# FAILURE MODE AND SENSITIVITY ANALYSIS OF GAS LIFT VALVES

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## ABSTRACT

*Gas-lifted oil wells are susceptible to failure through malfunction of gas lift valves. This is a growing concern as offshore wells are drilled thousands of meters below the ocean floor in extreme temperature and pressure conditions and repair and monitoring become more difficult. Gas lift valves and oil well systems have been modeled but system failure modes are not well understood. In this paper a quasi-steady-state fluid-mechanical model is constructed to study failure modes and sensitivities of a gas-lifted well system including the reservoir, two-phase flow within the tubing, and gas lift valve geometry. A set of three differential algebraic equations of the system is solved to determine the system state. Gas lift valve, two-phase flow, and reservoir models are validated with well and experimental data. Sensitivity analysis is performed on the model and sensitive parameters are identified. Failure modes of the system and parameter values that lead to failure modes are identified using Monte Carlo simulation. In particular, we find that the failure mode of backflow through the gas lift valve with a leaky check valve is sensitive to small variations in several design parameters.*

## Introduction

Gas lift is an artificial lifting method used to produce oil from wells that do not flow naturally. In gas-lifted wells, gas is injected through the well annulus and into the well tubing at a down-well location (as shown in figure 1). The gas mixes with the oil in the tubing, aerating the oil and causing it to rise to the surface.

To pass from the annulus to the tubing, the injection gas must flow through a valve called a gas lift valve. Gas lift valves are one-way valves that allow gas to pass through to the tubing but prevent oil from passing through to the annulus. Most valves contain a pressurized bellows and an internal check valve (see figure 2). The bellows opens when the injection gas is pressurized above a threshold value, and the internal check valve prevents oil from passing through the gas lift valve.

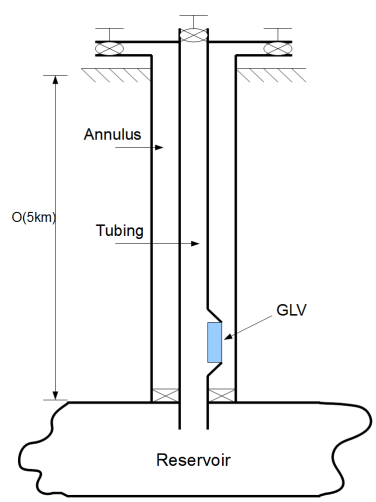


Figure 1. Schematic of oil well with gas lift valve (GLV). Top of figure represents sea floor.

Proper function of gas lift valves is very important for the safety of the well and surface operations. If hydrocarbons flow through the wrong path (i.e. backflow from the tubing into the annulus, through a gas lift valve leak), they can reach the wellhead and create an undesired accumulation of high-pressure combustible material. Wrong manipulation of surface valves,

procedures and accumulation of gasses is thought to have caused the 1988 accident on the Piper Alpha North Sea production platform, which led to an explosion and fire killing 167 men [14]. With offshore wells being drilled thousands of meters below the ocean in extreme temperature and pressure conditions, repair and monitoring of gas lift valves is becoming more difficult. Thus, it is important to understand which valve conditions lead to failure modes, and which valve parameters the failure modes are most sensitive to.

Several models have been developed for gas lift valves with experiments to back up predicted behavior [21], [9], [3], [5]. Basic sensitivity analysis has been conducted on the bellows position relative to temperature and pressure changes [19]. Several models have also been developed for the two-phase oil-gas flow inside the tubing [2], [1], [12], [7]. Commercial software systems such as PROSPER [15] and OLGA [17] are also available for analyzing artificial gas lift valves. However, no work has been published giving a full sensitivity analysis and failure mode analysis of the entire gas lift system (including gas lift valve, tubing, and reservoir). This systematic analysis is important because designers of new gas lift valves need to know which parameters are most important to consider in redesigning valves to be less susceptible to failure.

In this paper a quasi-steady state model is developed for the entire gas-lift system. Sensitivity analysis is performed on the model and sensitive parameters are identified. Failure modes of the system and parameter values that lead to failure modes are identified using Monte Carlo simulation.

Our goal in developing our own model and program was to gain a deeper insight into the physical mechanisms at work. This will allow us in future work to develop design improvements.

## Modeling Assumptions

### Valve

- The gas lift valve is injection pressure operated.
- A gas-filled bellows and spring are used in parallel.
- The bellows contains an incompressible gas dome.
- Side-forces on the bellows are small compared to the bottom force (by the small angle approximation for the folds in the sides of the bellows).
- No elastic deformation of bellows (also by the small angle approximation).
- The pressures at operation state will be such that the valve is in a quasi-steady state of completely open or completely closed. The transition between open and closed positions is not studied.

### Gas-Fluid Mixture Above Valve

- The gas-fluid mixture is assumed to be homogeneous and in a quasi-steady state.
- The pipe is assumed to be well-insulated and thus the gas-fluid mixture is at a constant temperature equal to reservoir temperature. (Future model iterations will include temperature dependence).

### Gas Inflow

- The injection gas pressure is set from the surface, and the mass flow rate of the injection gas into the tubing is dictated by the size of the valve opening.

