

WAYS OF ARRANGEMENT

– The Basic Operations of Form-making –

by

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Submitted to the Department of Architecture in Partial Fulfillment of the
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ABSTRACT

Making forms is essentially a matter of arranging things, and arranging things is essentially to establish spatial relations among selected elements. The thesis provides a minimal set of basic operations believed to be sufficient for constructing any given configuration. These basic operations can aggregate to make compound operations handy to designers. Both the basic and the compound operations are called 'arrangement moves'.

Two kinds of basic moves are distinguished: the generic moves, which construct only generic relations such as 'connection', 'separation', etc.; and the ordering moves, which are characterized by using virtual 'lines' as references in establishing spatial relations. A physical design is viewed as finding a correct arrangement that satisfies given constraints. Ordering moves are viewed as an operational foundation that makes such exploration of formal arrangement possible.

The thesis demonstrates that arrangement moves can describe any individual form by reconstructing it; arrangement moves can also describe any family of forms by formulating rules governing the form family. It is further demonstrated that the basic arrangement moves have inherent properties capable of constructing inference rules for perceiving spatial relations. Based on the fact that arrangement moves can sufficiently construct forms, representing rules of forms, and perceiving spatial relations, it is of particular interest to the development of a computational design system that can do arrangements, know form rules, and can check arrangements against rules.

Thesis Supervisor: N. John Habraken
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It can be clear-cut to mark the time when a Ph.D study is started and when it is finished in the registrar's office, but to a research project it can not be that precise to tell its beginning, nor its ending. In my case, I guess it all started five years ago in receiving a letter from Professor John Habraken who welcomed me coming back to MIT to pursue a long-developed interest in design theories.

First of all, I shall thank my dissertation committee from whom I have benefited in countless ways: John Habraken, for leading me toward building a kind of architecture one can only start from scratch; Aaron Fleisher, for his penetrating criticism and hard questions that I always enjoy contemplating; and Stanford Anderson, for exposing me to a stimulating landscape of history and philosophy. Over these years their care and help have been invaluable to me. I am particularly grateful to my thesis supervisor John Habraken who has read 'tons' of my drafts word by word. He deserves the credit of this contribution, while error or oversight, if any, shall be mine. I have learned from him much more than this dissertation can reveal.

During 1981-1984, Jamel Akbar, Mark Gross, and I shared the same office, which was the room to me both for intellectual inspiration and long-lasting friendship. I owe Mark Gross special thanks for saving several files of chapters from a panicky computer disaster; and for the good companionship of the adventure in the not-yet-authorized field of 'design theory and methods'.

For those who participated in the course 'Design moves: a language of form-making' I offered at MIT in fall, 1983, I shall thank them for their willingness and patience to do the class assignments based on the early results of this research. Some of their 'homework' is used as examples in Chapter Four of this dissertation.

I would also like to extend my gratitude to the following people: Paula Arai, Mark Gross, Stephen Kendall, who have proof-read and deliberately edited the text of one or two chapters of different versions of drafts; the Ashdown Group of Chinese Professionals in Design and Planning, Cambridge, who have shown continuous interests in my studies, and warmly invited me to present the work; Paula Arai and Chih-chien Wang, who participated as subjects in a small experiment of testing the use of arrangement moves; the research team of NSF's 'Design Games' project, 1985-1986, who made me understand many 'moves' involved in design other than the kind I described here; Akhtar Badshah, who generously offered the accessibility to the LaserWriter by which the final scrip was produced; Professor Chang-I Hwa at Boston University, for sharing enlightening discourses on many topics from philosophy to AI; Professor Chen-Tze Ho at Tunghai University, Taiwan, for his continued moral support since I was a college student.

My deepest debts are to my family, for everything.

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'Arrangement includes the putting of things in their proper places, and the elegance of effect which is due to adjustments appropriate to the character of the work.'

— Vitruvius, Book I, Chapter II,
'The fundamental Principles of Architecture'

Chapter 1

Introduction

§1.1 Operation

Things change when touched. Quantum physicists argue that the means adopted for observing things are not neutral to the things observed, because they disturb the observed.

Nature, they believe, is built in such a way that it will present different appearances to different means of observation. And consequently, searching for the 'ultimate' reality becomes a meaningless quest, not only for Physics, but perhaps also for all natural sciences. The point is well made. If this is a fact, then, I would like to interpret it as a fact about human knowledge, rather than a fact about Nature. However, almost paradoxically, another epistemological parallel appears on the opposite side: we cannot know the nature of things unless we touch them. That is, appropriate operations pertaining to examining the thing under observation should be defined, and applied as means of exploration. These operations are keys to the world of the unknown, without them we remain outside of it.

Let me make the point clearer by the example of natural numbers with which we are all familiar. Part of our mathematical knowledge is constituted by what we know about the properties of numbers. The theory of numbers is concerned with such questions as what properties numbers can have. The mathematician must ponder a methodological question before he/she starts the investigation: how can the properties of numbers be examined? That is, given a string of numbers, 1, 2, 3, 4, ..., how can we know their properties other than the fact that they are ordered as such? What are the necessary conditions that make knowing the properties of numbers possible? These are the kind of questions for which the mathematician should find at least one answer in order to do his business.

Numbers present no property other than the order of sequence unless we do something to them, and we cannot do anything without first defining some operations. It will be impossible, for instance, to gain the concept of odd and even numbers, unless

operations of summation and subtraction are defined in the first place. Without the operations of multiplication and division we cannot tell the prime number from the non-prime number; or the rational number from the irrational number. In other words, we would have known nothing about numbers, had such arithmetic operations as summation, subtraction, multiplication, and division, etc., not been available. The properties of numbers are expressed by their 'relations' to other numbers, usually in the form of theorems, and relations are established in terms of operations.

How do we know about built environments? How do we understand design? What constitutes our knowledge of form? By the same token, we would know nothing about the structure of the built environment unless we observe how it changes under the different operations we exert on it; we won't have a better understanding about design unless operations that make design possible are known; and we would know little about forms, if we were not equipped with operational means to explore them. Operations are epistemic devices by which human knowledge can be built. Our knowledge of the world, whether natural or artificial, can generally be viewed as a system which identifies and specifies the relations of things. Operations are powerful means to specify relations, or to put it another way, we employ operations to 'see' relations among things. This is the view I would like to adopt as the point of departure to study the nature of making forms.

§1.2. Relation

My objective is to investigate spatial relations. The thesis is this: one finite set of operations exists that can account for making any given form by constructing spatial relations among selected elements. The claim as such must sustain the challenge of the sheer fact that forms are infinite in number and various in appearance. These operations are presented in Chapter Two, and whether the claim can meet the challenge is discussed in Chapter Three, particularly the issues of completeness and sufficiency. Here I shall remark on the notion of 'relation' in conjunction to my claim. The points in this chapter are, I believe, of general interest as a way of seeing our humanly-constructed systems,

particularly formal systems, and I submit them only as background information, not as conditions for what follows in other chapters.

One apparent, but often overlooked characteristic of all humanly-constructed complex systems is that: if each system allows for a large number of variant configurations, then these variants can not all be completely different. That is, variants either share certain elements, or they share certain relations. But the point I would like to argue is this: if the system contains an infinite number of variants, it is the elements, not the relations, that give primary contribution to this infinity.

Let me offer an example of a spatial system. An arbitrary selection of elements consists of, say, a cat, a car, a can, and a cabbage. Merely having these elements is not enough to make any variant, spatial relations must be defined. Let's say there are two spatial relations: inside-of, and on-top-of. We can only have two completely different variants that share neither common elements, nor common relation, although we do have $4 \times 4 \times 2 = 32$ choices of pairs. That is, once one configuration is made, say, [cat inside-of car], then only one other candidate remains, i.e. [cabbage on-top-of can]. The third variant will inevitably share either an element, or the relation with one of these two. The only way we can add a new variant that is completely different from other variants is to introduce new pairs of elements, and new relations.

Now, the question comes: is it always possible to do so, so that the system can contain an infinite number of completely different variants? I see no trouble with bringing in new elements endlessly, but I fail to conceive that it is possible with relations, unless we include dimensional differences. I do see many cases that have a very large number of elements and very few relations; but I do not see one single case when the opposite is true. We all know that English words are made from 26 characters in the alphabet in various kinds of combinations, we must also notice that all combinations use only one operation to construct relations among characters, that is, concatenation -- stringing one character next to the other.

What all this implies, in the case of making forms, is that the spatial relations we need to know in order to construct configurations, or to perceive forms, are more limited in number than many creative designers are prepared to admit. It is easier to see many disparate configurations as a result of using different elements than as a result of using

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different relations. Let me offer an example. The pictures below show two walls of very different forms: a strip-coursed stone wall, and a masonry construction known to ancient Greeks as the 'cyclopean wall'. We see that the masonry with regular pieces has an orderly appearance in terms of the rhythmic repetition of course lines. The cyclopean wall, obviously, looks more chaotic. However, a closer look at the arrangement of stones in the cyclopean wall reveals a more subtle sense of order underlying the seemingly random appearance. We find that every edge of the irregular block 'conforms' to the edge of its adjacent block. The construction of the cyclopean wall requires that every piece of stone, though random in size and shape, has to fit tightly with adjacent ones to produce a continuous, closely knit surface without unfilled interstices. In antiquity well-trimmed, regular stones were used largely for sacred buildings. People with modest means could afford to build only with irregular blocks. When stones are cut regularly in shapes and size, they can be laid course by course much easier and faster. Nevertheless, the relation between two regular pieces is exactly the same as the irregular blocks of the cyclopean wall. It is the elements, not the relations, that make these two walls different.

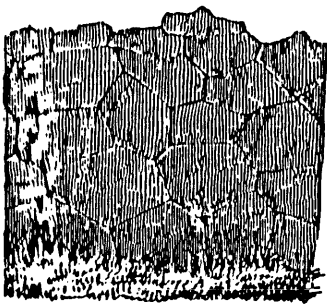


Figure 1.1.

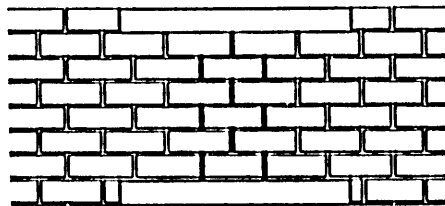


Figure 1.2.

This case also demonstrates the the possibility of generating a variety of configurations by using different elements but the same relations. If spatial relations are limited in kind, then the operations needed to establish spatial relations are also limited in number. The operation can be defined in terms of the relation it establishes. Therefore, these two different walls are constructed by the same operation since they share the same relation. It is based on this conviction that I embark on the thesis of this study.

§ 1.3. Order

Mankind made many things that have tangible forms, and they are made for certain uses. The usefulness of a thing can be conceived generally in two ways: 1). it is its tangible form that is useful, such as a cup, a chair, and a building. 2). It is its physical properties that make it useful, such as a stove that gives heat, a bulb that gives light, and a bell that gives sound. Making a form certainly has to do with the uses it performs. However, it is a mistake to think that forms can be created from meticulously analysing desired performance, although this once was, and perhaps still is, widely accepted as the designer's maxim. Knowing sound can best tell us how to test, not to generate, the form of the bell. It is our knowledge about certain materials that resonate to generate sound, and certain shapes that amplify and make sound audible, that makes the design of the bell form possible. I shall not delve too far into the subject of form/function dialectics, I only want to make the point that form-making involves many non-functional considerations, and the richest one in operational meaning is the sense of spatial ordering. It is also the one entertained most deeply in this enquiry.

I think that the involvement of non-functional concerns in design, in most cases, is simply a matter of fact. One example can suffice to demonstrate that form is not determined by function at all. The figures below show two circuit layout designs of a computer chip, one manually designed (Figure 1.3), the other machine generated (Figure 1.4), that satisfy exactly the same functional program.¹ If it is true that form 'follows' function, then these two designs should be at least very similar to one another, if not completely identical. Obviously, the functionalist claim fails here.²

It is also important to note that, as we can see, the hand-packed layout is more 'orderly' in its arrangement than the automatically generated layout. Modules are tightly packed by edge alignment and row conformation. (As a result, it is only 45% the size of the layout generated automatically.) It is obvious that such an orderly arrangement is not an accident, nor is this the 'natural' result of satisficing functional constraints, because the automatic design also satisfies functional constraints, but it does not have such an appearance of order. One reasonable explanation would be that such an orderly arrangement is the result of a kind of self-imposed act of form-making.

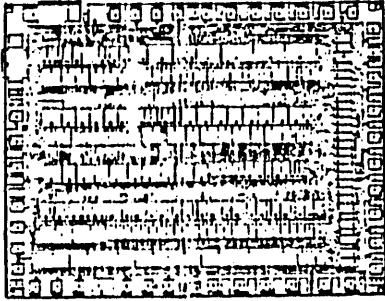


Figure 1.3.

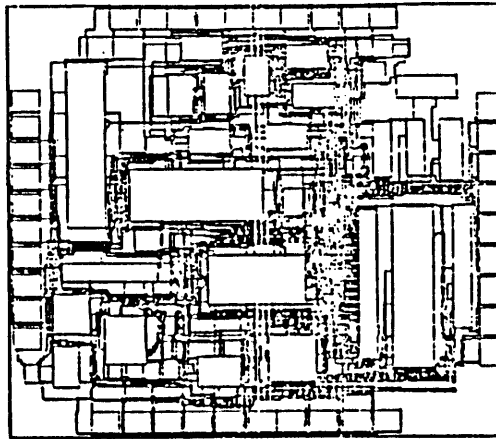


Figure 1.4.

There is no lack of other examples. It is, to give another one, functionally unclear why two headlights of a car are aligned despite the fact that they can be otherwise placed to achieve an equivalent performance of illumination. Certainly, such alignment is not accidental. Aesthetics can not be the only reason, nor is it the one we are interested here. In the case of computer chip design, there is no conceivable reason to make a 'beautiful' circuit layout which cannot be appreciated from the outside any way. There must be deeper reasons.

It is my conviction that such self-imposed 'ordering' means as the aforementioned examples of 'alignment', 'conformation', etc., are the kind of operations that can help us do spatial arrangement meaningfully. Form-making can not do without them. The modest goal of a physical design can be viewed as finding one 'correct' arrangement that can satisfy a given program of requirements. Merely placing elements here and there is not useful at all for exploring spatial arrangements. A useful exploration presupposes that we can 'characterize' spatial relations produced in one alternative from another alternative, otherwise they would all look the same, and the exploration would be meaningless. We can hardly tell the difference of arrangement of one group of pebbles from another, unless pebbles can be 'ordered' by specifying certain recognizable spatial relations. For instance, this is one string of pebbles called A; that is one string of pebbles called B, and pebble-string A conforms to pebble-string B. By adopting the word 'string', we no longer

see pebbles as randomly scattered pieces, but arranged along a line; and by 'conform', we see two strings related in a specific way.

Making forms is inextricably associated with principles of humanly-constructed order to guide our search. Without such order our work can only be random. The operations I proposed in this thesis manifest these principles of order, which help designers to explore formal arrangements as the stick helps the blind to explore his invisible surroundings.

§1.4. Synopsis

Operation is the viewpoint I adopt to undertake this enquiry; spatial relations are what I intend to explore; and the search for principles of humanly-constructed order explains my motivation. In the following chapters I shall restrict myself to more technical presentations, and engage in no further explanatory discussion.

Now I shall give a synopsis of this study. Basically this dissertation contains what is needed to answer two central questions: what operations are sufficient to make any arrangement? and of what use are these operations other than the making of arrangements? The operations are termed 'arrangement moves' because they provide the means to establish spatial relations in formal arrangements. In Chapter Two I introduce a system of such moves, and show that there is a minimal set of basic moves out of which many compound moves can be constructed.

Given this minimal set of basic moves and the claim of their sufficiency, immediately several related questions demand deliberation. Are these moves identifiable from observation? Can they be reduced as a compound of even more basic moves? Do we need these basic moves, or we can do as well without them? Do other spatial relations exist that cannot be construed by these basic moves? And in what sense can we know these basic moves are sufficient? These are the kind of questions I try to deal with in Chapter Three.

Then, there is the practical question: what are the uses of arrangement moves? The

last three chapters are devoted to this practical question. First, physical design is a process toward making specifications about form to be made. A specification is a description, and a form can be described in different ways. In Chapter Four, I demonstrate that arrangement moves can be used as a set of explicit terms to describe any configuration at any chosen level of detail, regardless of which manner of description is adopted.

Arrangement moves can also describe a family of forms, that is, a type, or a style. A type can generally be seen as a set of rules that specify a particular group of elements, as well as permissible relations among these elements. Rules of type represent the knowledge we have about the type. In Chapter Five I demonstrate this idea by formulating a set of rules of type for Wright's Usonian houses in terms of arrangement moves.

Some interesting properties embedded in arrangement moves are found that can be used for the formulation of inference rules for perceiving spatial relations. That is, it is possible to do logical reasoning on spatial relations merely based on descriptions expressed in terms of arrangement moves. This is of particular interests to construct the computer's capability to 'see' spatial arrangements. Based on the fact that arrangement moves are operationally sufficient to construct configurations, describe forms, and represent rules, in Chapter Six I will demonstrate how arrangement moves can be used to develop powerful computational design systems capable of checking design results against design rules. I also give, in the Appendix, a short review of the approach known as 'shape grammars' to discuss its descriptive power in rule representations as well as computational potentials in light of our current knowledge about arrangement moves.

To end this introductory chapter, I must also warn the reader as to what this study is not about. First of all, although I claim that form-making will be impossible without employing something like arrangement moves as operational means, this is not the sort of 'empirical' research that allows us to determine whether the move system provided here is indeed the system of operations people actually use in design practice, although I do believe they do. Whether my belief is supported by empirical evidence or not is irrelevant at this point. After all, we are interested in the competence, or the 'logic', behind form-making; not in explanations of design performance, or design behavior.³

Second, this research does not aim at making design tools. The arrangement move system at this stage of development falls short of providing any methodical technique that

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can help a designer to improve his/her work. If, as I believe, the arrangement moves offered are part of what designers do, that does not mean that knowing this theory will make them better designers, for the same reasons that knowing linguistic theories will not make English speakers speak or write better English.

(End of Chapter 1)

Footnotes to Chapter 1

1. This example is taken from the paper by Jiri Soukup, 'Circuit Layout', *Proceedings of The IEEE*, Vol. 69, No. 10, Oct. 1981.

2. By 'functionalists' I do not mean Louis Sullivan particularly, but rather those design researchers since 60's gathered under the banner of a new paradigm as represented by Christopher Alexander's book *Notes on The Synthesis of Form*, Harvard, 1964, which attempts to show a way of generating forms by structuring functional requirements. Such a functionalist approach is not, I think, at all what Sullivan meant by his maxim of 'form follows function'.

3. This point has been eloquently argued by Noam Chomsky in the linguistic domain. He separated the theory of linguistic competence from the theory of performance (the theory of language use) as two fundamentally different constructs. Any reasonable theory of performance must incorporate linguistic competences, but it must also incorporate many other components which are non-linguistical at all. See *Aspects of The Theory of Syntax*, MIT Press, 1965. Also see his 'A Note on The Creative Aspect of Language Use', *The Philosophical Review*, XCI, No. 3, 1982, PP. 423-434. Arrangement moves can account for what are produced as results of making forms; but do not account for the actual behaviors of making such forms.

Chapter 2

Ways of Arrangement

§ 2.1. Form

Form is bounded. Presumably, all physical entities have forms. Stones, trees, houses, and even a cloud in the sky present themselves with particular forms. But, 'water', although a physical entity, has no form by itself (at least at the macro-level). Water can have a form only if it is bounded externally. For instances, we can see the 'form' of the water only when it is in its solid state of ice; or when it is contained in a cup. Space has no form by itself either, it must be defined by other forms. We do not see the space called 'room' unless there are four pieces of walls framing it. Form is known by its boundaries.¹

The boundary claims what belongs to the form and what is not. To define a form is to draw its boundary. The world is a physical continuum consisting variety of matters that are not necessarily as discretely defined as we may take for granted. To see a form we must determine its boundary. We see stones, trees, houses, and a cloud in the sky, only if we choose to draw boundaries as such. There is no such thing as the absolute boundary of a stone, a tree, a house, or a cloud in the sky. Forms are our own making by deciding on their boundaries, and we have some long-established conventions to tell us how to draw them.

Thus, drawing boundaries is always a matter of interpretation and judgement. We may not argue the form of a given stone, tree, or house, but we may disagree about the blurred boundary of the cloud in the sky. While boundaries may not be clear and definite to beholders, this does not mean that therefore forms can not be defined. Should we not confuse the issue of interpretation with the issue of agreement on interpretation. We are free to define the boundary of a form as we like.

The boundary of a form is often seen as a closed line, such as a circle; or as a closed surface, such as a globe. But the outline of a single form may not be one continuous, closed line, or one continuous, closed surface. For instance, the plan of a courtyard house must have two concentric outlines, and a compound courtyard house can have even more.

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Every single, closed outline will divide the plane, or the space under observation into exactly two regions.² It is by boundary that the distinction of the inside and the outside is made; and it is by such a distinction that we can begin to talk about relations among forms, as we shall see later.

Forms are either spatial or physical, both are legitimate materials for form-making. When these spatial forms and physical forms are used in form-making, they are called *elements*. There are physical elements and spatial elements.

All forms are composed by elements, and all elements can be seen as forms composed by elements at a lower level. A table is a top piece with four legs supporting the corners; a top piece is a flat panel having six surfaces; each surface is a rectangle with certain dimensions, and so on. The hierarchy involved here is based on a part-whole relation. In principle, a form can have any number of part-whole levels as long as we like to continue decomposing it into smaller parts. We may observe another kind of vertical relations called 'dependency' hierarchy in our living physical environments.³ The levels involved in this hierarchy are not defined in terms of part-whole relations. Dependency hierarchy is based on the concept of dominance. The principle for determining dominance relations can be briefly described as this: given two related forms A, and B, if the change of A will affect B; but the change of B does not affect A, then A is at higher level than B, and B is dominated by A. For instance, the street dominates the houses along it; the structure of a house dominates the partitions inside it, because we can readily see that if we move the formers, the latter will inevitably be disturbed, but it is not necessarily true vice versa.

The part-whole assembly is a useful concept for making forms only if the notion of hierarchy is incorporated. Hierarchy is a way of dealing with complexity. Merely dissecting the whole into parts is not very helpful if parts are not organized. Let's see a simple example. The flower as given in Figure 2.1. consists of 8 elements: 1 pistil, 4 petals, 1 stalk, and 2 leaves. Arbitrary compositions of these elements will not make a flower. Some rules govern 7 pairs of relations: 1) four petals should connect to the pistil; 2) two leaves should connect to the stalk; and 3) the pistil should be on the top of the stalk.⁴ To make a correct composition of the flower with these 8 elements, we must choose 7 pairs of relations from the pool of at least 28 paired relations (excluding the

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symmetrical and the self-related relations, $(8 \times 8 - 8) / 2 = 28$). For a random selection, there are about one million possible combinations (i.e. $28! / 7! (28 - 7)! = 1,184,040$) in which we can find one right composition. But we can structure these 8 elements into a hierarchy so that 4 petals and one pistil make a corolla; 2 leaves and one stalk make a stem; and then the corolla and the stem make a flower. In this way, we only need to search the solution space of about six hundred possible combinations (i.e. $(3! / 2!) \times (10! / 4! (10-4)!) = 630$) to constitute a correct picture of the flower. Three levels are clearly distinguished in this case: petals, pistil, leaves, and stalk are at the bottom; the corolla and the stem are in the middle; and the flower is on the top. Hierarchy reduces complexity enormously.

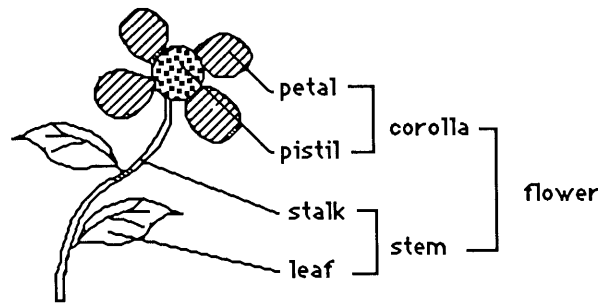


Figure 2.1.

Selection of elements is an important act in form-making. A same form may be made differently if its elements are selected under different interpretations. Take a simple A-B-A tartan grid as an example. The grid can be seen as made of one element, i.e. a straight line, first repeatedly placed in an A-B-A-B... rhythm in one direction, and then in another direction. The grid can also be seen as made out of three kinds of element, AxA square, BxB square, and AxB rectangle. First, we can make A-strings by AxA squares and AxB rectangles; B-strings by BxB squares and AxB rectangles. Then, A-strings and B-strings are connected in the A-B-A-B sequence that finally constitute the grid.

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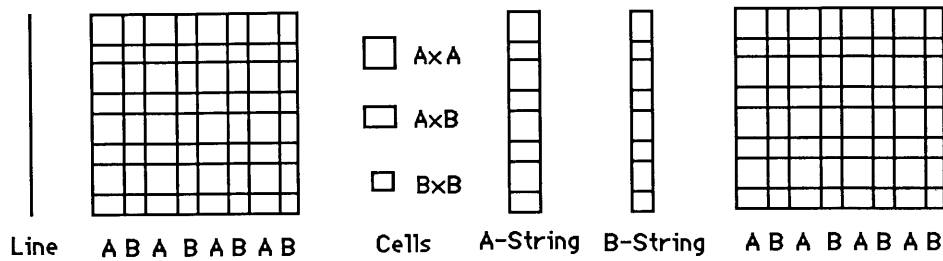


Figure 2.2.

In many cases, selection of elements is largely determined by firmly established conventions, such as the elements of the flower and the elements of the table. In other cases, selection of elements has to do with the means and procedures involved in form-making. Simple lines are more efficient than different sizes of cells if the grid is drawn on paper. However, if the grid is to be made as the pattern on a floor surface, then cells are perhaps more appropriate than lines. Generally speaking, elements are selected for our particular interests in form-making, as painters may choose colors, and architects may choose materials, to make their forms.

§ 2.2. System of Moves

Making forms is essentially a matter of arranging things; and arranging things is essentially to establish spatial relations among elements. From this point of view, form-making basically consists of two tasks: 1) selecting a set of elements by which forms are made, 2) arranging elements by establishing spatial relations. Let's call the acts of arrangement *arrangement moves*. When a set of elements are given, or selected, the question as to what forms can be made by these elements is determined by the arrangement moves applied. My objective is to define a minimal set of arrangement moves sufficient to make any formal arrangement.

One first remark: if there is only one element, there exists no relation at all. But we still can do something about it. Say, move it in different direction; rotate it with any angle; flip it over; and so on. Such acts are to be distinguished from arrangement moves,

therefore, they are called *physical moves* . Arrangement moves must deal with two or more elements.

2.2-1. Generic Moves

Let's begin with observations on some primitive arrangements. Given a form A, whether it is a physical element or a spatial volume, three simple properties can always be identified, by the definition of form:

- a). The boundary of A.
- b). The inside of A, with respect to the boundary.
- c). The outside of A, with respect to the boundary.

Therefore, given two elements, say, A and B, to relate B to A, the placement of B can produce only three possible situations:

- 1). B is outside of A
- 2). B is inside of A
- 3). B is partially outside and partially inside of A.

Note that the situation that one element has the same boundary as another element is ruled out here, because every element has its own boundary, and occupies a definite space exclusively. If two elements have their boundaries exactly coincident with each other, then they simply are the same element with two different names.

Corresponding to these three situations some spatial relations can be defined.

- 1). When B is placed outside of A, there are two cases.

- 1.1. B does not touch A, let's call this relation *separation*.
- 1.2. B touches A, let's call it *connection*.

- 2). If the two elements differ in size, and the larger one is a spatial volume, then we can put the smaller one inside the larger one. When B is placed inside of A, again, there are two cases.

- 2.1. A and B do not touch each other, we will call this relation *inside-separation*.
- 2.2. A and B are physically engaged, we will call the relation *inside-connection*.

- 3). Imagine that A is a spatial element, and B is placed in such a way that it is partially outside and partially inside of A, we will call this relation *overlap*.

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All these primitive spatial relations, as a class, are called *generic relations*, and the acts that establish such relations are called *generic moves*. They are generic because they are not specific enough to tell one instance from the other instance of a same move. For example, 'A inside-connects to B' can refer to almost infinite number of arrangements with the given A and B. To give a precise arrangement instruction to specify that 'A inside-connects to B in this particular way, and not in any other way', we need more operational devices and dimensional information.

Starting with the inside/outside distinction, no more generic spatial relations between two elements can be formulated other than the five listed here. The generic moves form a closed system, which is obviously from the way they are introduced.

To give a more formal treatment to these generic moves, we shall introduce the concept of *distance*, which is a primordial concept that we always use implicitly in arrangement. Distance is a spatial term, although it usually appears in a numerical form. Distance refers to the space that is bounded 'between' specified boundaries, say, 'the distance between the two houses is 40 foot'. We can talk about dimensions such as '40 foot' without referring to any state of spatial relation, but it is meaningless to talk about a distance without knowing from where to where it is to be measured. Being bounded in this way, the notion of distance acquires formal qualities. Saying that 'the distance between the two houses' is a generic spatial entity, and '40 foot' is its dimensional value, is same as saying that the house is a spatial entity, and '4000 cubic-foot' is its dimensional value. A distance has only one dimension while the house has three; but the distance, like the house, can be known spatially without specifying its dimension.

Spatial concepts like 'separation' and 'connection', in the way they are conventionally used, very much reflect spatial properties of the notion of distance. The distance between any two objects is greater than zero, if objects are separated; and equals to zero if connected. So, the notion of distance and the spatial relations of separation and connection can be mutually construed. As separation and connection apply to the inside/outside distinction of a form, we can make four variants: inside-separation, outside-separation, inside-connection, and outside-connection. In our move system, separation and connection are terms exclusively reserved for naming the outside spatial relations. Unfortunately, no simple, suitable terms are known for naming the situations of inside-separation and inside-connection, and therefore, we shall name them as such.

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Now we may define generic moves more formally. Given two elements A and B we can distinguish the following generic relations:

- 1). **Separation:** A and B are separated, if B is outside of A, and $dis > 0$.
- 2). **Connection:** A and B are connected, if B is outside of A, and $dis = 0$.
- 3). **Inside-separation:** B is inside of A, and $dis > 0$.
- 4). **Inside-connection:** B is inside of A, and $dis = 0$.
- 5). **Overlap:** A overlaps with B, if B is a spatial volume and part of A is inside of B, while other part of A is outside of B.

In the case of 'overlap', the concept of distance does not yield any new feature as far as the definition is concerned. This can be illustrated in the figures below. Figure 2.3-1 shows the case of overlapping in which a non-zero distance exists between A and B. Figure 2.3-2 shows another case in which the distance is reduced to zero while the generic relation is the same. In Figure 2.3-2 we may observe another distance between A and B in a different direction as shown in Figure 2.3-3. If this distance is reduced to zero, as shown in Figure 2.3-4, then we simply have the spatial relation of inside-connection between A and B, which is known already.

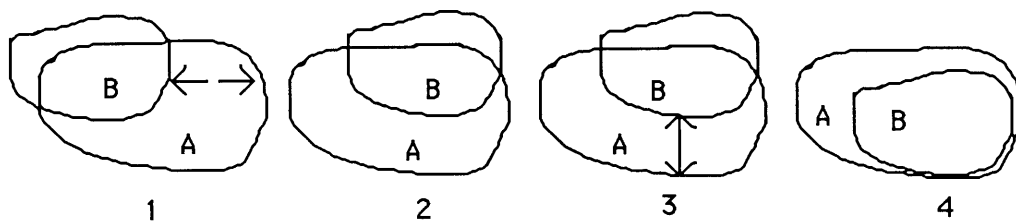


Figure 2.3.

I would like make a few terminological remarks. 'Overlap', although sometimes also refers to the spatial relation that one element is completely inside of another element in conventional conversations, here it will be exclusively used as a technical term only to account for the situation of 'partially inside and partially outside'. I will also use *containment* as a general term referring to both the relations of inside-connection and inside-separation. 'A contains B' (or B is contained in A) means that either B inside-connects to A; or B inside-separates from A. If B is in the spatial relation of

inside-separation to A, we will also say: B *inserts* in A; or A *includes* B. If B is in the spatial relation of inside-connection to A, we will also say: B *insert-connects* in A; or A *include-connects* B.

2.2-2. Ordering Moves

If we are given more than two elements, what can we do other than repeating those generic relations? I shall explore a few possible arrangements with which designers are familiar.

1). Let's introduce a line, straight or not, and place elements on the line to constitute a configuration, be it continuous or disjointed. Let's call this arrangement a *string*, and call the line on which elements reside a *stringing line*.

Based on the concept of string, now we begin to introduce another set of moves which can make more complex arrangements, and all these moves are about relations among strings.

2). We can make two strings, each string is made out of a set of discretely placed elements on a stringing line, and then to juxtapose one string on the other, that is, insert one string into the other string so that their stringing lines merge into one another, call this arrangement *coincidence*.

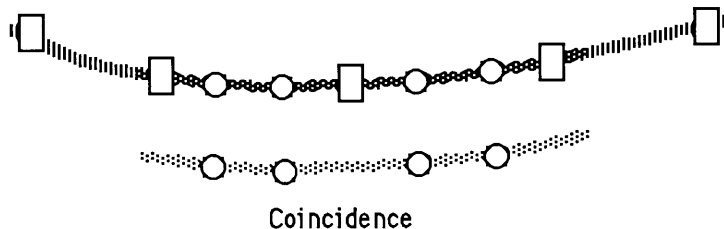


Figure 2.4.

3). We can make two strings, each has its own stringing line, and propose a common stringing line on which these two strings are placed so that two individual stringing lines merge into this common stringing line. Let's call this arrangement *alignment*.

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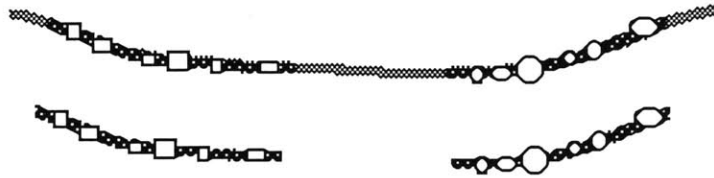


Figure 2.5.

4). We can arrange several strings that they all direct toward one point in a radial form, and call the arrangement *convergence*.

5). We can make two strings that one passes through the other from different direction, call it *cross*.

6). We can also make two strings that one just touches the other from different direction without crossing it, let's call it *abuttment*.

