type will be globally concave and the optimal investment for an office-motivated politician will be continuous.

Given the optimal policy of each politician type in the next period, define the value function of the voter from electing a politician from party θ as:

$$\hat{V}_\theta^{(1)}(\omega_1,\omega_2,v) \equiv V_\theta^{(1)}\left(\omega_1,\omega_2,x_{1\theta}^{(1)*}(\omega_1,\omega_2,v),x_{2\theta}^{(1)*}(\omega_1,\omega_2,v),v\right).$$

Taking the derivative with respect to v:

$$\frac{\partial \hat{V}_\theta^{(1)}}{\partial v} = \frac{\partial V_\theta^{(1)}}{\partial x_1}\left(\frac{\partial x_{1\theta}^{(1)*}}{\partial v}\right) + \frac{\partial V_\theta^{(1)}}{\partial x_2}\left(\frac{\partial x_{2\theta}^{(1)*}}{\partial v}\right) + \frac{\partial V_\theta^{(1)}}{\partial v},$$

where the right hand side is all evaluated at $x_{1\theta}^{(1)*}, x_{2\theta}^{(1)*}$. Recall that by definition:

$$V_\theta^{(1)}(\omega_1,\omega_2,x_1,x_2,v) \equiv W_\theta^{(1)}(\omega_1,\omega_2,x_1,x_2,v) - \beta\psi E\left[Pr\left(y'=\theta\right)|\ldots\right],$$

which implies that:

$$\frac{\partial V_\theta^{(1)}}{\partial x_i} \equiv \frac{\partial W_\theta^{(1)}}{\partial x_i} - \beta\psi\frac{\partial}{\partial x_i}\left(E\left[Pr\left(y'=\theta\right)|\ldots\right]\right).$$

When evaluating the above at $x_{1\theta}^{(1)*}, x_{2\theta}^{(1)*}$ since $\frac{\partial W_\theta^{(1)}}{\partial x_i} = 0$ we have:

$$\frac{\partial V_\theta^{(1)}}{\partial x_i} = -\beta\psi\frac{\partial E\left[Pr\left(y'=\theta\right)\right]}{\partial x_i}.$$

Putting this together we have:

$$\frac{\partial \hat{V}_\theta^{(1)}}{\partial v} = \frac{\partial V_\theta^{(1)}}{\partial v} - \beta\psi\left[\frac{\partial E\left[Pr\left(y'=\theta\right)\right]}{\partial x_1}\left(\frac{\partial x_{1\theta}^{(1)*}}{\partial v}\right) + \frac{\partial E\left[Pr\left(y'=\theta\right)\right]}{\partial x_2}\left(\frac{\partial x_{2\theta}^{(1)*}}{\partial v}\right).\right]$$

Next, taking the derivative with respect to net-valence:

$$\frac{\partial V_\theta^{(1)}}{\partial v} = \frac{\mathbb{I}(\theta)}{2} + \beta\frac{\partial E\hat{V}^{(2)}}{\partial v}.$$

Note that

$$\begin{aligned}
\frac{\partial E\hat{V}^{(2)}}{\partial v} &= \int_0^K \int_0^K \left[\int_{-\infty}^{v^{(2)*}} \delta \frac{\partial V_L^{(2)}}{\partial v'} f_v(v'|v) dv' + \int_{v^{(2)*}}^{\infty} \delta \frac{\partial V_R^{(2)}}{\partial v'} f_v(v'|v) dv' \right] \frac{d\eta_1 d\eta_2}{K^2} \\
&= -\frac{\delta}{2} \int_0^K \int_0^K \left[\int_{-\infty}^{v^{(2)*}} f_v(v'|v) dv' - \int_{v^{(2)*}}^{\infty} f_v(v'|v) dv' \right] \frac{d\eta_1 d\eta_2}{K^2} \\
&= \frac{\delta}{2} [1 - 2Pr(y' = L),]
\end{aligned}$$

or that

$$-\frac{\delta}{2} \leq \frac{\partial EV^{(2)}}{\partial v} \leq \frac{\delta}{2}.$$

This gives us that

$$\frac{\partial V_L^{(1)}}{\partial v} \leq -\frac{1-\beta\delta}{2} < 0 < \frac{1-\beta\delta}{2} < \frac{\partial V_R^{(1)}}{\partial v},$$

or that an increase in the net-valence always increases the value to the voter from electing a politician from the R-party more than from electing a politician from the L-party.