### Fluid Below Valve

- The fluid below the valve is pure oil (no water). This assumption is reasonable for new wells when little water is produced, but not for older wells which have higher water cuts [10].(Future model iterations will include non-zero water cuts).

### Reservoir

- The reservoir is assumed to be cylindrical with pure oil inflow to the tubing.
- The reservoir pressure is assumed to be known from other sources and to remain constant.

## Modeling Approach

Three constitutive equations of the fluid-mechanical system must simultaneously be satisfied: a differential equation of the well's pressure vs depth, an equation describing the oil mass flow rate from the reservoir into the tubing, and an equation relating the valve position to the pressure difference between the injection gas and the oil in the tubing.

### Pressure

The pressure change in the tubing is a result of hydrostatic and frictional pressure losses. Because the fluid is assumed to be in a quasi-steady state, there are no acceleration pressure losses. Thus the pressure drop equation in the tubing is

$$\frac{dp}{dz} = \rho(z)g + \frac{f(v(z))\rho(z)v^2(z)}{2D} \tag{1}$$

with the boundary condition of surface pressure (which is controlled at the wellhead). Here $p$ is pressure, $z$ is depth, $\rho$ mixture or liquid density, $g$ is gravity, $f$ is the friction factor, $v$ is the mixture velocity, and $D$ is the pipe diameter. This differential equation is applicable below and above the injection point. Below the injection point the density is the oil density while above

the injection point the mixture density is given by

$$\rho_{mix} = \frac{\rho_g \rho_l}{q\rho_l + (1-q)\rho_g} \tag{2}$$

where $\rho_g$ is the gas density, $\rho_l$ is the liquid density, and $q$ is the mixture quality, defined as the ratio of gas mass to total mixture mass [8].

The friction factor from equation (1) is determined by the Reynold's number of the fluid or mixture. The Reynold's number is a unitless measure of the ratio of inertial to viscous forces in a fluid and is given by

$$Re = \frac{\rho v D}{\mu} \tag{3}$$

where $\mu$ is the fluid viscosity. The flow is considered laminar for Reynolds numbers less than 2300 and turbulent for Reynolds numbers greater than or equal to 2300 [20]. For laminar flow the fluid friction factor is given by

$$f = \frac{64}{Re} \tag{4}$$

and for turbulent flow the friction factor is given by

$$f = \frac{1.325}{\ln^2(\frac{r}{3.7D} + \frac{5.74}{Re^{0.9}})} \tag{5}$$

where $r$ is the pipe roughness and $D$ is the pipe inner diameter [20].

## Oil Flow from Reservoir

Oil flow out of the reservoir and into the tubing is driven by a pressure difference between the reservoir and the bottom of the wellbore. This pressure difference is related to the oil mass flow rate by Darcy's Law,

$$\dot{m}_l = \frac{\rho_l h k_{well}(P_{res} - P_{bot})}{B\mu_l \ln(\frac{r_e}{r_w} + S)} \tag{6}$$

where $\dot{m}_l$ is the oil mass flow rate, $\rho_l$ is the oil density, $\mu_l$ the oil viscosity, $h$ the reservoir thickness, $B$ the fluid formation volume factor, $r_e$ the distance from the wellbore to the constant pressure boundary of the well, $r_w$ the distance from the wellbore to the sand face, $S$ the skin factor, $P_{res}$ the reservoir pressure, and $P_{bot}$ the well bottom hole pressure [6].

## Valve Position vs Flow and Pressure

The valve is modeled as an injection-pressure-operated pressurized bellows in parallel with a spring in tension. The bellows is connected to a pressurized dome of constant volume (see figure 2). The bellows itself is modeled as a series of frustums connected in an accordion-type fashion (see figure 3). If the temperature of the gas inside the bellows is assumed to remain approximately constant, then when the bellows compresses (ie when the gas lift valve opens), by the ideal gas law

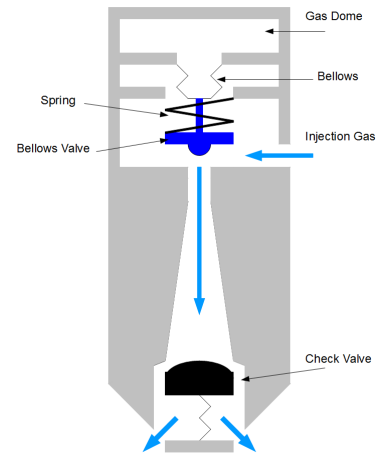


Figure 2. Gas lift valve model. Arrows represent injection gas flow

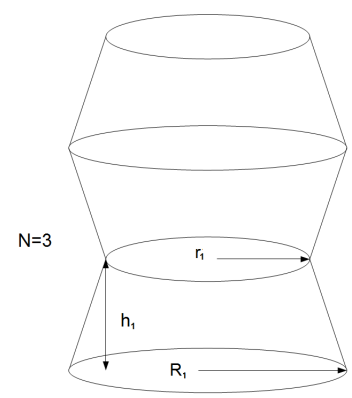


Figure 3. Frustum model of the bellows

$$P_{b1}V_{b1} = P_{b2}V_{b2} \tag{7}$$

where $P_{b1}$ is the initial bellows pressure, $V_{b1}$ is the initial bellows volume, $P_{b2}$ is the final bellows pressure, and $V_{b2}$ is the final bellows volume. The initial and final volumes are given by the frustum volumes. Thus, assuming the frustum radii remain constant and only the heights change, the total elongation or compression of the bellows is given by the difference in heights. This simplifies to