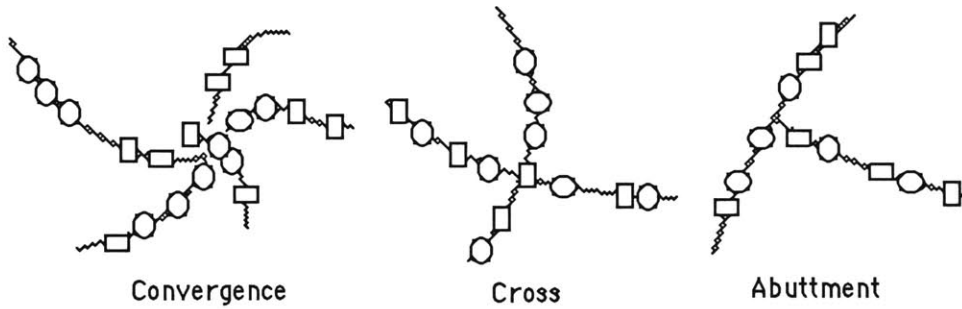


Figure 2.6.

7). We can make two strings parallel to each other in the sense that if one changes its direction, the other follows accordingly, and call this arrangement *conformation*.

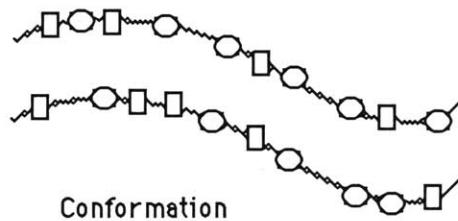


Figure 2.7.

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A string is a primitive organization of elements, which gives order to otherwise unrelated pieces here and there. Therefore, let's call all these acts of arrangement *ordering moves*. Here, we should notice that the string is a general concept, which is indeed one-dimensional. But the physical objects that form the string are not necessarily one-dimensional too; they can also be volumes, or surfaces.

How can these ordering moves be understood as one system rather than a sort of random collection? To give a systematic account of this second set of moves, the concept of 'stringing line' plays the central role. By definition, the string arrangement is impossible without a stringing line. The stringing line is projected whenever the string move is made. Such a line and the act of stringing can be viewed as born at the same time, and we can hardly explain one without the other. We can argue that the notion of line is the abstraction of a physical string as much as we can argue that the concept of distance is the abstraction of the state of 'separation'. The projected line, whether made visible or not, must be preconceived to guide the string move, otherwise it would become a random act. With the projected lines we can accurately control the act of placing elements by examining their relations with respect to the projected lines. I would like to claim that this relation of elements to lines is the basis for spatial arrangement. Ordering elements is indeed arranging them with respect to such lines.

The move string, as related to a single line, is the basic arrangement on which all other ordering moves are defined. All these ordering moves deal with relations about 'lines', whether they are stringing lines of a configuration, or *directional* lines suggested by the shapes of elements themselves, or any other specified reference lines. The line is therefore the base for defining ordering moves as the boundary of form is the base for defining generic moves. To structure the system of ordering moves we shall describe a world in which all possible relations between two lines are enumerated.

Given two strings, A and B, there are two general situations in which their stringing lines can relate to one another:

- a). A's stringing line and B's stringing line are co-linear, i.e. either A's stringing line is part of B's, or B's is part of A's; or they both are part of another line.
- b). A's stringing line and B's stringing line are not co-linear.

Corresponding to these two general situations, some spatial relations can be

observed between two strings.

1). In situation a) that A and B are co-linear, there are two cases.

1.1. String-A and string-B are arranged in such a way that either A's stringing line is part of B's; or B's is part of A's, such a relation is *coincidence*.

1.2. String-A and string-B are arranged in such a way that they both reside on another common underlying line, such an arrangement is *alignment*. (Note that the coincidence relation must be a kind of alignment; but alignment is not necessarily coincidence.)

In situation b) where A's stringing line and B's stringing line are not co-linear, there are two more situations: b-1). A's stringing line intersects with B's stringing line. b-2). A's stringing line does not intersect with B's stringing line. If only consider a stringing line to be a finite line, we can therefore say that it has two 'ends' and one 'body' between the two ends.

2). In the case of b-1) when two stringing lines intersect with each other, only three cases are possible:

2.1. String-A and string-B are arranged in such a way that one end of A's stringing line connects to an end of B's stringing line, this is *convergence*.

2.2. String-A and string-B are arranged in such a way that the end of A's stringing line connects to B's body, this is *abuttment*.

2.3. String-A and string-B are arranged in such a way that the body of A's stringing line intersects with B's body, this is *cross*.

3). If in situation b-2) where both A's and B's stringing lines do not intersect with each other, then they must be separated. In this situation, again, there are two cases: they are either parallel to each other; or they are not.

3.1. In the case that they are not parallel, then the two strings are simply in the generic relation of separation, which has already been defined, and such relation exhibits no quality of ordering whatsoever. We therefore consider only the case when they are parallel.

3.2. String-A and string-B are arranged in such a way that A's stringing line and B's are parallel to each other, this relation is *conformation*.

2.2-3. References

All ordering moves so far are shown as acts of establishing relations among strings, can they also apply to non-linear shapes like squares or globes? When we want these moves to be general means of arrangement, they certainly should be able to do so. To make these ordering moves generally applicable, we shall apply the concept of *reference* by which a more general definition can be formulated.

In a string arrangement, elements reside on the stringing line, but how exactly they are 'on' the stringing line is not the concern of the string move. For instance, given three elements, to make a string we first project a stringing line, then we place elements on the line without particular specifications on what sequence they are with each other; what distances between them, and how they connect to the stringing line. As long as elements are placed in line, i.e. on the underlying stringing line, they constitute a string. This is a *simple string* move, not unlike the generic move, such as separation, where two elements are in relation of separation as long as a positive distance is between them.

However, we do not use strings fruitfully unless we are more specific about the positions of elements relative to their stringing line. Such a positional specification can be given, for instance, by saying that elements are aligned by their centers, or by their edges. Or they are aligned in such a way that element-1 is placed on the line with its center; element-2 connects to the line with its edge; and element-3 touches the line with its corner. (See Figure 2.8.) Only when we refer to things like 'center', 'edge', and 'corner', can we give positional and dimensional specifications. Let's call things like 'center', 'edge', and 'corner' *arrangement references*.

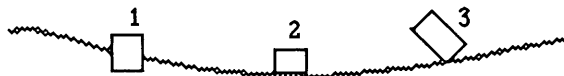


Figure 2.8.

An arrangement reference, or briefly reference, is any line, point, surface, or volume demarcated on a given element, or projected in the field of arrangement, to which the placement of element refers. For instance, the 'center' of a house can be a reference point to place the fireplace; its 'roof-ridge' can be a reference line to which the chimney abutts; its 'front-facade' can be a reference surface in which a Palladian widow is placed; and its

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'left-half' can be a reference volume in which a stair is contained. A stringing line of a string mentioned before is also a reference line. It not only serves to position elements that constitute a string, but also serves as a reference of a string when it is to be positioned as one piece relative to other strings.

We can mark as many references as we need in form-making; and we can define references in whatever way that fits our purposes of arrangement. References are usually defined according to conventions. For instance, an 'edge' of element usually means a segment of line where two surfaces meet. References can also be defined in an ad hoc manner. For instance, the 'center' of an element can simply be any given point of the element as long as it is defined as the center.

Each reference can be given a specific name. For instance, if we call the long-side edges of a rectangle 'side-edges'; and the short-side edges 'end-edges', then we can distinguish different references, such as End-edge-1, End-edge-2, Side-edge-1, Side-edge-2 etc., and therefore, End-edge and Side-edge become the family names of certain references. In the case that we want to refer only to one specific kind of reference, but not necessarily to any particular individual reference, these family names becomes useful. For instance, we may state: 'the end-edge of polygon-A should align with the side-edge of polygon-B', it does not matter which end-edge is aligned with which side-edge. If polygon-A has 3 end-edges and polygon-B has 4 side-edges, then there are 12 valid variants of this arrangement. Conceivably, if a very specific reference of each polygon is specified, then only one valid arrangement is possible. One family of references and another family of reference may also form a larger family, and have a large-family name. For instance, the end-edge family and the side-edge family can jointly make the 'edge' family. We may want to refer to this larger-family in some occasions that will give more arrangement variants. Thus, references can form a hierarchy of their own, and such an organization once established by convention is very useful for the sake of clarity and economy.

The 'sub-elements' of an element are often used as references in arrangement, such as the 'edges' of a polygon. But conceptually we should separate 'edges' as sub-elements of a polygon from edges as reference lines for arrangement. References are not physical elements, nor parts of elements as defined in a part-whole hierarchy, they exist only in a nominal sense. When we shift the reference of an element called 'central line', we do not

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change the element itself at all. References are meaningful only when they are defined, and used in the act of arrangement, while the sub-elements of an element can always be defined whenever we want to describe the element in terms of its constituent parts. When we say that building-A should be placed adjacent to building-B in such a way that their facades are aligned, 'facades' are references. When we say that building-A's facade should face building-B's facade, 'facades' are elements of arrangement, not references.

As we have shown, ordering moves like abutment, convergence, coincidence, conformation, and cross, are acts that establish relations between two or more strings. When these moves arrange strings, each string is arranged as a single piece, and the stringing line that underlies each string is referred as the reference. The spatial relations established by ordering moves between 'strings' can be conceived only in terms of the spatial relations established between 'stringing lines'. Without such reference lines, ordering moves are meaningless, and impossible. Therefore, we may conclude by saying that ordering moves must use references, and the references used must be lines, not points, nor surfaces, nor volumes. This is the character of ordering moves.

We shall also mention that some elements have certain inherent qualities that can be used as references. All long-narrow types of elements, such as walls and columns, have conspicuous 'directional qualities'. Directional qualities are also often used as references in arrangements. For instance, we often say that two walls are placed in the same direction, or in different direction, we implicitly refer to the *directionality* of wall. Many elements do not have natural directionality, such as a circle, or a square, unless it is given one by means of a reference line, which is always doable.

Now, not only can we 'converge' three sticks at one point by referring to their inherent directionalities, we can converge three globes at one point as well, if each globe is given a specified reference line to which the arrangement of convergence refers. For instance, we can postulate three stringing lines that converge, then place three globes on these lines. As long as reference lines of elements are specified, or reference lines for the placement of elements are postulated, ordering moves can apply to arrange elements by means of establishing relations among these reference lines.

An individual element, therefore, can always be considered as a string either in terms of its inherent directionality, or in terms of an imposed reference line. We can take one more step to make ordering moves applicable not only to strings, but also to individual

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elements regardless of their configurative characteristics. This is an important extension of the notion of ordering moves as general means for arrangement. The general formulations of ordering moves can be given as follows:

1). **String**: to project a line as a reference, and to place elements on this reference line.

2). **Alignment**: to place elements in such a way that every element has its reference on the same line.

3). **Coincidence**: to place elements in such a way that their reference lines contain one another.

4). **Convergence**: to place elements in such a way that all their reference lines connect to one same point.

5). **Abuttment**: to place elements in such a way that the end of one element's reference line connects the body of another element's reference line.

6). **Cross**⁵: to place elements in such a way that their reference lines pass through one another.

7). **Conformation**: to place two elements in such a way that their reference lines always remain parallel.

§ 2.3. Arranging Things

2.3-1. The Arranged and The Random

The system of moves is claimed to be sufficient for constructing any configuration, this makes it methodologically interesting. But, even if this claim is justified, are all configurations the results of applying the system of moves? Certainly not. As we know, random acts can also make configurations, but they do not qualified as arrangement moves. Again, given a configuration, how do we judge whether it is a result of arrangement moves, or merely a result of random acts? Even if we can observe the process of production, we still may not be able to determine whether the process incorporates random acts or not. We are in fact more interested in a different kind of

question: when do we call a form arranged? I shall, therefore, suggest a demarcation principle to distinguish an arranged configuration from a random configuration as this: any given configuration will be called an 'arrangement', (that is, seen as 'arranged'), if we can recognize arrangement moves in it. In other words, any given configuration, be it a pattern of pebbles on the beach, or a constellation of stars in the sky, will be conceived as arranged if we can 'reconstruct' it by, and only by, arrangement moves. Let's look at an example. The configuration given in Figure 2.9-1 is taken as a 'randomly' created configuration, but it can also be seen as the result of an arrangement as shown by Figure 2.9-2. That is, the picture can be construed by defining several reference lines, L1, L2,.. L5, to which positions of elements relative to one another refer. We readily see some moves employed to build relations among these elements: elements B, E, and D are aligned in L1 by their corners; C's side edge aligns with L2 which conforms to L1; A's corner, E's corner, and D's end-edge are aligned in L3; A's corner and E's edge are aligned in L4; B's side-edge aligns with C's end-edge in L5. From this perspective, we can see that this seemingly random configuration is arranged. The question as how this configuration is 'actually' constructed is no longer important, and is irrelevant to our argument that arrangement moves are sufficient to construct any configuration.

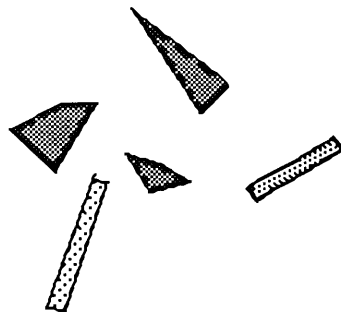


Figure 2.9-1.

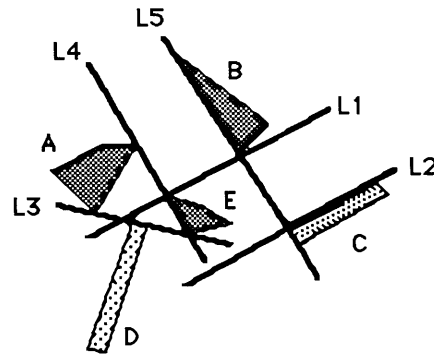


Figure 2.9-2.

2.3-2. Four Decisions to Describe an Arrangement

Now we face a practical question: how can the system of moves be used as a methodical construct that can do the job as claimed? To answer the question, let's give a synoptical

review of what we have said so far. First of all, any configuration can be seen as made of elements and relations among elements. This is a general definition of all systems, and not new here. Then, we propose a minimal set of acts called arrangement moves that can sufficiently establish all spatial relations that can possibly exist among elements. Although each arrangement move is specific in terms of the 'kind' of spatial relation it establishes, it is not 'unique' in the sense that, when applies to same elements, the result of a move can still admit large amount of possibilities. If we are going to reconstruct a given configuration exactly as it is, the methodical question would ask what other 'specifications' are needed in conjunction with arrangement moves.

Let's see an instance. Given a configuration made of two elements, A and B, that are separated by a certain distance (see Figure 2.10. below). We also observe that: A and B are aligned by centers, and A's <edge-3> conforms to B's <edge-1>. Now, what are 'minimal' set of specifications that can precisely construct the configuration as the given? Let's try a few constructions.

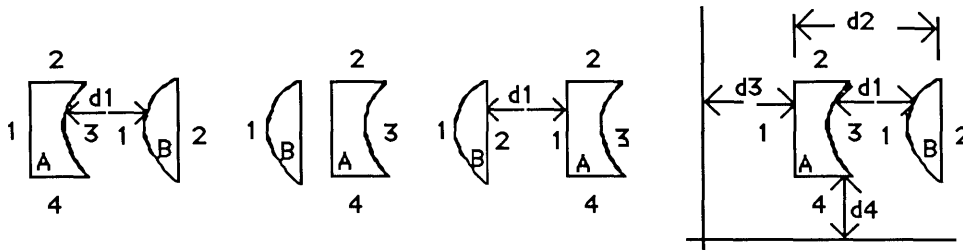


Figure 2.10-1 Figure 2.10-2 Figure 2.10-3

1. [A and B are separated, their centers are aligned, and A's <edge-3> conforms to B's <edge-1>]. The description is simple and clear, but admits almost an infinite number of valid interpretations. As we can see from Figure 2.10-1, although their spatial relations are correct, both the position of A to B (left or right of B), and the distance between them are unspecified.

2. [A and B are separated with distance d_1 , their centers are aligned, and A's <edge-3> conforms to B's <edge-1>]. Although this construction gives correct interpretation of both its relations and the distance, still we see that A and B may be in the wrong position relative to one another, as shown in Figure 2.10-2.

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3. [A is to the left of B, A's <edge-3> conforms to B's <edge-1>, and they are separated with distance d1]. Adding one more specification about the relative position of one to another (i.e. to the left of), we can now produce the exact configuration as the given one, see Figure 2.10-3. (As for the location of this configuration in the site, we can give more specifications concerning its relative positions as well as distances to the site, such as d3 and d4.)

These three interpretations can be seen as a series of steps building up constraints to exclude incorrect relations until they finally converges to a correct placement. The relative position, such as to-the-left-of or to-the-right-of, plays an important role in giving a sufficient instruction for reproduction. This is generally known as the *orientation* of an object, as demonstrated by the above example, and is a necessary consideration in arrangement. Although instruction [3] is a correct construction of the given configuration, it is not necessarily the only one. There may exist many equivalent instructions for producing a given configuration. But no matter how different these instructions are from one another, to give a correct construction, they must all consists of four decisions, as illustrated by the instruction [3]: element (i.e. A and B), relation (i.e. separation, and A's <edge-3> conforms to B's <edge-1>), orientation (i.e. to-the-left-of), and distance (i.e. d1). Therefore, any specific instruction of an arrangement must consist of four decisions:

1. Choosing elements.
2. Establishing relations by arrangement moves.
3. Determining orientation.
4. Specifying distance.

These four decisions together make a complete instruction for the act of arrangement, in which the relation specifies elements; the orientation specifies the relation; and the distance specifies the orientation. According to this 4-decision instruction we can give a general format for the expression of the act of arrangement:

Move-M (X<reference>, Y<reference>, orientation-R, distance-D)

This means that, given two elements X and Y, X's reference is in relation M to Y's reference so that X is at orientation R to Y, and with distance D to Y. For instance, the construction [3] mentioned above now can be expressed as this:

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Conform(A<edge-3>, B<edge-1>, left, d1)

Note that when apply the ordering moves like string, and alignment, the reference line to which elements are strung, or aligned, should also be specified. For instance:

Align<straight-line-L> (A's center, B's center, left, d1)

It is not necessary that every instruction for an act of arrangement should consists of all these four decisions. Specifications of elements and a move are always needed, but the orientation and the distance are sometimes optional, or sometimes can simply be dealt with by default. For instances:

Connect(A<edge-3>, B<edge-1>)

This move describes a configuration of two connected elements A and B, as shown in Figure 2.11-1 below. We find that both the specifications on orientation and distance are redundant. Because by the definition of 'connection', the distance between A and B must be zero; and by specifying A's edge-3 connects to B's edge-1, then, by default, A must be placed to the left of B.

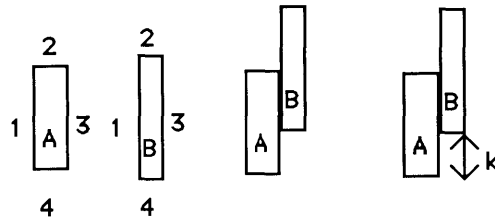


Figure 2.11-1 Figure 2.11-2

However, we shall also note that, in the same case, although we save the specifications of orientation and distance as such, we do need another set of specifications of orientation and distance in order to construct the exact arrangement as given. That is, A is below to B, and a distance should also be specified, say, between A's edge-4 and B's

edge-4, as illustrated in Figure 2.11-2 above. Therefore, a more rigid instruction will be this:

Connect(A<edge-3>, B<edge-1>, below, dis(A<edge-4>, B<edge-4>)=k)

2.3-3. Compound Moves

In some cases, to construct even a simple configuration we need more than one arrangement move. For instance, Figure 2.12-1 below presents an arrangement as result of following moves: A and B are placed in parallel to each other, B is to the right of A, their centers are aligned with a straight line perpendicular to their directionalities (call it 'perpen-line' for this case), and they are separated with a distance d. If we give this arrangement a name, say, Paral-centering, then it can be defined as a *compound move* combined by two basic moves:

Define: **Paral-centering**(A, B, left, d)

= Conform(A's directionality, B's directionality) and
Align<perpen-line>(A's center, B's center, left, d)

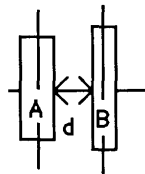


Figure 2.12-1

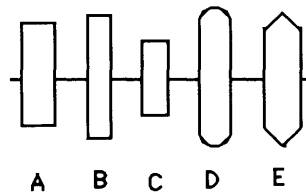


Figure 2.12-2

A compound move is an act of arrangement which establish more than one spatial relation at one time, (such as edge-conformed and center-aligned in the case of Paral-centering), thus it uses more than one basic arrangement move. A compound moves can also be defined by other compound moves. For instance, we can define a compound move called Centering-string by Paral-centering, which will generate the arrangement as shown in Figure 2.12-2.

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Define: **Centering-string**(A, B, C, D, E, left, d)

= Paral-centering(A, B, left, d) and
 Paral-centering(B, C, left, d) and
 Paral-centering(C, D, left, d) and
 Paral-centering(D, E, left, d)

Conceivably, we can change the value of distance to yield different arrangements, such as **Centering-string**(A, B, C, D, E, left, $d=0$) will produce the configuration as Figure 2.12-3; **Centering-string**(A, B, C, D, E, left, $d, 2d, 3d, 4d$) will produce the configuration as Figure 2.12-4.

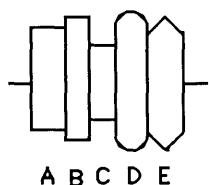


Figure 2.12-3

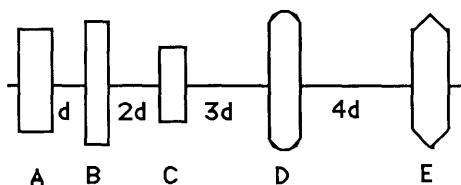


Figure 2.12-4

Variations can also be created by scrambling the sequence of elements, such as **Centering-string**(B, A, C, E, D, left, d) will produce configuration as Figure 2.12-5; and **Centering-string**(A, C, E, B, D, left, d) will produce configuration Figure 2.12-6.

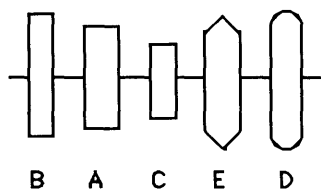


Figure 2.12-5

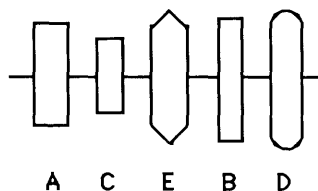
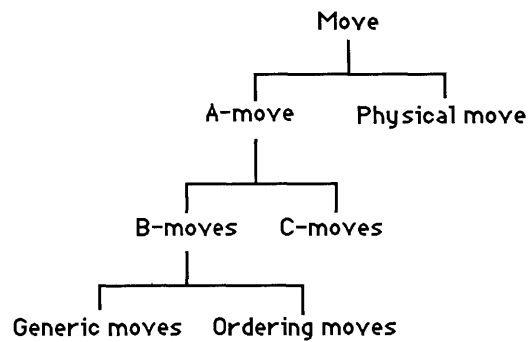


Figure 2.12-6

Once compound moves are made, they will facilitate as handy operations for arrangements. Although compound moves can be made out of other compound moves, all compound moves must ultimately be constructable by those basic moves defined in § 2.2.

Ways of Arrangement

Both compound moves and basic moves are arrangement moves, that is, the moves for arranging things, and they are distinguished from those physical moves as mentioned earlier. For convenience, I shall call arrangement moves 'A-moves', basic moves 'B-moves'; and compound moves 'C-moves'. A-move is a general term which consists both of B-moves and C-moves. The system of moves we introduced in this chapter can be summarized by the graph shown below.



(End of Chapter 2)

Footnotes to Chapter 2

1. I have a few more words about the issue of boundedness associated with our concept of form. In his lecture of 'Geometry and Experience' (Geometrie und Erfahrung) delivered to the Prussian Academy of Sciences in 1921, Albert Einstein proposed his cosmological model which is a finite-yet-unbounded three-dimensional universe. He succeeded in explaining, by analogy, how to visualize a two-dimensional continuum which is finite, but unbounded. The metaphorical model he employed is somewhat like this: there is a light on the surface of a sphere throwing shadow to a plane. The closer the point on the sphere is to the light, the more likely its shadow on the plane becomes infinitely great, which suggests that the plane is unbounded. Since the sphere is a finite space, and it has one-to-one correspondence to the plane, therefore, the finite-but-unbounded space is conceivable. However, understanding what the description of a form is meant does not necessarily make the 'visualization' of the form possible. Unbounded form is simply beyond human visualization, although may still within our imagination and comprehension. And such unbounded, metaphorical forms certainly are not the objects designers can use or make in their business of form-making.

2. There is a topological theorem called Jordan Curve Theorem that states: any simple closed curve in the plane divides the plane into exactly two regions, i.e. an inside and an outside. The proof of the theorem is based on another theorem about plane deformations: any simple closed curve in the plane can be reduced to a 'square' through the rubber sheet' kind of deformation. Since a square divides a plane into exactly an inside and an outside region, so does the closed curve as the result of its invariant property under the deformation. See *Turtle Geometry*, MIT Press, 1980, pp.182. This theorem does not apply when the boundaries of a form are not one simple curve, but consist of many closed curves.

3. See John N. Habraken, *Transformations of The Site*, Awater Press, Cambridge, Ma. 1982.

4. The structure of the flower is taken from Shaw's example in his article on 'Picture Grammars and Parsing', G.J. Dalenoart ed. *Process Models for Psychology*, Rotterdam University Press, 1973. The hierarchical analyses are mine.

5. The concept of 'cross' is intuitively understood here, some features are worth notice. The meaning of the word 'cross' given in the dictionary can be summerized as follows: (1) A position wherein one thing rests over another; to lay, or place one above the other (to cross over, to cross on), such as 'cross fingers'. (2) A device or emblem composed essentially of an upright bar traversed by a horizontal one; a transverse part of an object. More generally, extending, traversing from one side to the other; extending over; covering. (3) Running counter, mutually opposed; involving mutual interchange. These meanings indicate the morphological essence of the word 'cross' as the relation of two forms that are overlaped (lay over) in different directions (running counter) and cut through each other (traversing from one side to the other). In other words, to make a cross relation two conditions must be satisfied: (1) each form has overlapped and non-overlapped regions, and (2) the non-overlapped region has at least two discrete pieces. A relation fails to be called a cross in its ordinary sense if it fails to meet these two conditions. Two forms may overlap with each other but not necessarily cross each other if the non-overlapped part of any one form remains one continuous piece, which implies that it is not yet 'cut-through'. It is also interesting to note that we seldom find crossing relations in the natural world, 'cross' is perhaps uniquely belongs to the artificial world.

Chapter 3

Related Issues

Granted the system of arrangement moves, we now face some issues regarding the nature and the capacity of these moves. First, we shall ask, given an arrangement can we identify what moves are involved? This question is about the formation of the concept of arrangement move. Second, are those basic moves irreducible? The question is concerned with the possibility of the existence of even more basic acts out of which all arrangement moves can be constructed. Third, are these moves necessary? The question asks whether we can make formal arrangements as well without using these moves at all. Fourth, are basic moves complete? This is to ask whether there exist other moves that cannot be construed by the basic moves we formulate. Fifth, are these moves sufficient? That is, if these moves are sufficient, then they can construct any arrangement, or for any given arrangement it is always possible to find a set of arrangement moves to reconstruct it. This chapter is devoted to discussions of these questions.

§ 3.1. Identifiability

Are arrangement moves identifiable from a given configuration? This is not a technical question as how to observe moves in forms, but is an issue fundamentally relating to the formation of the concept of arrangement move. This issue is entangled with many other related concepts that must be clarified. First, we shall state that all arrangement moves are observable terms, not theoretical terms which are not directly observable¹. An arrangement move can be determined in observations because it is defined in terms of the resulting spatial relation that is empirically observable. Given a configuration made of two elements, A and B, such questions as: 'does A connect to B?', 'is A's edge aligned with B's edge?' can always have definite answers according to the definition of 'connection', and 'alignment'. There is no ambiguity at all.

Second, that arrangement moves are definable, and are defined in terms of observable spatial relations is one thing; the question as, given a configuration with identified elements, can we tell which move is applied is quite another thing. The former is a matter of definability; the latter identifiability. Identifiability is a concept nearly in reverse to definability. Definability is an issue pertaining to the empirical world, it will not become an issue in the 'rational' world (so to speak), such as Mathematics, in which we can always define what need to be defined regardless of their empirical relevance. Since arrangement moves operate in the empirical world, it is definable if it has empirical meaning in terms of observable measurements, and it is identifiable if it can be identified in the empirical world by the given measurements. But the identification as such always involves interpretations; and interpretations can hardly be unique. As non-theoretical terms, all arrangement moves must be directly identifiable from observations, but they are not necessarily 'uniquely' identifiable in the sense that a given relation may not exclusively admit only one single interpretation.

Take a configuration as shown in Figure 3.1 as an example, what is the move involved? We may say that A's edge-1 has been aligned with B's edge-1; it is equally valid to say that A's edge-2 conforms to B's edge-2. Here, we see that the selection of references to which spatial relations refer is important to interpretations, and we can not have any interpretation without such a selection. Since edge-1 and edge-2 are two different selections, we see different moves in the same given arrangement.

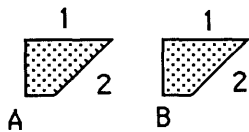


Fig. 3.1.

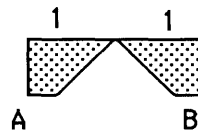


Fig. 3.2.

Usually when the selection of references is determined, the spatial relation can be uniquely identified, as the example shows above. However, there is an anomaly as shown in Figure 3.2. The state of spatial relation of this case can be identified as A's edge-1 is 'aligned' with B's edge-1; or as A's edge-1 'converges' to B's. edge-1. These two interpretations are not equivalent, although both select the same reference, i.e. edge-1. But

we shall notice that two different moves for the same configuration are possible only in the case that both moves are true. That is, A's edge-1 is aligned with B's edge-1, 'and' A's edge-1 also converges to B's edge-1. To mention only one relation constitutes only a partial description in this case. Once elements and their references of a given configuration are selected, arrangement moves can be identified unambiguously, despite that the given configuration may involve more than one move to produce it.

Third, it is important to note that spatial relations and acts that build the relations are not in one-to-one correspondance. A same spatial relation can be built by many different acts. Element A can connect to element B by the act of rotation, or by flipping, or first by rotation and then by flipping, or whatever. We do not care which action is actually taken to make A connect to B, and we will call any action the move of 'connection' as long as it establishes such a spatial relation. There is no need to identify which action is actually used as far as the spatial relation is concerned. To put it more generally: when we say move X, we mean 'to construct spatial relation X, or to build spatial relation X'. We are not particularly interested in the actual locomotive actions that are taken to construct, to build, to make the relation X. Despite the fact that 'move' is an action term, the emphasis is on the relation, not the action.

An arrangement move can be examined only from the result it produces. The 'move X' and the 'spatial relation X' are not distinguishable operationally², although distinguishable 'linguistically' in terms of the action word versus the non-action word. It is essential, therefore, to the concept of arrangement move that acts at the level of establishing spatial relations are always defined in terms of the spatial relations established, and this is exactly the way these moves are defined. Now we see the relation, and the distinction, between the physical 'act' and arrangement 'move': a move is a 'chunk' of physical acts that establish a particular spatial relation.

§ 3.2. Irreducibility

Are basic moves irreducible? If we consider B-moves as 'basic' operations for arranging

things, then, by definition, they can not be reduced to any other known operations. The question needs some qualifications. Let's begin with an observation on the similar issue in Physics. For centuries atoms were regarded as irreducible elements of matter. This claim was refuted in this century by the fact that scientists 'found' even 'smaller' particles by which atoms are constructed. Searching for irreducible, ultimate elements of matter, once constituted a reasonable goal for studies of Physics, it is no longer considered meaningful now. The assumption of the existence of the ultimate elements reflects the nature of our mental habits rather than the nature of reality.³ There is no such thing as the natural standard of irreducibility out there that eventually can be found as the 'true' criteria by which the basic can be distinguished from the compound. Both the basic and the compound are our own making.

It is perfectly alright to define a set of basic elements 'in the first place' without considering whether they are irreducible or not. Everything else in the system is thus considered compound if it can be constructed out of the basic elements. We will look for more fundamental entities only if, and usually when, the current base system fails to account for new facts found in the world under study. It is not unusual to see that one thing considered as the basic in one system becomes a compound product in another system⁴. Nothing is 'intrinsic' to the nature of a thing so that it should be viewed composite rather than elementary, or should be a primitive in lieu of being a derivable. It is 'extrinsic' criteria, such as simplicity, generality, that make theorists make such decisions. That is, for a system construction the selection of primitives largely depends on their accountability, which can be regarded as a matter of sufficiency or even efficiency, rather than irreducibility. The concept of natural irreducibility may be very misleading in this sense.

That basic moves are defined as the 'bases' for arrangements should be understood in the sense that all other identifiable arrangement moves can be replicated in these basic terms. If A, B, C, are defined to be the basic elements, the question of irreducibility is to ask whether they can sufficiently account for all other elements considered non-basic, but not to ask whether there could exist even more 'basic' elements A', B', C' to which A, B, C, can be reduced. Nevertheless, the notion of 'basic' is still a legitimate question. Given A, B, C, and D as elements of a system. Suppose we find that C, and D can be made out

of A, and B; but the reverse is not true. Then A and B are basic elements relative to C and D. Therefore, within a given system, and within the world in which the system emerges, one still can find some things which are more basic than others, both in the sense that the compound can be made by the basic, and in the sense that they all presuppose there is a common base.

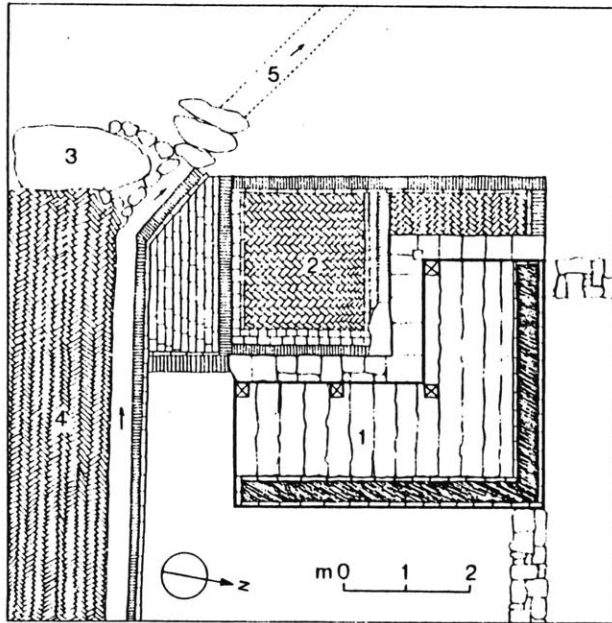


Figure 3.3.

