Faculty Working Papers

A PATH ANALYSIS MODEL OF THE
ADVERTISING-SALES RELATIONSHIP

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ABSTRACT

On the basis of the most commonly used mechanism linking advertising and sales--the so-called hierarchy of effects paradigm--the usual multiple equation models are contrasted with a path analysis model. The path analysis approach takes causal lags explicitly into account, and allows for direct effects as well as indirect effects--channeled through an intermediate variable--of advertising upon sales. This is especially valuable for a correct representation of the hierarchy of effects framework as is shown in an empirical example.

INTRODUCTION

Several approaches to the measurement of the strength of the advertising and sales relationship have been presented in the literature. The earliest efforts took the form of single equation regression models, with some explanatory variables, such as season, introduced in addition to advertising. An improvement over this simple approach was the introduction of explicit distributed lags, on the basis that there are theoretical reasons to assume that advertising has effects that extend over time. The latest development is the use of recursive and simultaneous equation models which take into account the fact that advertising decision rules often incorporate sales as one determinant.

Another approach, so far not tried in advertising-sales research, is path analysis. Developed in sociology for the analysis of complex interrelationships between explanatory and dependent variables, its aim is the exact portrayal of the causal links and their strength between the various variables. Since one of the basic mechanisms proposed as a mediator between advertising and sales--the so-called hierarchy of effects paradigm--comprises such a system of complex interrelationships, it seems appropriate to apply the path analysis approach to the disentangling of the causal links connecting advertising to sales.

After a presentation of the fundamentals of path analysis, the paper compares the technique briefly with the more common techniques in advertising research. Then an empirical application to advertising's effect upon market share (as a proxy for sales) and the linkages in the hierarchy is presented.

THE PATH ANALYSIS MODEL

The Basic Model

Several excellent presentations of the basic path analysis model exists in the literature. Thus, although no application has yet to our knowledge been made to the advertising effects issue, only a brief overview of the model will be given here. It is drawn mainly from Van de Geer [6] who also lists the basic references.

Assume we have a set of four variables, x_i, $i=1,2,3$, and y. Our theory tells us that x_1 and x_2 are independent of the other two variables and of each other. Similarly, our a priori notion is that x_3 is determined by x_1 and x_2, and in turn determines y. The y values, however, are also directly dependent upon x_1. Thus, we have the following two-equation system

$$x_3 = b_{31}x_1 + b_{32}x_2 + e_3 \qquad (1)$$

$$y = b_{y1}x_1 + b_{y3}x_3 + e_y \qquad (2)$$

where the relations are assumed to be linear, the intercept terms vanish on the assumption that we use standardized variables, and the e's represent equation error terms. Let us now interpret these errors as occurring because of unobserved or "latent" determining factors u_3 and u_y, respectively. Then the system can be written

$$x_3 = b_{31}x_1 + b_{32}x_2 + b_{3u}u_3 \qquad (3)$$

$$y = b_{y1}x_1 + b_{y3}x_3 + b_{yu}u_y \qquad (4)$$

The u's are assumed independent of each other and of x_1 and x_2.

The basic problem in path analysis is to find the weights b which describe the causal "paths" from the determining factors to the "dependent" or "endogeneous" variable y. The solution starts with the computation of the simple correlation coefficients (the r's) between all the variables. With the variables standardized we recognize that the variance of, say y, can be written

$$V(y) = y^1 y = 1 \qquad (5)$$

and the expectation of cross-product terms becomes, for example,

$$COV(y,x_1) = y^1 x_1 = r_{y1} \qquad (6)$$

where the variables are represented as observation vectors of order n.

If we now premultiply equation (3) by x_3 we get

$$1 = b_{31}r_{31} + b_{32}r_{32} + b_{3u}r_{3u} \qquad (7)$$

Then premultiplying (3) by x_1 we have

$$r_{13} = b_{31} + b_{32}r_{12} \qquad (8)$$

and finally by x_2 we derive

$$r_{23} = b_{31}r_{21} + b_{32}. \qquad (9)$$

We have then three equations for our desired b's--but since also r_{3u} is unknown, one more relationship is necessary in order to derive the estimates. This last equation is generated by a premultiplication of (3) by u_3, which gives

$$r_{3u} = b_{3u}. \qquad (10)$$

This last relation eliminates one unknown, and the three equations can be solved for the three b's.

In a similar fashion the estimates for the second relation (4) can be derived. Since the principles are the same, no further discussion will be presented here. The interested reader is referred to the Van de Geer reference [6].

