Mission Design for Compressive Sensing with Mobile Robots

Robert Hummel, Sameera Poduri, Franz Hover, Urbashi Mitra, Guarav Sukhatme

Abstract— This paper considers mission design strategies for mobile robots whose task is to perform spatial sampling of a static environmental field, in the framework of compressive sensing. According to this theory, we can reconstruct compressible fields using $O(\log n)$ nonadaptive measurements (where n is the number of sites of the spatial domain), in a basis that is "incoherent" to the representation basis [1]; random uncorrelated measurements satisfy this incoherence requirement. Because an autonomous vehicle is kinematically constrained and has finite energy and communication resources, it is an open question how to best design missions for CS reconstruction. We compare a two-dimensional random walk, a TSP approximation to pass through random points, and a randomized boustrophedon (lawnmower) strategy. Not unexpectedly, all three approaches can yield comparable reconstruction performance if the planning horizons are long enough; if planning occurs only over short time scales, the random walk will have an advantage.

I. INTRODUCTION

Mobile robots are increasingly being used to survey and map spatial phenomena for large-scale environmental monitoring applications. Examples include the NIMS system of tethered robots to map atmospheric conditions in forests [2], and robotic boats [3] and underwater gliders [4] to map chemical and physical properties of water. The success of these and other systems today means that soon groups of tens or even hundreds of robots will be deployed, and for longer missions than they are today. A fundamental question underlying such monitoring goals is: "Given some prior knowledge of the field of interest, how should robots move to gather maximum information, subject to energy and other constraints?" As is well-known, these planning problems can scale poorly even for one vehicle, and groups of vehicles executing a planned mission will likely have very significant connectivity and navigation requirements [5]–[10]. At the same time, recent results in compressive sensing (CS) [1], [11] show that under certain conditions it is possible to reconstruct fields using only $O(\log n)$ completely random measurements, where n is the number of sites in the discretized field. The major requirements are that the field is sparse in a basis, for example that its Fourier transform has only a few nonzero components, and that the sensing basis is incoherent with respect to the field basis. Indeed, most natural phenomenon are smooth and they admit sparse representations in the frequency or wavelet bases [12]; local spatial measurements tend to be incoherent with these bases.

Thus, using CS to reconstruct fields may provide a new opportunity for designing extremely simple but effective missions in autonomous vehicles. In addition to mitigating the path planning and navigation challenges mentioned above, this "lightweight" sampling approach has other advantages. For one, since it does not require the robots to sample at specific locations, sampling can easily be interleaved with other tasks. Further, it may be that effective multiple vehicle operations are no more complicated than a set of independent vehicles, each acting alone.

Our scope in this paper is to consider three basic mission design paradigms in the context of CS for a single vehicle. We compare a two-dimensional random walk, a travelling salesman problem (TSP) approximation to pass through random points (referred to here as "random TSP"), and a randomized boustrophedon ("mowing the lawn") strategy, subject to equivalent energy costs and budget; the value function is the CS reconstruction error. We consider in particular a discrete setting where the domain is a $2D$ grid with n sites, and assume the Discrete Fourier Transform (DCT) basis for sparsity/compressibility. The vehicle is assumed to be well-controlled, with minimal navigation errors.

Our contributions are as follows. We explicitly combine the constraints of energy costs with compressed sensing for the development of practical path planning. We further provide an optimized method for designing the randomized path which both satisfies the requirements for good field reconstruction as dictated by compressed sensing as well as explicitly takes the navigation costs into consideration. Finally, we compare our proposed strategy with the two schemes previously mentioned and find that for short horizons, our proposed scheme outperforms the other two with respect to energy consumption while maintaining a comparable field reconstruction accuracy.

This paper is organized as follows. Section II summarizes related work in the field of CS. Section III presents the CS background relevant to our work. Section IV provides the problem statement and mobility models. Section V presents and discusses our numerical simulations and results. Section VI concludes the paper.