$$E = \frac{V_D(P_{b1} - P_{b2})}{P_{b2}\frac{\pi}{3}(r_1^2 + r_1R_1 + R_1^2)} + \frac{Nh_1P_{b1}}{P_{b2}} - Nh_1 \tag{8}$$

where $V_D$ is the dome volume, $r_1$ is the inner frustum radius, $R_1$ is the outer frustum radius, $N$ is the number of frustums in the bellows, and $h_1$ is the height of each frustum.

Forces acting to open the valve are the injection gas pressure acting on the area of the bottom of the bellows and the oil pressure acting on the area of the bottom of the valve stem. Because the area of the bottom of the bellows is much larger than the area of the bottom of the stem, the valve is more sensitive to the injection pressure than the oil pressure. To determine the steady state position of the valve a free body diagram can be analyzed as given in Figure 4. By balancing the vertical forces on

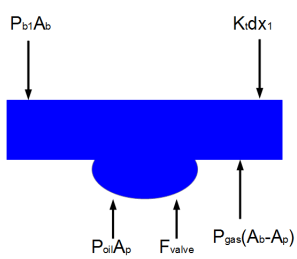


Figure 4. Bellows valve free body diagram

the bellows, in the closed position the valve position equation is

$$F_{valve} = P_{b1}A_b + K_t\delta x_1 - P_{gas}(A_b - A_p) - P_{oil}A_p \tag{9}$$

where $F_{valve}$ is the force between the stem and the valve, $A_b$ is the area of the bottom of the bellows, $K_t$ is the spring constant, $\delta x_1$ is the spring pre-stretch distance, $A_p$ is the area of the bottom of the stem, and $P_{oil}$ is the oil pressure at injection depth.

When the valve is open gas flows through the orifice and the valve stem is exposed to the gas pressure instead of the oil pressure. A new force balance yields

$$P_{b2}A_b + K_t(\delta x_2) = P_{gas}A_b \tag{10}$$

where $\delta x_2$ is the total length the spring is stretched, which is given by the equation

$$\delta x_2 = \delta x_1 + E \tag{11}$$

Combining equations (11), (10), and (8) yields the following quadratic equation the total spring stretch length

$$\begin{aligned} 0 = {} & (-K_t\chi)\delta x_2^2 + (K_t(\delta x_1 + Nh_1)\chi + K_tV_D + P_{gas}A_b\chi)\delta x_2 + \\ & + (Nh_1\chi + V_D)P_{b1}A_b - P_{gas}A_b((\delta x_1 + Nh_1)\chi + V_D) \end{aligned} \tag{12}$$

where $\chi$ is defined as

$$\chi = \frac{\pi}{3}(r_1^2 + r_1R_1 + R_1^2). \tag{13}$$

Solving for $\delta x_2$ yields

$$\delta x_2 = \frac{K_t(\delta x_1 + NH_1)\chi + K_tV_D + P_{gas}A_b\chi}{2K_t\chi} \pm \sqrt{\beta} \tag{14}$$

where $\beta$ is defined as

$$\begin{aligned} \beta = {} & \left(\delta x_1 + Nh_1 + \frac{V_D}{\chi} + \frac{P_{gas}A_b}{K_t}\right)^2 - 4\left(\frac{Nh_1P_{b1}A_b}{K_t} + \frac{V_DP_{b1}A_b}{K_t\chi}\right) - \\ & - 4\left(P_{gas}A_b\left(\frac{\delta x_1 + Nh_1}{K_t} + \frac{V_D}{K_t\chi}\right)\right) \end{aligned} \tag{15}$$

The quadratic equation has two solutions and the positive real solution is chosen. The position of the valve is then given by the elongation $E$ from equation (8).

## Injection Gas Flow

The flow of injection gas through the valve is modeled as orifice flow with the orifice area dependent on the valve position. When the valve is completely open the gas flows through an area equal to that of the valve orifice while when the valve is nearly closed the gas flows through only a small fraction of the same area.

The maximum gas flow rate is given by the compressible gas orifice flow equation:

$$\begin{aligned} \dot{m}_{max} = {} & \frac{C_d\pi d_p^2}{4\sqrt{(1-(\frac{d_p}{d_v})^4)}}P_{gas} \times \\ & \sqrt{\left(\frac{2M}{RT_{inj}}\frac{\gamma}{\gamma-1}\left(\left(\frac{P_{oil}}{P_{gas}}\right)^{\frac{2}{\gamma}} - \left(\frac{P_{oil}}{P_{gas}}\right)^{\frac{\gamma+1}{\gamma}}\right)\right)} \end{aligned} \tag{16}$$

where $C_d$ is the discharge coefficient, $d_p$ is the orifice diameter, $d_v$ is the total valve diameter, $M$ is the injection gas molar mass, $R$ is the universal gas constant, $T_{inj}$ is the injection gas temperature, and $\gamma$ is the gas specific heat ratio [20].