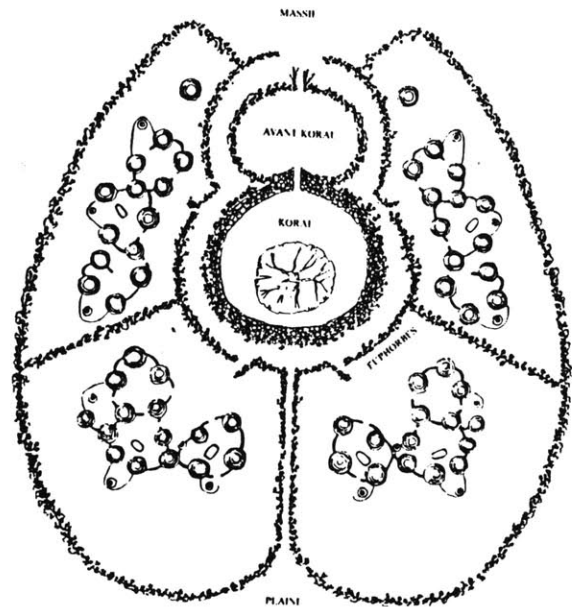


Figure 3.4.

Let's see an example from the ritual architecture in Panauti, Nepal. (See Figure 3.3.) The floor plan shows two L-shapes interlocked to achieve a perfect square-shape. Let's call the act that makes such a composition 'complement', which is an operation often used in many designs. Here we do not define 'complement' as a basic move, nor do we consider another widely-used arrangement 'symmetry' a basic move. In the case of Panauti floor plan, the 'complement' organization can be produced by 'aligning' the outer edges of two L-shaped forms, and also by 'conforming' their inner edges. A complementary arrangements can be produced by applying these two moves. The notion that two pieces are in complement with one another implicitly means the existence of an

configuration which is considered a complete 'whole', such as the 'square' in Panauti case. Therefore, the move complement can be seen as a reversed operation of dividing a whole into two parts, not unlike breaking a brick into two half-bricks. Two half-bricks can be recomposed into one piece of brick only if their 'breaking' edges conform to one another, and their adjacent edges are aligned. It will be difficult to explain the concept of complement without using the concepts of conformation and alignment. But alignment and conformation can be defined, and are defined, without using the concept of complement. In this sense, the complement act is a compound move, whereas alignment and conformation are basic.

Let's see the case of symmetrical arrangement. Symmetry is perhaps one of the primordial arrangements we can learn from our own body, and it is also a compound move. The primitive settlement of Dowayo in North Cameroun, Africa, (see Figure 3.4.) is a good example of symmetrical arrangement. The rule of symmetry dictates that, given a configuration, we can choose an axis against which the configuration on the one side can be mirrored onto the other side of the axis. Defining such a reference line as the mirroring axis is the central operation to make a symmetrical arrangement. Once an axis is defined, then, for any position on the one side of the axis, we can always find one, and only one, 'corresponding' position on the other side of axis. Two corresponding positions are equidistant to the axis, and the equidistance line is the reference line by which two corresponding positions are 'aligned'. Conceivably, the basic move for symmetrization is alignment, and usually a symmetrical arrangement consists of a number of such alignments.

It is also worth to remark that a symmetry arrangement emphasizes more on the equidistance of two corresponding positions rather than on the identity of two corresponding elements. For instance, statues on the two sides of a road in most geometrically designed Baroque gardens are seldom the same, but their positions are strictly determined with respect to the axis of the road. In the case of Dowayo settlement, the mirroring axis is quite obvious. Suppose one half (divided by the axis) of the settlement configuration is given, we can easily construct the whole picture by duplicating each identified element, and placing it at its corresponding position on the other side of axis so that it is aligned with the first one. Symmetrization thus can be construed as a

compound move made out of a series of basic moves 'alignment'. Again, we define alignment without using the concept of symmetrization, but we can hardly define symmetrization without alignment. In this sense, alignment is a basic move, and symmetrization is a compound move.

§ 3.3. Necessity

Here is a fundamental question: are basic arrangement moves necessary? The question can be understood to posit two conditions: 1). if they are necessary, then we can not do without them, and 2). if they are necessary, then no matter what operation systems are used for making all possible arrangements, they must manage to construct spatial relations specified by the basic arrangement moves.

Let's examine the first sense of the question. Arrangement moves are not indispensable if we can find another system which also provide operations that are sufficient to make all kinds of arrangement. Indeed, there exist a number of operational systems capable of making all kinds of configuration within the worlds they operate. For instance, the system of operations based on Cartesian geometry can produce any configuration. (And presumably there are many other kinds of geometry capable of doing that.) In such a system every single spatial point has an unique coordinate set, and therefore, simply by specifying coordinates, we can accurately produce any configuration. Since the Cartesian coordinating system provides an one-to-one mapping between configurations and coordinates so that all spatial relations are systematically transformed into dimensional relations. By virtue of this transformation every configuration has one unique numerical representation. However, doing an arrangement and making a configuration is not necessarily the same kind of act of design. We should bear in mind the conceptual distinction between an arrangement and a configuration by that a configuration is not necessarily a result of arrangement (as mentioned in §2.3-1). A Cartesian system and others like it are very powerful for calculations and productions, but very poor for designing in which the concern of positional relations precedes that of

absolute locations and dimensions. I would like to argue that, although configurations can be reproduced by the Cartesian system, they cannot be designed without using something like arrangement moves in the first place.

Let me give an example, then explain why. Given a complex picture, say, stars in the sky. Suppose that we recognize a figure in it, for instance, the constellation known as the Great Bear, can we explain what we see by means of coordinates? Admittedly, the coordinates can precisely locate every individual star within the Cartesian frame, but they do not tell us about the spatial relations among these stars. Therefore, we must employ other means than those coordinates to convey what we see as a figure. Generic arrangements do not help either, because generic relations are too general and too ubiquitous to distinguish one constellation from another. (In all constellations we always see such generic relations as connection, separation among stars.) Similarly, terms such as 'top', 'bottom', 'right', 'left', 'north', 'south', etc., are insufficient to establish the relations we want to convey, although they are necessary as directional references. Thus, we must define the configuration in other terms to make the figure conceivable.

When we explain what the Great Bear is like, I would guess that we project 'imaginary lines' to 'relate' otherwise randomly distributed sparkling dots. This is exactly the move we defined as 'string'. Presumably, seeing a constellation has to do with the act of 'stringing' the stars to make a figure, and then to test whether the figure can fit the reality or not. Although the string move is a necessary act to make figure-recognition possible, itself is not sufficient for such a recognition. Other acts are involved, such as selecting stars, and proposing a theme (the 'dipper' for example) as the figure to be recognized.

I like to make a more general statement about perceiving spatial relations. It is reasonable to believe that positional relations among elements are not conceivable until certain imaginary lines are postulated to 'register' one element to another with respect to these lines. For instance, we can 'read' a group of columns lined up to form a Greek peristyle, is presumably because we string them by imposing postulated lines that make us 'see' the peristyle form. Without stringing, there are only scattered columns here and there. In other words, unless we string elements with imaginary lines, no spatial relations exist except generic ones. This is the hypothesis I venture to claim concerning the

perception of spatial relations.⁵

The second sense of the necessity question is not about perception, but about generality. The basic arrangement moves are a system of operations for establishing spatial relations, or to put it another way, they represent a system of basic spatial relations. (Arrangement moves are defined only in terms of spatial relations, as discussed in § 3.1.) This system of spatial relations does not dictate which operation system that can implement it. Conceivably, there exist many 'implementation' systems that allow us to make spatial arrangements, such as different kinds of computational design systems. The second sense of the necessity question is understood as this: any implementation system capable of doing all possible arrangements must manage to do what the basic arrangement moves can do. More specifically, no matter what these implementation systems are, if they are sufficient for spatial arrangements, they must especially contain such operations of, or have capacity for, making strings and constructing relations among strings, other than many other formal manipulations. This claim can also be regarded as an empirical hypothesis. We can have confidence to make this statement only because it is assured by the answer to the first sense of the question. I do not submit this claim for further discussion, because, by its nature, it should be subject to empirical tests.

§ 3.4. Completeness

Are these basic arrangement moves complete? First of all we shall understand by what 'complete' means? If the question of completeness is viewed in the sense in which a hobby collector seeks to finish his collection, then we will never know the answer, and the question as such is rather uninteresting. The basic arrangement moves proposed here are not a random collection resulting from casual observations of designers' habitual acts, they are formulated as one system. Then the question is to ask: is this system in itself complete? Here, the meaning of completeness should be understood in the sense somewhat like this: if there are two parameters, and each parameter has two variables, then we will have only four variants that form a complete set, no more can be found within this

frame. That is, if the basic moves are complete, then no more other basic moves can be found within the system. When we talk about the issue of completeness, we must refer to a closed construct.

The system of basic arrangement moves consists of two sets: the generic moves and the ordering moves. Each set is constructed within its own frame of parameters. The generic moves are defined on the base of inside/outside distinction and the parameters of zero-distance and positive distance. As has been demonstrated in Chapter Two, § 2.2-1, five generic moves form a complete set within this frame. The ordering moves are defined within the world of string, i.e. an universe structured in terms of possible relations among lines. If we accept the partitioning structure (using parameters like intersections of the end, and the body of line) within which all possible relational situations are enumerated in the string world, we can conclude that ordering moves are also complete within the structure they are framed. Since demonstrations are already given in the last chapter, I need not repeat them here.

§ 3.5. Sufficiency

Are arrangement moves sufficient to make all kinds of arrangement? The fact that arrangement moves are identifiable, irreducible, necessary, and complete does not therefore grant that they are sufficient. Before trying to answer it, again, the question must first be understood. What does it mean that the set of moves are 'sufficient'? For what are they sufficient? In Logics, we learn that the 'sufficient conditions' for an argument mean a set of propositions that can facilitate to draw conclusions from premisses. Or, in order to 'infer' conclusions, such propositions are required as premisses, otherwise, it is 'insufficient' to conclude. For arrangement moves, the question of sufficiency can be put as this: to make all possible arrangements, what is the minimal set of moves we need? Once we have this minimal set, we 'need' no more, and therefore they are sufficient.

I would like to be more specific about what we mean by 'all possible arrangements'. An arrangement is an act, or a result of an act, of establishing spatial relations. In § 2.3-2,

Chapter Two, we mentioned that to give a sufficient description to an act of arrangement, four decisions have to be made: 1) selecting elements, 2) specifying relations, 3) determining orientations, and 4) determining distances. That is, to avoid being obscure in the instruction for implementing an act of formal arrangement, we must consciously make at least these four decisions. The sufficiency question confronted here, however, is not to ask whether these four decisions are enough to constitute a 'sufficient description', but to ask, given elements, orientations, and distances (which are always involved in any case), whether the basic arrangement moves are sufficient to account for all observable spatial relations or not.

The issue of completeness is concerned with the internal consistency of the move system, the issue of sufficiency is to test the system's power of performance. There is no single conclusive test that can do justice to the sufficiency claim, and it would be unwise to launch an exhaustive program to test all kinds of arrangements we can possibly imagine. On the other hand, we can at least briefly demonstrate what these basic arrangement moves can do in a few general categories of formal arrangement which cover some major areas of physical design: 1) creating spaces, 2). filling spaces, 3) extending, and 4) ending a deployment.

1). Creating spaces is an immediate objective of architectural design. It is also a generally important concern in many other physical designs. How is a space created? A simple piece of wall does not create a space, it merely indicates a boundary. Two pieces of discretely placed, but aligned walls do not create a space either. They simply make a longer boundary, or give a void gap in between. If two walls are 'dis-aligned', no matter in what manner, an elementary sense of space begins to emerge. We cannot make a simple space, such as a rectangular room, if four pieces of walls are all aligned on one line. Space is indicated, or defined, by an enclosure, and dis-alignment makes the formation of enclosure possible.⁶ Some enclosures give a strong sense of space, some do less, but the same principle of dis-alignment always applies. A box gives a rigidly defined simple space. If the box is 'destroyed', to use F. L. Wright's term, by recession, projection, sinking, raising, changing directions, etc., many additional spaces appear immediately. The acts of arrangements that create such spaces are basically the same, i.e. the simple act of dis-alignment.

2). But, inversely, how do we fill spaces? First we shall make a distinction that filling a space is not quite the same as covering a field, though results may look no different. Covering a blank field with chosen materials (like paving a floor) is one thing; to fill a given space defined within an existing configuration with materials is another thing. Speaking of operation, the latter kind of filling-in a space has to do with applying a configuration (or a single element) to the space, and it must conform to the boundary of the space. This certainly can be done by means of the conformation move.

3). Extension is basically a matter of continuously deploying elements. Repetitively using the string move and the alignment move (or other related compound moves), is perhaps the most effective way for such a deployment. Let's see a simple example in the case of making a grid pattern. First we can deploy the given elements to make a number of strings. We can then string these strings in a direction. All strings can be aligned by their centers, or by ends, or by any other specified reference. They can also be arranged to conform one another with certain distances. In the same manner, we can make another similar arrangement of strings, and then cross the two to achieve a grid pattern. Making a grid is operationally similar to extending a string; extending a continuous field is basically the same as making a grid.

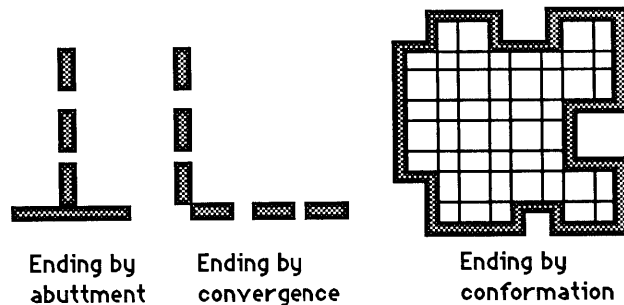


Figure 3.5.

4). How is a continuity to be ended? Ending is an important concern in design. If the column with the structure of base-shaft-capital looks more 'complete' than a bare shaft, it is perhaps due to the fact that the former is well 'ended'. To end a development by selecting different elements (or even colors) is a widely-used principle. Our concern here,

however, is to understand ending as a spatial arrangement, and to observe the spatial means of ending. Figure 3.5. shows a few simple examples of using abutment, and convergence as means for ending a directional development, and conformation as a way of ending a plane development. I believe that we will have no difficulty to find such examples in terms of real-world cases.

One more remark I like to make regarding the issue of sufficiency, which is: given a configuration and a set of moves, whether these moves are 'sufficient' or not to construct the given configuration partially depends on how we select elements, and of course selecting elements has to do with how to perceive a configuration as a single element. We know that a same configuration can be constructed differently if we select different elements, and therefore, different moves are in order. Let's use the simple A-B-A tartan grid as an example again. (See Figure 2. 2.) If we select the straight line for the element, the moves we need to construct the grid are 'conformation', and 'overlap'. If we select small cells like the AXA square, the BXB square, and the AXB rectangle for elements, then we need the moves 'string' and 'alignment'. The moves used in one case are insufficient for the other case simply because of different selections of elements.

The sufficiency question of basic arrangement moves is the question of the power of their performance. Nothing can be logically predetermined for this matter; nor is there such thing as the 'decisive' test to verify the claim. After all, the sufficiency test, by its nature, is a kind of question which can only be falsified, not verified.

(End of Chapter 3)

Footnotes to Chapter 3

1. The empirical meaning of theoretical term is given indirectly through the observables by which it is defined. Simon (1977) has argued and demonstrated that all theoretical terms are always eliminable under the conditions: if the theory is both 'finitely and irrevocably' testable. It follows, therefore, that theoretical terms are retained not for any 'intrinsic' reason of necessity; but for convenience, and perhaps convention. A theory is finitely and irrevocably testable, (i.e. FIT) according to Simon, means: 1) There exist at least a finite set of observations that disconfirm to the statements of the theory, if it is false. 2) Once the theory is disconfirmed by some set of observations, it is impossible to resume its validity by additional observations, that is, the test is irrevocable. See Herbert Simon, "Identifiability and The Status of theoretical Terms", in *Basic Problems in Methodology and Linguistics*, Dordrecht: D. Reidel Publishing, 1977, pp. 43-61. Simon and Groen also rendered a number of reasons to assert that all scientific theories are irrevocably testable theories, and they proved that if a theory is finitely and irrevocably testable, its theoretical terms are eliminable. See H. Simon, and G. Groen, "Ramsey Eliminability and the Testability of Scientific Theories", *British Journal of The Philosophy of Science* 24, pp. 367-380.

However, we shall not accept the notion of the finite-and-irrevocable testability uncritically. 1) It is not obvious at all which terms is theoretical, or is observable. If the observable means the measurable, then many methods of measurements are inextricably involved with some theoretical concepts. 2) Although FIT seem to be reasonable conditions for the testability of a theory, they are not necessarily the standards for 'evaluating' a scientific theory, and they might simplify the business of scientific enquiries. There are good reasons to believe that a theory is still valid and promising regardless of whether it can account for all available data, or even when it encounters with some negative evidence. To test the validity of a theory is a matter far more complex than Popperian philosophers would like to accept. For instance, Duhem proposed the thesis that: 'an experiment in Physics can never condemn an isolated hypothesis but only a whole theoretical group'. See Pierre Duhem, *Aim and structure of Physical Theory*, translated by P. Wiener, Princeton University, 1953. See Chapter VI: 'Physical Theory and Experiment'.

2. In his studies of the development of the child's imaginal representation, Piaget find that the figurative aspect (i.e. imaginal data) and the operational aspect (i.e. procedures) of thought are heterogeneous in general. However, one exception to this is that, in the realm of spatial manipulations, the image and the operation are homogeneous; they both are spatial. See Jean Piaget, 'The Spatial Image and Geometrical Intuition', in *Mental Imagery in The Child*, Basic Books, New York, 1971.

3. These reflections on modern Physics do not come from any particular single source, but I shall mention two articles that are close to the view presented here: John Archibald Wheeler, 'Bohr, Einstein, and the Strange Lesson of the Quantum', and Eugene Wigner, 'The Limitations of the Validity of Present-day Physics'. Both authors are renowned scientists. See Richard Q. Elvee, ed. *Mind in Nature*, Nobel Conference XVII, Harper & Row: San Francisco, 1982.

4. For instance, Hilbert's reformulation of Euclid's geometry axioms is the case. David Hilbert, in 1899, reformulated Euclid's axiom system. In his *Grundlagen der Geometrie*, for example, the notion of 'order' was defined by a set of axioms, which was never mentioned, but implicitly used to prove the existence of parallel lines in Euclid's *Elements*. In Euclid's formulation, 'order' is an undefined primitive, but it is a defined term in Hilbert's system.

5. Inadvertently, our necessity arguments extend to speculations on human perception. For curiosity, we may ask: is the string move learned from our spatial knowledge, such as Geometry, or is the string move a

Related Issues

primitive operation by which we begin to develop spatial knowledge? To this question, I suspect that the most 'primitive' ability of seeing meaningfully is to make a simple string. 'String' is perhaps the 'first' capacity we acquire to build 'formal order'. This speculation may be just a personal opinion, but there are a few clues. The once well-received empiricist hypothesis, which asserts that very young infants are anatomically incapable of seeing any form and pattern other than just blobs of light and dark, has been refuted by many experiments which, on the other hand, strongly support the assumption that human infants have some innate capability for perception. Experiments with dark-reared kittens indicate that depth perception matures independently of trial and error learning (See E. Gibson and R. Walk, 1960, "The Visual Cliff", in *Perception: Mechanisms and Models*, Readings from Scientific American, San Francisco: W. Freeman Company, 1972. Also see E. Hess, 1956, "Space Perception in The Chick", in *Perception: Mechanisms and Models*, Readings from Scientific American, San Francisco: W. Freeman Company, 1972.) Newly hatched chicks appear to have innate ability to perceive forms that are likely to be edible. And human infants, even from one to fourteen days old, appear to have particular interests to more complex patterns, facial patterns, and objects with depth (See Robert Fantz, 1961, "The Origin of Form Perception", in *Perception: Mechanisms and Models*, Readings from Scientific American, San Francisco: W. Freeman Company, 1972.)

I haven't found any experiment that is designed to extend these presumably innate capacities to include the ability of spatial ordering, which is an ability different from those needed for pattern perception and spatial-depth discrimination. But there are no lack of relevant examples. Experiments with 8 to 16 weeks old infants have shown that infants will continue to track the path of the moving object after it was actually stopped (Bower, 1971). I suspect that such path-tracking initiates the operation of postulating lines onto the visual world. (See T. Bower, 1971, "The Object in the World of the Infant", in *Recent Progress in Perception*, Readings from Scientific American, San Francisco: W. Freeman Company, 1976.)

In this experiment the fact that infants continued to look for the moving object after it stopped can be explained by three alternative hypothesis. 1). infants are unable to arrest head movement. This has been refuted by the circular tracking trial. 2). Bower interpretes the results as that the 8- to 16-week-old infants are not yet able to identify the stationary object and the moving object as actually being the same one. Therefore, infants kept on looking for the moving object. This is a possible interpretation for the behavior of continuation of path-tracking, but also a somewhat empty conclusion. 3). I am inclined to think that infants have 'natural' assumptions about the world by which a moving object will continue to move, while a stationary object will always remain stationary. When the moving object stops, (this fact being aversive to the 'natural' assumption), the infant continues tracking the path, and seems that she still expects the object keep moving in accord with her inborn view of the world. When infants grow, they have to adjust such views to conform to external experiences. In the same article, Bower also mentioned another experiment that is helpful to explain our hypothesis. The experiment measures the degree of surprise in infants, from 20 to 100 days old, at the reappearance or disappearance of an object after it has been covered by a moving screen for various periods of time. It shows that all age groups are surprised when the object does not reappear when the cover screen has been moved. This vindicates that the infant does have some kind of inborn assumption about the world on the one hand; it also shows, on the other hand, that infants have acquired perceptual capacity to know spatial concepts like 'front' and 'behind'. In the path-tracking experiment, leaving aside what the result means, the simple fact that infants can track different paths indicates that they know how to postulate 'lines' to test out their assumptions about the visual world.

Another experiment demonstrate that 8-week-old infants can actually postulate lines to complete broken triangles (See T. Bower, 1966, "The Visual World of Infants", in *Perception: Mechanisms and Models*, Readings from Scientific American, San Francisco: W. Freeman Company, 1972.) The completion experiment uses a wire triangle with a bar placed across it, thus interrupting the shape, as the conditioned stimulus. Four different triangular wires are used as test stimuli of which one is a complete

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triangle. The results show that this complete triangle elicited much more responses than others. This means, seems to me, that the line will continue its course to cross any interruption is an inbred assumptions about the world. Interpretations vary regarding these experiments, but It seems to strongly suggest that the spatial concept of 'line' has been acquired, and is used to anticipate, and to construct, 'orders' in the visual world which would otherwise be a chaotic mass of sensations. All these experiments do not provide conclusive evidence to resolve the nativism-empiricism controversy one way or another. Nevertheless, they provide good reasons to believe that it is 'innately' necessary to employ 'string' moves for establishing spatial orders. It seems that seeing spatial relation by postulating simple lines to order visual data, must be 'learned' at very early stage of perception development, if it is not an innate capacity in the first place.

6. We shall also note that creating a space does not mean the same as enlarging a space. A room is enlarged to have more space; a row of columns are removed to make the corridor wider; a big tree is dislodged to make the garden more spacious. In cases like these, we are primarily concerned with how to increase the sizes of original spaces which already have been created. Spaces are increased because solid things are moved out, and we gain the spatial volumes they occupied. But acts like making an opening in a wall, and using fences to claim a territory are indeed space-making. In these cases spaces have to be framed by their boundaries, and inevitably we have to consider how these boundaries shall be constructed. Obviously, if all edges are aligned, we can never make a space. The sense of space begin to appear only when some edges are not aligned. It seems not incorrect to say that operationally dis-alignment makes space-making possible.

Chapter 4

Form Reconstruction

Design gives specifications, and each specification is a description of form. Physical design can be seen as a continuous process of describing form, from one state to another state, to convey message to all parties involved. Among many ways of describing forms, reconstruction is the most useful one in design. Among many possible methods of reconstruction, two are generally applied: one specifies spatial relations among selected elements regardless of what procedure is taken to achieve them, call it relational description; the other specifies the sequence of acts to be followed to arrive at the desired result, call it procedural description. Again, for these two methods of reconstruction, different implementation systems, such as by means of coordinates, by physical actions, or by arrangement moves, etc., can be used. Some may be appropriate for one method, but awkward for another. The main purpose of this chapter is to demonstrate that arrangement moves are appropriate for both methods of reconstruction.

After discussing the idea of reconstruction as a way of describing forms, we will first present an example to show that a form, as a result of design, can be reconstructed by relational description in terms of arrangement moves. Since not all forms are results of design, we will then present another example which shows that a form can also be a product of a procedure of actions. Still such a 'procedural product' can be reconstructed by relational description in terms of arrangement moves. Third, we will demonstrate that, if we can figure out a procedure which can reconstruct a given procedural product, then arrangement moves maintain to be useful to describe such a procedure.

§ 4.1. Reconstruction as Description

Given a configuration, figuring out a description of it is one thing. Given a description, to generate the configuration it describes is another thing. The differences between

formulation of a description and generation of a configuration will be examined below; and will be used to separate the notion of construction from the notion of description.

1). A description presupposes that the image of the configuration it describes must exist beforehand, but a construction does not necessarily need a pre-existing image. A description must, by definition, always be a 'description-of' something. It would be meaningless to say that 'this drawing is a description' unless we know what the drawing intends to describe. The object to be described must precede its description. But we can still construct something without knowing 'what' is under construction. This is the case, for instance, when we implement a given set of instructions to produce a configuration, but are not informed with the result to be produced, because sometimes not only the actor, but also the instructor may not know the final result.¹ It is also the case when we are searching for, say, an appealing visual composition without committing to any known precedent. In all cases an object must be made in the first place before it can be described.

2). Describing a thing can simply by referring to another thing, such as 'Mary looks exactly like her mother'. This is perfectly legitimate as long as the reference is familiar to the audience, and it can be very powerful and elegant in some cases. In such a description, no elements nor operations are specified. However, elements and operations are not dispensable for a construction. It is simply impossible to make things without elements; and elements will not automatically constitute a thing by themselves, operations must be applied.

3). But to reconstruct a given form is a description. Construction can always be a way of description. To describe a 'circle', for instance, we can offer an instruction of actions which says: take a straight rope, fix one end and rotate the other end around the fixed point. Such a description is procedural, i.e. when to do what. Usually the sequence of action is consequential for a procedural description.

4). For any given description of a configuration, we may not always know an explicit way to make a reconstruction from it. For instance, it will be the case when using such a referential description as 'X is like Y' mentioned under the point 2). It will also be the case when giving a mathematical description of a shape, for another instance, we may have difficulty to find a way to construct it accurately with limited tools, say, by compasses and the ruler. Knowing the property of a geometry does not make the

construction of this geometry apparent. It is not without reasons that 'geometrical construction' appeared to be the topic for the first book in many Renaissance and Neo-classical Architectural treatises, such as Serlio's five books on Architecture.² In the recent development of Turtle Geometry,³ geometrical construction has now become a subject-matter of Mathematics in parallel to its 'descriptive' counterpart. The turtle is a little creature who can respond to a few simple 'action' commands, such as forward, backward, turn right, turn left, etc. When we give instructions in terms of the simple things this little builder knows, it can follow and can assist to make complex configurations, such instructions must be procedural.

5). But given a configuration itself, not a description of it, it is always possible that we can find a valid way to reconstruct it. In describing the genesis process of life forms, two perceptual models are observed by molecular biologists.⁴ One is called the Mont-Saint-Michel model; the other called the Mountain-stream model. The former is a way of labeling all cells with molecular markers according to which each cell will find its match-cells to be assembled. This is similar to the way the great abbey of Mont Saint Michel was built. Stones were shaped and marked on the mainland and reconstructed on the island according to the prescribed plan. The Mountain-stream model uses an analogy to the shape of a stream of water which is determined collectively by its simple initial conditions as well as by complex interactions with environmental constraints in the course as it moves down a mountainside. Thus, this model represents a kind of descriptions formulated by 'if-then' rules, which specify many situations on the one hand, and some corresponding 'moves' on the other. This Mountain-stream model is more dynamic, but also sequential in the sense that the final shape of the stream is to a large extent affected by its encounters with various situations in its downward path. That is, if the sequences of encounters change, its overall shape will be different.

We may, therefore, call the Mont-Saint-Michel approach a *relational description*, and the Mountain-stream approach a *procedural description*. As already mentioned, these two modes of description are frequently used in the business of describing forms, and in making forms as well.⁵

For these two approaches, some implementation systems, such as, among others, Cartesian coordinates, turtle actions, and arrangement moves, are available. For instance,

we can use Cartesian system to express a relational description of a given form; we can also provide a procedural description in terms of turtle's actions. We should notice, however, that not every implementation system is suitable for both approaches. Cartesian system, as we know, can hardly be useful in a procedural description; while turtle's actions are useless to specify spatial relations. On the other side, it is the major purpose of this chapter to show that arrangement moves are appropriate for both relational and procedural descriptions.

§ 4.2. The Relational

I would like first to present an example that stands for the reconstruction of form according to a relational description. The example is taken from an experiment about the representation of a spatial image as given in the floor plan of a building.⁶ The experiment asks students first to memorize one of several given floorplans, and then to recall the floorplan chosen to remember by drawing it. They are asked to present their reconstructions in step-by-step fashion. Let's see only one exercise more closely. Student-J chooses to reconstruct the floorplan of the Mosque of al-Juyushi in Cairo, Egypt. (See Figure 4.1. below).

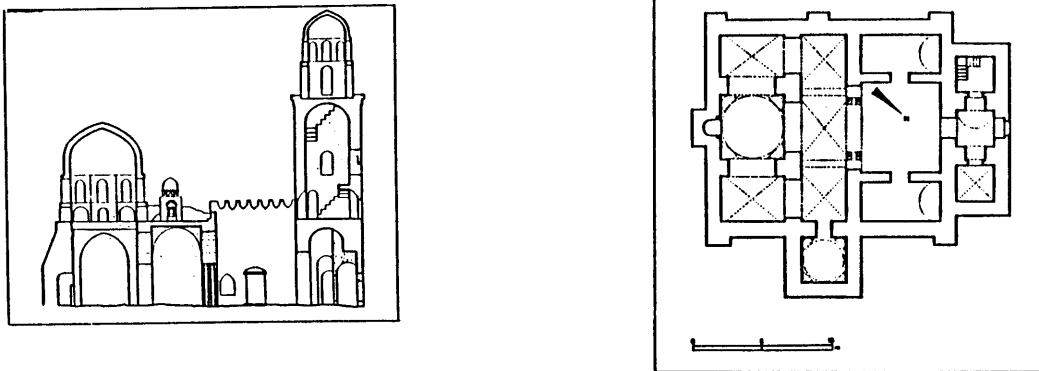


Figure 4.1.

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According to his presentation, the floor plan of Mosque al-Juyushi is reconstructed as follows:

1. 'I remember the plan as essentially square, with a vestibule "addition" and two apses-like protrusions in the wall, one at the front and one at a side. I know the walls are essentially of equal thickness everywhere, that the walls on the outside have pilaster-like protrusions on the four corners of the large square... that the vestibule is composed of three square rooms....that within the large square there are rectangular rooms with the same direction -- always counter to the direction of the vestibule... walls define the two rear rectangles, and columns and pilasters define the forward two...'

2. '...establishing the essential outline with two apses and vestibule...am aware of the continuous wall thickness, but must stop here because the vestibule is more complex...'

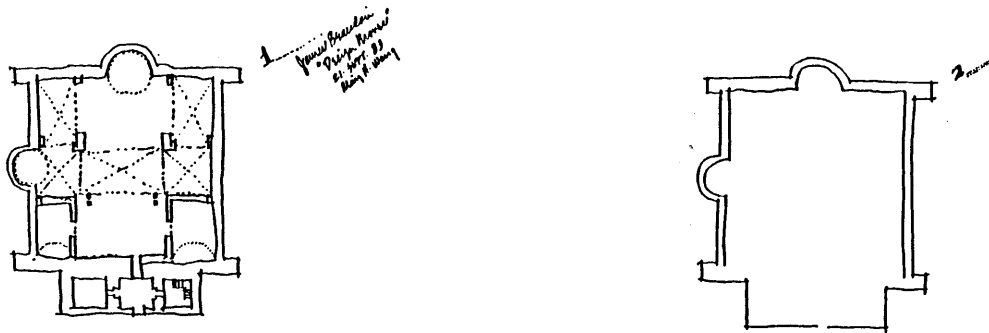


Figure 4.2.

3. '...attempting to reconstruct the vestibule, recalling that the center square the access control for the main part of the mosque, and for the two small square rooms...'

4. '...establishing the essential outline of the four corner rectangular rooms, as I see this move as being a "key" to establishing the essential proportions of the other divisioning...'



Figure 4.3.

Form Reconstruction

5. '... establishing more precisely in the floorplan the nature of the closures for each of the four rectangular rooms...'

6. '... drawing-in the two spatial columns near the two lower rectangular rooms, indicating the stairs, and indicating all the ceiling vaulting as I remember it...'



Figure 4.4.

From his representations, we see that student-J decomposed the plan basically into four parts:

1. A large square enclosure.
2. A rectangular vestibule.
3. Four rectangular rooms.
4. A long narrow three-bay arcade.

Each part also has a few associated elements, or can be decomposed into smaller elements. As we can see, the large square enclosure has two apse-like semi-circles sitting at the centers of two adjacent edges; the vestibule is composed of three small square rooms; of the four rectangular rooms, two are enclosed by walls, the other two are defined by arches. Structuring elements into a part-whole hierarchy conceivably helps to reduce the complexity of reconstruction.

Let's first define those directional relations such as 'parallel to', 'perpendicular to', and 'adjacent to', in terms of basic arrangement moves. To conform to our ordinary language, these terms are more clearer than those moves that define them.

- 1). 'A is *perpendicular to* B': A's directional line **abuts** to B's directional line at 90

Form Reconstruction

degree. It is a special case of the statement that A and B are *in different direction*.

2). 'A is *parallel to B*': A's directional line **conforms** to B's directional line. It is also known as A and B are *in same direction*.

3). 'A is *adjacent to B*': A **connects** to B; or A **overlaps** with B; or A **separates** from B, but no space other than an empty void or 'circulation path' exists between A and B.

Now, I shall represent the spatial relations among these four parts by means of Arrangement moves in accordance with Student-J's verbal descriptions.

a). The long-side of the rectangular vestibule *connects* to the large square enclosure at the edge without apse, and they are *aligned* by center.

b). Four rectangular rooms *insert-connect* to the larger square enclosure at four corners in such a way that their directionalities are *perpendicular to* the direction of the vestibule, and the two walled rooms are *adjacent* to the vestibule.

c). The three-bay arcade *insert-connects* to the large enclosure in such a way that it is in the direction *parallel* to the vestibule, and its directional center line is *coincident* with one of the larger enclosure's center lines.

d). Two column-pairs are *inserted* in the large enclosure (in the space defined by the two lower corner rooms, the vestibule, and the 3-bay arcade), and to *align* with the two lower corner rooms' short-edges that are *adjacent* to the arcade.