Relationship to Other Techniques

A brief account of the relations between path analysis and other techniques will be of interest in evaluating the potential of path analysis. Clearly, the approach provides much stronger a priori specification as compared to many other multivariate techniques. Accordingly, it is closer to econometric techniques than to much of behavioral statistics. The model encompassed by equations (1) and (2) can clearly be seen as a recursive system, estimable by the usual least squares techniques. The difference between path analysis and recursive systems is in fact small, and consists mainly of the more explicit consideration of indirect effects upon an endogenous variable offered by the path analysis. For example, the linkage between y and x_2 might seem nonexistent if only equation (2) is evaluated, whereas the indirect effect of x_2 via x_3 might in fact be great--in path analysis, the total impact of an earlier variable upon a later one is simply determined by a summation of the separated indirect and direct effects. Thus, the effect of x_1 upon y in system (3) and (4) is computed as $b_{y1} + b_{y3}*b_{31}$. Again, however, the recursive system contains the same basic information, although not as clearly articulated.

The path analysis does not allow two-way causal directions between endogenous variables. The causation can flow from one endogenous variable to the next, and, over time, there can be feedback from one to the other, but contemporaneous loops are ruled out. For this reason, path analysis offers no help with simultaneous systems. As is well known, the simultaneity is often a reasonable approximation of the advertising-sales relationship [1] and where that is the case path analysis is inappropriate. The problem hinges ultimately on the specification of the "error" terms or "unobserved" variables and is thus connected with the identification problem. No exhaustive treatment can be given here, but the role of identifying predetermined variables should be pointed out. In the path analysis model, it suffices often to have one single exogenous variable which will guide the whole system. When the appropriate model is simultaneous, several exogenous variables are needed to identify the separate causal relations. The interested reader is referred to Blalock's treatment [2].

Since much research into the advertising-sales relationship has utilized a distributed lag approach, a brief comment is warranted. The appropriateness of a distributed lag model is in most respects independent of which of the earlier models are chosen. It is only when the distributed lag affects the causal links in the system that care has to be exercised. Thus, for example, it might be that advertising has an immediate effect upon customers awareness, and after some time will affect other cognitive structures, such as liking, and after some further time has passed it will finally reach purchase behavior. Such a process--partially incorporated into the so-called hierarchy-of-effects paradigm [3]--might appear as a distributed lag effect

although it should properly be viewed as different simple lags for different measures. In the application of the path analysis presented below, a polynomial distributed lag approach is followed rather than the usual Koyck model [5]. In the estimation runs made to choose the appropriate lag, some efforts were made to account for such simple lags, but no conclusive evidence emerged supporting their existence. Since the majority of the literature in the area supports a distributed lag approach, we followed their stance for the present paper.

AN EMPIRICAL APPLICATION

The Data

An empirical application of the path analysis model to the advertising-sales relationship is presented here. The data used come from [4] where an extensive discussion of the original source can be found. In brief, monthly observations of total brand advertising (in dollars) and repeated survey data on the brands are available. The survey questions were directed to about 600 product users--the product used here is hair spray--and covered their brand awareness, liking, and preferences, as well as brand trial, last and repeat purchases. From the response frequencies we derived the different indicators as sample proportions of heavy users. The variables used, besides advertising, are the proportion of heavy users aware of the brand, the proportion liking it, the proportion rating the brand higher than or as high as any other brand (acceptance), the proportion rating the brand higher than any other brand (preference), and the proportion buying the brand last time (purchase). This last measure stands for a type of "market share among heavy users" variable in this particular application, and represents our sales measure. Thirteen monthly data points were available for each brand.

The Models

The path analysis approach was used to formulate two alternative causal structures linking the variables. In both, advertising was seen as exogenous, not determined by the other variables, including purchases. This assumption was based upon the short time span involved, allowing no feedback into the next period's advertising decision making. The hierarchy variables, excepting purchases, were all seen as simultaneously determined, since the survey questionnaire literally forced such simultaneity (by directing all questions about the hierarchy variables to the present situation). Purchase, on the other hand, was seen as determined by lagged hierarchical variables, since the "present" purchase in fact was the one made last time. To account for the possibility of a feedback from purchase to the hierarchy, one alternative model incorporated the last period's purchases as a predetermined variable in addition to the advertising.

The lagged effects of advertising were incorporated by using polynomial distributed lags. The particular form of the lag was chosen after several alternatives had been tried [4]. It incorporated two constraints, effectively setting the impact of advertising to zero after five periods. With the constraints, the advertising variable became simply a moving average of five periods, with weights 1.0, 8/5, 9/5, 8/5, and 1.0. The maximum impact from advertising occurs accordingly in period t-2, or two months after insertion.