Robert Hummel is a master's student in the Department of Mechanical Engineering at the Massachusetts Institute of Technology, Cambridge, MA 02139 USA. `hummel@mit.edu`

Sameera Poduri is a postdoctoral research associate in the Computer Science Department at the University of Southern California, Los Angeles, CA 90089 USA. `sameera@usc.edu`

Franz Hover is an assistant professor with the Department of Mechanical Engineering at the Massachusetts Institute of Technology, Cambridge, MA 02139 USA. `hover@mit.edu`

Urbashi Mitra is a professor with the Department of Electrical Engineering at the University of Southern California, Los Angeles, CA 90089 USA. `ubli@usc.edu`

Guarav Sukhatme is a professor with the Department of Electrical Engineering and the Department of Computer Science at the University of Southern California, Los Angeles, CA 90089 USA. `gaurav@usc.edu`

II. Related Work

The benefits of compressive sensing are being vigorously studied in sensor networks for data gathering and compression, and recent work has explored the selection of sparse, local projections [13], [14]. With mobile robots, en-route compression is not relevant because the energy spent on storing and communicating data is negligible compared to the energy spent on motion. The main challenge in the robot sampling scenario is rather that the spatial dependence of robot motion will not provide uniformly random measurements as desired for compressive sensing. Recently, Mostofi et al. [15] considered a field reconstruction problem in which compressive sensing is used for cooperative mapping with robot groups using properties of wireless transmission; however, vehicle path planning is not addressed. As stated in the introduction, our interest is in vehicle mobility models which are *lightweight* in terms of robot cooperation and navigation, can accurately reconstruct the field of interest, and include vehicle kinematic constraints. Due to the use of compressed sensing-based strategies, we retain the feature of collecting a small set of measurements as exploited in [15].

III. CS Methodology

We present some key results from compressive sensing [1], and lay out our particular scenario.

A. Orthonormal Measurement and Sparsity Bases

Let $\Psi \in \mathbb{R}^{n \times n}$ be an orthonormal basis in which the signal, f, has a concise or *sparse* representation. Suppose $f = \Psi x$ for $x \in \mathbb{R}^n$. Formally, x is *S-sparse* if it has at most S non-zero elements. Further, x is *compressible* if for x_S obtained from x by keeping only the S largest terms, the loss in signal energy, $\|x - x_S\|$ is "small". Many natural phenomenon are compressible in domains such as frequency, wavelets or total variation [12] . Let $\Phi \in \mathbb{R}^{n \times n}$ be another orthonormal basis, and let $y^1, y^2, \cdots y^m$ be the measurements obtained by projecting f on vectors $\phi_1, \phi_2, \cdots, \phi_m \in \mathbb{R}^n$ i.e. $y^i = \langle f, \phi_i \rangle$. In the usual robot sampling scenario, sensing is performed in the spatial domain, so that the measurement basis functions are delta functions in space. The domain grid is vectorized into a $n \times 1$ column, and each measurement is an $n \times 1$ column of zeros, with a single 1 at the measurement location. Φ is orthogonal, a permutation of the $n \times n$ identity matrix. We are interested in cases where the number of samples is significantly less than the dimension of the field i.e. $m << n$; this is encoded into R, an $m \times n$ matrix which pre-multiplies Φ. Thus we have $y = R\Phi f$, with $f = \Psi x$.

B. Incoherence and Reconstruction

Incoherence is a critical requirement for CS, and captures how dissimilar a pair of sparsity and measurement bases are; the idea is that local information in one basis will be spread out in the other basis. Incoherence is defined as

$$\mu(\Phi, \Psi) = \sqrt{n} \max_{1 \leq j,k \leq n} \|\langle \phi_j, \psi_k \rangle\| .$$

For a $2D$ field that is sparse in the DCT basis and is sampled spatially as described above, $\mu(\Phi, \Psi) = 2$, a constant value which satisfies the CS requirement. For an S-sparse field, the reconstruction problem can now be formulated as the convex optimization program [1]

$$\min_{\tilde{x} \in \mathbb{R}^n} \|\tilde{x}\|_{\ell_1} \text{ subject to } y = R\Phi\Psi x .$$

In words, this optimization is finding the set of coefficients of $\tilde{x}$ which is consistent with the measurements, y, and has the minimum ℓ_1 norm.