These parts and their spatial relations can be conceived as the set of statements representing the mosque student-J knows. Since there is no time limitation, subjects can take as much time as they need, and can, in principle, remember as many details as they want. How well the subject can remember the given floorplan correctly is not the point here, the point is to see how a complex form is reconstructed as an arrangement of spatial relations among the pieces they selected. We see that this arrangement can be constructed by means of Arrangement moves. It is not surprising that the subject could not remember many positional details, nor the exact shape of element. He selected what he understood to be the essence of the building.

From the example of reconstructing the floorplan of the mosque, we see a few interesting features:

Form Reconstruction

1). the configuration to be reconstructed is determined by its relational specifications among elements, and is independent to the procedure of reconstruction. In the case of Student-J's reconstruction we can use the same elements and follow the same relations, but proceed in a different sequence. For instance, first we may locate the three-bay arcade, then connect four rectangular rooms to it, and then include them with the large square enclosure, and finally attach the vestibule. Conceivably, no matter what construction sequences are taken, it will affect only the building schedule, not the built form. Nevertheless, the sequence of moves may be important as a matter of preference.

2). As mentioned earlier that a same configuration can be constructed by different selections of elements and relations, here we may see such an example. In the reconstruction of the mosque Student-B chose to begin with the distribution of interior spaces rather than the exterior outline. (See Figure 4.5. below). He stated: '..remembered as a number of spaces and their interrelationship, with a nine-space grid as the starting point. Note: nine space geometry is common to Islamic architecture.' Apparently, Student-B chose spatial elements rather than material elements because, to him, the internal spatial organization is more essential and important than the external appearance to a mosque.

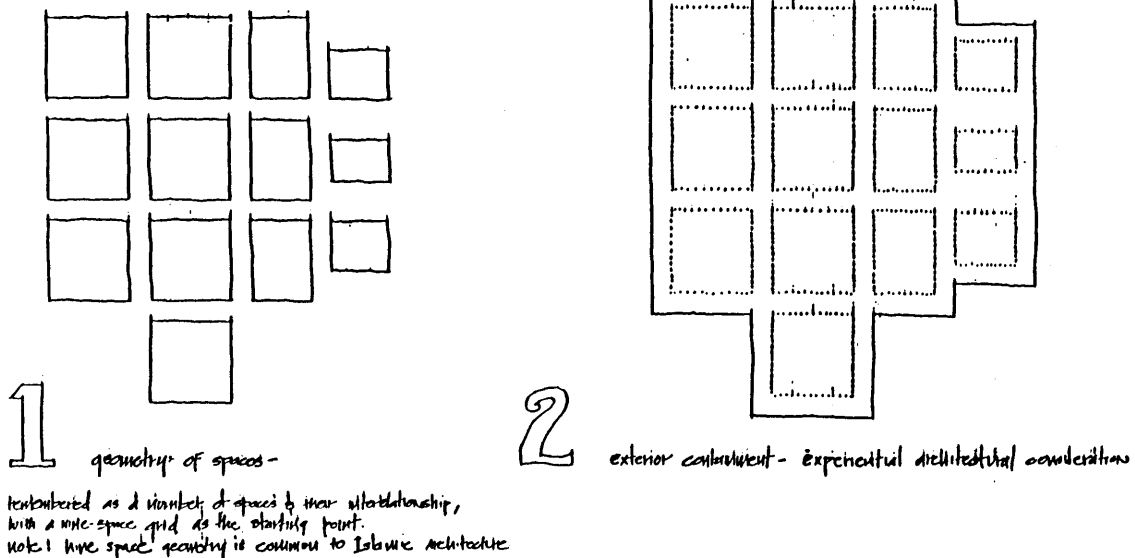


Figure 4.5.

When adopting relational description to reconstruct a given form, again, selection of element is an important act. Once the elements are determined, their relations more rather than less become definite. In all cases, arrangement moves are undoubtedly useful for specifying these relations.

§ 4.3. The Procedural

Selecting elements and specifying their relations are indispensable in design. There exist many forms that are not necessarily designed as such, they are produced simply by following certain sequences of actions. Let's see an example (see Figure 4.6 below). It is a configuration constructed by an insect builder.⁷ First, let me describe how this form is build according to scientific observations. It is a net constructed by the moth larva, an insect known as a silkworm. They belong to the species of filter feeders that hatch in aquatic environments like lakes, streams. These insects, with in-built knowledge of architecture, spin threads to weave fine-meshes to trap food as their means of survival. The larva will first build a shelter with materials binded by its own silk, to face up-stream with an open end. At this open end it then builds the structure member of the net: a hooplike oval frame, and followed by the net-knitting.

The actual process it takes to build the net is based on an procedural rule, observed by the German entomologist Werner Sattler, which is strikingly simple. As long as it knows this simple rule, the insect can build its architecture easily without necessarily having any image-plan beforehand. The rule says nothing more than a repetition of the figure-eight movement, i.e. the larva spins its threads following the path somewhat like our hand movement in writing the figure '8' (see Figure 4.6. below).

More descriptively, the first thread is made close to an edge of the frame. The larva then continues to move to an approximately similar position on the other side of the frame relative to the starting point, and spins the second strand to intersect with the first near the end. It then moves back to the side near the first position to complete the first figure-eight

Form Reconstruction

movement, and to begin the second figure-eight movement, and so on. To complete the net, observed by Werner Sattler, it takes the larva about 'eight' minutes, an interesting coincidence.

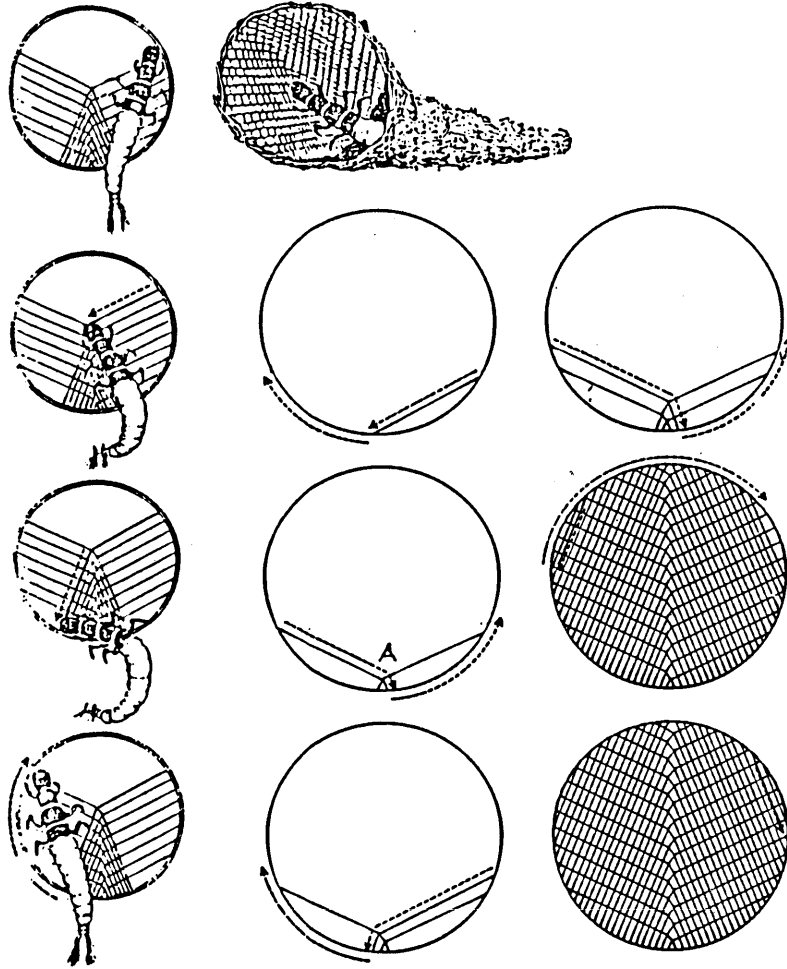


Figure 4.6.

Given the completed net as shown in Figure 4.6, how do we describe it? Although the net is built, as observed, by a sequence of actions, we can still describe it in terms of relational description, with or without knowing the actual way the larva builds it. I will demonstrate below (in Figure 4.7.) that arrangement moves are appropriate means for such an undertaking.

Form Reconstruction

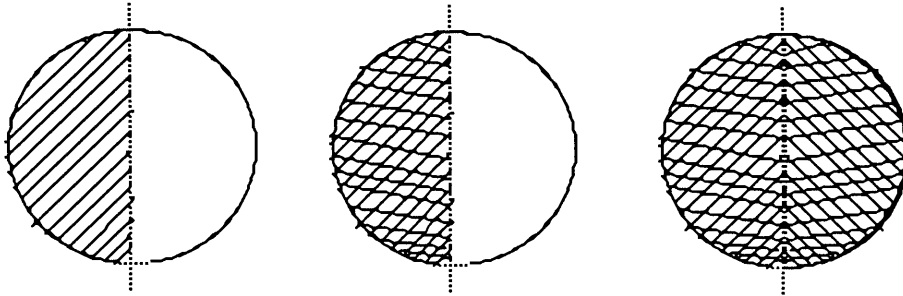


Figure 4.7.

1). First, choose the simple straight line as the basic element, and divide the frame into two halves, H_1 and H_2 , by a central line C .

2). Make a string of parallel lines of different length, L_1, L_2, \dots, L_n , (i.e. lines conform one another) in one direction, say, from the top down, with a constant distance d . Call this string S_1 , which can be represented in terms of arrangement moves as this:

$$S_1 = \text{String}(L_1, L_2, \dots, L_n, \text{top}, d) \text{ and} \\ \text{Conform}(L_1, L_2, \dots, L_n).$$

3). Place S_1 in H_1 such that it *insert-connects* to the edge of the frame on the one side, and to the central dividing line with a particular angle Ω on the other side. The connecting points to the central dividing line C are labeled $P_i, i = 1, 2, \dots, n$. That is:

$$\text{Insert-connect}(S_1, H_1) \text{ and} \\ \text{Abutt}(L_1, C\langle P_1 \rangle, \Omega) \text{ and} \\ \text{Abutt}(L_2, C\langle P_2 \rangle, \Omega) \text{ and} \\ \dots \\ \text{Abutt}(L_n, C\langle P_n \rangle, \Omega)$$

4). Make another string of parallel lines S_2 , then place it also in H_1 so that it will

Form Reconstruction

cross S_1 , and each of its lines abutts to the central line C at the same set of points P_i with the angle Δ to the central line. That is:

**$S_2 = \text{String}(k_1, k_2, \dots, k_n, \text{top}, d)$ and
 $\text{Conform}(k_1, k_2, \dots, k_n)$.**

Then,

Insert-connect (S_2, H_1) and

Cross (S_2, S_1) and

Abutt($k_1, C\langle P_1 \rangle, \Delta$) and

Abutt($k_2, C\langle P_2 \rangle, \Delta$) and.....

Abutt($k_n, C\langle P_n \rangle, \Delta$)

5). On the H_2 side, we shall repeat what we do on the H_1 side to make a symmetrical arrangement with respect to the central dividing line. That is to say, there are two sets of intersecting lines, S_3 and S_4 , both *insert-connect* to the H_2 side of the frame, and to the central line at point P_i with angle Ω and Δ respectively.

Now, the larva's net is reconstructed.

Given a net as such, however, if we discover a simple procedure of actions that can produce it, then we can offer a more elegant description than the relational one just presented above. Now, given the procedural description of the net as the entomologist observed, can arrangement moves be useful to present it? I shall give a demonstration below (see Figure 4.8.) to show that the answer is positive. According to the procedural description, it is quite obvious that as long as we can built up the configuration of the first 'figure-8' movement, the rest actions can all be described on this base.

Before we start, a few technical notes are in order for this exercise.

1). All ordering moves are based on underlying lines that are not necessarily visible. If an ordering move is superscribed with an asterix symbol, it means that the move will produce a 'visible' line, which is the element for constructing the net. For instance, the move $\text{String}^*(P_1, P_2)$ means making a straight line segment from point P_1 toward P_2 that

physically connects the two points. (We know there is only one such a straight line for any two given points in Euclidean space.)

2). Instead of giving two points, if we know the starting point P_1 , the direction of angle Ω , and the distance d , we can also construct such a line segment as $\text{String}^*(P_1, \Omega, d)$.

3). To produce a visible line S , we can also start with a selected point, and proceed in a chosen direction until it is stopped by abutting to an object O . In this case we can express in this way:

let $S = \text{String}^*(P_1, \Omega)$ and
 $\text{Abutt}^*(S, O)$
 or, $\text{Abutt}^*(\text{String}^*(P_1, \Omega), O)$

4). On the other hand, the move $\text{String}\langle L \rangle (P_1, P_2)$ means that we can find a point P_1 such that P_1 and P_2 are on the same stringing line L , which could be a real line, or just an imaginary line. By the same token, the move $\text{Align}\langle L \rangle (P_1, P_2)$ means the two points P_1 and P_2 are aligned with respect to the reference line L .

Now we are ready to begin our procedural description of the larva's net in terms of arrangement moves.

1). Make directional distinctions of right and left, top and down, in the given frame F with respect to two reference lines X and Y across the center of the frame. X and Y are perpendicular to each other.

2) Let the starting point be on the right side of the frame, and name it $P_1\langle r \rangle$. From this point, let's choose a direction defined by the angle Ω , and then move straight until it hits the frame F . The line thus constructed is named $S_1\langle r \rangle$. That is:

$S_1\langle r \rangle = \text{String}^*(P_1\langle r \rangle, \text{right-}\Omega)$ and
 $\text{Abutt}^*(S_1\langle r \rangle, F)$

Form Reconstruction

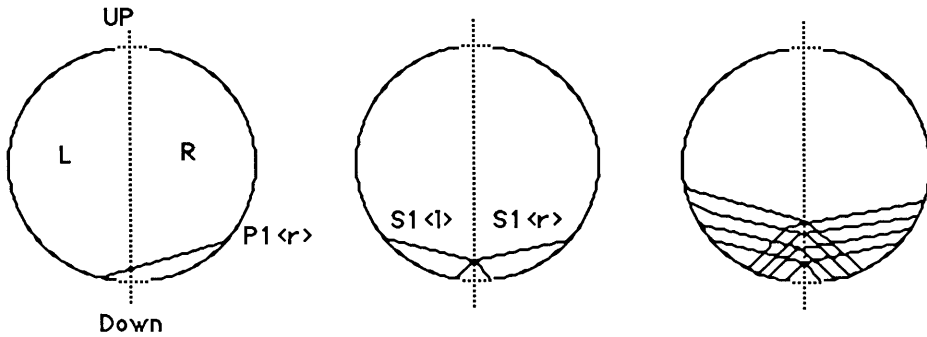


Figure 4.8.

3). Go to the left side, find the point $P_1\langle l \rangle$, which is a point symmetrically corresponding to $P_1\langle r \rangle$, in the sense that they are aligned with respect to a reference line L_1 which is perpendicular to the central vertical line Y (or parallel to the central horizontal line X). From $P_1\langle l \rangle$ make another string $S_1\langle l \rangle$ in symmetry to $S_1\langle r \rangle$ with respect to Y , and cross $S_1\langle r \rangle$ at point Y_1 . That is, this step consists of two sub-steps. First, find the point $P_1\langle l \rangle$:

Align $\langle L_1 \rangle$ **(** $P_1\langle l \rangle$ **,** $P_1\langle r \rangle$ **)** and **Conform** $\langle L_1, X \rangle$

Then, make the string $S_1\langle l \rangle$:

$S_1\langle l \rangle =$ String* $\langle P_1\langle l \rangle, \text{left-}\Omega \rangle$ and
Abutt* $\langle S_1\langle l \rangle, F \rangle$ and
Cross $\langle S_1\langle l \rangle, S_1\langle r \rangle, Y_1 \rangle$

4). Now we have constructed the first 8-figure pattern of the net. Base on this pattern we can follow the same procedure to produce more strings, each one *conforms* to the previous one alternately from right to left. To do this, we need to define a general step that can find the next starting point for making a new 8-figure pattern. Given a point P_i on the one side of the frame F , to find a point P_{i+1} upward from P_i with a distance μ on the

same side of the frame, we can use such move as this:

String<F>(P_{i+1}<o>, P_i<o>, top, μ)

in which o = r, if the last string starts from left side;

o = l, if the last string starts from right side.

String<F> means that the move string uses F (the frame) as its stringing line.

Once we know how to find the point P_{i+1}, then we can make new thread S_{i+1} from this point that will conform, at distance μ, to the last thread S_i starting from the point P_i. That is:

**S_{i+1}<o> = String*(P_{i+1}<o>, Ω) and
 Conform* (S_{i+1}<o>, S_i<o>, μ) and
 Abutt* (S_{i+1}<o>, F)**

5). Repeat this process, and stop when the last string's length is shorter than the first string we begin with.

We notice that in the given complete net (see Figure 4.6.) S₁<r> is bended at the intersecting point Y₁ when S₁<l> comes to connect it. This phenomenon should be understood as the result of the deformation due to the elastical property of the material when two threads physically intersected. This should not be seen as a purposeful spatial arrangement. Had the thread been a 'rigid' body, such a deformation would not have happened. Therefore, the procedure we described above is still sound and valid, despite the fact that the process itself does not take care of this feature of deformation. We are confident to claim that no matter the elements are rigid or elastic, arrangement moves remain useful to any procedural description for reconstructing the net, though results may not be the same if the elements used have different physical properties.

To summarize, the procedural description of the larva's net in terms of arrangement moves consists of two subprocesses: the SET-UP, which constructs the first 8-figure pattern; and the FOLLOW-UP, which continues to finish the net on the basis of the first 8-figure pattern.

SET-UP ($P_1\langle r \rangle$, Ω , X , F)

$S_1\langle r \rangle = \text{String}^*(P_1\langle r \rangle, \text{right-}\Omega)$ and
 $\text{Abutt}^*(S_1\langle r \rangle, F)$

$\text{Align}\langle L_1 \rangle(P_1\langle l \rangle, P_1\langle r \rangle)$ and $\text{Conform}(L_1, X)$

$S_1\langle l \rangle = \text{String}^*(P_1\langle l \rangle, \text{left-}\Omega)$ and
 $\text{Abutt}^*(S_1\langle l \rangle, F)$ and

FOLLOW-UP (P_i , μ , F)

$\text{String}\langle F \rangle(P_{i+1}\langle o \rangle, P_i\langle o \rangle, \text{top}, \mu)$

$o = r$, if the last string starts from left side;
 $o = l$, if the last string starts from right side.

$S_{i+1}\langle o \rangle = \text{String}^*(P_{i+1}\langle o \rangle, \Omega)$ and
 $\text{Conform}^*(S_{i+1}\langle o \rangle, S_i\langle o \rangle, \mu)$ and
 $\text{Abutt}^*(S_{i+1}\langle o \rangle, F)$

The SET-UP process takes four arguments: the starting point $P_1\langle r \rangle$, the angle of direction Ω , the horizontal reference line X , and the frame F . Given values of these variables, the first 8-figure pattern can be constructed. The FOLLOW-UP process takes three arguments: a last starting point P_i , the distance μ between two adjacent points on the frame, and the frame F . Repeat this process until it meets the stopping condition. Now, we can give a more general description of this process as follows⁸:

LARVA'S-NET ($P_1\langle r \rangle$, Ω , μ , X , F)

SET-UP ($P_1\langle r \rangle$, Ω , X , F)

REPEAT FOREVER

STOP: if $S_i \leq S_1$

FOLLOW-UP (P_i , μ , F)

To conclude this chapter, I would like to make a few remarks. First, the fact that all reconstructions must be carried out by procedures does not therefore mean that they all are 'procedural'. The distinction between the relational and the procedural lies in different emphasis: the former is concerned with structuring spatial relations among elements,

regardless of what procedures are taken to establish such relations. The latter is primarily looking for a step-by-step sequence which plays a determining role to the result of reconstruction.

Second, the relational is congenial to the so called top-down approach in which the general spatial layout is contemplated prior to the considerations of detail. The reconstruction of the mosque plan serves as one case of this approach. The procedural, on the other hand, has some bearings on the bottom-up approach in which the overall picture is the result of incremental aggregation of small pieces. The larva's net serves an example of this approach.

Third, it is possible that any given configuration can always be reconstructed in terms of the relational description, regardless it is a design result or a procedural product. To make a configuration we must take a certain procedure; but once it is generated, it can always be reconstructed in terms of relational arrangements. We have demonstrated this point clearly by giving a relational description to the larva's net, which is characteristically a procedural product according to the empirical observation. Any given configuration can also be reconstructed by a procedural description, which seems to be an underlying assumption of the turtle geometry. But, again, we should not confuse acts of production with acts of *designing*, in which a procedural description can hardly be a useful working method.⁹

Physical designs also involve many parties: there are designers, builders, programmers, and users, etc, and design is basically about describing forms to others. People may choose not only different implementation systems of description (such as coordinates, actions, moves), but also different modes of description (such as the relational versus the procedural). Here, we have no intention to judge the merits of different implementation systems, nor different modes of description. The purpose is to show that arrangement moves are capable of not only making forms, but also describing forms by means of reconstruction, whether in relational mode, or in procedural.

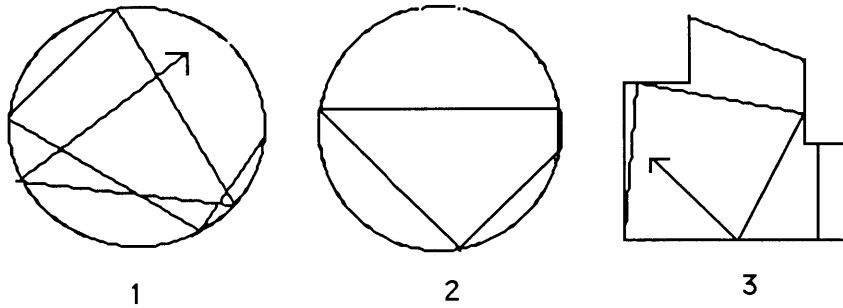
(End of Chapter 4)

Footnotes To Chapter 4

1. One salient characteristic of the procedural description is that its final pattern is not foreseeable until the process is terminated. The 'final' configuration presents itself gradually in the process, which is not preconceived at the outset. Let me provide an example serving to demonstrate the point. A circle is given as a frame within which a configuration can be made according to the set of instructions as follows:

- a). Select any point on the frame; proceed from the point in any direction chosen toward the inside of the frame to produce a line.
- b). When reach the frame, stop and make right turn.
- c). The first right turn is 90 degree, then the next right turn will be 100 degree. The turning angle increases by the increment of 10 degree.
- d). Stop when the turn becomes 360 degree.

Can we tell the completed picture from these instructions? Figure-1 at below shows an incompleting picture after 6 turns, which still give us no clear idea as what it looks like when it reaches its final state.



We also learn that a same procedure can generate radically different results simply by changing the value of procedural variables. Figure-2 at above shows the result of the same procedure as Figure-1, but uses 45 degree, instead of 10, as the increment at each turn. This nature of procedural description has been more formally and thoroughly investigated by the Turtle Geometry.

Initial conditions also play a significant role in determining the final result of a procedure. For instance, Figure-3 at above shows the picture of 6 moves exactly the same as those used in figure-1, except the given frame and the initial selection of direction are different. Conceivably, the final results of these two constructions will not be same. This nature is best presented in John Conway's 'Life game'. See John H. Conway, 'Life Game', *Scientific American*, Oct. 1970. Life game is about the growth and decline of the population of given cells. Each cell is in a finite number of 'states' which are influenced by its neighboring cells. A set of 'transition' rules are applied simultaneously to all existing cells, and change their states. The pattern changes as the game proceeds, but the next stage of development only has to do with the immediate last stage. Although the life pattern is in constant flux, the next development is completely predictable when the rules are known. In this sense, the life pattern is fatally predetermined by the spatial distribution of each cell at the beginning. Shall we say, the destiny is already embedded in its initial formation.

2. From the Renaissance onward the audience of architectural treatises changed gradually from patrons to the artisan class, and as results, methods of measuring and surveying became increasingly a central concerns

Form Reconstruction

of the authors. Examples are many. Here I shall mention only one reference in which this topic is discussed with a broader scope. Rudolf Wittkower 'English Literature on Architecture', in *Palladio and English Palladianism*, Thames & Hudson, New York, 1974.

3. Harold Abelson and Andrea diSessa, *Turtle Geometry*, MIT Press, Cambridge, Ma. 1980.

4. Gerald M. Edelman, 'Cell-Adhesion Molecules: A Molecular Basis For Animal Form', *Scientific American*, April, 1984.

5. Claude Bragdon, an American architect in the early of this century, observed two kinds of architecture:

"When we come to consider architecture through the world and down the ages, we find it bisected by a like inevitable duality: either it is organic, following the law of natural organisms; or it is arranged, according to some Euclidian ideal devised by proud-spirited man. In other words, it is either cultivated, like the flower; or it is cut, like the gem."

See Claude Bragdon, 'The Language of Form', in *Roots of Contemporary American Architecture*, ed. Lewis Mumford, Reinhold:New York. Original from Six Lectures on Architecture, by R. A. Cram, T. Hastings, C. Bragdon, Chicago, 1917. According to Bragdon's demarcation, the Gothic is organic; the Renaissance is arranged. Bragdon polarized two kinds of architecture to make his point in supporting the philosophy of the emerging Modern Movement. The original resource of those principles such as material authenticity, structural rationality, and functional identity, enthusiastically embraced by the Modern architects, is ascribed, by Bragdon, to the Gothic architecture. Bragdon's ideological point, shared by many of his contemporaries like Sullivan and Wright, is quite clear when he conceptually separated the organic from the arranged. Actually, these two types of form have been noticed by many people in different observations. Our concern in this connection is not ideological, but technical. Drawing on the analogy of the flower and the gem, we can see a given form as been made in such a process as a flower grows; we can also see the same given form as been made by such a process as a gem is shaped according to an imposed image. Bragton's view, on the other hand, seems to imply that the organic and the arranged are distinguished on the basis of formal characteristics. Here I prefer to see the arranged and the procedural as two 'working methods', instead of two styles of forms. Any given form, whether natural or artificial, can be analysed in these two ways.

6. The experiment is taken from one of the exercises of the course *Design Moves: A Language of Form-making*, instructed by Ming-Hung Wang, Department of Architecture, MIT, Fall, 1983.

7. Richard Merritt, J. Bruce Wallace, 'Filter-feeding Insects', *Scientific American*, 1981.

8. This should not be seen as a piece of 'computer program' that can run in the machine. I am doing here nothing more than a procedural analysis intended to make it operationally explicit in terms of arrangement moves.

9. Both the relational and the procedural can be used for construction tasks in addition to reconstructions. However, the procedural approach is less powerful for constructions, because, as pointed out in Footnote 1, we can hardly know what it will produce not until at the end of the process. It is also difficult to agree that the procedural approach of making-form is useful in designing, which usually involves many 'prescriptions' of the desirable spatial relations to be made.

Chapter 5

Rule Representation

We have shown that any given configuration can be described by arrangement moves in terms of reconstruction, one might ask: are these moves also useful to describe a family of forms, i.e. a type? A type can be viewed as a family of form governed by a system of rules. The question therefore is to ask whether the arrangement moves proposed are capable of formulating rules of any given form type. This chapter is devoted to the demonstration of using arrangement moves for rule formulation. The chapter basically consists of three parts: I will begin with a general discussion regarding the issue of rules in design, and with the introduction of some technical preliminaries to be used later in rule representations. Second, a rule system of a type formulated by arrangement moves will be presented. Wright's Usonian house type is chosen as the study case. Finally, the Usonian rules formulated are tested against a real example.

§ 5.1. Design and Rule

Designing is not a random behavior, it must be guided by certain rules. Wright had words on this: "style is important. A style is not. There is all the difference when we work with style and not for a style."¹ Wright may not have conceived his, say, Usonian houses as a particular kind of building style, however, since he was deliberately working 'with style', he must have followed certain rules. If we see that a style is a particular way of doing things, then the rules by which we can work 'with style' in physical designs must be about three things: 1) selecting a particular kind of elements, 2) arranging elements in a particular way, and 3) following a particular kind of procedures. The first two determine the style of a design result; the third develops a style of designing (not the style of a design result). Thus, we can distinguish at least two major kinds of rules: the *form rules*, which are about determining elements and their relations; and the *procedural rules*, which are about

sequences of doing things. A form rule, therefore, is either to tell how to select elements; or to specify how elements relate to one another spatially. A procedural rule usually can be expressed in the if-then format: the if-side specifies a situation, the then-side specifies actions to be taken under such a situation. In physical design both the situation and the action are related to form rules.

5-1.1. Design Rules

Before dealing with the theme of the chapter as how can arrangement moves be used to represent rules, I would like to discuss two main reasons for this undertaking first.

1). For designing: design is involved with rule-making. Some rules are given as programmes, or building codes; some are agreements made among participants regarding specific concerns brought into the project under design. There are rules as shared conventions that need no explicit formulations. Rules, however, can be made only if they can be clearly formulated, and to formulate them coherently we need a powerful descriptive system, i.e. a set of vocabulary capable of expressing rules used in design.

2). For understanding design: in designing rules are made and followed, therefore, knowing procedural rules helps to understand processes of design; and form rules help to understand design intentions as well as design results. Our knowledge about a design process can be seen as a set of rules governing the generation mechanism of a design. Our knowledge about a form type can be seen as a set of rules that can account for all its instances. Knowing rules means that we can give a rigorous formulation of them. (However, there are tacit understandings, as Michael Polanyi argued, that we can not formulate. Since we can not formulate, then I guess there is no point to try to discuss it.) To formulate, we need a descriptive system.

Rules involved in design are either form rules or procedural rules. As far as rule formulation is concerned, there is no difference between representing rules for designing (i.e. to do design), and representing rules to account for a design (i.e. to understand a design). Presumably, both form rules and procedural rules apply arrangement moves, because their contents are primarily about how one form is spatially related to another form, be it a description of a situation, or a specification of an action to be taken in that situation. Seeing in this light, one exercise on the formulation of rules, whether about a

form type, or about a design procedure, should suffice to demonstrate the capability of arrangement moves for rule representations. We chose to formulate a set of form rules defining a building type, and we chose F. L. Wright's Usonian houses as the test case, because such rules are more readily testable with the help of empirical data.²

5-1.2. Form Type

Why is that describing a type of form has to do with the formulation of rules? We may take it for granted, but the reasons are not self-evident. Consider this question: how can a type be described? To present all of its instances does not describe the type, but only begs the question. A type consists of a particular collection of forms that constitute a 'family'. Within the family, all forms bear resemblance to one another, but they are not identical. To describe a type therefore means that we have to manage the balance between constraints and choices: the former ensure the similarities; the latter offer freedom of being different. Rules allow us to do so. Rules can leave choices open basically by two kinds of expression: 1). in the format of 'alternatives', such as: 'the house can be only one-storey, or two-storey, or three-storey high'; and 2). in the format of 'threshold', such as: 'the house should not be higher than three stories.' Both expressions give some constraints and provide same range of options. One manner may be more economical and effective than the other in certain circumstances.

If we claim that we know a type, say, the Roman basilica, the Greek temple, or the Chinese courtyard house, then we must be able to 'tell' its elements and the relations among the elements, by which we can replicate any given instance of the type. Form rules are about selection of elements and arranging their relations. As we have mentioned, there are two kinds of elements involved in physical design: the spatial elements and the material. Each element has a name, and can be defined by specifications regarding its shape, dimensions, etc. (Usually the spatial element is named in terms of its 'use', such as kitchen, garage, etc.) By this distinction, given a type of form at least two kinds of rules can be formulated: the spatial rules and the material rules.

In the spatial rule system, relations among elements are often specified in terms of function, that is, in the case of architectural design, which activity should be adjacent, or not adjacent to which others. This is usually the way we talk about 'functional' relations

among rooms/spaces. Functional relation certainly is not the only relation among spaces, there also has, for instance, aesthetical one. I shall limit my formulation of the Usonian spatial rules only to functional relations for the simple reason that, according to his own words, Wright's concern of spatial relations is mainly functional.

When spaces relate to each other 'functionally', and are grouped to form larger spaces, they are in a part-whole hierarchy. On the other hand, relations among elements in the material rule system are basically about connection: how one piece of material connects to another. If materials at different levels are engaged, their relations are bounded within a dependency hierarchy.

To begin formulating the rules of Usonian type, I would like to introduce some technical preliminaries for the sake of economy as well as clarity in rule presentations. These preliminaries consists of three parts: the technical terms for naming references, a list of compound moves to be used, and the convention for structuring rules.

5-1.3. Reference terms

We are used to seeing a periphery area and a center area in any given form. We may also see a margin area between the center and the periphery, and a corner area as shaped by two edges, etc. We use these terms in ordinary conversations regarding spatial relations. Rules can be stated more efficiently if we can refer to terms like them.

As observed, most spatial elements of Usonian houses are of a rectangular shape. We will call the short edges of a rectangle *ends* or *end edges*; the long edges are called *sides* or *side edges*. The side edge also indicates the rectangle's directionality, that is, the *directional line* is a reference line drawn parallel to the side edge. A part of an edge may also be specified as the 'window' edge, or the 'door' edge, where the window or the door is found. The window or door edge must be left unobstructed when it is connected to other edges.

For the economy of description, each spatial element of the Usonian house type will be subdivided into small regions. We divide a rectangular spatial element into three *sectors* along its length: (right) end sector, centre sector, and (left) end sector; and three *zones* along its width: (top) side zone, centre zone, and (bottom) side zone. The size of each

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sector or zone is determined either by a particular default assignment, or by division lines specified in each case. It is understood that dimensional variations do not alter the organizational characteristics of a form.

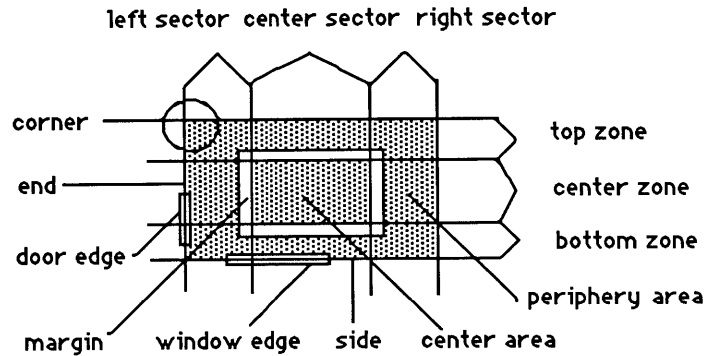


Figure 5.1.

When two related rectangles are considered, some specific references are worth notice. As they engage with each other, they are either overlapped, or connected. If they are connected by sharing an edge, we will call this connecting edge *joining edge*; and call the end sector (zone) containing the joining edge *joining sector (zone)*. We can also use the joining sector (zone) as a reference, and call the edge of any engaged space *conjoining edge* if this edge immediately connects to, or is included within, a joining sector (zone). The union of two or more joining sectors (zones) is called *joining area*. Symmetrically, there are *open sector* and *open edges* for a space as opposed to its joining counterparts.