A.2 Extending the time horizon

When extending the time horizon I focus on the case that the politician's problem in each period is globally concave. In every period t, any convexity in the continuation value is bounded, and there exists a discount factor β^t in each period small enough to guarantee global concavity. Finally setting a discount factor $\overline{\beta} = \min \beta^t$ will guarantee global concavity in every period. The following proposition shows that there is an upper bound on the optimal amount of investment a politician will choose.

PROPOSITION 7 *In any equilibrium it must be that $x_{i\theta}^* \leq \omega_i$ or that no elected politician ever invests more in either issue than the severity of that issue.*

Proof. This follows directly from assumptions on utility and transition which make $x_{i\theta} > \omega_i$ strictly dominated by $x_{i\theta} = \omega_i$. Suppose a politician invests $x_{i\theta} > \omega_i$ in one issue and arbitrary $x_{j\theta}$ in the other issue. Since $\omega_i' = \max\{\omega_i - x_i, 0\} + \eta_i$ the distribution of ω_i' is the same for $x_{1\theta} > \omega_i$ and $x_{1\theta} = \omega_i$ and the continuation value for the politician remains the same when decreasing $x_{i\theta}$ to ω_i. However, since $C_{i\theta}(x_{i\theta})$ is increasing in $x_{i\theta}$, and $\rho(s_i)$ is constant for $s_i \leq 0$, by then decreasing $x_{i\theta}$ to equal ω_i increases current period utility without having any effect on the future. Therefore, $x_{i\theta} > \omega_i$ is strictly dominated by $x_{i\theta} = \omega_i$ for $i = 1, 2$ and $\theta \in \{L, R\}$, and neither politician ever invests more than the severity of an issue. ■

The following proposition says that in a model with an arbitrary time horizon T given an initial state (the initial severity of issues and net-valence), there is a bound on all future states.

PROPOSITION 8 *Given an initial state* $\{\omega_1, \omega_2, v\}^0$ *in a finite game of length* T *the set of possible future states is bounded.*

Proof. Given an initial starting severity of an issue ω_i^0, the minimum severity of this issue in all future states is trivially zero, since by construction the severity cannot be improved beyond zero. However, the maximum severity of the issue is less trivial, since while the maximum stochastic worsening from one period to the next is bounded by K, the elected politician always has the option to deliberately worsen ("sabotage") the severity of the issue without bound. Thus, to bound the severity of an issue from above it is necessary to show that there is an upper bound on the amount of "sabotage" (i.e. a lower bound on x_i) that can be optimal. To do this consider the possible incentive for a politician to choose $x_i < 0$.

The only incentive a politician has to choose $x_i < 0$ is if it increases the probability of a successor being elected in the next period. The highest marginal benefit from decreasing x_i from $x_i = 0$ occurs if it results in the probability of a successor being elected going from zero to one, in which case the marginal utility from decreasing x_i can be at most ψ. Since by assumptions on the utility function the social cost of investing is increasing at an increasing rate in $|x_i|$, eventually the cost of investing will outweigh any potential benefit to the politician. Let $\underline{x}_{i\theta}$ be the policy on issue ω_i such that the marginal social cost of implementing policy is equal to the value of successor election ψ or the value of x_i such that $\frac{\partial C_\theta}{\partial x_i} = \psi$. Therefore, given an initial severity of an issue $\omega_i^{(0)}$ the severity of this issue in all subsequent periods $0 \leq t \leq T$ must be bounded by:

$$\omega_i^{(t)} \in \left[0, \omega_i^{(0)} + T(\underline{x}_{i\theta} + K)\right],$$

where $\underline{x}_{i\theta}$ is defined above.