To model the flow when the valve is in an intermediate position between completely closed and completely open the flow is assumed to asymptotically approach the maximum flow rate value. This asymptotic behavior can empirically be modeled with an arctangent curve.

$$\dot{m}_{gas} = \dot{m}_{max} \frac{2}{\pi} \arctan\left(\frac{x_1}{x_2} \tan\left(\frac{y\pi}{2\dot{m}_{max}}\right)\right) \tag{17}$$

where $x_1$ is the valve position and $y$ is the flow rate when the valve position is at the value $x_2$.

## Solving the Equations

**Input Parameters** The table below lists all input parameters that must be specified for this model.

**Solution Algorithm** The three constitutive equations for pressure, valve position, and oil mass flow rate can be completely satisfied if the well bottom hole pressure is known. In this algorithm the bottom hole pressure is guessed and the three equations solved to yield a pressure profile of the well. The differential pressure equation has no analytical solution, so the Runge-Kutta numerical solution technique is used. The bottom hole pressure guesses have a lower bound of the hydrostatic pressure of the well if filled with pure gas above the injection point and pure oil below it. The upper bound is the reservoir pressure. With each guess, a surface pressure is determined for that guess and a curve of model surface pressure vs input bottom hole pressure is made. The surface pressure in reality will be either atmospheric pressure (if the well is open at the top) or a known pressure if a pressure-regulating device is used at the well head. Thus the bottom hole pressure guess that yields the known surface pressure is used.

## Comparison with Experimental Data

To check the validity of the model, a pressure profile predicted by the model can be compared to pressure data from an actual well. Pressure surveys of two wells were provided by Chevron for comparison. The model takes 35 input parameters but not all of these parameters are given in the well pressure surveys. Nominal values are initially assumed for these remaining parameters, and the values are optimized within parameter ranges to yield a closer model match with the data. Figure 5 shows a pressure profile for a 2750 meter well with data taken between reservoir depth well head depth. Figure 6 shows a pressure profile for a 1000 meter well with data taken between reservoir depth and well head depth. Optimized parameter values are given in the table below.

| | | |
|---|---|---|
| 1. | $P_{gas}$ | Injection Gas Pressure |
| 2. | $P_{res}$ | Reservoir Pressure |
| 3. | I | Injection Depth |
| 4. | L | Well Depth |
| 5. | D | Pipe Diameter |
| 6. | B | Formation Volume Factor |
| 7. | h | Reservoir Thickness |
| 8. | $r_e$ | Reservoir Radius |
| 9. | $r_w$ | Wellbore Radius |
| 10. | k | Reservoir Permeability |
| 11. | S | Skin Factor |
| 12. | $K_t$ | Bellows valve spring constant |
| 13. | $r_1$ | Bellows inner Radius |
| 14. | $R_1$ | Bellows outer Radius |
| 15. | N | Number of Bellows Frustums |
| 16. | $dx_1$ | Initial Spring Stretch |
| 17. | $h_1$ | Initial Frustum Height |
| 18. | $V_D$ | Dome Volume |
| 19. | $P_{b1}$ | Initial Bellows Pressure |
| 20. | $\gamma$ | Gas Specific Heat Ratio |
| 21. | $C_d$ | Orifice Discharge Coefficient |
| 22. | $\mu_L$ | Oil Viscosity |
| 23. | $\rho_L$ | Oil Density |
| 24. | T | Temperature |
| 25. | $\mu_g$ | Gas Viscosity |
| 26. | r | Pipe Roughness |
| 27. | $P_{surf}$ | Surface well pressure |
| 28. | $K_c$ | Check valve spring stiffness |
| 29. | $A_b$ | Outside area of bellows bottom |
| 30. | $A_s$ | Area of stem of bellows valve |
| 31. | $A_d$ | Inside area of bellows bottom |
| 32. | $A_o$ | Area of orifice bottom |
| 33. | y | Maximum check valve spring length |
| 34. | $y_0$ | Initial check valve spring length |
| 35. | d | diameter of obstacle/debris |

The magnitude and shape of the modeled pressure profiles are reasonably close to the actual pressure profile. Pressures agree within 10 percent along the entire curves.

## Parameter Sensitivity Analysis

The first step to improve the design of the gas lift valve is to understand the influence each input parameter value has on the

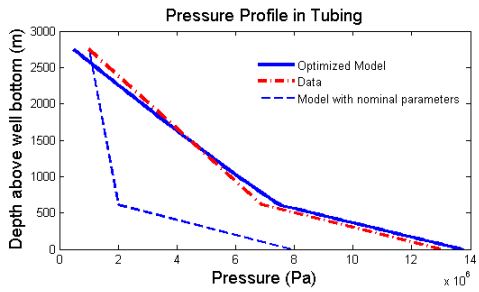


Figure 5. Pressure profile for 2750m well. Data taken between reservoir depth and surface.