A key theorem is of CS states that with if the coefficient sequence x of f in the basis Ψ is *S-sparse*, the reconstruction is exact with high probability if $m \geq C\mu^2(\Phi, \Psi)S \log n$ for some positive constant C, and the measurements are taken in the Φ domain uniformly at random. This implies that if the incoherence is constant, then the number of measurements needed for exact reconstruction is $O(\log n)$.

C. RIP and Measurement Techniques

For signals which are not strictly sparse, but only compressible, i.e. they have rapidly decaying coefficients, an extension is needed to the above theorem. This extension states that for compressible signals that obey the *Restricted Isometry Property (RIP)*, the reconstruction quality decays gracefully. Informally, a measurement system Φ, Ψ, R satisfies RIP of order S if $R\Phi\Psi$ approximately preserves the Euclidean length of S-sparse vectors. This is equivalent to saying that the columns of $R\Phi\Psi$ are approximately orthogonal. If a compressible signal satisfies RIP then CS reconstruction algorithms can be used with high confidence. However, CS algorithms are generally applied with a class of random projection matrices constructed using Gaussian and Bernoulli variables that are known to satisfy RIP and therefore lead to efficient reconstruction [1], [11]. Such random projections are not suited for our problem of sampling using robots because the motion constraints make gathering of independent random measurements infeasible. Therefore it is desirable to find new projection matrices that 1) result in efficient CS reconstruction and 2) can be constructed efficiently by mobile robots. In general, it is not easy to show that structured projections satisfy the RIP, although there have been recent efforts directed towards the design of deterministic projection matrices [16]–[18]. However, these approaches directly control the structure of the sensing matrix $A (= R\Phi\Psi)$ whereas in our case, we can only control the measurement matrix $R\Phi$. To the best of our knowledge, there are no techniques for structured random construction of the measurement matrix, especially given the motion constraints of robots.

IV. Setup for Mission Comparison

A. Energy Budget and Costs

In order to constrain the motions and the number of measurements that the robots can take, an energy budget with associated costs was developed to mimic real-world systems. Costs were included for linear motion, direction changes, and measurements, given by

$$E = \alpha \sum_{i=1}^{m} L_i + \beta \sum_{i=1}^{m} (\Delta\theta_i)^2 + \gamma m$$

where E is the expended vehicle energy, α is the linear motion cost, β is the direction change cost, γ is the measurement cost, L_i is the length of step i, $\Delta\theta_i$ is the magnitude of the direction change before step i, and m is the number of measurements. We forego a separate communication cost, assuming that it could be included as a piece of the total measurement cost.

Several scenarios were investgated. One scenario is that of expensive movement and cheap measurement, such as when measuring a temperature field. Another scenario is that of measurement costs on a par with movement costs, such as with biological sampling [19] or measurements that require active stationkeeping for an extended period. Also, direction change costs were varied to simulate differences in vehicle dynamics and domain size; sharp turns with streamlined vehicles will greatly reduce speed, thereby increasing the amount of energy consumed. The initial vehicle energy was based on the size of the domain. The energy was set to allow for a fixed number ($O(10)$) of traversals of the space, along with the associated $90°$ turns required for a complete boustrophedon pattern, if no measurements were taken. The mission design must then trade off movement for measurement. These scenarios are discussed in more detail in Section V-A. Additional scenarios not discussed here include consideration of obstacles and complicated domain geometry, non-uniform information distributions, and measurement duration and its effect on information quality.

B. Mobility Models - Mission Design

1) Random Walk: The random walk mobility model is designed to satisfy the compressive sensing requirement of uniform random measurement locations while enforcing vehicle kinematic constraints. The vehicle begins in a randomly chosen location and a measurement is taken. After each measurement i, a joint probability mass function (PMF) $P(\Delta\theta, L)$ is sampled for a direction change $\Delta\theta_i$ and step length L_i. The vehicle follows this trajectory and takes a measurement at the end of each step and then samples the PMF again. Fig. 1(a) shows a typical random walk tour. The step-by-step model is illustrated in Fig. 2. We assume instantaneous direction changes, a fair assumption when domain size is large compared to vehicle speed. Domain boundary constraints are enforced by projecting the desired path back into the domain, with the angle of reflection equal to the angle of incidence. McNish [20] investigates reflection angles designed to achieve probabilistically uniform coverage; however, that work applies only to circular domains with direction changes only at the boundaries. Further work in reflection strategies may be warranted.