Lastly, since net-valence is independent of the issues and the actions of the politicians and voter, it immediately follows that given an initial net-valence $v^{(0)}$ in a game of finite length T net-valence in all subsequent periods $0 \leq t \leq T$ it must be bounded by $v^{(t)} \in v_0 \pm T\overline{v}$. Therefore, given initial values of $\omega_1^{(0)}, \omega_2^{(0)}, v^{(0)}$ the severity of both issues and net-valence in all subsequent periods are bounded. ■

Given an initial severity of issues ω_i^0 and net-valence advantage $v^{(0)}$ we can bound the states which can occur in the future. If the set of possible states is bounded, the concavity that enters the problem of each politician through the probability of re-election is also bounded in each period. Let $\beta^{(t)} > 0$ be the discount factor that in period t guarantees that the problem of the politician is globally concave in that period. If we let $\overline{\beta} = \min \beta^{(t)}$ then as long as the discount factor satisfies $\beta \leq \overline{\beta}$ then the problem the politician from each party θ is globally concave in every period and there will exist a unique continuous optimal investment for office-motivated politicians. To extend the model for a finite but arbitrary long time horizon I solve the problem numerically, recursively and numerically verify that this condition holds. The optimal policies and value functions converge relatively quickly ($T < 30$).

A.3 Functional forms and parameter values

Where necessary, such as some of the numerical examples in the two-period model as well as for the numerical solution of the longer time horizon, I will assume the following functional forms. I assume quadratic dis-utility from issue severity:

$$\rho(s) = s^2$$

For the social cost function I assume quadratic costs with differing marginal cost parameters:

$$C_L(x_1, x_2) = \underline{c}x_1^2 + \overline{c}x_2^2 \qquad\qquad C_R(x_1, x_2) = \overline{c}x_1^2 + \underline{c}x_2^2$$

With $\overline{c} > \underline{c} > 0$. In addition, where needed I use the following numerical values to find numerical solutions:

$$\underline{c} = 0.2, \qquad \overline{c} = 0.8, \qquad K = 10, \qquad \overline{v} = 20, \qquad \beta = 0.5, \qquad \delta = 0.2, \qquad \psi = 200$$

APPENDIX B

APPENDIX FOR CHAPTER 2

B.1 Model estimation details

B.1.1 Derivation of the estimation equation

Assume the utility function has the following quadratic form:

$$
\begin{aligned}
U(y_t, y_{t-1}, p_t, z_t, e_t) =&(a_1 + a_{13}p_t + a_{331}p_t^2)y_t + a_{11}y_t^2 + \\
&a_2 y_{t-1} + a_{22}y_{t-1}^2 + a_3 p_t + a_{33}p_t^2 + a_4 z_t + a_{44}z_t^2 + a_5 e_t + a_{55}e_t^2 + \\
&a_{12}y_t y_{t-1} + a_{14}y_t z_t + a_{15}y_t e_t + a_{24}y_{t-1}z_t + a_{25}y_{t-1}e_t + a_{45}z_t e_t,
\end{aligned}
$$

where each a_i, a_{ii}, a_{iii} represent a scalar. In the notation each number in the subscript of a scalar represents a variable in the term that the scalar is multiplied by. For example, a_{331} is the coefficient on $p_t^2 y_t$ or the third variable of the function squared times the first. The first line above represents the way in which current playtime y_t enters and how lagged performance p_{t-1} affects the marginal utility of playing. The second line includes the first and second power terms of the remaining variables and the third line includes the interactions of variables.

Returning to the individual's maximization problem:

$$
\max_{\{y_t\}_{t=1}^{\infty}} E\left[\sum_{t=1}^{\infty} \beta^{t-1} U(y_t, y_{t-1}, p_t, H - y_t, e_t)\right]
$$

By expanding the summation we can see in what terms y_t appears:

$$\max_{\{y_t\}_{t=1}^{\infty}} \dots + \beta^{t-1} E_{p_t}\left[U(y_t, y_{t-1}, p_t, H - y_t, e_t)\right] + \beta^t E_{p_{t+1}}\left[U(y_{t+1}, y_t, p_{t+1}, H - y_{t+1}, e_{t+1})\right] + \dots$$

The first order condition is:

$$\begin{aligned} E_{p_t}\left[U_1(y_t, y_{t-1}, p_t, H - y_t, e_t)\right] - E_{p_t}\left[U_4(y_t, y_{t-1}, p_t, H - y_t, e_t)\right] \\ + \beta E_{p_{t+1}}\left[U_2(y_{t+1}, y_t, p_{t+1}, H - y_{t+1}, e_{t+1})\right] = 0. \end{aligned} \tag{B.1}$$