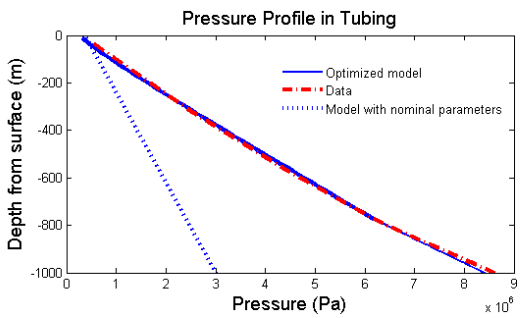


Figure 6. Pressure profile for 1000m well. Data taken between reservoir depth and surface.

important output parameters like oil mass flow rate and valve position [13]. The input parameters that are the most sensitive to changes represent areas for potential design improvement. For example, if small changes in the bellows pressure lead to large changes in the valve position, then the bellows should be examined as an area for design modification. One way to determine which parameters are the most sensitive is to make a plot of output parameter change vs input parameter change with respect to nominal values for a range of input parameter changes. To compare parameters with different magnitudes of nominal values, the percentage change in output can be compared to the percentage change in input. In figure 7 the change in valve position from a nominal starting position is plotted against changes in individual input parameters. When the curve for a given parameter is flat at 0 percent output change this means the output is not sensitive to that parameter, while if the curve has a nonzero slope then the output is sensitive to the input change. In this case if the valve position changes by -100 percent, this means the valve completely closes. A discontinuity in the graph where the slope is nearly vertical represents a sharp change in valve position from open to closed as opposed to a gradual change. This could be caused by the input parameter crossing a threshold value which would immediately close the valve.

| Parameter | Range of Values | Source | Fit Values Fig 5, Fig 6 * denotes known value |
|---|---|---|---|
| $P_{gas}$ | $10^5 - 10^7$Pa | [4] | 6.9x$10^6$*, 6x$10^6$ |
| $P_{res}$ | $10^7 - 10^9$Pa | [4] | 3x$10^8$, 3x$10^9$ |
| I | 100-$10^4$ m | [4] | 2140*, 1002* |
| L | 100-$10^4$ m | [4] | 2750*, 1067* |
| D | 0.02-0.2 m | [4] | 0.1016*, 0.0889* |
| B | 1 | [6] | 0.9, 1.1 |
| h | 10-100 m | [6] | 190, 200 |
| $r_e$ | 100-1000 m | [6] | 340, 410 |
| $r_w$ | 0.02-0.2m | [6] | 0.1, 0.1 |
| k | $10^{-15} - 10^{-13}$ $m^2$ | [6] | 2x$10^{-13}$, 2.1x$10^{-13}$ |
| S | 0-1 | [6] | 0.001, 0.001 |
| $K_t$ | $10^3 - 10^5$ N/m | [4] | 1.7x$10^4$,1.5x$10^4$ |
| $r_1$ | 0.01-0.02 m | [4] | 0.01, 0.012 |
| $R_1$ | 0.01-0.02 m | [4] | 0.017, 0.015 |
| N | 5-20 | [4] | 12, 15 |
| $dx_1$ | $10^{-5} - 10^{-3}$m | [4] | 7x$10^-4$, $10^{-3}$ |
| $h_1$ | 0.001-0.01 m | [4] | 0.006, 0.005 |
| $V_D$ | $10^{-3} - 10^{-5}$ $m^3$ | [4] | 5x$10^{-4}$,5x$10^{-4}$ |
| $P_{b1}$ | $10^4 - 10^7$ Pa | [4] | 4x$10^5$,4x$10^5$ |
| $\gamma$ | 1-2 | [20] | 1.0, 1.3 |
| $C_d$ | 0.1-1 | [20] | 0.7, 0.6 |
| $\mu_L$ | 0.01-0.1 Pa-s | [18] | 0.1, 0.1 |
| $\rho_L$ | 800-1000 kg/$m^3$ | [18] | 1000, 850 |
| T | 300-400 K | [18] | 350*, 332* |
| $\mu_g$ | $10^{-6} - 10^{-5}$ Pa-s | [20] | 9x$10^{-6}$, 1.7x$10^{-5}$ |
| r | $10^{-5} - 10^{-4}$ | [20] | 5x$10^{-5}$,5x$10^{-5}$ |
| $P_{surf}$ | $10^5 - 10^7$ Pa | [4] | $10^6$,$10^6$ |
| $K_c$ | $10^5 - 10^7$ N/m | [4] | 2x$10^5$, $10^5$ |
| $A_b$ | $10^{-4} - 10^{-3}$ $m^2$ | [4] | 6x$10^{-4}$, 5x$10^{-4}$ |
| $A_s$ | $10^{-6} - 10^{-3}$ $m^2$ | [4] | 2x$10^{-4}$,3x$10^{-6}$ |
| $A_d$ | $10^{-4} - 10^{-3}$ $m^2$ | [4] | 9x$10^{-4}$,7x$10^{-4}$ |
| $A_o$ | $10^{-4} - 10^{-3}$ $m^2$ | [4] | 6x$10^{-4}$,5x$10^{-4}$ |
| y | 0.01-0.1 m | [4] | 0.1, 0.15 |
| $y_0$ | 0.01-0.1 m | [4] | 0.3, 0.3 |
| d | 0-0.01m | [4] | 0*,0* |

Figure 7 and additional plots for the remaining input parameters show show that the valve position is sensitive to the parameters $\gamma, C_d, r_1, R_1, N, h_1, V_d, P_{b1}, P_{gas}, P_{res}$, D, and B.