The random walk PMF is optimized as described in Section IV-E. An important feature of the random walk is that it requires no time or measurement horizon. It simply continues until the available energy is expended or some other mission parameter is satisfied. The use of an optimized PMF to generate the random walk is one of the major contributions of this work.

2) Random TSP: The random TSP mobility model is designed to use uniform random measurements that will fully satisfy the measurement requirement for compressive sensing. A specific number, m, of measurement locations are randomly chosen from the domain. A tour of these points is then generated using a TSP approximation. The expected energy consumption is computed, and if it is 'close' to the available energy, the path is used. Otherwise m is modified to better match the available energy and another set of points and a tour are generated. For our comparisons, we use the nearest neighbor technique (NNTSP). Fig. 1(b) shows a typical random TSP tour.

3) Radomized Boustrophedon: The randomized boustrophedon mobility model is comprised of back-and-forth motions traversing the space. The deterministic boustrophedon (commonly referred to as a "lawnmower" trajectory) is a widely used measurement strategy for coverage of a space [21]; here we introduce randomness for the purposes of employing compressive sensing. The path is determined based on the the desired number of measurements, with the location of the lengthwise traversals chosen randomly. The measurement locations are then chosen randomly along this path. Additionally, the ratio of energy spent on movement and measurement is configured to match that of the random walk to provide a fair and accurate comparison between the two strategies. Fig. 1(c) shows a typical boustrophedon.

C. Mobility Models - Algorithm Complexity

Complexity of mission planning algorithms is a major factor for large numbers of vehicles or measurements. The random walk and boustrophedon calculations have a cost which is linear in m. For the random TSP, the NNTSP approximation we use here is also linear in m; however, there is no approximimation factor with the NNTSP. Certainly a more efficient strategy could be designed – a minimum spanning tree gives an approximation factor of two with $O(m^2)$ computational cost, while a matching step will give an approximation factor of $3/2$ with $O(m^3)$ cost [22].

D. Horizon Planning

Generating a pre-planned mission relies on a prediction of vehicle energy expenditure. Stochastic variables, such as ocean currents or unknown obstacles can cause unexpected changes in mission cost. When the vehicle has an energy surplus or deficit, the vehicle time is not being used as efficiently as possible. These inefficiencies can result in a waste of support vehicle and personnel expenditures or a lost opportunity for the robot to perform other tasks. The unplanned random walk model does not suffer from this problem; its one-step horizon enables the use of all available energy. Differences in expected and actual energy expenditure, however, do affect the random TSP and boustrophedon models' planning strategies. A common practice is to split a mission into a set of phases each with a shorter horizon.

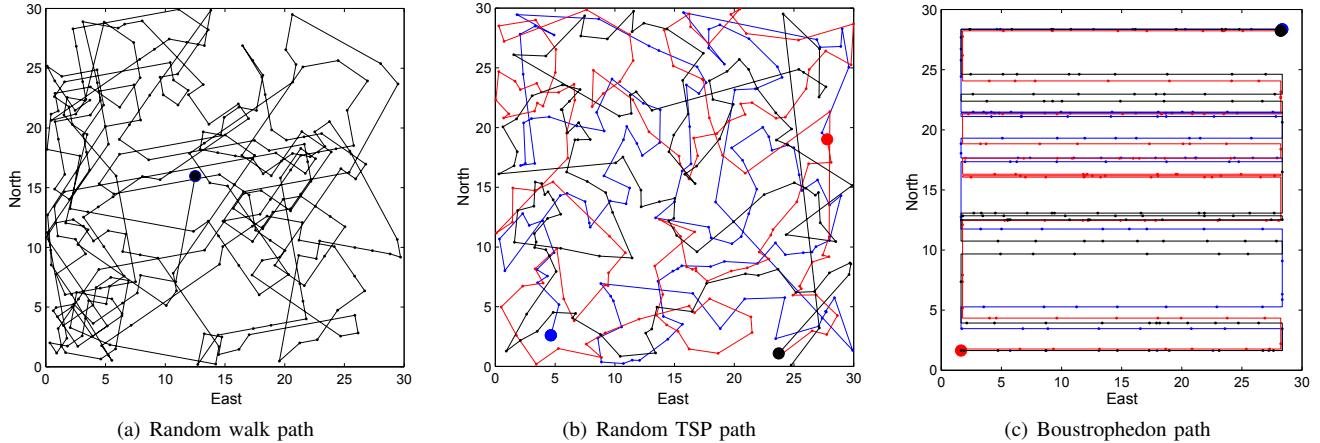(a) Random walk path   (b) Random TSP path   (c) Boustrophedon path