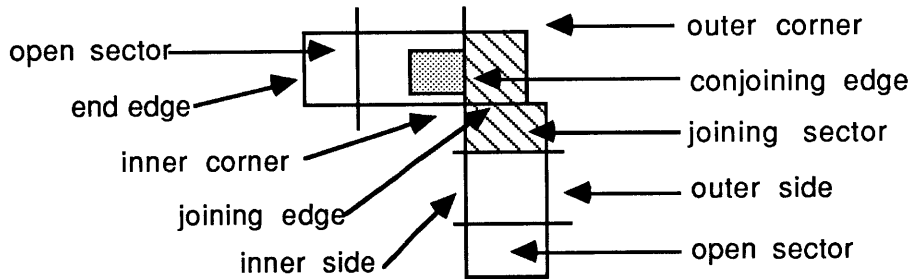


Figure 5.2.

If two engaged rectangles form a L-shape, we can further specify that the L-shape, as a whole, has two ends; an *inner side* and an *inner corner* as opposed to the *outer side* and the *outer corner*. A corner is seen as an area with a size determined in ad hoc, or by a default assignment.

Finally, we should be careful not to confuse these reference terms with names of the 'sub-elements' of an element.

5-1.4. Compound Moves

Two sets of compound moves that will be frequently used later in demonstrations are to be defined here: one is about directional relations, such as 'in front of', and 'face to'; and conjunctive relations, such as 'connected by', 'separated by'. These moves are worth formulation because they are part of our ordinary language that can convey meaning more efficiently. The other is about some particular ways of composition, such as, when two rectangles connect, they have one edge aligned, or they have two edges aligned. All these two categories of relations are defined by B-moves with some directional and/or dimensional specifications.

I shall begin to define the C-moves concerning directional relations between two elements A and B; and those about element C in conjunction with A and B.

- 1). 'A is *in front of* B: A is **adjacent** to B's 'front' side, provided that, B's front side, or back side, has been specified in the first place. (Note that 'adjacent' has been defined in Chapter 4 by B-moves.)
- 2). 'edge-A *faces* edge-B': edge-A is **adjacent** to edge-B, and edge-B is **included** within the perpendicular reference lines drawn from edge-A toward edge-B. Usually 'facing' is a spatial relation referred only to edges (surfaces). If we say that 'edge-A faces the north', then the perpendicular reference lines drawn from edge-A should direct toward the north.
- 3). 'A and B is *connected by* C': A and B are **separated**, and A **connects** to C; C also **connects** to B.
- 4). 'A and B is *separated by* C': A and B are **separated**, and C **inserts** between A and B such that C is **adjacent** to A, and at the same time is **adjacent** to B as well.

Let's now define some C-moves concerning compositions of two reclinear elements. These C-moves are defined by generic B-moves (i.e. connection, separation, containment, inside-connection, and overlap) in conjunction with some ordering B-moves, such as alignment, coincident, and conformation. These compositional C-moves have no particular bearings on the structure of Wright's Usonian house type, but they can be seen as a scenario of 'logically' possible compositions for the orthogonal deployment consistently used by Wright. As can be vindicated by the orthogonal grids he adopted in most of his designs, these C-moves are indeed parts of the technical rules Wright used for compositions.

When a functional rule states that 'space A connects to space B', only limited 'moves' we can choose to establish such a relation within the constraint of orthogonal deployment. We provide the catalog of these moves for the convenience of demonstration. Later when we test our Usonian rules by making a real instance of the Usonian house type, some of these moves will be applied. To make the demonstration rigorous, not only each rule used should be specified, and also each move of arrangement.

Connection-1: Two rectangular spaces **connect** to each other, each space has one non-connecting edge **aligned**.

Connection-2: Two rectangular spaces are **connected** without any non-connecting edge **aligned**.

Connection-3: Two rectangular spaces are **connected**, each space has two non-connecting edges **aligned**.

Connection-4: Two spaces are **connected**, one space has two adjacent edges **conforming** to the corner of the other space, but there is no other edge **aligned** between these two spaces.

Connection-5: Two spaces are **connected**, one space has two adjacent edges **conforming** to the corner of the other, and there also have one pair of edges **aligned** between these two spaces.

Connection-6: Two spaces are **connected**, one space has two adjacent edges **conforming** to the corner of the other, and there also have two pairs of edges

Rule Representation

aligned between these two spaces.

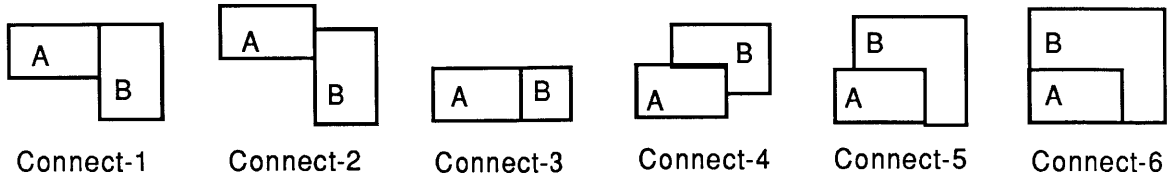


Figure 5.3.

Separation-1: Two rectangular spaces are **separated** from each other with one edge **aligned**.

Separation-2: Two rectangular spaces are **separated** without any edge **aligned**.

Separation-3: Two rectangular spaces are **separated** with two edges **aligned**.

Separation-4: Two spaces are **separated**, one space has two adjacent edges **conforming** to the corner of the other space, but there is no other edge **aligned** between these two spaces.

Separation-5: Two spaces are **separated**, one space has two adjacent edges **conforming** to the corner of the other space, and there also have one pair of edges **aligned** between these two spaces.

Separation-6: Two spaces are **separated**, one space has two adjacent edges **conforming** to the corner of the other space, and there also have two pairs of edges **aligned** between these two spaces.

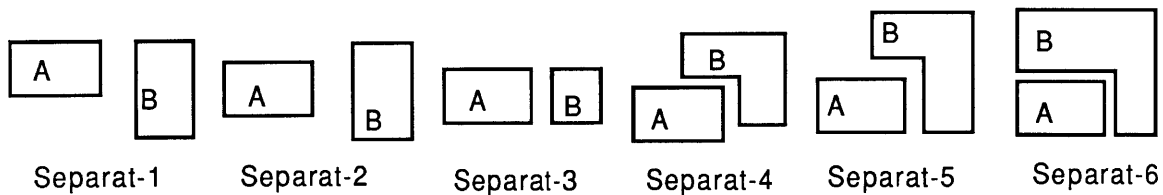
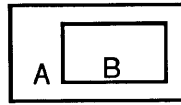


Figure 5.4.

Inclusion-1: One rectangular space **includes** another rectangular space, and they have at least one pair of edges that are **conformed**.

Rule Representation

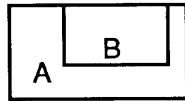


Inclusion-1

Figure 5.5.

Insert-connect-1: One rectangular space **includes** another rectangular space, and they have only one pair of edges **coincident**.

Insert-connect-2: One rectangular space **includes** another rectangular space, and they have only two pairs of adjacent-edges **coincident** at corner.



Insert-connect-1



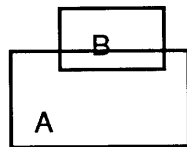
Insert-connect-2

Figure 5.6.

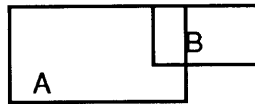
Overlap-1: One rectangular space **overlaps** with another rectangular space from one edge, and at least one pair of edges are **conformed**.

Overlap-2: One rectangular space **overlaps** with another rectangular space, and they have only one pair of edges **aligned**.

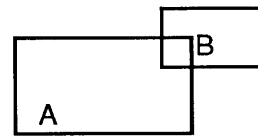
Overlap-3: One rectangular space **overlaps** with another rectangular space at corner, and at least one pair of edges are **conformed**.



Overlap-1



Overlap-2



Overlap-3

Figure 5.7.

Although most of these C-moves are applicable to the composition of material element too, here we limit them only to deal with relations between spatial elements. We

do not deliberately coin a 'name' for each of these C-moves (they are 'numbered' instead), because they are used primarily for this particular study, although they do possess certain qualities as general arrangement moves.

5-1.5. Structuring Rule System

Except those rules concerning the selection of elements and grids, the basic sentence of each rule is expressed in the format as this:

element-X is in relation-R to element-Y (with certain orientational or dimensional specification)

For instance, 'the corridor **conforms** to the edge of the bedroom wing'.

All spatial relations are specified by A-moves (which will be printed in bold letters). Each rule is made up of one, or several basic sentences connected by 'and'. Since there are two systems of rules, the spatial and the material, we shall distinguish them by 'S-rules', which stands for the rules about spatial elements; and 'M-rules' about material elements. All rules are numbered.

One rule may have some 'alternatives' in terms of same relations among different elements. That is, by the same relation-R, element-X can relate to element-Y; and can also relate to a different element-Z. We will code them by Rule#-A, and Rule#-B. For instances:

S-Rule 10-A: the fireplace is inserted in the living room's end sector, and its opening edge faces the inside of the living room.

S-Rule 10-B: the fireplace is inserted in the living room's outer side zone, and its opening edge faces the inside of the living room.

For each rule, or each alternative of a rule, there may also have a few 'versions' in terms of different relations between same elements. That is, if X relates to Y, then X and Y can be in relation-P, or in relation-Q. Different versions of a rule will be coded by Rule#-A.1, Rule#-A.2, etc. For instances:

S-Rule 10-A.1: the fireplace is included in the living room's end sector, and its opening

edge faces the inside of the living room.

S-Rule 10-A.2: the fireplace insert-connects to the living room's end edges, and its opening edge faces the inside of the living room.

When apply a rule, we can always chose one of its alternatives, or one of different versions. If a rule has only different 'versions', not 'alternatives', then they will be denoted as Rule#.1, Rule#.2, etc. For instances:

S-Rule 7.1: the loggia is included in the joining area of the body and the tail.

S-Rule 7.2: the loggia insert-connects to the joining area of the body and the tail.

§ 5.2. Usonian Rules

"A modest house, this Usonian house, a dwelling place that has no feeling at all for the 'grand' except as the house extends itself in the flat parallel to the ground. It will be a companion to the horizon.... Withal, this Usonian dwelling seems a thing loving the ground with the new sense of space, light, and freedom - to which our U.S.A. is entitled" (Wright, 1954, pp.81-82)

The word 'Usonian', according to Wright, was coined by Samuel Butler, a novelist who believed this name to be better a title for the nation than the United States of America. Wright warmly embraced the vision and used the name 'when needing reference to our own country or style'. Through numerous 'low-cost' houses built from 1938 to 1954, Wright developed the Usonian house type which he thought was the true expression of American life. This style of house is different in many aspects from his earlier prairie style. The differences, as I see, are based on different rule systems adopted.

The Usonian house type is chosen for this study because it provides a good example of Wright's organic philosophy of spatial composition, which seems to resist explicit analyses. Formulating rules that can account for such organic organizations is therefore difficult, and also challenging. There lacks no serious attempt to explicate the Usonian type, and a rigorous rule system has also been formulated by people along the line of 'shape grammars' (which I will discuss in the Appendix). Rules can be formulated from various perspectives. To describe a type is always a matter of interpretation, and I have no

Rule Representation

intention to advocate the rule system proposed here as the genuine knowledge of Usonian houses. The main purpose, as said, is to demonstrate the descriptive power of arrangement moves. In this light I would like to make a few general remarks at the beginning about the business of rule formulation for a form type.

First, no claim of 'completeness' is made here. It is always possible, in principle, to add more rules to those presented in this study. We would prefer fewer rules rather than more rules as far as elegance is concerned. The question of completeness, and the question of elegance as well, is not a very relevant issue regarding rule formulation.

Second, whether the rules are 'essential' or trivial to represent the Usonian house type is, I am afraid, a matter of opinion that cannot be seriously debated unless we stand on a shared ground in terms of scope and purpose in the first place.

Third, where there is a rule, there is an exception. We may find that certain rules are applicable to most, but not all, Wright's Usonian houses. In such a situation, if the rule system is required to conform to all cases, then there are only two choices: either make 'exceptional' rules, or make 'exceptional' cases. The idea that the rule system should account for all cases is, seems to me, relatively uninteresting. In order to be accountable for all cases we may end up with a set of rules that are perhaps too general to characterize any given style at intelligible level. To the other extreme, very likely we may find that there is no such thing as an Usonian house, but only different individual houses. In this study, therefore, I shall only present rules that can characterize those houses I believe to be typically Usonian, not to account for every single house built by Wright in the period of 1938-1954. It follows that the validity test of the rule system will be this: all houses (set A) generated by the system of rules will be definitely 'Usonian'; but not all Usonian houses (set B) conform to all rules (i.e. set B \geq set A).

In the follows I will first describe the spatial rule system, which is a set of specifications about how spaces are adjacent to one another. Then, I shall present the material rule system which specify ways of physical connection among material pieces. To formulate these rules Wright's own analyses will be consulted (and his words quoted) whenever available.

5.2-1. Spatial Rule System

Selection of spatial elements

S-Rule 1: Every Usonian house is made by selecting spatial elements from the following

list:

B = (single) Bedroom (1 or 2)
b = bathroom (2)
Car = Carport (1)
D = Dining space (1)
F = Fireplace (1 or 2)
H = Hallway (1 or as needed)
L = Living room (1)
Pl = Planter (1 or as needed)
St = Storage (1)
tl = tool room (1)
U = Utility room (1)

MB = Master bedroom (1)
Ct = Coat room/Coat closet (1 or 2)
cd = corridor/gallery (1 or as needed)
E = Entry (1)
G = Garden (1 or as needed)
K = Kitchen (1)
Lg = Loggia (optional)
S = Study room (optional)
T = Terrace (1 or as needed)
t = trellis (1 or as needed)
W = Working room (optional)

Each space/room has a name, which is abbreviated, and its quantity is given in the parentheses. 'Optional' means zero or one; 'as needed' means as many as needed. (Note that the range of the size of each space/room is not given here because it is not so much relevant to our concern of spatial relations, although it is not difficult to be specific about it.)

S-Rule 2: an individual spatial element must be either a square or a rectangle, and a compound elements must be an orthogonal polygon.

Grid

Wright has continuously used various kinds of grid for spatial as well as material positioning since his very early designs, it is no exception in Usonian houses. The Usonian grids are not only the devices for space planning, some of them also have been actually built into concrete 'floormats' for the ease of construction. Most grids use the module of 2 foot by 4 foot rectangle; there are a few in squares (e.g. Goetsch-Winkler house, Okemos, Michigan, 1939); and hexagons (e.g. Vigo Sundt project, Madison, 1941). One-directional deployment of spaces on a grid will produce a linear organization; two-directional deployment will generate a L-shaped, or a cruciform, that can be found in Wright's designs very often.

Rule Representation

For a rectangular grid, its direction is defined to be the direction of its rectangular module. (For a rectangle, its direction line runs along its length.) Here we see a rule about the grids used in Usonian houses, and a rule concerning the deployment of spaces on a grid.

S-Rule 3: an orthogonal grid is always used, mostly used is the 2'X4' rectangular grid. If a square grid is used, the module is 4'X4'.

S-Rule 4: when the spatial element is placed on the grid, at least one of its edges should be coincident with the grid line.

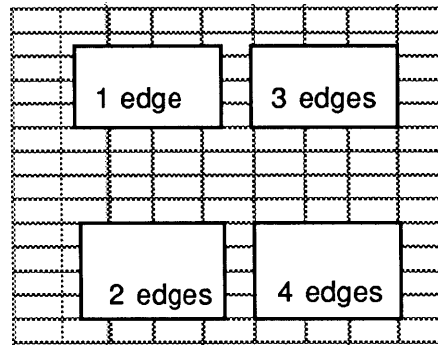


Figure 5.8.

This rule also means that, for a rectangular spatial element, there are four ways of placing it on a grid: 1). The space has all its edges coincident with grid lines. 2). The space has one edge that is not coincident with the grid line. 3). The space has two edges that are not coincident with grid lines. 4). The space has three edges that are not coincident with grid lines.

Polliwog: the plan structure

After elements and grids are known, we may now look at relations among elements. Let's first look at the level of the overall organization of Usonian house. Wright had explicit words on this topic:

Rule Representation

"As you see from the plans, Usonian houses are shaped like polliwogs - a house with a shorter or longer tail. The body of the polliwog is the living room and the adjoining kitchen - or workspace - and the whole Usonian concentration of conveniences. From there it starts out, with a tail: in the proper direction, say, one bedroom, two bedrooms, three, four, five, six bedrooms long; provision between each two rooms for a convenient bathroom. We sometimes separate this tail from the living room wing with a loggia - for quiet, etc.; especially grace. The size of the polliwog's tail depends on the number of children and the size of the family budget. If the tail gets too long, it may curve like a centipede. Or you might break it, make it angular. The wing can go on for as many children as you can afford to put in it. A good Usonian house seems to be no less but more adapted to be an ideal breeding stable than the box." (pp.165)

The curving 'centipede' I hardly find, most Usonian houses are polliwogs. The tail is either aligned with the body, or is turned 90 degrees to form a L-shape with the body. Before discussing rules about the body and the tail, there is a rule regarding the placement of the body with respect to the grid.

S-Rule 5: If a rectangular grid is used, the body is always deployed in the same direction as the grid.

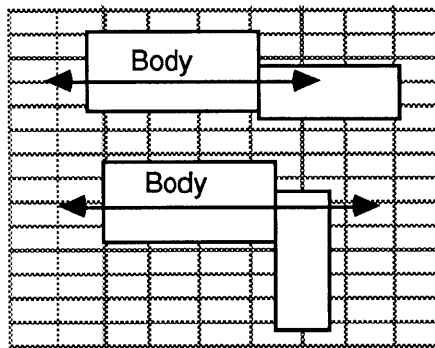


Figure 5.9.

According to Wright's description, the basic parts of the Usonian house are the body and the tail, and the two have to be joined. (I therefore ignore those cases with separated body and tail.) The tail does not connect to the body unrestrictedly, as one may notice that Wright specifically pointed out that "from there it starts out with a tail", in which 'there' clearly means the place where the kitchen is located. Let this place be named as the *kitchen region* for the sake of convenience. The size of the kitchen region is adjustable in accord with its contents. Since the kitchen region is the place where the body and the tail 'join',

Rule Representation

and therefore, one can also specify the joining sector and the joining edge by referring to this region, if it is made known in advance.

S-Rule 6: to establish a relation between the body and the tail, choose one of the following rules:

*6-A.1: the tail is placed **perpendicular** to the body, and tail's joining edge connects to body's joining sector.*

*6-A.2: the tail is placed **perpendicular** to the body, and tail's joining sector overlaps with body's joining sector.*

*6-B.1: the tail is placed in the **same direction** to the body, and tail's joining edge connects to body's joining sector.*

*6-B.2: the tail is placed in the **same direction** to the body, and tail's joining sector overlaps with body's joining sector.*

S-Rule 7: to place the loggia, choose one of the following rules:

*7.1: the loggia is **included** in the joining area of the body and the tail.*

*7.2: the loggia **insert-connects** to the joining area of the body and the tail.*

The L-shape organization also has to do with the site planning in which the garden life is concerned: "We will have a good garden. The house is planned to wrap around two sides of this garden." (pp. 77)

About the house layout on the site with respect to lighting Wright also had a few words: ".. the south side of the house the 'living' side. Ordinarily the house should be set 30-60 to the south, well back on its site so that every room in the house might have sunlight some time in the day." (pp. 165)

The edges of site can be named in terms of four orientations: north edge, south edge, east edge, and west edge (presumably it is a regular lot with four edges). That the garden must be placed between the inner side of the house and the south edge of the lot seems to be the picture Wright had in mind, as far as I can conjure up from what he has said. Now we come up with the rules regarding the plan organization.

Rule Representation

*S-Rule 8: the house is **included** in the site, and the body's side edge faces to the south at 60-degree angle.*

*S-Rule 9: the garden is **included** in the space defined by the inner side of the house and the south edge of the site.*

Living

"We must have as big a living room with as much vista and garden coming in as we can afford, with a fireplace in it, and open bookshelves, a dining table in the alcove, benches, and living room table built in; a quiet rug on the floor. Convenient cooking and dining space adjacent to if not a part of the living room." (pp. 77)

The living room is the major part of the body. As one can immediately recognize, Wright's concept of living 'room' is not at all a conventional 4-wall enclosure. The room is a big continuous space flowing from one corner to another by which many functionally separated places are integrated into one organic whole. Technically speaking, the basic operation to create a space as such is overlapping. When A overlaps with B; B overlaps with C; their boundaries blur, and as a result, they cannot be definitely distinguished from one another any more. It is meaningless, however, to say that spaces are 'overlapped' unless they are individually identified first. Fireplace, kitchen, dining, and studying (if provided) are the spaces to be integrated with the 'living space' (as a single spatial element) to form the large living room which, according to Wright, is almost identical to the generic space he called the body. Therefore, all rules concerning the body should be applicable to the living room as well. Since the kitchen is a fairly complex cluster that demands a set of rules on its own right, I shall discuss it separately. The rules in the follows consider only such spaces as fireplace, dining, and studying in relation to the living room.

S-Rule 10: apply one of the following rules to place the fireplace:

*10-A.1: the fireplace is **included** in the living room's end sector, and its opening edge faces the inside of the living room.*

*10-A.2: the fireplace **insert-connects** to the living room's end edges, and its opening edge faces the inside of the living room.*

Rule Representation

*10-B.1: the fireplace is **included** in the living room's outer side zone, and its opening edge faces the inside of the living room.*

*10-B.2: the fireplace **insert-connects** to the living room's outer side edge, and its opening edge faces the inside of the living room.*

*10-C1: the fireplace **insert-connects** to the study room's edge, and its opening edge faces the inside of the study room.*

*10-C2 the fireplace **overlaps** with the study room's edge, and its opening edge faces the inside of the study room.*

S-Rule 11: to establish relations between the dining and the living, apply one of these rules:

*11-A: the dining space **overlaps** with the living room's inner side zone, and is **adjacent** to the body's joining sector.*

*11-B: the dining space is **included** in the living room's joining sector, and is **adjacent** to the living's inner side edge.*

*11-C: the dining space **connects** to the living room's conjoining side, or its inner side edge.*

S-Rule 12: apply one of these rules to place the study room, or the family room, or the workshop space:

*12-A: the study room, or the family room, or the workshop space **connects** to living room's open end edge.*

*12-B: the study room, or the family room, or the workshop space, **overlaps** with living room's open sector.*

Kitchen cluster

"In the Usonian plan the kitchen is called the workspace and identified largely with the living room. As a matter of fact, it becomes as alcove of the living room" (p.175).

"Convenient cooking and dining space adjacent to, if not a part of, the living room. This space may be set away from the outside walls within the living area to make work easy....; thus connection to dining space is made immediate without unpleasant features and no outside wall space lost to the principle rooms." (p.174)

The kitchen is considered by Wright as a part of the large living room, and Ideally, it should be 'included' within the body (i.e. in the relation of inside-separation to the body that includes it). In many Usonian houses, however, the kitchen is not positioned completely away from external walls. We shall therefore reserve the possibility of making the kitchen in the relation of inside-connection to the living.

As one can observe in many cases, the kitchen and many other spaces, such as the storage, utility room, workshop, dining, and sometimes the hallway and the fireplace, are closely grouped into a cluster. The spaces of such cluster contains vary from design to design. Primarily, the kitchen or the *kitchen cluster* occupies the body's joining sector in two ways: it is either overlapped with this sector, or is completely included within it. The kitchen cluster uses the tail's joining sector only secondarily, and of course, it all depends on how a joining sector is defined. To put it more safely, the kitchen or the kitchen cluster is located in the joining area of the body and the tail.

S-Rule 13: apply one of these rules to place the kitchen or its cluster:

*13-A.1: The kitchen or the kitchen cluster is **included** in the joining area of the body and the tail.*

*13-A.2: The kitchen or the kitchen cluster **insert-connects** to the joining area of the body and the tail.*

*13-B.1: The kitchen or the kitchen cluster is **included** in the body's joining sector.*

*13-B.2: The kitchen or the kitchen cluster **insert-connects** to the body's joining sector.*

*13-C.1: The kitchen **overlaps** with Living's joining edge.*

*13-C.2: The kitchen **connects** to Living's joining edge.*

Wright also had descriptions about relations within the kitchen cluster:

"There are steps leading down (from the kitchen proper) to a small cellar below for heater, fuel, and laundry, although no basement at all is necessary if the plan permits" (p. 174).

"..the utility stack has, economically standardized and concentrated within it, all appurtenances of modern house construction... . This enlarged hollow chimney - about 6

Rule Representation

by 8 feet on the ground - is accessible from the coat-room and so placed that only one short run of horizontal pipe or wire to the study is necessary." (p. 178)

" The bathroom is usually next (to the utility room) so that plumbing features of heating, kitchen and bath may be economically combined." (p.174)

For economical reasons (which were one major concern for the design of Usonian houses) many rooms/spaces are grouped around the kitchen to share utilities, and form the kitchen cluster as the result. The kitchen cluster includes many rooms and spaces, such as storage, tool, working, and entry, that have no conceivable functional requirement to be adjacent to the kitchen, I am inclined to think this as the result of two other factors brought into design. One is the idea of what Wright called the 'concentration of conveniences' that concentrates all 'supporting' spaces in one region to arrive at a clear, tripartite spatial organization: living-sleeping-supporting. The other factor has to do with the program of physical construction (as we shall see later in discussing the material rule system) in which the kitchen cluster is the 'hinge' area where the physical structures of both the body and the tail converge. Conceivably, in such a hinge area many wall elements are needed to respond beams coming from different directions, and as a result, spaces/rooms defined by these wall elements are also 'clustered' within this hinge area, although they need not to be adjacent as such in functional sense.

S-Rule 14: any one of the following spaces: fireplace, utility, working, storage, and dining, can be placed according to one of these rules:

*14-A: it can **connect** to the kitchen, but not at its opening edge.*

*14-B: it can be **separated** from the kitchen or the kitchen cluster **by the corridor or the hall.***

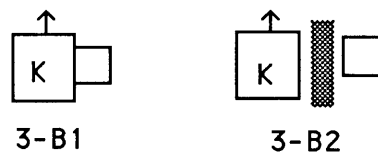


Figure 5.10.

Entry cluster

The entry to the house is usually located near the outer corner of the joining area, if the

Rule Representation

house is L-shaped. Obviously the carport shall be close to the entry for both reasons of convenience of circulation and economy of construction. Carport can share the roof and walls with the house, and should not be built as a garage, of which Wright antagonized the idea:

"A garage is no longer necessary as cars are made. A carport will do, with liberal over-head shelter and walls on two sides. Detroit still has the livery-stable mind. It believes that the car is a horse and must be stabled."³

The carport relates not only to the entry, but also to the living space in such a way that it should be located at the outer corner of the body so as not to block the view from the living room toward outside. In addition to the carport some other spaces such as the tool room, closets, and planters, are related to the entry too, and they together can form a cluster.

It will be inconvenient if the tool room is separated from the carport by another room or space. It is also expensive to build an independent tool room at the outside alone. If the tool room is at the outside, it should be included within the carport in such a way that its walls can support the carport's roof. Otherwise, it should either be inside of the entry; or should connect to the entry from the outside.

A closet for coats should be included in, or connect to the entry. There may be as many closets as needed in an Usonian house, they are either related to the entry/hallway/corridor region, or to bedrooms.

Planters are usually, if not always, located at both the inner and the outer corner of the house, they also relate to outside terraces. Occasionally we may find planters inside the house, but this can be seen as an exception rather than as a rule.

S-Rule 15: to establish the relations between the entry, or the entry cluster and the body, one of the following rules can be applied:

*15-A: the entry or the entry cluster **insert-connects** to the outer edge of the joining sector of the body, and the entry's door edge is **coincident** with the joining edge.*

*15-B: the entry or the entry cluster **connects** to the outer edges of the joining sector of the body, and the entry's door edge should **face** toward the outside.*

*15-C: the entry or the entry cluster **overlaps** with the joining sector of the body,*

Rule Representation

and the entry's door edge directly faces toward the outside.

S-Rule 16: to place a closet one of the following rules can be applied:

*16-A.1: the closet **connects** to entry/hallway/corridor, and the closet's door edge should **face** the inside of the entry/hallway/corridor.*

*16-A.2: the closet **insert-connects** to entry/hallway/corridor, and the closet's door edge should **face** the inside of the entry/hallway/corridor.*

*16-B.1: the closet **connects** to any space, but not at its door or window edge.*

*16-B.2: the closet **insert-connects** to any space, and closet's door edge should **face** the inside of the space.*

S-Rule 17: one of these rules can be applied to place the toolroom:

*17-A.1: the toolroom **connects** to the entry, and toolroom's door edge may **face** toward the outside.*

*17-A.2: the toolroom **insert-connects** to the entry, and toolroom's door edge may **face** toward the outside.*

*17-B.1: the toolroom is **included** in the side zone, or the end sector of the carport, and its door edge should **face** the center area of the carport.*

*17-B.2: the toolroom **insert-connects** to the side zone, or the end sector of the carport, and its door edge should **face** the center area of the carport.*

S-Rule 18: one of the following rules can be applied to place a planter:

*18-A.1: a planter can **connect** to any edge of the house from outside, but not to any door on the edge.*

*18-A.2: a planter can **overlap** with the house, but not at where doors locate.*

*18-B.1: a planter can **connect** to any edge of any terrace.*

*18-B.2: a planter can **overlap** with any terrace.*

S-Rule 19: one of the following rules can be applied to place the carport:

*19-A.1: the carport **connects** to the entry or the entry cluster, and it should be placed in **perpendicular** to the body.*

Rule Representation

*9-A.2: the carport overlaps with the entry or the entry cluster, and it should be placed in **perpendicular** to the body.*

*9-B: the carport is **separated** from the entry, but they should be **connected** by the outside walkway.*

Tail: the bedroom wing

The tail is a string of bedrooms and bathrooms. The end of the string that joins the body is called the tail's 'joining edge', the other end opens to the outside is called the tail's 'open end edge'. In the joining sector of the tail we usually find the hallway or the corridor, while the open end is often terminated with the sector occupied by the master bedroom, and incidentally by the study room or other working spaces. A long corridor, within 4'-8' width range, conforms to the side edge of the string that its one end always connects to, or overlaps with the hallway at the joining sector, and its other end connects to the edge of the room at the open end sector.

S-Rule 20: to make a tail, one of the following rules can be applied:

*20-A: **string** single bathrooms, bedrooms, and the master bedroom with at least one edge **aligned**. The string is terminated by the master bedroom at the open end, and the hallway/corridor at the joining end.*

*20-B: **string** single bathrooms, bedrooms, and the master bedroom with at least one edge **aligned**. The string is terminated by the study room at the open end, and the hallway/corridor at the joining end.*

S-Rule 21: to place a corridor, one of these rules can be applied:

*21-A.1: The corridor **conforms** to one of the bedroom string's side edges, and the corridor's one end **connects** to the hallway/corridor at the string's conjoining end, and the other end **connects** to the edge of the room at the string's open end.*

*21-A.2: The corridor **conforms** to one of the bedroom string's side edges, and the corridor's one end **overlaps** with the hallway/corridor at the string's conjoining end, and the other end **connects** to the edge of the room at string's open end.*

Rule Representation

Wright explicitly stated that "The single bathroom, for the sake of privacy, is not immediately connected to any single bedroom" (p.174), Here, 'connected' should be read as 'directly accessible from', if I understand it correctly. The move 'connect' as we formally defined here is a different matter. As a rule, a single bedroom is connected either with another single bedroom, or master bedroom, or with the single bathroom. The master bedroom has its own bath either included within it and connected to the edges, or connected from the outside, and a private corridor is provided for access between these two rooms if they are not directly accessible through an inside door. They may well be overlapped as generic spaces.

S-Rule 22: one of the following rules can be applied to establish relations between the master bedroom and the master bathroom:

*22-A.1: the master bathroom **insert-connects** to the corner of the master bedroom.*

*22-A.2: the master bathroom **connects** to the master bedroom.*

*22-A.3: the master bathroom **overlaps** with the master bedroom.*

S-Rule 23: to place a single bathroom one of these rules can be applied:

*23-A.1: the single bathroom's side edge **connects** to the single bedroom's side edge.*

*23-A.2: the single bathroom is **separated** from the single bedroom by the corridor/hallway.*

S-Rule 24: to place a single bedroom one of these rules can be applied:

*24-A: the single bedroom's side edge **connects** to another single bedroom's side edge.*

*24-B: the single bedroom's side edge is **adjacent** to the master bedroom's side edge.*

Outside

The last set of spatial rules are about locations of the garden, terraces, planters, and trellis.

Rule Representation

Both terraces and the garden are located at the inner side of the house. One terrace should be large enough to accommodate outdoor activities and should connect to the inner side, and/or the window edges of the living room. Moreover, its directional line should conform to (i.e. be parallel to) that of the living space. Small terraces can serve as outside paths or transitional patios. Trellises are usually connected to the open ends of both the body and the tail; or to the window edges of the living room. These are all outdoor elements, we shall make it explicit that they all relate to the house from the 'outside'.

S-Rule 25: the major terrace's side edge connects to body's inner side.

S-Rule 26: the small terrace's side edge connects to any edge of the house.

S-Rule 27: to place a trellis one of the following rules can be applied:

27-A: the side edge of the trellis connects to the body's open end edge.

27-B: the side edge of the trellis connects to the tail's open end edge.

27-C: the side edge of the trellis connects to the window/door edge of the living room.

Roof

The roof defines space, and therefore we consider the roof as a space. The roof of the Usonian house is often a complex composition consisting of several layers. Basically, the lower-level roof appears at door-top height like a continuous 'deck', above that is a raised clerestory band which in turn is covered by the *top roof*. Due to different heights of rooms, and each room having a *deck roof* as well as a top roof, the result is rather complex. Usually, these layers of roofs are arranged, roughly in a descending order, in this way: the highest roof over the kitchen and fireplaces, the roof over the body, the roof over the tail, the over-head for the carport, and deck roofs.

There are reasons for laying the highest roof over the kitchen and fireplaces: "The kitchen being one of the places where smells originate, we made it the ventilating flue of the whole house by carrying it up higher than the living room. All the air from the surrounding house is thus drawn up through the kitchen itself." (p. 175) In both cases of

the body and the tail, the highest roof of each space is usually raised above its center zone, and thus provides clerestory windows which introduce light into the interior.

From the floor plan these separated roofs cannot be distinguished, still, certain rules are applied. There are two general principles involved: The roof line aligns with the wall line; and the roof line is set off the wall line toward outside with a distance which is determined by the sun orientation and natures of the space under its cover.

S-Rule : to decide the boundaries of a roof, one of these rules can apply:

28-A: The roof line conforms to the exterior wall lines with a certain distance.

28-B: The roof line coincides with the outer-most exterior wall line.