Because performance today only effects the marginal utility of playing today, performance only shows up in the first term in the first order condition. Consider the first term of equation B.1:

$$\begin{aligned} E_{p_t}\left[U_1(y_t, y_{t-1}, p_t, H - y_t, e_t)\right] =& a_1 + a_{13} E_{p_t}\left[p_t\right] + a_{331} E_{p_t}\left[p_t^2\right] + 2a_{11} y_t \\ &+ a_{12} y_{t-1} + a_{14}(H - y_t) + a_{15} e_t. \end{aligned}$$

Since $E_{p_t}[p_t] = p_{t-1}$ and $E_{p_t}[p_t^2] = (p_{t-1})^2 + \sigma_\eta^2$ we can rewrite:

$$\begin{aligned} E_{p_t}\left[U_1(y_t, y_{t-1}, p_t, H - y_t, e_t)\right] =& a_1 + a_{13} p_{t-1} + a_{331}(p_{t-1})^2 + a_{331}\sigma_\eta^2 + 2a_{11} y_t \\ &+ a_{12} y_{t-1} + a_{14}(H - y_t) + a_{15} e_t. \end{aligned}$$

The expectation is degenerate over the second and third term in the equation B.1 and these terms are:

$$\begin{aligned} U_4(y_t, y_{t-1}, p_{t-1}, H - y_t, e_t) =& a_4 + 2a_{44}(H - y_t) + a_{14} y_t + a_{24} y_{t-1} + a_{45} e_t \\ U_2(y_{t+1}, y_t, p_t, H - y_{t+1}, e_{t+1}) =& a_2 + 2a_{22} y_t + a_{12} y_{t+1} + a_{24}(H - y_{t+1}) + a_{25} e_{t+1}. \end{aligned}$$

Putting the terms together in the first order condition and collecting like terms we have:

$$2\left[a_{11}-a_{14}+\beta a_{22}+a_{44}\right]y_t+\left[a_{12}-a_{24}\right]y_{t-1}+\left[\beta a_{12}-\beta a_{24}\right]y_{t+1}+a_{13}p_{t-1}+a_{331}p_{t-1}^2+$$
$$+\left[a_{15}-a_{45}\right]e_t+\beta a_{25}e_{t+1}+a_1-a_4+a_{331}\sigma_\eta^2+\beta a_2+\left[a_{14}+\beta a_{24}-2a_{44}\right]H=0.$$

Solving for y_t we get the following demand function:

$$y_t=\theta y_{t-1}+\beta\theta y_{t+1}+\theta_1 p_{t-1}+\theta_2 p_{t-1}^2+\theta_3 e_t+\theta_4 e_{t+1}+K, \tag{B.2}$$

where the coefficients are

$$\theta=\frac{a_{12}-a_{24}}{A}\qquad \theta_1=\frac{a_{13}}{A}\qquad \theta_2=\frac{a_{331}}{A}\qquad \theta_3=\frac{a_{15}-a_{45}}{A}\qquad \theta_4=\frac{\beta a_{25}}{A},$$

with constants $A=2\left(a_{14}-a_{11}-\beta a_{22}-a_{44}\right)>0$ and $K=a_1-a_4+a_{331}\sigma_\eta^2+\beta a_2+\left(a_{14}+\beta a_{24}-2a_{44}\right)H$.

B.1.2 Derivation of the estimation equation with learning

To simplify the problem suppose there are no quadratic terms of p_t in utility function. Or that one period quadratic utility can be written as:

$$\begin{aligned}U(y_t,y_{t-1},p_t,z_t,e_t)=&(a_1+a_{13}p_t)y_t+a_{11}y_t^2+a_2y_{t-1}+a_{22}y_{t-1}^2+\\&a_3p_t+a_4z_t+a_{44}z_t^2+a_5e_t+a_{55}e_t^2+a_{12}y_ty_{t-1}+\\&a_{14}y_tz_t+a_{15}y_te_t+a_{24}y_{t-1}z_t+a_{25}y_{t-1}e_t+a_{45}z_te_t.\end{aligned}$$