Fig. 1. Typical robot paths. The random walk path begins at a random location and samples a joint PMF for heading change and distance after each measurement. The random TSP path is designed in three phases, each using some fraction of the available vehicle energy. A nearest neighbor tour connects randomly chosen points for each phase. The boustrophedon path is designed by choosing the number of traversals and measurements to result in a desired ratio of measurement and movement energy expenditure. The traversal and measurement locations are then chosen randomly. The boustrophedon path is also designed as a set of independent phases.
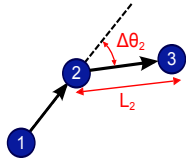

Fig. 2. Illustration of the random walk. At each measurement location, a new step length L_i and a heading change $\Delta\theta_i$ are chosen from a joint PMF.

A TSP path planned in this manner will suffer from a lower efficiency, as there are fewer points to connect with each phase of the mission, but this strategy ensures that at least one path will be completed, providing some assurances that most of the domain will have been visited. With this in mind, we design the random TSP and boustrophedon missions as a series of phases that combine into the complete path. In the specific example below we use three phases which each consume 33% of the vehicle energy.

E. Random Walk Optimization

A particle swarm optimization (PSO) algorithm is used to design the PMF for the random walk. The PSO is performed as in [23], which is a comprehensive learning variant [24]. PSO was used because it is well-suited to multi-modal functions where local optima may exist. The PSO is performed as follows: each position in the joint PMF is represented by several distinct 'particles.' Each particle moves throughout the design space in response to the current best evaluation of the fitness objective function, along with an additional influence from pseudo-random noise. In this case, the objective function is the reconstruction error achieved with a random walk tour using the PMF. Movement and measurement costs will affect this performance, so a separate optimization was run for each set of costs. As expected, with high movement costs the optimization returns a PMF which places more weight on shorter step lengths.

V. SIMULATION EXPERIMENTS

We have tested the proposed sampling strategies through extensive simulations in MATLAB. The domain is a 2D grid with n grid points. The field is taken from measurements of sea surface temperature. Each of the sampling strategies mentioned above was compared under three different scenarios of vehicle movement and measurement costs. Each of these scenarios is meant to simulate differences in real-world operating conditions, such as vehicle mobility, type of measurement, and domain size.

A. Measurement and Movement Cost Scenarios

1) Scenario 1: Inexpensive Measurements and Large Domain: For this scenario, the linear motion cost, α, is set to one, and the direction change cost, β, and measurement cost, γ, are set to 0.01. These settings simulate a vehicle which is taking cheap measurements, such as temperature or optical imaging. Additionally, the small turning cost simulates a domain which is large in comparison to the vehicle turning radius, or a highly maneuverable vehicle. Fig. 3 (left) shows the simulation results for Scenario 1. The top plot show energy expended as a function of measurement number. The bottom plot shows reconstruction error. For this plot, reconstruction error is computed successively as measurements are added.