5.2-2. Material Rule System

To the design of a house, its spatial organization and material composition are always complementary. We have seen how the spatial structure of Usonian house is given by the particular selection of spatial elements; and by the arrangements of these elements according to their functional rules. Now I would like to observe the rules about organizing physical elements, which have to do with Wright's notion of the 'destruction of the box'.⁴

As we can tell from their appearance, all Usonian houses are not box-like, but they can be seen as derived from the box through a 'destruction process'. According to Wright's theory, the first destruction is to remove side walls from corners because corners are not the most economical places for positioning vertical supports. By doing this, the corner is also set free to let space in, or let go there. The second destruction is to move walls inward to a certain distance measured from the box edge because, as such, the resulting cantileverage would lessen actual spans. The third destruction is to reduce unnecessary mass of the continuous wall, and to create instead several pieces of independent 'screen' walls. "They are separate supporting screens, any one of which may be shortened, or extended, or perforated, or occasionally eliminated",⁵ and certainly can be rotated in different directions. It is interesting to find that the same principles of destruction apply not only to walls, but to the roof and the floor as well. Both the roof and the floor can be decomposed into several layers of sub-planes, each as an independent piece that moves freely.

Rule Representation

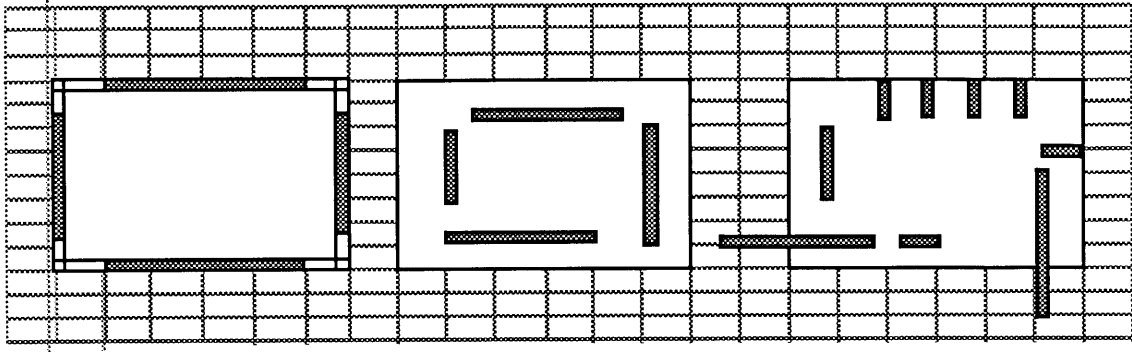


Figure 5.11.

By destroying the box, space flows. In this light we grasp the operational meaning of the words Wright frequently said: "spaces flow from inside out, and from outside in". To capture the configurational characteristics of the Usonian house, these principles of positioning walls, roofs, and de-materializing corners should be the central part of the material rule system of the type. To formulate these rules, a few words worth our attention at beginning:

First, these material rules should be used in conjunction with the spatial rules because, in the way they are made, they can not operate alone. Walls or columns are materials to define spaces, and in principle the kind of space to be defined should be conceived prior to the placement of materials. The Usonian material rules are not about what spaces are to be made at where, but about how they can be bounded by physical elements.

Second, therefore, if material rules can be used to find ways to deal with one space, they can deal with all spaces. According to S-Rule 2, all spatial elements in Usonian type are virtually rectangular, therefore box-like. The material rules must be able to destruct the box when they are applied. For space-construction (or 'destruction') purpose, we should give, according to its dimensional specifications, each space has a maximum, and a minimum boundary within which a range of possible positions are available for placing materials. (But I will not give such a range for each space here.)

Third, the material rule system is not about 'materials' per se. It is not our concern

Rule Representation

whether the wall is brick, the column is concrete, and the trellis is wood or not. I shall restrict material rules only about 'wall' elements and their connections. By wall elements we mean those physical elements used as space dividers as we can read them from the floorplan. Issues such as: wall elements may have different heights, and may consist of different kind of subelements, we do not deal with here because they do not help to define the type. Wall elements are differentiated in terms of the roles they play in the dependency hierarchy. The rules about their physical connections are based on this differentiation.

Fourth, which material element should be selected for making which space is an important concern to the Usonian type, but not our concern here. Such consideration I think belongs to another kind of 'functional' rules. We do not care whether a room is defined by level-1 elements or level-4 elements, but we do care if level-1 elements, or level-4 elements are used, there are certain rules about their connections that should be followed. By the same token, which rule should be selected for a given space so that a particular configuration can be made to accommodate windows, doors, or whatever, is still a 'functional' consideration of a different kind, which we do not give rules here.

Element Selection

M-Rule 1: 4 levels of wall elements are used: Element-A is a support wall which plays the structural role of carrying the weight of the roof. Element-B is a partition wall which separates the inside from the outside, but is not necessarily used for such separation only. Element-C is also a partition wall used only in interior. Element-D is a fence wall which does not attach to the ceiling, and is used only as a space definer either inside or outside.

Element-A is the highest level element and element-D is the lowest. Higher level elements can also play the role of lower level elements, but not the other way round. Given any Usonian house, all these elements can be identified and distinguished, because all such criteria as support bearing, inside/outside distinction, and free-standing fence are applicable without ambiguity. There are elements, however, which play roles pertaining to two or more levels, such as a wall that is not only an interior partition wall, but also a bearing wall. In such a case, we should identify it by its higher level role.

Although we can distinguish wall elements at different levels, there is a technical

Rule Representation

question: how do we define a 'piece' of wall which runs, as a physical continuum, on one level? Thanks to the orthogonal configuration of Usonian house, we can define the wall element as any straight physical string at one level, which may vary in width, length, and height. By this definition any corner must be made by at least two wall elements.

Placement on the grid

M-Rule 2: one of the following rules can be applied to place a material element on the grid:

2-A: the element has one side coincident with a grid line.

2-B: the element's central line is coincident with a grid line.

*2-C: the element is **included** within a grid band, and its side is **parallel** to the band.*

*2-D: the element's both sides are **coincident** with two adjacent grid lines of one band. (In other words, its width equals to one grid band module.)*

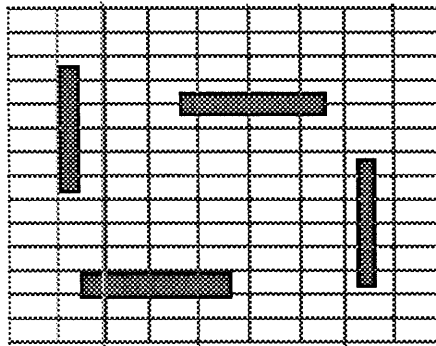


Figure 5.12.

Defining spaces

M-Rule 3: Given a spatial element with a maximum, and a minimum boundary, to place a material element in relation to these boundaries so as to define the given space, one of the following rules can be applied:

*3-A: the element can be placed in such a way that its side edge is **coincident** with any space boundary line.*

*3-B: the element can be placed in such a way that it **overlaps** with any space boundary line, but its directional line is **parallel** to the boundary.*

Rule Representation

*3-C.1: the element is **included** within the boundaries, and its directional line is **parallel** to the boundaries.*

*3-C.2: the element is **included** within the boundaries, and its directional line is **perpendicular** to the boundaries.*

*3-D.1: the element **insert-connects** to the area between max/min boundaries with its one end edge, and its directional line is **perpendicular** to the boundaries.*

*3-D.2: the element **insert-connects** to the area between max/min boundaries with both end edges, and its directional line is **perpendicular** to the boundaries.*

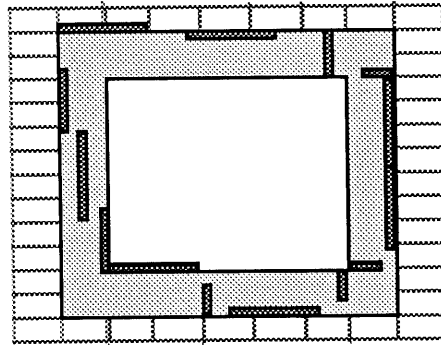


Figure 5.13.

These rules can be demonstrated by the figure shown above. Two concentric rectangles, representing the maximum and the minimum boundaries of a given space, define the margin in which wall elements can be placed in many different ways. According to these rules the space can become a conventional box, or the box can be deliberately destroyed.

Relations of connection

Here are some rules regarding relations among the elements on different levels. We only formulate rules about the relations when elements are physically connected, not about the cases when they are separated. Because if they are separated, they relate only to the space boundaries, which can be dealt with by the aforementioned M-Rule 3.

*M-Rule 4: elements of all levels can **converge** to one another at ends.*

Rule Representation

*M-Rule 5: except A-elements, elements on the same level can **abutt** to one another.*

*M-Rule 6: if two elements on different levels are engaged, the lower level element can **abutt** to higher level, but not vice versa.*

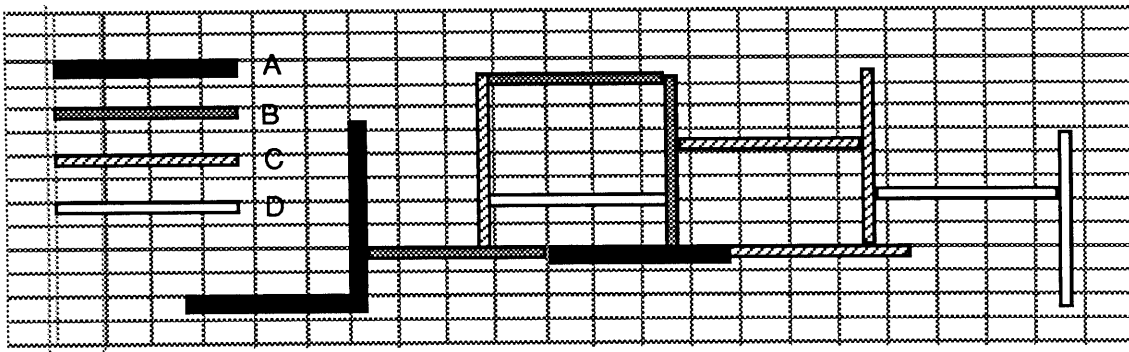


Figure 5.14.

All these connection rules about elements on different levels can be demonstrated by the figure shown above. In which, we find: a string can be made by all kinds of elements; lower level elements can abutt to the higher level; higher level elements do not abutt the lower level; the highest level elements can only converge at ends, but do not abutt; other levels can do both.

§ 5.3. Rule Testing

Now we shall see analyses of an Usonian house as to demonstrate the validity of the rules. Rosenbaum's house (1948, Alabama, see Figure 5.15) is chosen for the test case. Both the elements and the relations of the house will be identified according to the rules formulated. That is, any relation observed from the given house plan must be constructable by our rules. As a way of analysis, we will 'generate' the house piece by piece by applying the rules. The selection of a starting point will affect the selection of the rules to be applied.

Rule Representation

The same house can be generated by applying different rules, or using the same rules but in different sequences. It will be easier to arrange spaces according to the functional rules first, and then realize the spaces by the material rules. Therefore, the following demonstrations consist of two parts: in the first part, I will generate generic shapes according to the spatial rule system with fairly precise dimensional considerations. And based on this result, in the second part, I will generate the physical structure of the same house according to the material rule system.

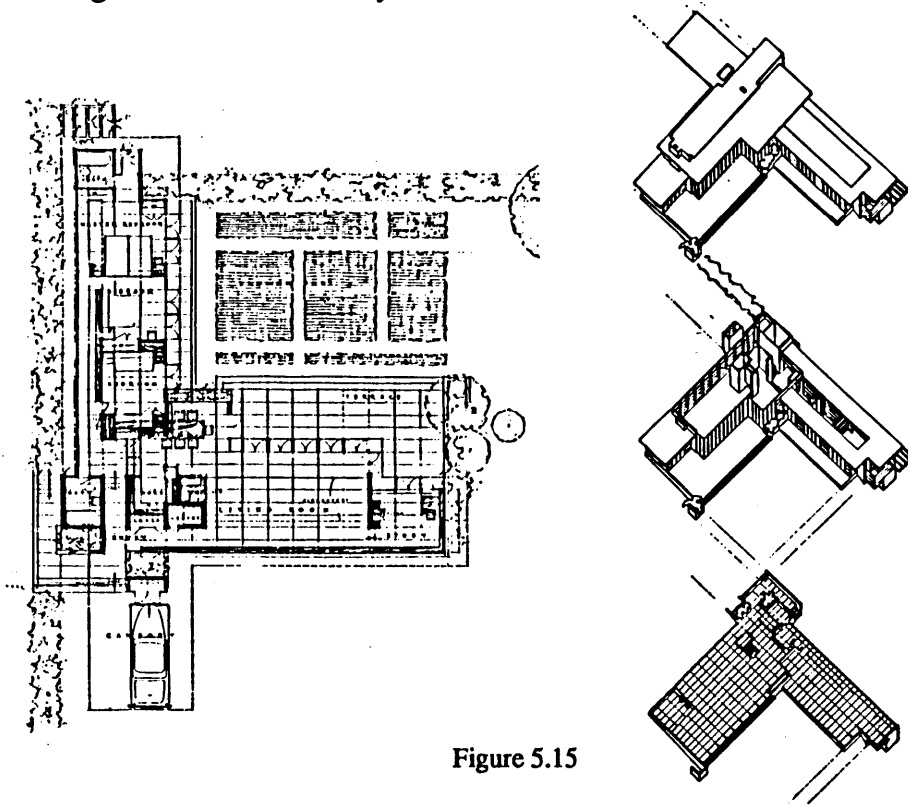


Figure 5.15

The first part of the demonstration takes 29 steps, except the initial step-0 to set up the grid and to select elements. Each step is specified basically by two acts: 1). apply a spatial rule to establish a functional arrangement. 2). apply a C-move to interpret such an arrangement. For example:

2. Kitchen § Living (S-Rule 13-C.2, Connection-2)

means that the step-2 is to relate the kitchen to the living room ('§' symbol stands for any

Rule Representation

formal relation) according to S-Rule 13-C.2, and such relation is interpreted by the move Connection-2. There are cases, however, in which a space is placed with respect to more than one other space at same time, then the step should actually make more than one move. For instance:

3. Fire § Living (S-Rule 10-A.2, insert-connect-1), and Fire § Kitchen (S-Rule 14-A, Connection-1).

Step-3 is to place the fireplace by relating it not only to the living room, but also to the kitchen. Two relations should be established in one step. Such a 'multiple-step' will be taken whenever we feel it is necessary to do so to better interpret of the arrangement.

In our demonstration of the space organization we shall start by the living space and the kitchen, which is the heart of the Usonian house, then followed by making other parts of the body, and then the tail. Terraces, planters, and trellises are added last. (Note that the roof lines will not be demonstrated here to keep the illustration simpler.)

0. Start: select spatial elements (S-Rule 1), and
 make a 2x4 grid (S-Rule 3).
1. Living § Grid (S-Rule 5-A, S-Rule 4).
2. Kitchen § Living (S-Rule 13-C.2, Connection-2).
3. Fire § Living (S-Rule 10-A.2, insert-connect-1) and
 Fire § Kitchen (S-Rule 14-A, Connection-1).
4. Utility § Kitchen (S-Rule 14-C, Overlap-3).
5. Entry § Living (S-Rule 15-B, Connection-1).
6. Closet § Entry (S-Rule 16-A.2, insert-connect-1).
7. Dining § Living (S-Rule 11-A, Overlap-3), and
 Dining § Kitchen (S-Rule 14-A, Connection-2).
8. Study § Living (S-Rule 12-A, Connection-1).
9. Fire § Study (S-Rule 10-C.1, insert-connect-1).
10. Bedroom § Hallway (S-Rule 20-A, Connection-1), and
 Tail § Body (S-Rule 6-A.2, Overlap-3).
11. Corridor § Bedroom-string (S-Rule 21-A.2, Connection-5).
12. Bath § Single-bed (S-Rule 23-A.2, Separat-1).

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- 13. Bath § Master-bed (S-Rule 22-A.2, Connection-1).
- 14. Closet § Bedroom (S-Rule 16-B.2, insert-connect-2).
- 15. Closet § Master-bed (S-Rule 16-B.1, Connection-1).
- 16. Closet § Corridor (S-Rule 16-A.1, Connection-2).
- 17. Closet § Hall (S-Rule 16-A.2, insert-connect-1).

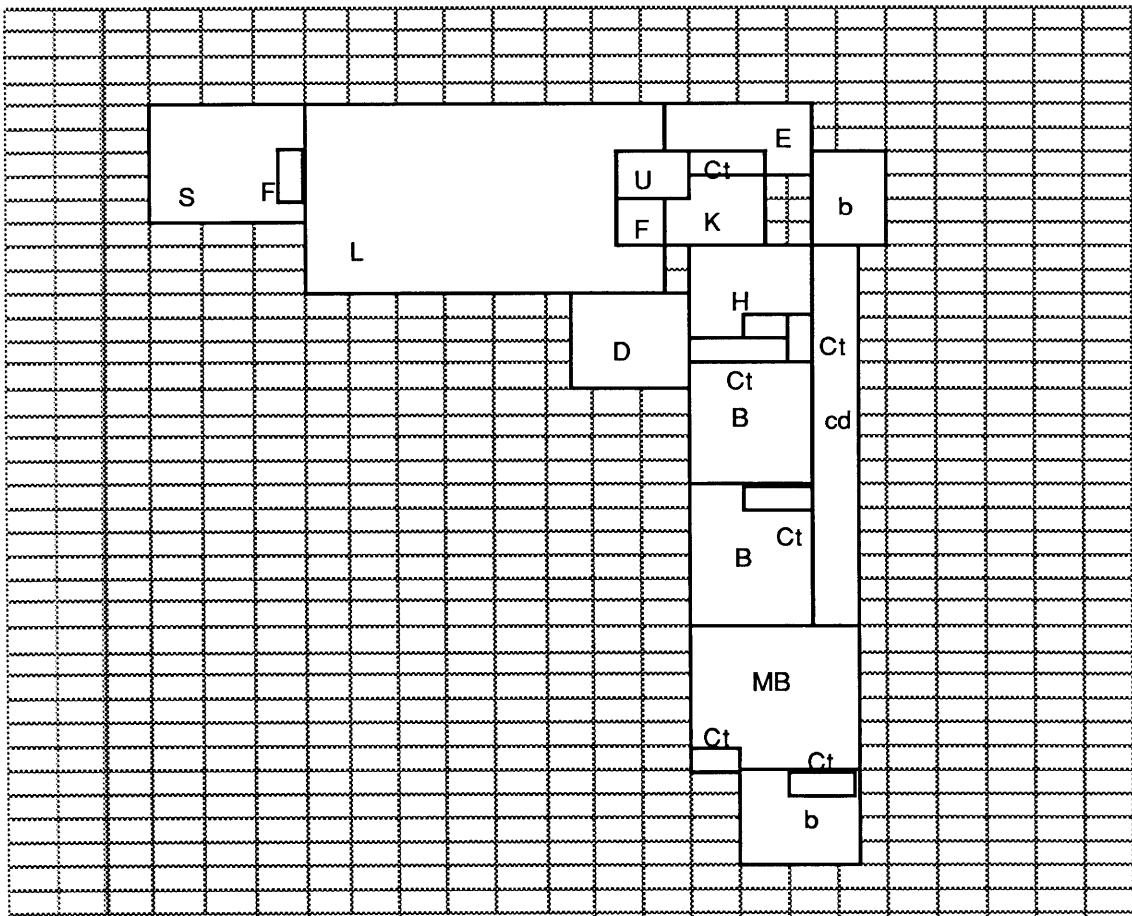


Figure 5.16.

- 18. Carport § Entry (S-Rule 19-A.1, Connection-2).
- 19. Toolroom § Carport (S-Rule 17-B.1, Inclusion-1).
- 20. Terrace-1 § Body (S-Rule 25, Connection-5).
- 21. Terrace-2 § Tail (S-Rule 26, Connection-2).

Rule Representation

25. Planter-3 § Dining (S-Rule 18-A, Connection-3).
 26. Planter-4 § Terrace (S-Rule 18-B.1, Connection-1).
 27. Trellis-1 § Tail (S-Rule 27-B, Connection-1).
 28. Trellis-2 § Body (S-Rule 27-A, Connection-1).
 29. Trellis-3 § Body (S-Rule 27-B, Connection-5).
- (End)

The second part of the demonstration is to identify the placements of physical elements and their connections according to material rules. Step-by-step analyses are not necessary in this case because once the six material rules are well understood, their applications is quite obvious. Other than defining spaces, physical elements must play a role in support of floors and roofs. I shall, therefore, proceed from identifying those elements supporting the highest roof to those supporting the lower roofs.

There are basically three layers of roofs in this particular house.⁶ The highest roof is located at the center zone of the body, naturally, it is to be supported by walls within this zone. We also find that some spaces are placed in this zone: fireplaces, kitchen, utility, closet, and bathroom, which have to be physically defined. Thus, according to M-Rule 1, we see that A-elements are selected to carry the load of the roof, and are placed to define these spaces primarily according to M-Rule 3-A. Most connections are based on M-rule 4.

The second layer roof basically covers the whole area of the body. Under the cover are those 'flowing' spaces such as living, dining, hall, and entry. Since these spaces are usually seen as an integrated whole, no sharp boundary shall be drawn between them. For the 'character' of such a space, wall elements are basically in the form of small piers located only on the periphery. Immediately below this roof are roofs which cover the study room, and the carport due to their sunken floors relative to the living room. The study is defined by placing wall elements on the boundary that supports its own roof. The toolroom is included within the carport. It is defined by those elements which can also support the cantilever roof of carport. Here we see that M-Rule 3-A and M-Rule 3-D.1 are particular used for space definitions.

Rule Representation

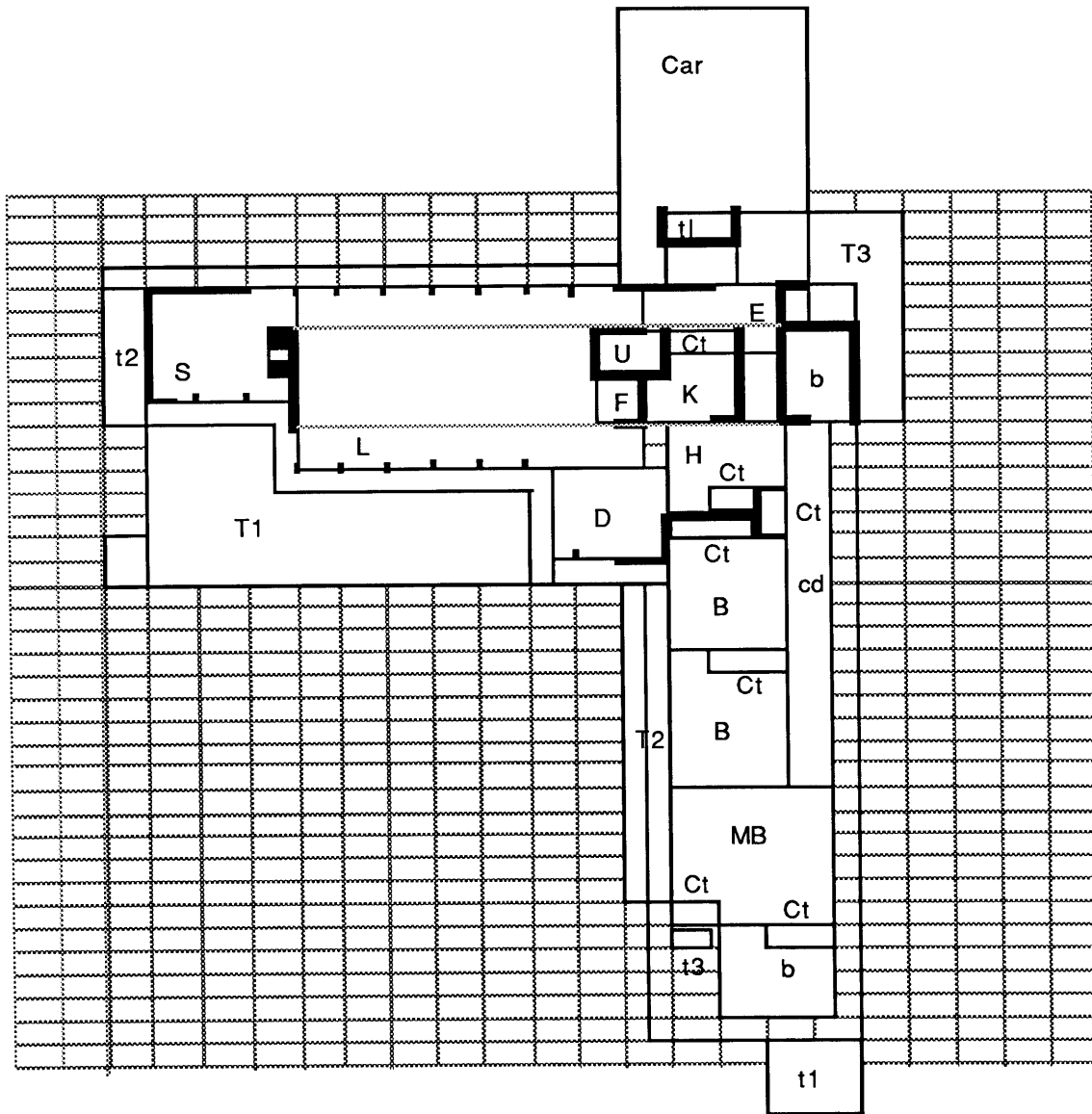


Figure 5.18.

The roof over the tail also has two layers: the deck level and the top level. Similar to the case of the body, the top roof covers the center zone of the tail, and is supported by the walls of bedrooms below. The deck roof is supported by elements primarily distributed along the outer edges of those spaces under cover. Again, the space definitions basically follow M-Rule 3-A and M-Rule 3-D.1; while physical connections are according to M-Rule 4 and M-Rule 5.

Rule Representation

Finally, we shall observe those wall elements without roof-bearing function, which are inserted primarily for the sake of defining spaces. B-elements are placed along the outer edges of the house; C-elements are used primarily for enclosing closets; and D-elements define planters and small terraces. We find that M-Rule 3, M-Rule 5, and M-Rule 6 are applied to define the spaces, and to connect to the existing wall elements.

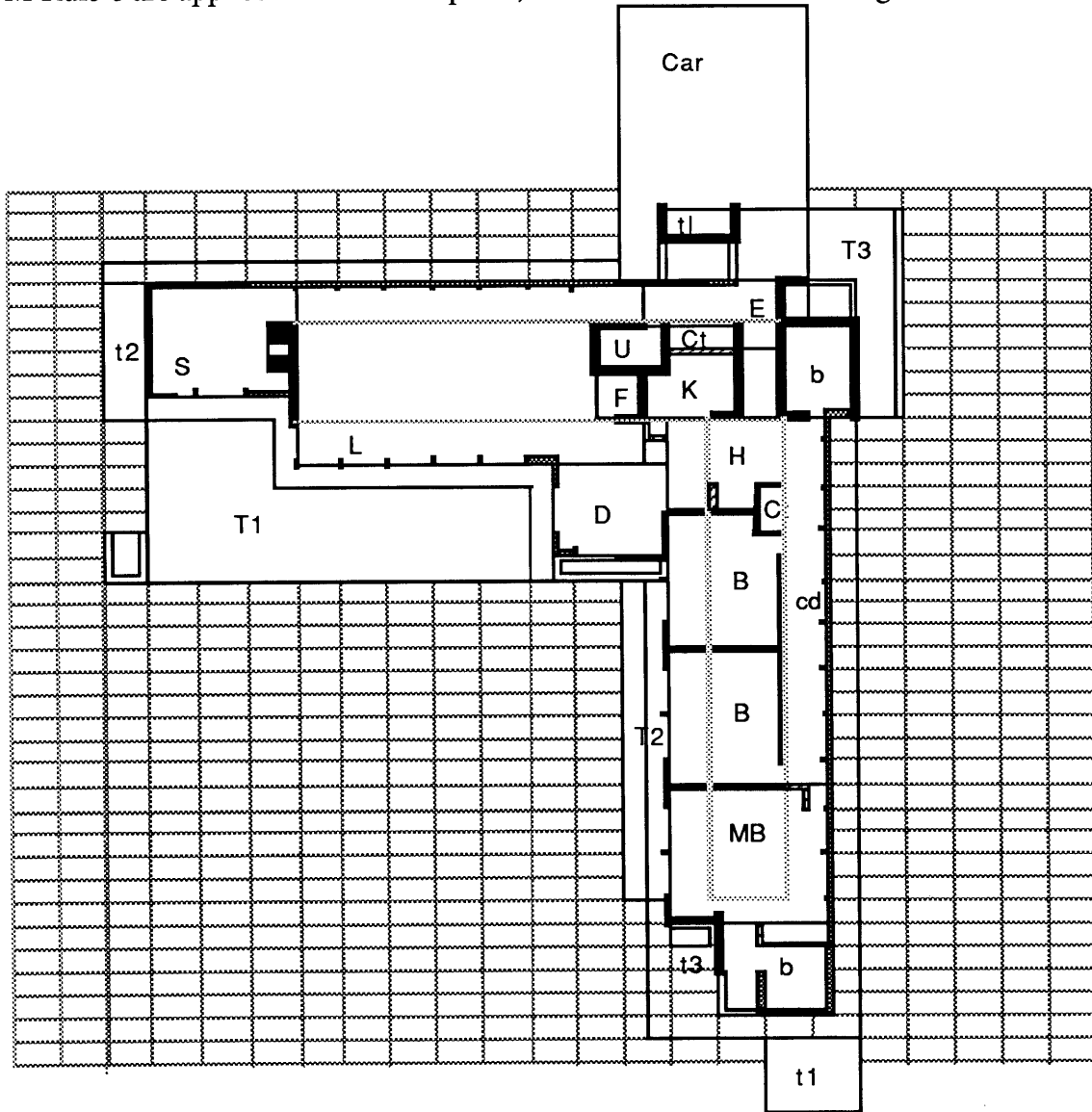


Figure 5.19.

I would like to give a few final remarks to conclude this chapter.

1). Most Usonian rules are about spatial organization in terms of quantity. Each spatial rule is basically to specify an adjacency relationsbetween a pair of selected spaces. From the given list, there are about, roughly, 20 different spaces for any Usonian house. We can expect $20 \times 20 = 400$ rules to account for a systematic description of paired relations (leaving aside many alternatives for each rule). Here we provide no more than 30 rules, and believe them to be sufficient to describe the type. Because not every space is related to every other space, but only relates to a few spaces. The house is made by the body and the tail; the body contains kitchen, living, and entry; the tail contains bedrooms and bathrooms, and so on. As mentioned earlier, this is a part-whole hierarchy. Again, we see the power of the hierarchical structure.

2). In the last section we produced an Usonian house in a step-by-step fashion, which was aimed at testing the validity of our Usonian rules formulated in terms of arrangement moves. The way of generation follows the bottom-up process that the house is the composition of many single pieces. If we understand the part-whole hierarchy of the spatial organization, we may do the same in the top-down manner as well. Because these rules do tell us how one level of cluster relates to another level of cluster, such as the body and the tail; and how small clusters can be found within a larger cluster, such as the kitchen cluster and the entry cluster within the body.

3). We must confess that knowing these Usonian rules alone is not sufficient to 'design' the house. Although in the given demonstrations the procedure of producing the house is very clear, the step-by-step procedure itself is not part of the Usonian rules, nor can it be considered as a generation process. It provides nothing more than a 'checking list' by which we examine each pair of spatial relation observed from the given case against the rules we formulate. When and how to apply what rule has to do with many other concerns, such as circulation, dimensional fitting, which have no direct bearing on a particular form type. And such rules draw on a variety of knowledge, such as structural engineering and problem-solving, which are not about formal relations. What we are describing are rules of a type of forms, not rules of designing. As mentioned at the beginning, although rules of forms are not rules of designing forms, they are nevertheless helpful to 'understand' designing.

4). The Usonian rules must first be formulated by Wright himself. Our formulation (or, interpretation) is not even the second one. About 30 years ago, House and Home magazine outlined 32 Usonian rules (they called 'ideas') to advocate this 'national' style.⁷ More recently, explicit Usonian rules have been proposed by researchers employing the linguistical model of generative grammars. These 'grammatical' Usonian rules are featured by their 'graphical' presentations and their generative formulations, which have their power, but also some drawbacks. (I will discuss this approach in more detail in Appenedix.) The grammatical Usonian rules are basically a procedural description of the type, ours are relational. We could give a procedural description of the type by arrangement moves as well, which would combine type-description and instance-generation as one set of rules. However, as we shall see in Appendix, there are reasons not to do so. After all, the main purpose of this chapter is to demonstrate the capability of arrangement moves in the business of rule formulation in general, not to advocate a particular rule type.

(End of Chapter 5)

Footnotes to Chapter 5

1. F. L. Wright, *The Natural House*, Horizon: N. Y. 1954, pp. 69. All other quotations taken from the same book will not be further specified except by giving page numbers behind the quotes.

2. Judging from his words, by 'work with style' Wright perhaps meant procedural rules rather than form rules, since he didn't consider 'a style' important. However, there is no way we can be sure about this, because there simply have too few records about his working process. On the other hand, most of his design results are well documented, from which we do see specific styles of forms, such as prairie houses and Usonian houses, among others, that have been manifested in his designs. A style can be the result of form rules, as well as procedural rules.

3. See *An American Architecture: Frank Lloyd Wright*, E. Kaufmann ed. Horizon: N.Y. 1955, pp. 82.

4. Ibid., pp. 75-85. Also see discussion by H. Allen Brooks, 'Frank Lloyd Wright and The Destruction of The Box', *Journal of Society of Architecture History*, Vol. 38, No.1, 1979.

5. See *An American Architecture: Frank Lloyd Wright*, E. Kaufmann ed. Horizon: N.Y. 1955, pp. 75-85.

6. I am particularly referring to the analytical diagram as appeared in Sergeant's article, which shows the complex roof structure of the Rosenbaum house. See John Sergeant, 'Woof and Warp: a spatial analysis of Frank Lloyd Wright's Usonian houses', *Environment and Planning B*, 1976, vol.3, pp.211-224.

7. The original article appeared in the issue of September, 1956, pp. 136-141. *House and Home Magazine*. Also see John Sergeant, *Frank Lloyd Wright's Usonian Houses: The Case For Organic Architecture*, Whitney, New York, 1976. pp. 157-158.

Chapter 6

Spatial Inference

Despite the fact that drawings can be made at any level of detail on screens, none of the present-day CAAD (Computer-Aided Architectural Design) systems understand what is drawn in the sense that machines can not 'see', and therefore, can not tell spatial relations, such as 'the loggia faces the backyard'. This chapter shows a way of building the machine's visual competence to perceive spatial relations through reasoning processes based on properties of the basic arrangement moves. The idea of this approach is that arrangement moves used for making forms are also the data needed for spatial inference. Suppose a CAAD system adopts arrangement moves as its commands for spatial manipulation, then, when moves are applied in designing, they would be, at the same time, the data needed for spatial reasoning. No additional inputs, nor data translations are required. As we know that arrangement moves are capable of describing form types, representing rules (including building codes), therefore, as long as these rules are formulated in terms of moves, and are programmed in the machine, it would be possible for the machine to check design results against rules.