Under the functional form assumed above the first order condition with respect to y_t of the problem in 2.11 is:

$$U_1^{(t)} + U_4^{(t)} + \beta U_2^{(t+1)} + \sum_{i=t+1}^{\infty} (\beta b_1)^{i-t} U_3^{(i)} = 0, \tag{B.3}$$

where $U^{(t)} = U(y_t, y_{t-1}, p_t, H - y_t, e_t)$. The first three terms are similar to what we had in the equation B.1 of the main model. The addition of the the infinite sum as a fourth term represents the effect of playing today on performance in every future period (because playing today affects performance tomorrow and performance persists). Solving equation B.3 for y_t we have:

$$y_t = \theta y_{t-1} + \beta\theta y_{t+1} + \theta_1 p_{t-1} + \theta_2 p_{t-1}^2 + \theta_3 e_t + \theta_4 e_{t+1} + K + \sum_{i=t+1}^{\infty} \beta^{i-t} \alpha y_i, \tag{B.4}$$

where $\beta^{i-t}\alpha > 0$ captures the marginal effect of playing in period t on future period i through learning. Note that the coefficients in this equation are not the same as the coefficients in the regular model, equation B.2. To estimate equation B.4 we would need to include as explanatory variables playtime in each and every future period. Including all future values of playtime is not feasible, and so I consider two approximations of this final term that will allow me to estimate demand.

The first approximation method assumes $y_i = y_{t+1}$ for all $i > t+1$. This reduces the final term in equation B.4 to $\alpha \left(\frac{\beta}{1-\beta}\right) y_{t+1}$. There are two drawbacks with this approach. Since y_t is generally increasing over time this will on average underestimate the value of learning for individuals who continue to play. At the same time, for individuals who completely stop playing in some future period, this approach may overestimate the value of learning. The second approximation method assumes $y_i = 0$ for all $i > t+1$. This reduces the final term in equation B.4 to $\alpha\beta y_{t+1}$. This approach will consistently underestimate the value of learning for all individuals who play beyond the next period.

Both approximation methods lead to the same demand equation with slightly different interpretations of α:

$$y_t = \theta y_{t-1} + \beta(\theta + \alpha)y_{t+1} + \theta_1 p_{t-1} + \theta_2 p_{t-1}^2 + \theta_3 e_t + \theta_4 e_{t+1} + K \tag{B.5}$$

This demand function is also consistent (i.e. not an approximation) with the above model if individuals are forward-looking but only consider the benefits of learning in the immediately following period.

The main advantage of equation B.5 is that by comparing the estimated coefficients on y_{t-1} and y_{t+1} for a fixed discount rate β it is possible to impute an estimate of the value of learning.

B.1.3 Instrumental variables

In section 2.4 I am able to reject the hypothesis that a non-IV panel regression is consistent. To deal with the endogeneity of the model I use the average playtime and skill of friends as instruments for an individual's own playtime. Table B.1 shows the results of the first stage of a 2SLS estimation. Friend playtime in the current period and performance in the previous period are significant in explaining an individual's playtime.

B.2 Game background information

Some of the following game background information is adapted from the Valve-supported Official Team Fortress 2 Wiki at `http://wiki.teamfortress.com/`

Team fortress 2 (TF2) was originally released on October 10, 2007 as part of a bundle called the Orange Box which cost \$49.99. In the first year alone the Orange Box sold over 3 million retail copies Remo (2008). On April 9, 2008 it was released as a stand alone game for \$29.99. In 2009 the price of TF2 was reduced to \$19.99. Beginning on June 23, 2011 TF2

Table B.1: First stage of 2SLS, Dependent Variable y_t

	Fixed effects		Pooling	
	(i)	(ii)	(iii)	(iv)
y_t^F	0.205***	0.228***	0.273***	0.288***
	(191)	(164)	(259)	(214)
p_{t-1}^F	-	0.00045***	-	0.000943**
		(2.59)		(5.76)
R^2	0.0213	0.0218	0.0359	0.0338
F	36476	13486	67308	23033
n	129904	104406	129904	104406
T	65	64	65	64
N	1807386	1317359	1807386	1317359

Significance levels: * = 10%, ** = 5%, *** = 1%, t-statistics in parentheses.

became "Free-to-play", meaning the game became freely available to anyone and ongoing development is supported by in-game microtransactions, or the purchasing of in-game items for money (vanity hats, weapons, etc).