There are apparently three types of sensitivities:

- Small changes in input have little effect but a threshold change causes the valve to close: $\gamma, C_d, P_{gas}$, D, B. This could be caused, for example, by the bellows spring bottoming out.
- Input changes result in roughly proportional changes in valve position: $r_1, R_1, N, h_1, V_d, P_{b1}$. These parameters directly affect the pressure on the bellows valve and will thus directly affect the valve position.

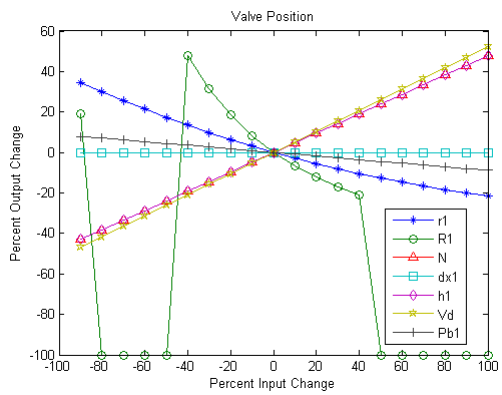


Figure 7. Percentage change in valve position vs percentage change in input parameters with respect to nominal starting values

- Input changes have no almost no effect on valve position: $\mu_L, \mu_G, \rho_L, dx_1, h, r_e, r_w, k, S, K_t$,$P_{res}$, I, L. Most of these parameters will effect the oil pressure, but because the oil pressure acts on a much smaller area of the bellows valve than the injection gas pressure, these parameters have very little effect on the valve position.

Similar plots can also be made for other output variables such as the oil mass flow rate (see figure 8). This figure and figures for the remaining input parameters show that the oil mass flow rate is sensitive to $\mu_L, \rho_L, \gamma, h, k, r_e, r_w, B, D, and P_{res}$. Figure 9 is a plot of the gas injection rate sensitivities and this and additional plots for the remaining input parameters show that the injection gas mass flow rate is sensitive to T, $\gamma, R_1$,$P_{gas}$, I, D, and $P_{res}$.

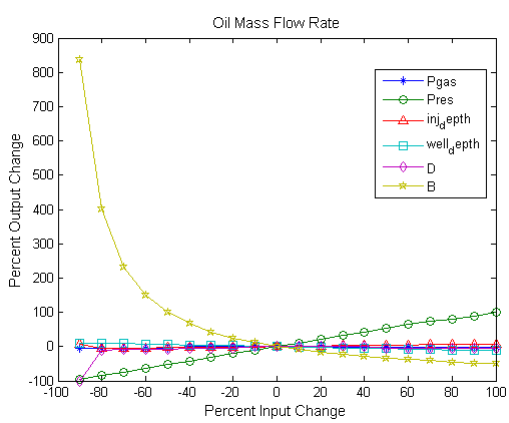


Figure 8. Percentage change in oil mass flow rate vs percentage change in input parameters with respect to nominal starting values

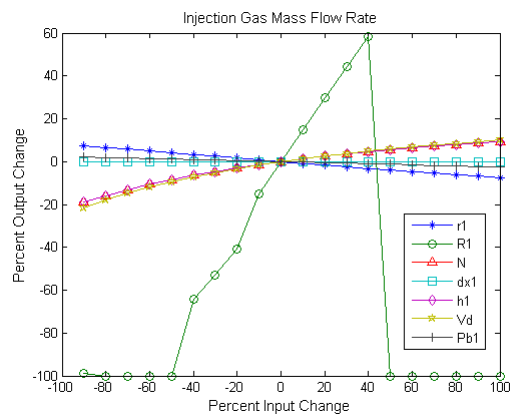


Figure 9. Percentage change in injection gas mass flow rate with respect to nominal flow rate vs percentage change in input parameters with respect to nominal values

## Failure Modes

The main mode of failure of the system is non-closure of the one-way check valve leading to oil flow into the annulus. From figure 4 the valve opens when $Fvalve$=0 and the opening forces are greater than the closing forces, given by

$$P_{oil}A_p + P_{gas}(A_p - A_b) > P_{b1}A_b + K_t d_{x1} \tag{18}$$

When the valve opens and the oil pressure is greater than the gas pressure, oil will flow into the annulus if the one-way check valve fails to close. Non-closure of the check valve is mainly caused by the following:

- Debris stuck in main valve or check valve
- Incorrect injection gas pressure. If injection gas pressure is higher than valve opening pressure but less than oil pressure, this would cause the valve to open and oil would flow into the annulus. If a sensitive parameter of the system is modeled with an incorrect value then the model may miscalculate the required injection pressure for optimal flow.
- Bellows pressure too low. Valve could remain open.
- Corrosion of valve stem (main valve or check valve) to prevent uniform contact with orifice. This allows oil to leak into the annulus.