§ 6.1. Test Forms Against Rules

If the design machine, such as the CAAD system, is not only the surrogate of pencil-and-paper, but must also be equipped with the capabilities of checking what designers do against what designers intend to do, or ought not to do, then, we have a specific question to be answered: how shall we make the design machine 'perceive' spatial relations established in the course of design so that they can be checked against rules it knows? In my vision, a good CAAD system should be built on the distinction that the designers do the generation tasks, and the machine checks and evaluates. We will have an intelligent computer for architectural design, for instance, if it has acquired knowledge

about building codes and types of form, so that it can check design results. To do this, the computer should be able to

- 1). provide operations that can produce any configuration with given elements,
- 2). understand rules to which design results should conform, and
- 3). be able to 'see', in the sense that it can relate design results to rules.

These three capacities can be built independently using three different sets of 'vocabulary'. The perception system may not know how configurations are constructed; the construction operations need not to concern about how rules are represented; and the program of rule representation may have no idea about how design results are 'seen' by the perception system. If these three systems have to communicate with one another to make design-testing possible, then there is a technical question about the translation of information from one system to another. For instance, if the machine's graphic operations are based on the Euclidean transformations like translation, rotation, reflection; the rules it knows are written in terms like 'at-right-of', 'on-top-of'; and its perception system uses Cartesian coordinates, then two 'interpreters' are in order: one translating between construction and perception; the other translating between perception and rules. That is, the results of construction should be translated into terms that the perception system can understand, otherwise it can not 'see' the results; and what the perception system 'sees' should be translated into terms that the rule system can understand, otherwise the results can not be checked. This also means that, whether such translations are conducted inside the machine or outside of it, a considerable amount of energy for data input and processing is needed.

Admittedly, designing interpreters to account for the task of translations is a very complex task (though the idea is straightforward). One might ask, however, would it be possible to find a set of terms that are sufficient to construct configurations, powerful enough to represent form rules, and also have perceptual capacities? The answer is that this is indeed possible. Arrangement moves are believed to be adequate to:

- 1). construct configurations, given selected elements (as demonstrated in Chapter Four),
- 2). formulate rules about spatial relations (as demonstrated in Chapter Five).

This chapter is devoted to explore the capacity of arrangement moves to meet the third requirement: 'perceiving forms'. We should make ourself clear that this is not an

attempt to study machine vision in general. Technically speaking, the perception problem we face here is the problem of data translation: how to make the message read in design results 'comparable' with the message written in rules? All messages we are concerned with are about spatial relations. As have been shown, arrangement moves are able to provide an identifying description of any given configuration, but such a description itself needs not be unique. That is, similar spatial relations can be expressed by arrangement moves in different, but equally valid ways. Then, given two descriptions, say, one read from design results, the other written as a rule, how can we judge that they are equivalent or not? This is the kind of question we must deal with.

Let me propose an approach. Given two descriptions, D1 and D2, expressed by arrangement moves. If both D1 and D2 describe the same given configuration, and D1 and D2 are two different descriptions in the sense that they use different moves, then these two sets of moves must have certain properties of 'equivalence'. We know that D1 and D2 are equivalent simply because D1 and D2 refer to the same given configuration, not because we know *a priori* that these two set of moves are equivalent. However, If we make such equivalences of moves as 'a priori', we can 'logically deduce' that D1 is equivalent to D2 merely from their descriptive sentences without necessarily referring to the 'real' configuration any more. This is the way to reach the heart of the matter.

We do find, indeed, that there are some invariable relations among the basic arrangement moves that can be used to infer spatial relations, and therefore, can serve as the building blocks to construct the machine's perception system. Based on the input moves, the machine, if it knows these inference rules, would be able to tell much more spatial relations than those merely input by moves.

§ 6.2. Propositional Calculus for Arrangement Moves

In what follows, we shall explore some a priori relations among basic arrangement moves that can constitute spatial inference rules. We shall limit ourselves only to formulate such inference rules in terms of the commutative, transitive, 'projective', and distributive

relations. For the sake of expressing these rules economically, each move is denoted by a symbol.

$A + B$ A connects to B.

$A >< B$ A separates from B.

$A >> B$ A is in relation inside-separation to B.

$A >| B$ A is in relation inside-connection to B

$A \# B$ A overlaps with B

$A \neg B$ A and B are in a string.

$A \parallel B$ A is aligned with B.

$A \setminus B$ A is coincident with B.

$A * B$ A converges to B.

$A -| B$ A abutts to B.

$A \times B$ A crosses B.

$A \approx B$ A conforms to B.

6.2-1. Commutative

As we have seen, relations between two elements A and B can be symmetrical, or asymmetrical. 'A conforms to B' is equivalent to 'B conforms to A'; 'A connects to B' is identical to 'B connects to A'. Such a symmetrical relation is known as the commutative property of an operation. But moves like 'containment', 'abuttment', and are asymmetrical. For instance, 'A insert-connects to B' does not mean that 'B insert-connects to A', and 'A is included in B' is not the same as 'B is included in A'. For more clear presentations, let's make distinctions:

- 1). Let 'A is included in B' be named as 'A inserts in B', denoted $A >> B$. In other words, B includes A, denoted $B << A$.
- 2). In the same way, if B insert-connects to A, denoted $B >| A$, it is tantamount to say that 'A include-connects to B', denoted $A |< B$.
- 3). If A abutts to B, denoted $A -| B$, then from B's standpoint, we shall say 'B borders A', denoted $B |- A$.

Some commutative relations of generic moves and ordering moves are found as follows (each rule is numbered with the letter 'C' that stands for 'commutative'):

- C1. $A + B = B + A$ (Connection is commutative)
- C2. $A \gg B = B \gg A$ (Separation is commutative)
- C3. $A \# B = B \# A$ (Overlap is commutative)
- C4. $A \ll B \neq B \ll A$ (inclusion is not commutative)
- C5. $A |< B \neq B |< A$ (Include-connection is not commutative)
- C6. $A \gg B \neq B \gg A$ (Insertion is not commutative)
- C7. $A >| B \neq B >| A$ (Insert-connection is not commutative)
- C8. $A \ll B = B \gg A$ (Inclusion and insertion are exchangeable)
- C9. $A |< B = B >| A$ (Insert-connection and include-connection are exchangeable)
- C10. $A \neg B = B \neg A$ (Simple string is commutative)
- C11. $A || B = B || A$ (Alignment is commutative)
- C12. $A \setminus B = B \setminus A$ (Coincidence is commutative)
- C13. $A * B = B * A$ (Convergence is commutative)
- C14. $A -| B \neq B -| A$ (Abuttment is not commutative)
- C15. $A |- B \neq B |- A$ (Border is not commutative)
- C16. $A -| B = B |- A$ (Abuttment and border are exchangeable)
- C17. $A \times B = B \times A$ (Cross is commutative)
- C18. $A \approx B = B \approx A$ (Conformation is commutative)

6.2-2. Transitive

Now let's consider some transitive relations among arrangement moves, which ask: if $A \S B$, and $B \S C$; does the relation $A \S C$ hold? We find that inclusion and insertion are the only two moves that are transitive in generic moves, and they are also exchangeable. (But note that two other exchangeable generic moves inside-connection and include-connection are not always transitive.) In ordering moves, only alignment, coincidence, and conformation hold transitive relations. The transitivity of all other arrangement moves we have formulated so far cannot be determined by themselves without additional information.

(Each rule is numbered with the letter 'T' that stands for 'transitive'.)

- T1.** $A \ll B, B \ll C, \longrightarrow A \ll C$ (Inclusion is transitive)
- T2.** $A \gg B, B \gg C, \longrightarrow A \gg C$ (Insertion is transitive)
- T3.** $A \parallel B, B \parallel C, \longrightarrow A \parallel C$ (Alignment is transitive)
- T4.** $A \setminus B, B \setminus C, \longrightarrow A \setminus C$ (Coincidence is transitive)
- T5.** $A \approx B, B \approx C, \longrightarrow A \approx C$ (Conformation is transitive)

6.2-3. Projective

If $A \S B$, and $A \partial C$, in which \S and ∂ are two different moves, then what relation might exist between B and C ? This is known here as the 'projective' relation because no directly associated relation between B and C is in the given statements. Many projective relations can be derived from given relational statements in 'negative' form, which will be denoted by attaching symbol '~' in front of the relation. That is, whether C is related to B as such is uncertain without further information, but that C is NOT related to B as such is logically determinable. For instance, If A connects to B ; and A also inserts in C , then it is quite certain that B can not include or include-connect to C ; nor can B connect to, or separate from C . But B might overlap, cross, insert in, or insert-connect to C . Projective relations always involve three elements, and two different moves among them. These rules can be systematically scrutinized, and the results provide bases for translations between two equivalent, but differently expressed spatial relations. (Again, the letter 'P' associated with rule numbers stands for 'projective'.)

P1. If $A \gg B$,

and $A \gg C$, nothing can be ascertained.

(P1.1) and $A + C, \longrightarrow B \sim \ll C, B \sim |< C.$

(P1.2) and $A \# C, \longrightarrow B \sim \ll C, B \sim |< C.$

(P1.3) and $A >| C, \longrightarrow B \sim \ll C, B \sim |< C.$

(P1.4) and $A \gg C, \longrightarrow B \sim \ll C, B \sim |< C.$

(P1.5) and $A |< B, \longrightarrow B \gg C.$

(P1.6) and $A \ll B, \longrightarrow B \gg C.$

P2. If $A + B$,

and $A + C$, nothing can be ascertained.

(P2.1) and $A >< C$, $\rightarrow B \sim >> C$, $B \sim >| C$.

(P2.2) and $A \# C$, $\rightarrow B \sim << C$, $B \sim |< C$.

(P2.3) and $A >| C$, $\rightarrow B \sim >< C$, $B \sim << C$, $B \sim |< C$.

(P2.4) and $A >> C$, $\rightarrow B \sim >< C$, $B \sim + C$, $B \sim << C$, $B \sim |< C$.

(P2.5) and $A |< B$, $\rightarrow B >< C$, $B + C$.

(P2.6) and $A << B$, $\rightarrow B >< C$.

P3. If $A \# B$,

and $A \# C$, nothing can be ascertained.

(P3.1) and $A >< C$, $\rightarrow B \sim >> C$, $B \sim >| C$.

(P3.2) and $A + C$, $\rightarrow B \sim >> C$, $B \sim >| C$.

(P3.3) and $A >| C$, $\rightarrow B \sim >< C$, $B \sim + C$, $B \sim << C$, $B \sim |< C$.

(P3.4) and $A >> C$, $\rightarrow B \sim >< C$, $B \sim + C$, $B \sim << C$, $B \sim |< C$.

(P3.5) and $A |< B$, $\rightarrow B >< C$, $B + C$.

(P3.6) and $A << B$, $\rightarrow B >< C$.

P4. If $A >| B$,

(P4.1) and $A >< C$, $\rightarrow B \sim >> C$, $B \sim >| C$.

(P4.2) and $A + C$, $\rightarrow B \sim >< C$, $B \sim >> C$, $B \sim >| C$.

(P4.3) and $A \# C$, $\rightarrow B \sim >< C$, $B \sim + C$, $B \sim >> C$, $B \sim >| C$.

(P4.4) and $A >| C$, $\rightarrow B \sim >< C$, $B \sim + C$, $B \sim << C$, $B \sim >> C$.

(P4.5) and $A >> C$, $\rightarrow B \sim >< C$, $B \sim + C$, $B \sim << C$, $B \sim |< C$.

(P4.6) and $A |< B$, $\rightarrow B << C$, $B |< C$.

(P4.7) and $A << B$, $\rightarrow B << C$.

P5. If $A >> B$,

(P5.1) and $A >< C$, $\rightarrow B \sim >> C$, $B \sim >| C$.

(P5.2) and $A + C$, $\rightarrow B \sim >< C$, $B \sim + C$, $B \sim >> C$, $B \sim >| C$.

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- (P5.3) and $A \# C$, $\rightarrow B \sim > < C$, $B \sim + C$, $B \sim >> C$, $B \sim > | C$.
(P5.4) and $A > | C$, $\rightarrow B \sim > < C$, $B \sim + C$, $B \sim > | C$, $B \sim >> C$.
(P5.5) and $A >> C$, $\rightarrow B \sim > < C$, $B \sim + C$.
(P5.6) and $A | < B$, $\rightarrow B << C$, $B | < C$.
(P5.7) and $A << B$, $\rightarrow B << C$.

P6. If $A | < B$,

and $A | < C$, nothing can be ascertained.

- (P6.1) and $A > < C$, $\rightarrow B > < C$.
(P6.2) and $A + C$, $\rightarrow B > < C$, $B + C$.
(P6.3) and $A \# C$, $\rightarrow B \sim << C$, $B \sim | < C$.
(P6.4) and $A > | C$, $\rightarrow B > | C$, $B >> C$.
(P6.5) and $A >> C$, $\rightarrow B >> C$.
(P6.6) and $A << B$, $\rightarrow B \sim >> C$, $B \sim > | C$.

P7. If $A << B$,

and $A << C$, nothing can be ascertained.

- (P7.1) and $A > < C$, $\rightarrow B > < C$.
(P7.2) and $A + C$, $\rightarrow B > < C$.
(P7.3) and $A \# C$, $\rightarrow B \sim << C$.
(P7.4) and $A > | C$, $\rightarrow B >> C$.
(P7.5) and $A >> C$, $\rightarrow B >> C$.
(P7.6) and $A | < B$, $\rightarrow B \sim << C$, $B \sim | < C$.

If these projective rules of generic moves are displayed in a 7x7 matrix, we will find that they are symmetrically distributed if all relations of insertion and inside-connection are converted to relations of inclusion and include-connection from one side to the other side along the diagonal axis of the matrix. We also notice that except the overlap relation, all other generic relations can be 'projectively' derived.

Let's now look at projective relations among ordering moves. Since ordering moves usually refer to specified references, let's denote $A \langle e \rangle$, in which $\langle e \rangle$ is the name of the

reference defined on element A. Again, most projective rules of ordering moves are expressed in negative form too, but they are even more symmetrical distributed along the diagonal axis of the matrix, on which if they are displayed.

P8. If $A\langle e \rangle \times B\langle e \rangle$,

(P8.1) and $A\langle e \rangle \setminus C\langle e \rangle \rightarrow B\langle e \rangle \sim \setminus C\langle e \rangle, B\langle e \rangle \sim \parallel C\langle e \rangle, B\langle e \rangle \sim^* C\langle e \rangle, B\langle e \rangle \sim \approx C\langle e \rangle.$

(P8.2) and $A\langle e \rangle \parallel C\langle e \rangle \rightarrow B\langle e \rangle \sim \setminus C\langle e \rangle, B\langle e \rangle \sim \parallel C\langle e \rangle, B\langle e \rangle \sim^* C\langle e \rangle, B\langle e \rangle \sim \cdot C\langle e \rangle, B\langle e \rangle \sim \approx C\langle e \rangle.$

(P8.3) and $A\langle e \rangle * C\langle e \rangle \rightarrow B\langle e \rangle \sim \setminus C\langle e \rangle, B\langle e \rangle \sim \parallel C\langle e \rangle.$

(P8.4) and $A\langle e \rangle \cdot C\langle e \rangle \rightarrow B\langle e \rangle \sim \setminus C\langle e \rangle, B\langle e \rangle \sim \parallel C\langle e \rangle.$

(P8.5) and $A\langle e \rangle \approx C\langle e \rangle \rightarrow B\langle e \rangle \sim \setminus C\langle e \rangle, B\langle e \rangle \sim \parallel C\langle e \rangle, B\langle e \rangle \sim \approx C\langle e \rangle.$

(P8.6) and $A\langle e \rangle \times C\langle e \rangle \rightarrow B\langle e \rangle \sim \setminus C\langle e \rangle, B\langle e \rangle \sim \parallel C\langle e \rangle.$

(P8.7) and $A\langle e \rangle \vdash C\langle e \rangle \rightarrow B\langle e \rangle \sim \cdot C\langle e \rangle, B\langle e \rangle \sim^* C\langle e \rangle, B\langle e \rangle \sim \parallel C\langle e \rangle.$

P9. If $A\langle e \rangle \setminus B\langle e \rangle$,

(P9.1) and $A\langle e \rangle \setminus C\langle e \rangle \rightarrow B\langle e \rangle \setminus C\langle e \rangle.$

(P9.2) and $A\langle e \rangle \parallel C\langle e \rangle \rightarrow B\langle e \rangle \parallel C\langle e \rangle.$

(P9.3) and $A\langle e \rangle * C\langle e \rangle \rightarrow C\langle e \rangle \dashv B\langle e \rangle, \text{ or } C\langle e \rangle * B\langle e \rangle.$

(P9.4) and $A\langle e \rangle \dashv C\langle e \rangle \rightarrow B\langle e \rangle \dashv C\langle e \rangle, \text{ or } B\langle e \rangle \times C\langle e \rangle.$

(P9.5) and $A\langle e \rangle \approx C\langle e \rangle \rightarrow B\langle e \rangle \approx C\langle e \rangle.$

(P9.6) and $A\langle e \rangle \times C\langle e \rangle \rightarrow B\langle e \rangle \sim \setminus C\langle e \rangle, B\langle e \rangle \sim \parallel C\langle e \rangle, B\langle e \rangle \sim^* C\langle e \rangle, B\langle e \rangle \sim \approx C\langle e \rangle.$

(P9.7) and $A\langle e \rangle \vdash C\langle e \rangle \rightarrow B\langle e \rangle \sim \setminus C\langle e \rangle, B\langle e \rangle \sim \parallel C\langle e \rangle, B\langle e \rangle \sim \cdot C\langle e \rangle, B\langle e \rangle \sim \times C\langle e \rangle, B\langle e \rangle \sim \approx C\langle e \rangle.$

P10. If $A\langle e \rangle \parallel B\langle e \rangle$,

(P10.1) and $A\langle e \rangle \setminus C\langle e \rangle \rightarrow B\langle e \rangle \parallel C\langle e \rangle.$

(P10.2) and $A\langle e \rangle \parallel C\langle e \rangle \rightarrow B\langle e \rangle \parallel C\langle e \rangle.$

- (P10.3) and $A\langle e \rangle * C\langle e \rangle \rightarrow B\langle e \rangle \sim\setminus C\langle e \rangle, B\langle e \rangle \sim\parallel C\langle e \rangle,$
 $B\langle e \rangle \sim\approx C\langle e \rangle, B\langle e \rangle \sim\text{---} C\langle e \rangle.$
- (P10.4) and $A\langle e \rangle \text{---} C\langle e \rangle \rightarrow B\langle e \rangle \sim\setminus C\langle e \rangle, B\langle e \rangle \sim\parallel C\langle e \rangle,$
 $B\langle e \rangle \sim\approx C\langle e \rangle, B\langle e \rangle \sim^* C\langle e \rangle.$
- (P10.5) and $A\langle e \rangle \approx C\langle e \rangle \rightarrow B\langle e \rangle \approx C\langle e \rangle.$
- (P10.6) and $A\langle e \rangle X C\langle e \rangle \rightarrow B\langle e \rangle \sim\setminus C\langle e \rangle, B\langle e \rangle \sim\parallel C\langle e \rangle,$
 $B\langle e \rangle \sim^* C\langle e \rangle, B\langle e \rangle \sim\text{---} C\langle e \rangle, B\langle e \rangle \sim\approx C\langle e \rangle.$
- (P10.7) and $A\langle e \rangle \text{---} C\langle e \rangle \rightarrow B\langle e \rangle \sim\setminus C\langle e \rangle, B\langle e \rangle \sim\parallel C\langle e \rangle,$
 $B\langle e \rangle \sim^* C\langle e \rangle, B\langle e \rangle \sim\text{---} C\langle e \rangle, B\langle e \rangle \sim\text{---} C\langle e \rangle,$
 $B\langle e \rangle \sim\approx C\langle e \rangle.$

P11. If $A\langle e \rangle * B\langle e \rangle,$

- (P11.1) and $A\langle e \rangle \setminus C\langle e \rangle \rightarrow B\langle e \rangle \text{---} C\langle e \rangle,$ or $B\langle e \rangle * C\langle e \rangle.$
- (P11.2) and $A\langle e \rangle \parallel C\langle e \rangle \rightarrow B\langle e \rangle \sim\setminus C\langle e \rangle, B\langle e \rangle \sim\parallel C\langle e \rangle,$
 $B\langle e \rangle \sim\approx C\langle e \rangle, B\langle e \rangle \sim\text{---} C\langle e \rangle. .$
- (P11.3) and $A\langle e \rangle * C\langle e \rangle \rightarrow C\langle e \rangle * B\langle e \rangle.$
 (if converge at the same point).
- (P11.4) and $A\langle e \rangle \text{---} C\langle e \rangle \rightarrow B\langle e \rangle \text{---} C\langle e \rangle.$
 (if abut at the same point).
- (P11.5) and $A\langle e \rangle \approx C\langle e \rangle \rightarrow B\langle e \rangle \sim\setminus C\langle e \rangle, B\langle e \rangle \sim\parallel C\langle e \rangle,$
 $C\langle e \rangle \sim\approx B\langle e \rangle.$
- (P11.6) and $A\langle e \rangle X C\langle e \rangle \rightarrow B\langle e \rangle \sim\setminus C\langle e \rangle, B\langle e \rangle \sim\parallel C\langle e \rangle.$
- (P11.7) and $A\langle e \rangle \text{---} C\langle e \rangle \rightarrow B\langle e \rangle \sim\setminus C\langle e \rangle, B\langle e \rangle \sim\parallel C\langle e \rangle,$
 $B\langle e \rangle \sim^* C\langle e \rangle, B\langle e \rangle \sim\text{---} C\langle e \rangle, B\langle e \rangle \sim\text{---} C\langle e \rangle.$

P12. If $A\langle e \rangle \text{---} B\langle e \rangle,$

- (P12.1) and $A\langle e \rangle \setminus C\langle e \rangle \rightarrow C\langle e \rangle \text{---} B\langle e \rangle,$ or $C\langle e \rangle X B\langle e \rangle.$
- (P12.2) and $A\langle e \rangle \parallel C\langle e \rangle \rightarrow B\langle e \rangle \sim\setminus C\langle e \rangle, B\langle e \rangle \sim\parallel C\langle e \rangle,$
 $B\langle e \rangle \sim\approx C\langle e \rangle, B\langle e \rangle \sim^* C\langle e \rangle.$
- (P12.3) and $A\langle e \rangle * C\langle e \rangle \rightarrow C\langle e \rangle \text{---} B\langle e \rangle.$
 (if converge at the same point).

(P12.4) and $A\langle e \rangle \text{---} | C\langle e \rangle \text{---} \rightarrow B\langle e \rangle \setminus C\langle e \rangle$, or $B\langle e \rangle \times C\langle e \rangle$.
(if abut at the same point).

(P12.5) and $A\langle e \rangle \approx C\langle e \rangle \text{---} \rightarrow B\langle e \rangle \sim \setminus C\langle e \rangle$, $B\langle e \rangle \sim || C\langle e \rangle$,
 $C\langle e \rangle \approx B\langle e \rangle$.

(P12.6) and $A\langle e \rangle \times C\langle e \rangle \text{---} \rightarrow B\langle e \rangle \sim \setminus C\langle e \rangle$, $B\langle e \rangle \sim || C\langle e \rangle$.

(P12.7) and $A\langle e \rangle \text{---} | - C\langle e \rangle \text{---} \rightarrow B\langle e \rangle \sim \setminus C\langle e \rangle$, $B\langle e \rangle \sim || C\langle e \rangle$.

P13. If $A\langle e \rangle \approx B\langle e \rangle$,

(P13.1) and $A\langle e \rangle \setminus C\langle e \rangle \text{---} \rightarrow C\langle e \rangle \approx B\langle e \rangle$.

(P13.2) and $A\langle e \rangle || C\langle e \rangle \text{---} \rightarrow B\langle e \rangle \approx C\langle e \rangle$.

(P13.3) and $A\langle e \rangle * C\langle e \rangle \text{---} \rightarrow C\langle e \rangle \sim \setminus B\langle e \rangle$, $C\langle e \rangle \sim || B\langle e \rangle$,
 $C\langle e \rangle \approx B\langle e \rangle$.

(P13.4) and $A\langle e \rangle \text{---} | C\langle e \rangle \text{---} \rightarrow C\langle e \rangle \sim \setminus B\langle e \rangle$, $C\langle e \rangle \sim || B\langle e \rangle$,
 $C\langle e \rangle \approx B\langle e \rangle$.

(P13.5) and $A\langle e \rangle \approx C\langle e \rangle \text{---} \rightarrow B\langle e \rangle \approx C\langle e \rangle$.

(P13.6) and $A\langle e \rangle \times C\langle e \rangle \text{---} \rightarrow B\langle e \rangle \sim \setminus C\langle e \rangle$, $B\langle e \rangle \sim || C\langle e \rangle$,
 $B\langle e \rangle \approx C\langle e \rangle$.

(P13.7) and $A\langle e \rangle \text{---} | - C\langle e \rangle \text{---} \rightarrow B\langle e \rangle \sim \setminus C\langle e \rangle$, $B\langle e \rangle \sim || C\langle e \rangle$,
 $B\langle e \rangle \approx C\langle e \rangle$.

6.2-4. Distributive

If the expression $A \S (B \partial C)$ means element A is in spatial relation \S to a compound form of $(B \partial C)$, what are the spatial relations we can tell between A and B, A and C? This is the question concerning the distributive properties of arrangement moves. Let's begin with a few conventions and definitions. In the following expressions, $A = \{B, C, D\}$ means that A is a compound form made by elements B, C, D.

Notation: $X = \{A, B, C\}$, X is a form consisting of elements A, B, and C, let @X represents any non-empty sub-set of X.

For instance, @X could be A, or B, or C, or {A, B}, or {A, C}, or {B, C}, or {A, B, C}. To put it in the general form, @{Fi, i=1, 2,...n}, which means any one of the possible non-empty combinations of Fi.

We use parentheses () to group elements and relations into sub-sets. For instance, the expression A § (B § (C ∂ D)) presents different levels of assembly: (C ∂ D) is a configuration, (B § (C ∂ D)) is a larger configuration, and (A § (B § (C ∂ D))) is an even larger configuration.

The logical connective 'and', denoted '&', is also used for connecting a set of relations to express the description of a compound configuration. Note that the symbol & is only used between two strings of relations, it should not be used to link singular elements. For instance, the expression X & Y is acceptable if both X and Y are compound forms, X = (A ∂ B ∂ C), Y = (L § M § N). That is: X & Y = (A ∂ B ∂ C) & (L § M § N). If X and Y are singular elements that will not be further decomposed, then the expression X & Y can always be eliminated.

Formula-1: $(A § B) \& (B § C) \& (C § D) \& \dots = A § B § C § D § \dots$

That is, we can rewrite a group of pairly-related elements in the form of a string of relations. For instance, given a compound configuration made of elements A, B, C, D,..., in which A connects to B; and B connects to C; and C connects to D; etc., this compound configuration can be straightly expressed as (A + B) & (B + C) & (C + D) &...., and according to our definition, this is equivalent to the expression A + B + C + D + ...

Now, we can formulate two inference rule concerning the distributive relation:

D1: Two forms related, X § Y, and X is a compound configuration, X = (A ∂ B ∂ C ∂ ...), § and ∂ can be any basic move, then

$$\begin{aligned} X \text{ § } Y &= (A \text{ ∂ } B \text{ ∂ } C \text{ ∂ } \dots) \text{ § } Y \\ &= (A \text{ ∂ } B \text{ ∂ } C \text{ ∂ } \dots) \text{ \& } (@X \text{ § } Y). \end{aligned}$$

D2: Two forms related, X § Y, and Y is a compound configuration, Y = (A ∂ B ∂ C ∂ ...), § and ∂ can be any basic move, then

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$$\begin{aligned} X \S Y &= X \S (A \partial B \partial C \partial \dots) \\ &= (X \S @Y) \& (A \partial B \partial C \partial \dots) \end{aligned}$$

For instance, $(A + B + C + \dots) + D = (A + B + C + \dots) \& (@\{A, B, C, \dots\} + D)$. That is, if D is to connect to a group of connected elements, it could be connected to any one of its sub-group.

If $X \S Y$, in which both X and Y are groups of elements, say $X = (A \partial B \partial C \partial \dots)$; $Y = (L \partial' M \partial' N \partial' \dots)$, then according to Rule D1 and D2:

$$\begin{aligned} X \S Y &= (A \partial B \partial C \partial \dots) \S Y \\ &= (A \partial B \partial C \partial \dots) \& (@X \S Y) \\ &= (A \partial B \partial C \partial \dots) \& (@X \S (L \partial' M \partial' N \partial' \dots)) \\ &= (A \partial B \partial C \partial \dots) \& (@X \S @Y) \& (L \partial' M \partial' N \partial' \dots) \\ &= (A \partial B \partial C \partial \dots) \& (L \partial' M \partial' N \partial' \dots) \& (@X \S @Y) \end{aligned}$$

This result can be expressed as:

Formula-2: $X \S Y = X \& Y \& (@X \S @Y)$, in which $X = (A \partial B \partial C \partial \dots)$; $Y = (L \partial' M \partial' N \partial' \dots)$, and \S, ∂, ∂' can be any basic move.

Now, in more general sense, if there are a string of compound forms $P_1 \S P_2 \S P_3 \S P_4 \S \dots P_n$ relating to one another, each compound configuration is made up by a set of related elements: $P_j = (X_{j1} \partial X_{j2} \partial X_{j3} \partial \dots)$, $j = 1, 2, 3, \dots, n$, then according to Formula-1 and Formula-2:

$$\begin{aligned} P_1 \S P_2 \S P_3 \S P_4 \S \dots &= (P_1 \S P_2) \& (P_2 \S P_3) \& (P_3 \S P_4) \& (P_4 \S \dots) \\ &= (P_1 \& P_2 \& (@P_1 \S @P_2)) \& (P_2 \& P_3 \& (@P_2 \S @P_3)) \& \\ &\quad (P_3 \& P_4 \& (@P_3 \S @P_4)) \& (P_4 \& \dots) \end{aligned}$$

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$$\begin{aligned}
 &= P_1 \& P_2 \& (@P_1 \& @P_2) \& P_2 \& P_3 \& (@P_2 \& @P_3) \& \\
 &\quad P_3 \& P_4 \& (@P_3 \& @P_4) \& P_4 \& \dots \\
 &= P_1 \& P_2 \& P_3 \& P_4 \& \dots \& (@P_1 \& @P_2) \& \\
 &\quad (@P_2 \& @P_3) \& (@P_3 \& @P_4) \& (@P_4 \& \dots) \& \dots
 \end{aligned}$$

This expression remains generally true if § is any one of the generic moves or ordering moves. Therefore, Rule D3:

$$\text{D3. } P_1 \& P_2 \& P_3 \& P_4 \& \dots = P_1 \& P_2 \& P_3 \& P_4 \& \dots \& (@P_1 \& @P_2) \& (@P_2 \& @P_3) \& (@P_3 \& @P_4) \& (@P_4 \& \dots) \& \dots$$

in which $P_j = (X_{j1} \& X_{j2} \& X_{j3} \& \dots)$, $j = 1, 2, 3, \dots, n$, and

$$\& = \{+, >, >>, >|, <<, <|, \#, X, \backslash, ||, *, -|, |- , \approx\}$$

This rule can be useful to parse a complex expression into simple statements connected by the logical connection &. Let's take an arbitrary expression: $(A + B >> C) >> (D >| E) << F \# G$. Suppose this expression intends to describes a configuration Δ made of elements A, B, ...G. To examine whether the description is true or false we can parse the expression into a string of paired relations, and then examine the truth value of each paired relation. According to Formula-1 and Rule D3:

$$\begin{aligned}
 (A + B >> C) >> (D >| E) << F \# G &= ((A + B >> C) >> (D >| E)) \& \\
 &\quad ((D >| E) << F) \& (F \# G) \\
 &= (A + B >> C) \& (D >| E) \& \\
 &\quad (@\{A,B,C\} >> @\{D,E\}) \& \\
 &\quad (D >| E) \& (@\{D,E\} << F) \& (F \# G) \\
 &= (A + B) \& (B >> C) \& (D >| E) \& \\
 &\quad (@\{A,B,C\} >> @\{D,E\}) \& \\
 &\quad (@\{D,E\} << F) \& (F \# G)
 \end{aligned}$$

That is, a string of moves and elements can always be parsed into paired relations connected by the sign '&'. The left-hand side expression is true as a description of the configuration Δ only if the right-hand side is also true.

Given a compound form X, in some cases $@X$ is identifiable by the natures of the moves involved. For instance:

$$\begin{aligned}(A \gg B) + C &= (A \gg B) \& (@\{A, B\} + C) \\ &= (A \gg B) \& (B + C)\end{aligned}$$

If $(A \S B \S C \S \dots) + P$, and $\S = \{ \gg, >|, \ll, |<, \}$, then we can identify $@\{A, B, C, \dots\}$ in this expression with respect to the move '+' and to the form P. Because moves like insertion, insert-connection, inclusion, and include-connection, will yield one, and only one containing enclosure, which is determinable. For such a group of forms $(A \S B \S C \S \dots)$, the move 'connection' can relate only to one form which is the containing enclosure.

For an expression like $(A \S B \S C \S \dots) \S P$, we find that it can be even more specific in determining $@\{A, B, C, \dots\}$, i.e. the non-empty sub-set of $\{A, B, C, \dots\}$, if \S represents any one of the ordering moves, than if \S represents any generic move. For instance:

$$\begin{aligned}(A\langle e \rangle X B\langle e \rangle -| C\langle e \rangle) \approx D\langle e \rangle &= (A\langle e \rangle X B\langle e \rangle) \& (B\langle e \rangle -| C\langle e \rangle) \& \\ &(@\{A\langle e \rangle, B\langle e \rangle, C\langle e \rangle\} \approx D\langle e \rangle)\end{aligned}$$

For $@\{A\langle e \rangle, B\langle e \rangle, C\langle e \rangle\} \approx D\langle e \rangle$, we know that $D\langle e \rangle$ must 'conform' to one, and only one of $\{A\langle e \rangle, B\langle e \rangle, C\langle e \rangle\}$. Which one? Let's see a way by which it can be determined.

- 1). If $D\langle e \rangle \approx A\langle e \rangle$, then it is impossible that $D\langle e \rangle \approx C\langle e \rangle$, since according to Rule 8.4, we know that if $(A\langle e \rangle X B\langle e \rangle) \& (B\langle e \rangle -| C\langle e \rangle)$, then, either $A\langle e \rangle \sim \setminus C\langle e \rangle$, or $A\langle e \rangle \sim || C\langle e \rangle$ will be true.
- 2). It is certainly that $D\langle e \rangle \approx B\langle e \rangle$, because $A\langle e \rangle X B\langle e \rangle$.
- 3). By the same token, if $D\langle e \rangle \approx B\langle e \rangle$, then it is impossible that $D\langle e \rangle$ will conform to $A\langle e \rangle$; nor $C\langle e \rangle$. And so on so forth.