In addition to the two path analysis models, two recursive models incorporating a similar causal structure were estimated for comparison. Two brands were run, one relatively new and another well established. The models and the results are presented in Figures 1 and 2.

FIGURE 1
HIDDEN MAGIC

	I	II	III	IV
P_{13}	.933	.933	.933	.933
P_{14}	-.066	-.078	-.066	-.089
P_{15}	.148	.149	.118	.113
P_{16}	.512	.227	.603	.118
P_{17}	.781	1.22	1.808	1.12
P_{25}			.142	.133
P_{26}			1.053	.625
P_{27}			.295	.928
P_{34}	1.023	1.035	1.023	1.10
P_{45}	.844	.842	.743	.756
P_{46}	.035	.753	-.093	.681
P_{47}	.557	.697	.947	.779
P_{56}	-.189	-.206	-.765	-.595
P_{57}	-1.942	-1.745	-2.631	-2.379
P_{67}	.489	.476	.198	.202

FIGURE 2
VO-5

	I	II	III	IV
P_{13}	.441	.441	.441	.441
P_{14}	-.058	-.074	-.058	-.088
P_{15}	-.255	-.255	-.263	-.266
P_{16}	.341	.371	.545	.375
P_{17}	.803	.845	.776	.887
P_{25}			.145	.107
P_{26}			-.531	-.026
P_{27}			-.532	-.262
P_{34}	.760	.796	.760	.496
P_{45}	.924	.924	.794	.844
P_{46}	.358	.358	1.002	.372
P_{47}	.669	.504	1.867	.651
P_{56}	.270	.270	.316	.276
P_{57}	-.199	-.201	-.828	-.146
P_{67}	-.670	-.664	-.971	-.670

KEY

Subscripts

1 = Advertising (t)
2 = Purchase $(t - 1)$
3 = Awareness (t)
4 = Like (t)
5 = Acceptance (t)
6 = Preference (t)
7 = Purchase $(t + 1)$

(P_{13}, for example, is path from advertising to awareness, i.e., the effect of advertising on awareness.)

I = Path Analysis w/o $P(t - 1)$
II = Recursive w/o $P(t - 1)$
III = Path Analysis w $P(t - 1)$
IV = Recursive w $P(t - 1)$

The Results

In analyzing the results, let us first compare the path analysis models with the recursive formulations. As would have been expected, both approaches yield similar results in the early stages of the hierarchy-- here the advantages of the path analysis make little difference. Proceeding towards the purchase variable, however, we can see how the separation of causal influences possible in the path analysis approach yields added insights into the working of the hierarchy. For example, in Figure 1 it seems that advertising affects liking slightly negatively, but when the indirect effect through awareness is accounted for in the path analysis, the net effect is positive. Similarly, the small negative effect of acceptance on purchase is counterbalanced by the positive influence via preference.

Comparing the path analysis runs for the two different brands, some interesting features emerge. For the newly introduced brand (Figure 1), the effect of advertising upon awareness is much stronger than for the well established brand (Figure 2). On the other hand, the relationship visavis purchase is the opposite, with the established brand showing a stronger positive effect. With the exception of the preference-to-purchase link, the established brand exhibits a generally stronger advertising effect in the later stages of the hierarchy, with the new brand showing the greater effect early. This would seem to indicate that as a brand becomes established in a market, advertising can be directed more effectively towards purchase related output, whereas for a new brand the advertising will have to focus upon earlier stages in the hierarchy.

The preference-to-purchase link seems to go against this very plausible picture, however, since it is slightly negative for the established brand and strongly positive for the new brand. The explanation here might be that for the established brand preferences have been quite firmly developed, and changes in purchases will occur quite independently of preferences. The small negative impact might come about through deal offers of the brand which might temporarily increase purchases but lead to slightly negative preference reactions among loyal customers. The new brand, however, will attract purchases continuously from the pioneer customers who develop a preference for the brand, and these customers might in addition generate favorable word-of-mouth communication.

This interpretation of the preference discrepancies is supported by the differences between the two brands when the effect of lagged purchases is incorporated. Thus, the established brand shows a negative relationship between lagged purchases and preferences, as well as a weak negative relationship between past and present purchases. For the new brand, both of those relationships are strongly positive.

CONCLUSION

Although there are other interesting results depicted in Figures 1 and 2, the main findings quoted should be sufficient to indicate the potential of path analysis in the analysis of the advertising and sales relationship. If the aim of such an analysis is the exploration of possible causal relationships, the specification approach followed in path analysis seems very logical and relatively easy to carry out. Using path analysis as a complement to simultaneous systems where the relationships are uni-directional in any one time period would seem to offer more insights into the causal mechanism than the usual recursive modeling approach.

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