2) Scenario 2: Inexpensive Measurements and Small Domain: For this scenario, α and β are set to one, and γ is set to 0.01. Here, direction changes play a larger role, and we refer to this as the small domain effect; it comes from having a less maneuverable vehicle or a domain which is small in comparison to the vehicle turning radius. The small domain imposes a greater turning cost because large direction changes will significantly slow the vehicle. Additionally, a small domain will impose more severe limitations on the vehicle path that may decrease the efficiency of path planning
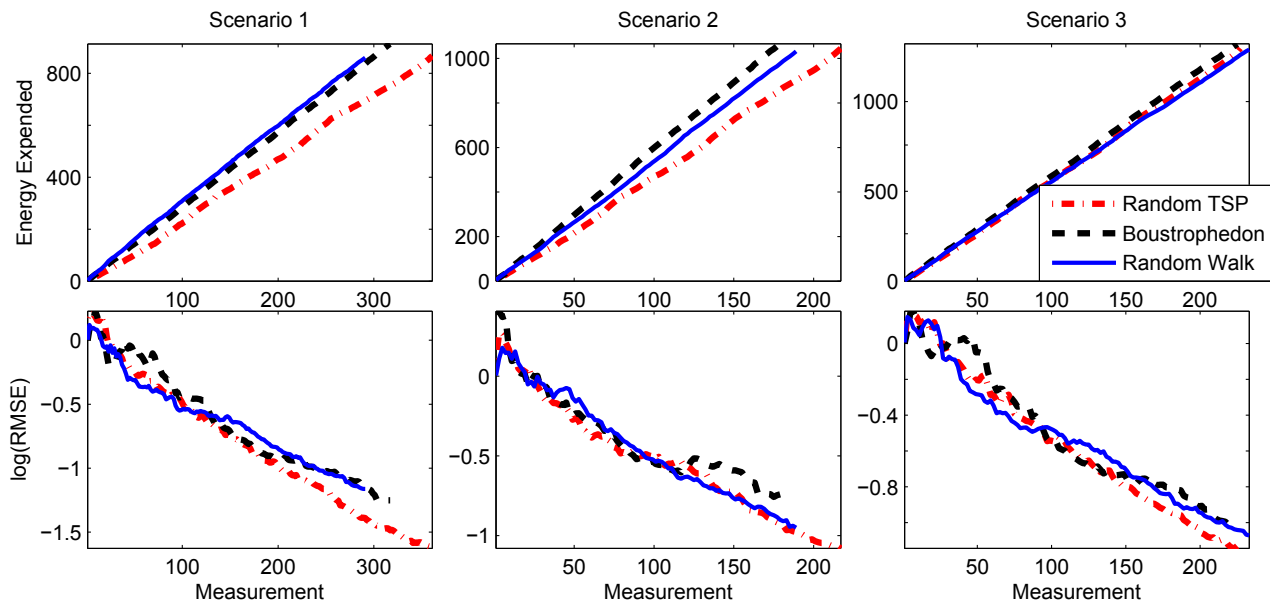
Fig. 3. Energy expenditure and reconstruction error as measurements are taken during a mission. Each sampling strategy is given the same initial energy. Scenario 1: Inexpensive Measurements and Large Domain. Scenario 2: Inexpensive Measurements and Small Domain. Scenario 3: Expensive Measurements and Small Domain. In Scenario 1, the random TSP is more efficient, taking more measurements and also achieving a better reconstruction error. In Scenario 2, the random TSP is again more efficient and achieves a better final reconstruction error; however, for a given number of measurements, the random walk perfoms equally well. In Scenario 3, all strategies are similarly efficient, and here the random TSP is superior given the same number of measurements. The boustrophedon performes similarly to the random walk in Scenarios 1 and 3. In Scenario 2 the random walk is superior to the boustrophedon.

algorithms. Fig. 3 (center) shows the simulation results for Scenario 2.

3) Scenario 3: Expensive Measurements and Small Domain: For this scenario, α, β and γ are set to one. These settings simulate a vehicle taking expensive measurements, such as biological samples in a small domain. Fig. 3 (right) shows the simulation results for Scenario 3.

B. Discussion

Results in each scenario above show that, for a given number of measurements, the random TSP strategy marginally outperforms the 'ophedon and random walk. Likely, this performance difference can be attributed to the uniformly random measurements taken by the random TSP, as opposed to the quasi-random measurements of the other strategies.