That is, $@\{A\langle e \rangle, B\langle e \rangle, C\langle e \rangle\}$ is usually identifiable with respect to the ordering move it

operates.

The spatial inference rules as presented above can be proved with formal rigor as 'theorems' if an axiomatic system is provided, but such proofs need separate investigations of their own.

§ 6.3. Examples

I would like to demonstrate the inference abilities of the arrangement moves by a few examples. The spatial perception capacities that can be provided by arrangement moves at this stage of exploration are basically three kinds:

- 1). Given two descriptions of relations among same elements, are they equivalent?
- 2). If A relates to B; B relates to C; C relates to D; etc., can we tell the relations between any two elements that are not directly related as the given? Such as, A connects to B, B aligns with C, C inserts in D, what are the relations that might exist between A and C, or B and D?
- 3). Given a configuration, if a new element is to be added to establish a new relation to the configuration, can we 'infer' what moves should this element make?

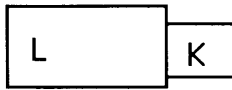
The first category is already demonstrated by those commutative rules. Therefore, two examples will suffice to demonstrate spatial reasoning of the other two categories. We shall use only simple cases from the construction of the Usonian house according to the Usonian spatial rules formulated in the last chapter.

Let's see the first example. Given a configuration, how to infer spatial relations of the unknown from the known? Suppose three moves are made to set up relations between Kitchen(K) and Living(L); Bath(B) and Kitchen; and Fireplace(F) and Living in the design of Usonian house. That is:

$(K + L: Lr\backslash Kl, d(Lt Kt)=d1)$

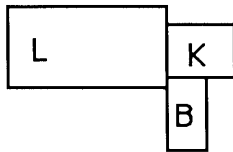
(i.e. connect Kitchen to Living, L's right edge coincident with K's left edge, and the distance from the top of L to the top of K equals to d1.)

Spatial Inference



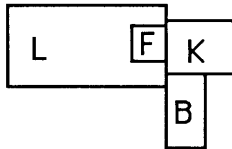
(B + K: $B \wedge K_b, B_l \parallel K_l$)

(i.e. connect Bath to the Kitchen, B's top edge coincident with K's bottom, and B's left edge aligns with K's left).



(F >| L: $F_r \setminus L_r, d(F_b L_b) = d_2$)

(i.e. insert-connect Fireplace to Living, F's right edge coincident with L's right edge, and the distance between the bottom of F to the bottom of L equals to d_2).



Now, we should ask what are the relations between:

- 1). L and B?
- 2). F and B?
- 3). F and K?

In the follows we will demonstrate that these three relations can be determined purely on the basis of 'reasoning' from input statements without necessarily 'seeing' the configurations.

1). Relation between L and B.

$B_l \parallel K_l, L_r \setminus K_l$ are known.

According to commutative properties: $B_l \parallel K_l = K_l \parallel B_l$;

$L_r \setminus K_l = K_l \setminus L_r$.

Spatial Inference

According to Rule-P9.2: if $Kl \setminus Lr$, and $Kl \parallel B1$, then $Lr \parallel B1$, that is, Living's right edge aligns with Bath's left edge.

2). Relation between F and B.

Since $Fr \setminus Lr$ is known, $Lr \parallel B1$ is also known from 1), and it is commutative that $Fr \setminus Lr = Lr \setminus Fr$.

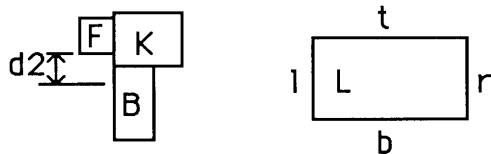
According to Rule-P10.1: if $Lr \parallel B1$, and $Lr \setminus Fr$, then $Fr \parallel B1$, that is, Fireplace's right edge aligns with Bath's left edge.

3). Relation between F and K.

$Fr \setminus Lr$, $Lr \setminus Kl$ are known, and $Fr \setminus Lr = Lr \setminus Fr$.

According to Rule-P9.1: if $Lr \setminus Kl$, and $Lr \setminus Fr$, then $Fr \setminus Kl$, that is, Fireplace's right edge coincides with Kitchen's left edge, therefore, Fireplace connects to the Kitchen.

Let's see another example which asks: if such and such relations are to be held, what move should be made? Using Usonian design again, suppose K connects to F; B connects to K, and we know $Fr \setminus Kl$, $Kl \parallel B1$, $Kb \setminus Bb$. If L is a rectangle with given dimensions, and is to be place in such a way that L will connect to K, and include F, and $d(Fb \setminus Lb) = d2$, then what is the move of L? The move seems apparent to the human designer, but it can be inferred through a long course of reasoning according to the inference rules. The process is the follows:



First, we should determine generic move.

a). Since F is to be included in L, there are only two generic moves for L to achieve this goal, either $L \ll F$, or $L \lfloor F$.

Spatial Inference

b). If $L \ll F$, and $L + K$, then, according to Rule-P7.2, $F \succ K$. This result, F separates from K , contradicts to the given fact that $Fr \setminus Kl$, i.e. F connects to K .
Therefore, $L \ll F$ is not the right move.

c). If $L \mid F$, and $L + K$, then, according to Rule-P6.2, either $F \succ K$, or $F + K$. From above we know that $F \succ K$ is not right, but $F + K$ is possible.
Therefore, $L \mid F$ is the right generic move.

Second, we should determine ordering move, if possible.

d). By the definition of $L \mid F$, it means $L\langle e \rangle \setminus F\langle e \rangle$. Since $L + K$, let $L\langle e \rangle \setminus K\langle e \rangle$, in which $\langle e \rangle$ is any edge of the right, left, top, bottom.

Now the task is to determine which edge of L should connect to which edge of F and K .

e). According to Rule-P9.1: $L\langle e \rangle \setminus F\langle e \rangle$, and $L\langle e \rangle \setminus K\langle e \rangle$, then $F\langle e \rangle \setminus K\langle e \rangle$.

Since $Fr \setminus Kl$ is known, we can assign $F\langle e \rangle = Fr$, and $K\langle e \rangle = Kl$. Now the move will be: $(L \mid F: L\langle e \rangle \setminus Fr, L\langle e \rangle \setminus Kl, \text{ and } d(Fb \setminus Lb) = d2)$.

But, still the question as what is $L\langle e \rangle$ should be answered.

f). Since $d(Fb \setminus Lb) = d2$, it means $Fb \approx Lb$. From the property of the rectangle, we know $Fb \sim \setminus Fr$. Therefore, $Lb \sim \setminus Fr$. And we know that $Lt \approx Lb$, therefore, $Lt \sim \setminus Fr$.

So, $L\langle e \rangle \neq Lb$, and $L\langle e \rangle \neq Lt$.

g). Now, we have two choices: either $L\langle e \rangle = Lr$, or $L\langle e \rangle = Ll$.

If $L\langle e \rangle = Ll$, $Ll \setminus Fr$, then $L \sim \setminus F$. This result contradicts to the previous result $L \mid F$.

h). Therefore, $L\langle e \rangle = Lr$. That is, the correct move is $Lr \setminus Fr$, and $Lr \setminus Kl$.

In conclusion, the move for L will be

$(L \mid F: Lr \setminus Fr, Lr \setminus Kl, d(Fb \setminus Lb) = d2)$.

I like to conclude this chapter with two short remarks. First, these inference rules are about B-moves, but many rules may be written in C-moves as we have shown in the last

Spatial Inference

chapter, then, are these inference rules applicable in those cases? I think that there is no difficulty regarding this matter, except we should take trouble to parse those C-moves in terms of B-moves. Alternatively, we may seek establishing inference rules at the level of C-moves. Second, if the machine knows also geometrical theorems, its capabilities for spatial inferences would certainly increase beyond what the arrangement moves can render here. For instance, when one circle 'contains' another circle and their boundaries are 'conformed', then it can infer that these two circles' centers must be 'coincident'. It is certainly plausible and advisable to increase the machine's knowledge base to increase its perceptual power for visual inference, but it is simply beyond the scope of this study.

(End of Chapter 6)

Appendix

Review of Shape Grammars

'Every house worth considering as a work of art must have a grammar of its own. "Grammar", in this sense, means the same thing in any construction - whether it be of words or of stone or wood. It is the shape-relationship between the various elements that enter into the constitution of the thing' -- Frank Lloyd Wright¹

The idea that buildings are architecturally structured like sentences is hardly new. Serious research based on this view is, however, only done recently. The most representative study along this line is the work done on 'shape grammars', which stands out as another approach to deal with something very close to what the arrangement moves do. This appendix is a short comparison of this approach with the one presented here concerning such topics as form description, rule representation, and type recognition.

§ A.1. Shape Grammars

Shape grammars deal with 'shapes', shapes are representations of physical forms. As a formal generation process, a shape grammar basically consists of two sets of rules: spatial construction rules, and rules of transformation. This division seems to follow Chomsky's proposal in 1965, later known as the Standard Theory, in which syntactic structures essentially consist of two set of rules: the base rules that generate the 'deep structure', and the transformation rules that apply to the base rules and yield the 'surface structure'. The spatial construction rules of the shape grammar are made up by the following components.

- 1). A set of labelled shapes. A labelled shape is a shape with a set of associated points called 'spatial labels', usually by symbol • (and the symbol of a small circle) to indicate how, where, and when to apply rules.
- 2). A set of rules concerning the spatial relations between labelled shapes.
- 3). A set of 'state labels' (not 'spatial labels') associated with rules, not with shapes, to

indicate the state a shape must be in when a rule is applied. All state labels are denoted by natural numbers, or by the symbol F (the final state symbol).

4). an initial shape (denoted by symbol 'I') must be defined.

The spatial transformational rules consist of two components: 1). shape change rules that transform an initial set of spatial relations into a final set of spatial relations, and 2). state change rules that transform an initial set of state label pairs into a final set of state label pairs. (Examples of grammatical rules will be given later when we represent the Usonian grammar by arrangement moves.)

Shape grammars were developed by Stiny and Gips in the early 70's with many technical borrowings from linguistic theories and other related research.² There has not been any fundamental change in shape grammars since its inception, although applied research on different cases has expanded considerably over the last decade. Changes in shape grammars are primarily about terminology and classification. For instance, 'standard shape grammars' and 'parametric shape grammars' are distinguished in the sense that the former have fixed spatial relations; the latter allow for dimensional variations by using 'state' labels to exercise external controls. Those external controls and transformational rules are recently developed as one system of instructions called 'description functions', which are claimed to be the mechanisms to ascribe meaning to designs, and can translate grammars from one design to another (Stiny, 1985).

§ A.2. Usonian Shape Grammar

Rules about Usonian houses have also been formulated by shape grammars, which include both Frank Lloyd Wright's earlier prairie houses (Koning and Eizenberg, 1981) and his later Usonian houses (Knight, 1983). Knight argues that the later Usonian rules are results of transformations of the earlier prairie rules, and he deliberately demonstrates such grammatical transformations. 26 spatial construction rules are proposed as the design language of Wright's prairie houses. 17 shape change rules and 19 state change rules are used to transform the prairie grammar into 18 Usonian grammatical rules. Taking the fireplace as the initial shape, these 18 rules for Usonian grammar are made of 3 rules about

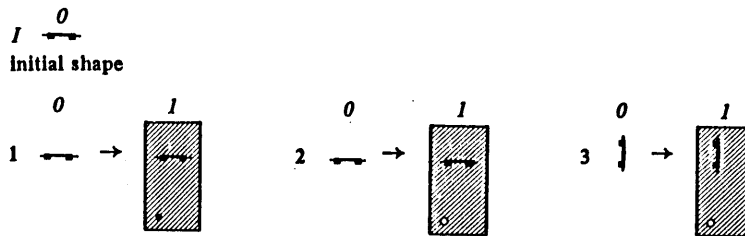
adding the living (the body) to the fireplace; 5 rules about adding the bedrooms (the tail) to the living to form the basic shape, called 'core unit', of the house; and 10 rules about optional 'extensions' (which mean spaces like workshop, study room, family room that can be converted into bedroom in the future) both to the living wing or the bedroom wing. In addition to these 'core unit' rules for the general layout of the Usonian house, Knight also formulates 24 'ornamentation rules' for more detailed treatment. This ornamentation set has 2 rules about adding spaces to the 'hinge' of the core unit; 8 rules about the ways two shapes overlap; 7 rules about adding secondary fireplaces in any spaces; 4 rules for adding spaces to the extensions; and finally, 3 rules about indenting exterior corners concerning the 'destruction of the box'.

In total, the core unit rules plus the ornamentation rules, there are 42 rules that constitute the Usonian grammar. All rules are, of course, graphically presented with some labels. Some rules are purely about adding, extracting, and changing labels (such as Rule 11, 12, 17, 18, 19, 29, 30, 36), which have nothing to do with the essences of Usonian houses, but are simply modelling techniques. And some pairs of rules are identical, therefore one is redundant if we ignore the differences of labels (such as Rule 1, 2; Rule 9, 10; and Rule 17, 18).

Can these grammatical rules be represented in terms of arrangement moves? Here we attempt to represent Usonian grammar only as a set of form rules, not to account for its generative mechanism, which can be modelled, I believe, by a separate set of procedural rules. Those redundant, label-manipulation grammatical rules will not be dealt with, since they have nothing to do with Usonian house whatsoever. To represent the Usonian grammar, we will first interpret what each grammatical rule means, and then to translate it in terms of arrangement moves (in *italic*). We will also look through our Usonian rules to find, and to list within the parentheses, if there exists any one corresponding to the grammatical rule under interpretation.

To show how this work is done, I like to give an example. The Usonian grammar starts with the fireplace, the first three rules are about different ways of placing the fireplace in the living space. (See Figure below). The initial shape is a 'fireplace'. The grammar Rule 1 and Rule 2 are spatially identical except they use different spatial labels (the black dot and the white dot at corners of rectangles). The numbers, 0 and 1, on the

top of shapes are 'state labels'. Both rules say that if the fireplace is in state-0, then it can change to state-1 by applying rules. The grammar Rule 3 is an alternative to place the fireplace in different direction. We can express, for instance, the first two graphical grammar rules by A-moves in the sentence as this: 'The fireplace *inserts* in the living space at its end sector in such a way that the side edge of the fireplace *conforms* to the end edge of the Living, and the fireplace's opening *faces* the inside of the living.'



Corresponding to these first two grammatical rules, the S-Rule 10-A.1 in Chapter Five states: 'the fireplace is *included* in the living room's end sector, and its opening edge *faces* the inside of the living room.' In our Usonian rule system, such a description as 'the side edge of the fireplace *conforms* to the end edge of the Living' is redundant because, since all spatial elements are rectangular (S-Rule 2), and they are all placed on the grid orthogonally, (S-Rule 3 and S-Rule 4), their edges will inevitably conform to one another. This example shows the way of representing the Usonian grammar in terms of arrangement moves, and the results are given in the follows.

REPRESENTATION OF USONIAN GRAMMAR BY ARRANGEMENT MOVES

Shape Grammar Rule 1 and Rule 2: The fireplace *inserts* in the living space at its end sector in such a way that the side edge of the fireplace *conforms* to the end edge of the Living, and the fireplace's opening *faces* the inside of the living. (S-Rule 10-A.1)

Shape Grammar Rule 3: The fireplace *inserts* in the living space at its end sector in such a way that the side edge of the fireplace *conforms* to the side edge of the Living, and the fireplace's opening *faces* the inside of the living. (Note that it is not clear to judge from the shape grammar rule whether the fireplace's opening faces the

inside of the living, or faces toward outside. But in our formulation, it must face the inside.)

(S-Rule 10-B.1)

Shape Grammar Rule 4: the bedroom wing (the 'tail') *overlaps* with the living space (the 'body'), and its side edge *aligns* with the end edge of the living.

(S-Rule 6-A.2)

Shape Grammar Rule 5: the bedroom wing (the 'tail') *connects* to the living space (the 'body'), and its side edge *aligns* with the end edge of the living.

(S-Rule 6-A.1)

Shape Grammar Rule 6: the bedroom wing (the 'tail') *separates* from the living space (the 'body'), and its side edge *aligns* with the end edge of the living.

(No S-Rule deals with the separated body and tail.)

Shape Grammar Rule 7: the bedroom wing (the 'tail') *connects* to the living space (the 'body'), and its end edge *aligns* with the side edge of the living.

(S-Rule 6-A.1)

Shape Grammar Rule 8: the bedroom wing (the 'tail') *separates* from the living space (the 'body'), and its end edge *aligns* with the side edge of the living.

(No S-Rule deals with the separated body and tail.)

Shape Grammar Rule 9. to 18: an additional room such as the family, the study, the workshop, can *connect* either to the open end of the body, or to the tail, with its side edge *aligned* with the side edge of the body, or the tail. (Note that Rule 11, 12, 17, 18, are label changing rules, and the rest can be stated as one rule.)

(S-Rule 12-A, and S-Rule 20-B)

Shape Grammar Rule 19. & 20: the study, or the workshop can *connect* to the outer corner of the joining sector of the living space and the bedroom wing, and have one of its edge *aligned* with the outer side of the house.

(No such a S-Rule)

Shape Grammar Rule 21. to 25: any two adjacent spaces can be *overlaped* with one edge *aligned*. (To change the relation from connection to overlapping)

(No such a S-Rule, but the effects can be achieved by M-Rule 3)

Shape Grammar Rule 26. to 28: any two adjacent spaces can be *connected* with one edge *aligned*. (To change relations from overlapping to connection)

(No such a S-Rule, but the effects can be achieved by M-Rule 3)

Shape Grammar Rule 29 to 35: a secondary single-hearth or double-hearth fireplace can *inside-connect* to any room, and its opening should *face* the inside of the room.

(S-Rule 10-C.1, the secondary fireplace only used in the study room.)

Shape Grammar Rule 36 to 39: an additional space *connects* to the side of the extended space (such as the workshop, the family, the study, and the future bedroom) with its edge *aligned* with the end edge of the extended space.

(No such a S-Rule, but the effects can be achieved by M-Rule 3)

Shape Grammar Rule 40 to 42: two exterior corner edges 'change directions' (i.e. *in different directions* with respect to their original directions) toward inside of the room, then they *converge*, or *abut* to each other.

(No such a S-Rule, but the effects can be achieved by M-Rule 3)

As far as formal relations are concerned, all these grammatical rules can be represented by arrangement moves. For most grammatical rules we can also find corresponding Usonian rules of our formulation, except a few disagreements about the essence of the style (e.g. Rule 6, Rule 8.), which is not at all an unusual thing. There are many grammatical rules pertaining to a sort of 'transformation' rules, such as Rule 21 to Rule 28, and Rule 36 to Rule 42. All these transformations can be dealt with by Usonian M-Rule 3 represented by arrangement moves, which is about the 'destruction of the box'. In grammatical representations, we find that 'connection' is the most important generic relation, and 'alignment' is the often used ordering move. Notably, many rules in our formulations are not observed by the Usonian shape grammar. As we can see from demonstrations, the Usonian grammar seems to be more interested in simulating the shape outlines of Usonian houses, and not much in the spatial relations among individual rooms or spaces. But we shall not discuss this any further, since the primary goal for this section is to test the descriptive power of arrangement moves for representing the contents of the grammatical rules, not to debate the validity, or the fidelity of the Usonian grammar per se.

§ A.3. Representing Constraints

First of all, many constraints concerning the qualities of form as those given in building codes, or design requirements, can hardly be properly represented by the formalism of shape grammar, simply because these constraints are either 'shapeless', or are applicable to all kinds of shape. For instances, 'A must be in front of B', 'all rooms must be accessible'. Most designs must deal with constraints as such, and eventually must be

checked by them. If these constraints can't be known in the first place, they can't be checked. Of course, shape grammars can offer a shape rule about 'A is in front of B', but this is possible only when the shapes of both A and B are known, which is unlikely if A and B are something yet to be designed; and can be of any shape. Here we see that a 'name' is much more powerful than a 'picture'.

One might argue that description functions in shape grammars can serve the purpose of representing certain general spatial relations. But the description function, as the way it is formulated almost like a 'footnote' to the grammatical rule, has no independent function without associating with a shape rule. In the case of the arrangement move representation, as long as we can define the spatial relation 'in-front-of' by arrangement moves, the constraint 'A must be in front of B' can be easily formulated in a single sentence without giving any 'shape interpretation', which would be a 'design' by itself. Giving constraints and designing are separate things.

§ A.4. Computational Capabilities

Shape grammars are explicit algorithms that perform arithmetic calculations with shapes, but there are no good reasons for them to be explicit as such. One would like to think that explicit shape grammar rules are ready for computational implementation, or application, but this is an illusion, I am afraid. The fact is that perceiving 'shape' by machine is far more complicated than one would expect. It is a formidable task to interpret these graphical rules into computable representations. Although a shape grammar interpreter has finally been developed (Krishnamurti, 1981), spatial ambiguities cause much trouble for implementation on computers. To meet this difficulty, an alternative, less ambiguous (and also less powerful) formalism had to be made for easy computations (Stiny, 1982a). There is also an attempt to relate shape grammars with semantical processing (Gero, 1985), the result is not very promising.³

Due to the difficulty that one shape can be interpreted in many different ways, the authors of shape grammars establish a theoretical connection between the concept of shape and the logician's idea of 'individual', which is a definite entity without definite parts

(Goodman, 1951; Stiny, 1982b). Not surprisingly, sum, difference, and product, are three operations both for Stiny's arithmetic of shapes and for Goodman's calculus of individuals. But this philosophical connection is not helpful. Shape ambiguity is not a virtue, but a difficulty as far as computational interpretation is concerned. And the problem, after all, lies in the picturesque presentation all shape grammars adopt.

§ A.5. Recognizing an Instance of a Style

Undoubtedly, the beauty of the shape grammar, or any other generative grammar, is that it not only grasps the structure of a style, but can also generate many valid variations of it. But then the question is: how can the shape grammar of a style be used to tell whether a given configuration is an instance of the style or not? One might expect this to be very easy, but again, this is an illusion. We find that the generative formalism of shape grammars is not very advantageous in this, as well as in many other aspects. Let me explain. To tell an instance of a style can be seen as a task of pattern-recognition, which consists of three subtasks: a). Analyze the given form into elements. b). Observe spatial relations among these elements. c). Check these elements and relations against the rules describing the type of the form in question. For shape grammars and alike, these problems mean the following: a). Identify primitives from the given form. b). Infer grammatical rules from identified primitives. c). Construct these primitives into structural description according to the grammar.

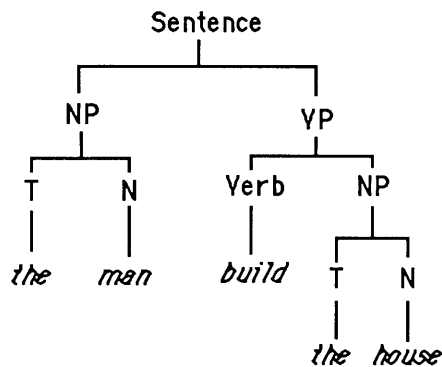
If such a structural description can be constructed by grammatical rules, the form is recognized as an instance of the style; otherwise it is not. The process of constructing a structural description according to grammars is called 'parsing'. Although parsing is technically complicated, the most difficult part of the problem is still to identify the primitives of the grammar from the given form (Shaw, 1972, 1973). If the construction 'history' of the form is known, the identification of primitives is no problem when we use arrangement moves, because such identifications are automatically established in the course of construction. Since a shape is an 'individual' which has a definite entity without definite parts, it is difficult for shape grammars to define proper primitives so that

grammatical rules can be applied. To be fair to the comparison, let's say all spatial primitives are given.

Before we examine the problem of recognizing an instance of a style by shape grammars, something must be said beforehand about two important features of shape grammars: the picture-like presentation of rules, and the generative-mode of formulation of rules. As a rule-governed activity, the recognition of pictures in many ways resembles the determination of correct sentences. The determination of the grammatical correctness of a given sentence requires the parsing of the sentence according to syntactic rules. Parsing produces the structural description of sentence, i.e. it presents how words in a given sentence relate to one another. For example, the diagram below is the result of parsing the sentence "The man builds the house".

The parsing uses the following rules⁴:

1. Sentence \rightarrow NP + VP
2. NP \rightarrow T + N
3. VP \rightarrow Verb + NP
4. T \rightarrow the
5. N \rightarrow man, house, car, etc.
6. Verb \rightarrow build, paint, design, etc.



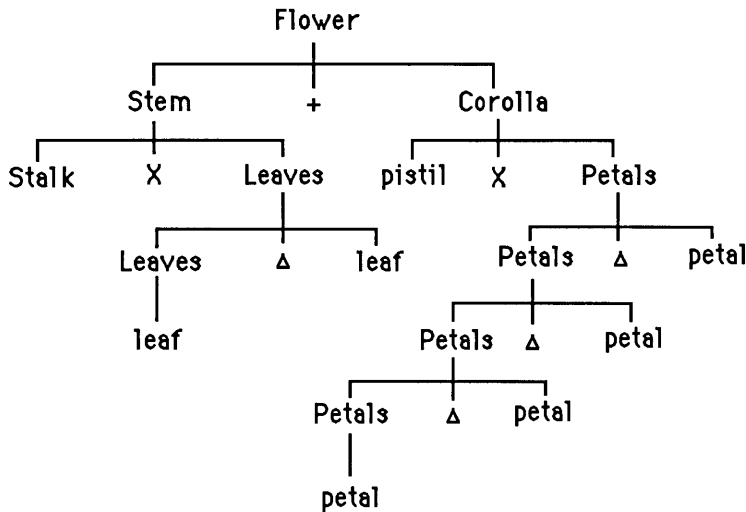
Note that to change from 'build' to 'builds' a transformation rule has to be applied. We may generate a sentence like "the car paints the man", which is nonsense, but grammatically correct.

We like to draw attentions to two important features of this grammatical formalism - the rewriting format and the concatenation operation. The power of the generative grammar lies in the recursive property of the rewriting format, $X \rightarrow Y$, which means "rewrite X as Y" (in context-free grammars). The only important operation to establish relations among a string of symbols is 'concatenation', represented by the plus sign '+', and occasionally by the hyphen '-', or simply a wider spacing. (Chomsky, 1957, pp.109) Since linguistic models intend to represent utterances, i.e. strings of 'sounds' arranged in temporal sequences, the only important relation for any string of utterance is the 'next-of' relation. Therefore, although grammatical rules themselves are complex, the relational operation for constructing sentences is extremely simple: to link one word next to the other, that is, concatenation. Unfortunately, spatial relations are much more complicated. In dealing with spatial relations, picture grammars have to find ways of defining spatial operations. For instance, Shaw (1973) gives a very simple grammar describing the image of a flower uses three spatial relations: *connect around the perimeter*, by symbol x; *connect to the same entity*, by symbol Δ ; *connect at the end*, by symbol +. The flower grammar and the structural description of a flower as the result of parsing are given as below. (The sign '/' means 'or', also see Figure 2.1. in Chapter 2).

1. Flower \rightarrow Stem + Corolla
2. Stem \rightarrow Stalk x Leaves
3. Leaves \rightarrow leaf / Leaves Δ leaf
4. Corolla \rightarrow pistil x Petals
5. Petals \rightarrow petal / Petals Δ petal

Picture grammars are grammars about pictures, but grammars themselves need not be picture-like, and in many cases they better not be picture-like. For instances, the 'flower grammar' is about the construction of the 'picture' we call flower, but all rules are a string of symbols made up by 'names' of elements and 'names' of relations, which are not pictures. The pictorial presentation has its visual economy (one picture is worth a thousand words), but it become cumbersome when pictorial rules have to be described in propositional terms to represent constraints, or for computation, as mentioned above. The

advantage of non-picture presentations is that all rules can be processed as propositions without taking trouble to interpret visual data about spatial relations. But none of the rules in shape grammars is expressed in terms similar to those in the 'flower' grammar which uses no 'shape' at all.



As result, shape grammars have no clear way of expressing the parsing of a given form, which is essential to syntactical approach to form-recognition. The spatial label • attached to shapes in shape grammars is not a spatial relation, it is only used as a mark to indicate the location for shape connection. For instance, we see the label • at the lower-left corner of the living space in the Usonian shape grammar Rule 1 (See the Figure above), the fireplace is positioned with respect to this reference, and when another space connects to the living, it is also positioned with respect to this reference. The meaning of this lable is similar to the distinction of the 'opening end', the 'joining zone', etc. In a sense, such a pictorial rule of shape grammar only visually 'displays' spatial relations, but does not specifically 'describe' it. Description needs a kind of vocabularies like arrangement moves that can account for the spatial relations under consideration.

To parse a given form, say, the floorplan of the Rosenbaum house, is to decompose the form into elements and relations held by elements, that is, to produce the 'structural description' of the form. To recognize that Rosenbaum house is an Usonian house we must match the structural description of this house with a stored general

description of the Usonian house. Due to the lack of spatial terms (which can be seen as spatial 'concepts'), shape grammars can present the "derivation" of a given configuration in a step-by-step fashion, but are difficult to parse a given configuration to arrive at a structural description. Even when the given configuration is broken down into elements, still this does not constitute a structural description because lack 'terms' to specify spatial relations among elements. For instance, suppose that all those operation symbols, +, x, Δ, in the diagram representing the parsing of the flower are taken away, then the diagram will no longer be a structural description of the flower, but is merely a taxonomical break-down of the flower.

Even when it is acceptable that the picture-like shape rules themselves are the 'icons' of the spatial terms like connection, separation, alignment, etc., the structural description is still unattainable. Many shape rules have attached spatial labels •, and state labels. When giving a configuration to test whether it is an instance of a style or not, the configuration is at its 'final state' in which no such symbol • exists; nor any state label of number. Grammatical rules deal with many transformations between non-final state shapes, while the given instance must be in the final state, i.e. no labels are associated with it. Although some shape grammar rules are specifically about deleting such labels, but I have not seen any rule about adding the labels. Can we add them arbitrarily? If shape grammar rules can not be applied correctly without these labels, then the parsing will be difficult if there is no rule to tell us how to add the labels to the given instance under test. To recognize an instance of a style by shape grammars is therefore very difficult, if not impossible.

Since the generative mechanism is the only powerful device shape grammars possess, it seems that the way to recognize a given Usonian house is first to generate Usonian houses, and then to see if there is an identical one. This is a kind of goal-directed heuristical search problem which attempts to find, within a large number of paths, a particular 'route' leading to the generation of the house in question. It is difficult to write a workable algorithm for such an task. We can quickly judge "build man the house the" is not a sentence without trying to see whether such a sentence can be generated by all phrase structure rules or not. Similarly, we can determine that the Rosenbaum house is an Usonian house simply by checking the test house against Usonian rules formulated by

something like arrangement moves. We need not to generate and to search many Usonian variants, which could be enormously large in number. Testing and generation can be separated, and to use generation as a way of testing is unnecessarily complex and unconvincing.

We have demonstrated, at least in principle, that the arrangement move approach to rule-making can produce as valid Usonian houses as those generated by Usonian shape grammars. We also argued that rules represented by arrangement moves are more flexible than shape grammars both in terms of rule-revision and in terms of alternative generation procedures, because we can separate form rules from procedural rules. We further argued that form rules represented by arrangement moves can deal with the problem of form-recognition with ease while shape grammars will encounter difficulties. Since rules formulated by arrangement moves are neither 'shape' rules, nor 'generative' rules, the simple rule-by-rule checking the given instance can be carried out easily, and very efficiently if done by parallel processes. The major reason that such rule-by-rule checking is possible for arrangement move representations, and less likely for grammatical representations is this: each spatial relation between elements observed in the given instance can directly find corresponding rules regarding these elements, and therefore the observed relation can be checked against rules. But such direct correspondences between instances and rules do not exist in shape grammars.

Many built forms have been analysed by shape grammars so far, and it is likely that many others will be processed in the same fashion in the future. Shape grammars are claimed to be generally useful to many applications from processing meaning to designing, but their true power, in my view, is their abilities to generate variations of a type, presumably by means of computers. But, here too we will encounter great difficulties unless we can overcome those drawbacks just mentioned with confidence.

(End of Appendix)

Footnotes to Appendix

1. See the article by Frank Lloyd Wright 'Grammar: the house as a work of art', in *The Natural House*, Horizon Press, 1954. PP. 181.

2. Syntactical approaches have been applied to picture recognition in computer sciences since the 60's (Kirsch, 1964; Narasimhan, 1964), when formal theories of languages were available. Since then many grammars of shapes, such as 'picture grammars' (Shaw, 1972), 'web grammars' (Pfaltz and Rosenfeld, 1969; Rosenfeld and Milgram, 1972), were developed, which, chronologically, become the immediate predecessors of 'shape grammars'. All these grammars borrow from Chomsky's syntactical theory which itself is in debts to Post-Kleene's recursive function in mathematical logic (Post,1943, 1944; Kleene, 1956).

3. The attempt to deal with 'semantics' in shape grammars was made by introducing PROLOG programming language (Gero and Coyne, 1985). It is hard to see any convincing argument regarding this matter other than a brief suggestion: to infer X from Y, some necessary assumptions S1, S2, ...should be inserted to make logical inferences possible. Of course, these voluntary assumptions represent knowledge not mentioned explicitly in grammars, but are useful for designing. What we actually see in this research are largely demonstrations of the idea that shape grammars can be implemented by programs written in PROLOG language. This is a rather uninteresting idea. Moreover, the result is not very powerful. To mention only one instance: to match a given configuration to the form on the left-hand side of the grammatical rule so as to activate the right-hand side, the program can permit a match only if the configuration is one of four 90-degree rotations of the left-hand side form (Gero and Coyne, 1985, pp.359). Such a matching process is obviously incompetent, unless the shape grammar is restricted to orthogonal organizations. The incompetence dose not lie in the programming language, but in the picture-like presentation of rules. (See discussions in the text.) The best of what can be said about this topic is that PROLOG has some in-built reasoning capacities, as long as the 'missing' information can be sufficiently supplied, semantical inference is possible. This is the kind of thing John McCarthy called the 'commonsense' logic, but real questions arise: how do we know what relevant information is missing in a design? How large should this information base be? and how will this information be structured? None of these questions have been addressed at any generally interesting level in the approach adopting PROLOG programming.

4. The derivation of the sentence 'the man builds the house' according to the rules can be given in the following:

Sentence → NP + VP

Sentence → T + N + VP

Sentence → T + N + Verb + NP

Sentence → the + N + Verb + NP

Sentence → the + man + Verb + NP

Sentence → the + man + build + NP

Sentence → the + man + build + T + N

Sentence → the + man + build + the + N

Sentence → the + man + build + the + house

The example uses the standard English phrase structure rules, see Noam Chomsky, *Syntactic Structures*, Hague: Mouton,1957.

References to Appendix

(Mostly those works are listed that are referred to in the Appendix)

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