## Multi-Factor Failure: Monte Carlo Simulation

Failure will likely be a result of a configuration of multiple parameters, and it is thus informative to vary multiple parameters simultaneously in a Monte Carlo simulation to see which configurations lead to system failure [16]. For this simulation MATLAB software is used with code generated by the authors. Each of the 35 parameters is assigned a random value from a

uniform distribution within +/- 90 percent of the nominal value, with a mean of the nominal value. The set of differential algebraic equations is then solved for this sample of input parameters, and if system failure results then the sample input parameter values are recorded. Histograms are made of individual parameter values that resulted in a failure configuration. For this simulation 250,000 samples were taken.

Of these samples the parameters that had non-uniform histogram distributions at failure were pipe diameter, injection depth, injection gas pressure, reservoir pressure, and bellows outer diameter. Figure 10 shows that injection depth tends to be higher when the system fails while bellows outer diameter tends to be lower at system failure. Injection gas pressure and pipe diameter tend to be lower while reservoir pressure tends to be higher. Some parameters do not have definite correlations at failure. For example, bellows pressure values are uniformly distributed at failure so no correlations can be inferred. In each sample it is also possible for there to be relationships between pairs of parameters that lead to system failure. For the same Monte Carlo simulation the correlation coefficients between every pair of parameters was calculated for samples that led to system failure (see figure 11). The correlation coefficient, $r$ is defined as

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{\left[n\sum x^2 - (\sum x)^2\right]\left[n\sum y^2 - (\sum y)^2\right]}} \tag{19}$$

where $n$ is the number of data points (in this case the 40000 failure configurations of the 250000 samples taken), $x$ is the set of values of one parameter that lead to failure, and $y$ is the set of values of a second parameter that lead to failure [11]. A correlation coefficient of zero means no correlation between the two parameters at failure while a coefficient of 1 or -1 means direct correlation between the two parameters at failure. Figure 11 shows that all correlation coefficients are less than 0.3, meaning that no pairs of parameters are highly correlated. However, some relationships can be determined from the 6 parameter pairs with correlation coefficients greater than 0.1. Figure 12 shows contour plots of failure frequency at differnet parameter pair values. A plot of parameter pair values was divided into a 10x10 grid and the number of failures in each grid square counted to generate the contour plots. These plots show regions of high and low failure probability. The cross markers in each plot signify approximate locations with least failure probability. For example, the lower right plot shows that failure is most likely for wellhead pressure greater than $10^6$ Pa with injection gas pressure less than $2x10^6$ Pa. This could be because a higher wellhead pressure means that the oil at injection depth is at a higher pressure, and if the injection gas pressure is low than the oil pressure could exceed the gas pressure. This could lead to oil passing into the annulus in the event of a leaky check valve.

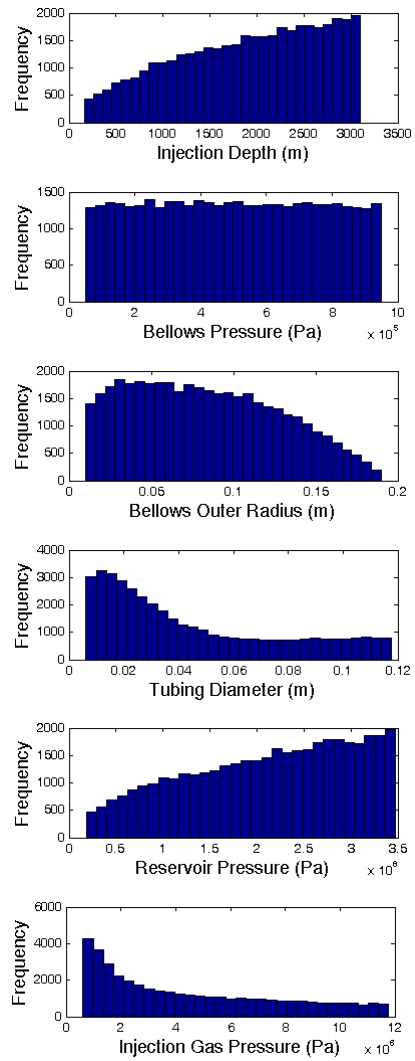


Figure 10. MC simulations. Histograms of bellows pressure, bellows radius, tubing diameter, reservoir pressure, and injection gas pressure at failure.

From the upper left plot, failure is most likely for gas specific heat ratio around 1.5 with pipe diameter less than 0.02 m. The upper right plot shows that failure is most likely for injection gas pressure less than $4x10^6$ Pa with pipe diameter less than 0.04 m. The plot of pipe diameter versus reservoir pressure shows that failure is most likely for reservoir pressure greater than $2x10^8$ Pa with pipe diameter less than 0.02 m. From the middle right plot failure is most likely for pipe diameter less than 0.02 m with any wellhead pressure, and from the bottom left plot injection depths greater than 2000 m with injection gas pressure less than $2x10^6$ Pa has the highest likelihood of failure.