A larger influence on final reconstruction error, and what we will discuss from here on, seems to come from differences in the number of measurements taken with each mission type. The boustrophedon mission was designed to match the random walk's ratio of movement energy to measurement energy, so a similar number of measurements is taken with the two strategies, thus resulting in a similar reconstruction error. The random TSP strategy, however, becomes more efficient as more measurements are taken. Fig. 4 shows the average step length as a function of the number of measurements for the random walk in each of the cost scenarios, and for a single instance of the random TSP (the TSP approximation does not depend on vehicle costs). The random walk shows a constant step length, independent of mission duration. This constant is equal to the expected

value of the step length from the optimized PMF for each scenario. The random TSP, however, shows a step length which decreases with increasing mission duration. As the step length of the random TSP mission gets shorter, the random TSP is able to take more measurements, resulting in a better reconstruction. A mission design strategy would seek to maximize the number of measurements taken, and thereby minimize the recontruction error. Looking again at Fig. 4, we conclude that for missions of short duration, i.e., to the left of the intersection for a particular scenario, the mission would benefit from the random walk, whereas, with long missions, to the right of the intersection, the mission should utilize the random TSP.

VI. CONCLUSIONS

We have investigated three mission design strategies for compressive sensing of spatial environmental features using mobile robots. Our random walk strategy includes a novel addition to classical compressed sensing: we optimize the probability mass function of the walk in order to consider motion constraints. The random walk strategy is lightweight and scalable with regards to path planning and is efficient at low numbers of measurements. The random TSP strategy outperforms the random walk at larger numbers of measurements, but an additional computational cost must be incurred to design a TSP tour. The boustrophedon strategy performs similarly to the random walk and is more computationally manageable than the random TSP method; however, it is still more expensive than the random walk. We establish that these measurement strategies combined with compressive
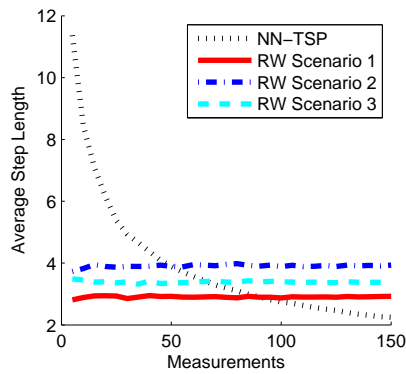
Fig. 4. Number of measurements versus average step length. The step length of the random walk is constant and equal to the expected value of the optimized PMF. The step length of the random TSP decreases as mission duration increases. Results averaged over 200 trials.

sensing provide an efficient way to reconstruct compressible natural fields.

REFERENCES

[1] E. J. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Transactions on Information Theory*, vol. 52, no. 2, pp. 489–509, 2006.

[2] R. Pon, M. A. Batalin, J. Gordon, A. Kansal, D. Liu, M. Rahimi, L. Shirachi, Y. Yu, M. Hansen, W. J. Kaiser, M. Srivastava, G. Sukhatme, and D. Estrin, "Networked infomechanical systems: A mobile embedded networked sensor platform," *2005 Fourth International Symposium on Information Processing in Sensor Networks*, pp. 376–381, 2005.

[3] G. S. Sukhatme, A. Dhariwal, B. Zhang, C. Oberg, B. Stauffer, and D. A. Caron, "Design and development of a wireless robotic networked aquatic microbial observing system," *Environmental Engineering Science*, vol. 24, no. 2, pp. 205–215, 2007.

[4] E. Fiorelli, N. E. Leonard, P. Bhatta, D. A. Paley, R. Bachmayer, and D. M. Fratantoni, "Multi-auv control and adaptive sampling in Monterey Bay," *IEEE Journal of Oceanic Engineering*, vol. 31, no. 4, pp. 935–948, 2006.

[5] E. Fiorelli, P. Bhatta, and N. E. Leonard, "Adaptive sampling using feedback control of an autonomous underwater glider fleet," in *Proc. 13th Int. Symposium on Unmanned Untethered Submersible Tech*, 2003, pp. 1–16.

[6] D. O. Popa, A. C. Sanderson, R. J. Komerska, S. S. Mupparapu, D. R. Blidberg, and S. G. Chappel, "Adaptive sampling algorithms for multiple autonomous underwater vehicles," *2004 IEEE/OES Autonomous Underwater Vehicles*, pp. 108–118, 2004.