In TF2 there are three basic modes of play. In *capture the flag* both teams simultaneously attempt to obtain a briefcase of intelligence from the enemy team's base and return it to their own base while preventing the opposing team from doing the same. The *control* mode varies but generally requires capturing and holding particular point or points on the map and holding it for a desired time. In the *payload* mode one team works to escort a rail cart carrying a bomb along a track through a series of checkpoints, eventually detonating the bomb in the other team's base. The other team has to defend their positions and prevent the cart from reaching the end within a set amount of time

In TF2 player choose to play as one of nine classes. The classes are designed to have strategic complementarities and encourage teamwork. For example, the *heavy* and *medic* classes are weak on their own but can be unstoppable together. At the same time the classes are designed to have a comparative advantage against each other in a non-transitive, or "rock-paper-scissors" way: each class has strengths versus some classes and weaknesses versus others. This structure prevents any one class from dominating the others and gives an advantage to a team that plays a mix of classes rather than all playing the same class. The nine available classes can be grouped into three roles.

The first role is *offense*. Players that choose offense are usually responsible for directly attempting the goal of the map (i.e. capturing the flag or capturing a control point). The *soldier* is a basic all-purpose character with a rocket launcher. The *pyro* is armed with a flamethrower that can set other players on fire. The *scout* is a fast, agile character that can capture points twice as fast.

The second role is *support*. Players that choose support are responsible for supporting the players that are directly attempting a goal, either by healing them or providing cover. The *medic* is armed with a "medigun" to heal teammates, and can make teammates temporarily invulnerable or enhance their firepower. The *sniper* is equipped with a laser sighted sniper rifle to attack enemies from afar. The *spy* has covert tools, such as a temporary cloaking device, an electronic sapper to sabotage Engineers' structures, and a device that gives him the ability to disguise as other players.

The third role is *defense*. Players that choose defense are usually responsible for defending the goals of the map (i.e. preventing the capture of the flag or securing a control point). The *demoman* is armed with a grenade launcher and a sticky-bomb launcher. The *heavy* can sustain more damage than any other class and can put out an immense amount of firepower, but is slowed down by both his own size and that of his minigun. The *engineer* is capable of building a number of structures to support his team: a sentry gun to defend key points, a health and ammunition dispenser, and a teleporter system.

A player earns points by completing certain actions in the game. Some actions are available only to certain classes. An individual can earn points by "killing" an opposing team member, assisting a teammate with "killing" an opposing team member, or capturing or defending a key strategic point. In addition to these basic ways, there are also many other class-specific ways to earn points (for example, healing another player, which is something only the medic can do).

B.3 Additional Data Statistics

B.3.1 Specialization index

In TF2 there are nine different classes that a player can choose to play. To capture this I define an index of heterogeneity for specialization as:

$$p = \sum_{k=1}^{9} \left(\frac{\text{playtime for class } k}{\text{total playtime}} \right)^2$$

This index will take values from $p = 1/9$ to $p = 1$. A value of $p = 1$ represents complete specialization or that the individual played only one class the entire week. A value of $p = 1/9 \approx 0.111$ represents complete diversification, or that the individual played all classes equally.

B.3.2 Herfindahl index

The Herfindahl index measures the size of one firm or individual in relation to the entire market or population. In this context the Herfindahl index provides a measure of how influential an individual in determining the total accumulated points of all players. This is important because individuals freely enter and exit the panel and if a small number of players remain in the panel for a long period of time they may have a significant effect on the aggregate level of accumulated points. If the Herfindahl index is large, then this means that a small number of individuals are disproportionately contributing points to the total accumulated points of all players. To calculate the index, I use the total number of accumulated points for each individual for their entire appearance in the panel:

$$H = \sum_{i=1}^{N} \left(\sum_{t=1}^{T} s_{it} y_{it} \right)^2 = 0.0000246$$

The Herfindahl index is small, typically anything less than 0.01 is considered highly competitive, this suggests that the behavior of one individual has little effect on the total accumulated points of all players.

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