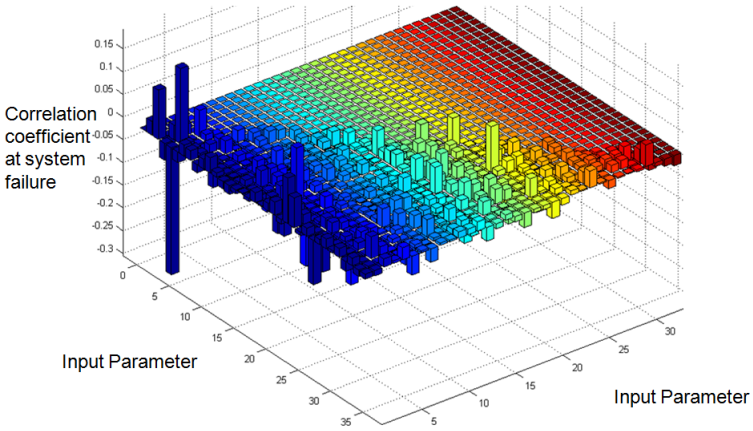


Figure 11. Correlation coefficients between two parameters at failure.

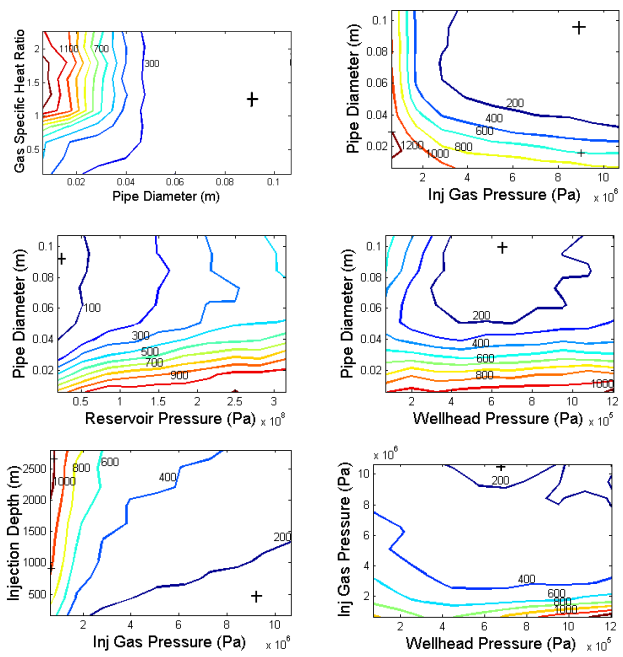


Figure 12. Contour plots of failure frequencies of input parameter value pairs. Input parameters with correlation coefficients greater than 0.1 are plotted. 40,000 failures were sampled.

## Conclusion/Discussion

A model for a quasi-steady-state gas-lift well system including a gas lift valve has been developed and compared to experimental well data. Sensitivity analysis was performed on the model and sensitive parameters identified. Gas lift valve position was found to be sensitive to injection gas specific heat ratio, orifice discharge coefficient, bellows inner and outer radii, number of bellows frustums, initial frustum height, gas dome volume, bellows pressure, injection gas pressure, reservoir pressure, pipe diameter, and fluid volume formation factor. This means that small changes in any one of these parameters will result in roughly proportional changes in bellows valve position. Oil mass flow rate was found to be sensitive to oil viscosity and density, injection gas specific heat ratio, reservoir thickness and permeability, reservoir radius, wellbore radius, fluid formation volume factor, pipe diameter, and reservoir pressure. Injection gas mass flow rate was found to be sensitive to injection gas temperature and specific heat ratio, bellows outer radius, gas pressure, injection depth, pipe diameter, and reservoir pressure.

These sensitive parameters represent areas for potential design improvement as well as problem areas in the event of unknown or uncontrollable variations.

System failure modes and parameter values that lead to system failure are identified using Monte Carlo simulation. Failure of the system is defined as a configuration where oil will pass into the annulus in the event of a leaky check valve.

In the Monte Carlo simulation all parameters are varied simultaneously. Results show that at system failure, pipe diameter, bellows outer radius, and injection gas pressure tend to be lower than nominal values while injection depth and reservoir pressure tend to be higher than nominal values. This means that a well-system is more susceptible to failure with these input parameter values.

In the second scenario, variations in pairs of parameters are examined using the Monte Carlo simulation results. As before, all parameters are varied simultaneously and the values are recorded if the resulting configuration is a system failure mode. The correlation coefficient is calculated for all pairs of parameters at failure. Results show that no two parameters are highly correlated at failure, but injection gas pressure and injection depth, as well as several other parameter pairs, are slightly correlated at failure. Failure frequency contour plots show which parameter values of these pairs lead to higher failure frequencies.

Based on the sensitivity and failure mode analyses, the most important parameters to consider when redesigning a gas lift valve and well system to be less susceptible to failure are the bellows outer radius, injection gas pressure, and tubing diameter.

## Future Work

The next stage of this project is to use the results of these analyses to improve the safety and reliability of gas lift valves. The first goal is to make the system less susceptible to errors in bellows preload, either through an improved preloading process or through a design change. The sensitivity and failure analyses results show that the bellows radius is an important parameter to consider in a design change, while other parameters such as number of frustums and dome volume are less sensitive but still important.

The next goal is to develop a postitive locking mechanism that will prevent product from backing up in to the annulus in

the event of non-closure of the check valve. A prototype of this mechanism will be constructed and the results of the sensitvity analyses will aid in the testing of the prototype. For example, the first prototype may be a scaled-up version for ease of machining, and the sensitivity results will determine how accurately testing results on the scaled-up version can be compared to results for an actual-size version.


## Acknowledgements

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