[7] B. Zhang and G. S. Sukhatme, "Adaptive sampling for estimating a scalar field using a robotic boat and a sensor network," *Proceedings of the 2007 IEEE International Conference on Robotics and Automation, Vols 1-10*, pp. 3673–3680, 2007.

[8] N. K. Yilmaz, C. Evangelinos, P. F. J. Lermusiaux, and N. A. Patrikalakis, "Path planning of autonomous underwater vehicles for adaptive sampling using mixed integer linear programming," *IEEE Journal of Oceanic Engineering*, vol. 33, no. 4, pp. 522–537, 2008.

[9] A. Singh, A. Krause, C. Guestrin, W. Kaiser, and M. Batalin, "Efficient planning of informative paths for multiple robots," in *IJCAI'07: Proceedings of the 20th International Joint Conference on Artifical Intelligence*, 2007.

[10] A. Singh, R. Nowak, and P. Ramanathan, "Active learning for adaptive mobile sensing networks," *IPSN 2006: The Fifth International Conference on Information Processing in Sensor Networks*, pp. 60–68, 2006.

[11] D. L. Donoho, "Compressed sensing," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289–1306, 2006.

[12] M. S. Crouse, R. D. Nowak, and R. G. Baraniuk, "Wavelet-based statistical signal processing using hidden Markov models," *IEEE Transactions on Signal Processing*, vol. 46, no. 4, pp. 886–902, 1998.

[13] W. Wang, M. Garofalakis, and K. Ramchandran, "Distributed sparse for refinable random projections approximation," *Proceedings of the Sixth International Symposium on Information Processing in Sensor Networks*, pp. 331–339, 2007.

[14] S. Lee, S. Pattem, M. Sathiamoorthy, B. Krishnamachari, and A. Ortega, "Spatially-localized compressed sensing and routing in multi-hop sensor networks," *Geosensor Networks*, vol. 5659, pp. 11–20, 2009.

[15] Y. Mostofi and P. Sen, "Compressive cooperative sensing and mapping in mobile networks," *2009 American Control Conference*, pp. 3397–3404, 2009.

[16] R. DeVore, "Deterministic constructions of compressed sensing matrices," *Journal of Complexity*, vol. 23, no. 4-6, pp. 918–925, 2007.

[17] L. Applebaum, S. Howard, S. Searle, and R. Calderbank, "Chirp sensing codes: Deterministic compressed sensing measurements for fast recovery," *Applied and Computational Harmonic Analysis*, vol. 26, no. 2, pp. 283–290, 2009.

[18] P. Indyk, "Explicit constructions for compressed sensing of sparse signals," in *Proceedings of the 19th Annual ACM-SIAM Symposium on Discrete Algorithms*. Society for Industrial and Applied Mathematics, 2008, pp. 30–33.

[19] C. Scholin, S. Jensen, B. Roman, E. Massion, R. Marin, C. Preston, D. Greenfield, W. Jones, and K. Wheeler, "The Environmental Sample Processor (ESP) - an autonomous robotic device for detecting microorganisms remotely using molecular probe technology," 2006, pp. 1–4.

[20] M. J. McNish, "Effects of uniform target density on random search," Master's thesis, Naval Postgraduate School, 1987.

[21] H. Choset and P. Pignon, "Coverage path planning: the boustrophedon cellular decomposition," in *International Conference on Field and Service Robotics*, 1997.

[22] N. Christofides, "Worst-case analysis of a new heuristic for the travelling salesman problem," Carnegie-Mellon University, Tech. Rep., 1976.

[23] R. A. Hummel, J. A. Taylor, and F. S. Hover, "Numerical optimization of generative network parameters," in *ASME International Mechanical Engineering Congress and Exposition*, 2010, to appear.

[24] J. J. Liang, A. K. Qin, P. N. Suganthan, and S. Baskar, "Comprehensive learning particle swarm optimizer for global optimization of multimodal functions," *IEEE Transactions on Evolutionary Computation*, vol. 10, no. 3, pp. 281–295, 2006.