THE LUMINOSITY AND ENERGY DEPENDENCE OF PULSE PHASE LAGS IN THE ACCRETION-POWERED MILLISECOND PULSAR SAX J1808.4−3658

Jacob M. Hartman[1,2,3], Anna L. Watts[4], and Deepto Chakrabarty[1].
Accepted by ApJ

ABSTRACT

Soft phase lags, in which X-ray pulses in lower energy bands arrive later than pulses in higher energy bands, have been observed in nearly all accretion-powered millisecond pulsars, but their origin remains an open question. In a study of the 2.5 ms accretion-powered pulsar SAX J1808.4−3658, we report that the magnitude of these lags is strongly dependent on the accretion rate. During the brightest stage of the outbursts from this source, the lags increase in magnitude as the accretion rate drops; when the outbursts enter their dimmer flaring-tail stage, the relationship reverses. We evaluate this complex dependence in the context of two theoretical models for the lags, one relying on the scattering of photons by the accretion disk and the other invoking a two-component model for the photon emission. In both cases, the turnover suggests that we are observing the source transitioning into the "propeller" accretion regime.

Subject headings: binaries: general — stars: individual (SAX J1808.4−3658) — stars: neutron — stars: rotation — X-rays: binaries — X-rays: stars

1. INTRODUCTION

Accretion-powered millisecond pulsars (AMPs) provide a unique laboratory for studying the process of disk accretion onto magnetic stars (Psaltis & Chakrabarty 1999 and refrences therein). The $\sim10^8$ G stellar magnetic field truncates the accretion disk only a few neutron star radii above the surface, channeling the infalling matter toward the magnetic poles. The resulting X-ray–emitting hot spots and accretion shocks are modulated by the stellar rotation to produce the observed pulsations. Such compact accretion geometry poses a particularly interesting challenge. We want to understand the details of the interaction between the magnetic field and the disk, and the effects that result from the proximity of different emitting regions. The accretion-powered 2.5 ms pulsar SAX J1808.4−3658 is a particularly good system for this type of study: in addition to having multiple outbursts, it spans orders of magnitude in accretion rate. In this paper we report our study of the energy-dependence of pulsations from this source and show that it gives new insight into these problems.

SAX J1808.4−3658 was the first detected AMP (Wijnands & van der Klis 1998), and it remains the best studied. Persistent pulsations have been detected throughout the four ~1 month outbursts from this source that have been observed by the *Rossi X-ray Timing Explorer* (*RXTE*) during 1998–2005. The timing of these pulsations establishes a 2.01 hr binary orbital period (Chakrabarty & Morgan 1998) that is increasing on a timescale of $P_{\rm orb}/\dot{P}_{\rm orb} = (66 \pm 4) \times 10^6$ yr, possibly due to the ablation of its low-mass companion during X-ray quiescence (Hartman et al. 2008, hereafter H08; Di Salvo et al. 2008). The presence of X-ray pulsations during the peak accretion rate and the long-term spin-down of this source provide a tight constraint on its magnetic field: $B = (0.4$–$1.5) \times 10^8$ G (H08; Psaltis & Chakrabarty 1999). A distance of 3.4–3.6 kpc is estimated from thermonuclear X-ray burst properties (Galloway & Cumming 2006).

The outbursts from SAX J1808.4−3658 had similar light curves, which we divide for convenience into four stages (see Fig. 2 of H08 for a schematic): a rise from quiescence lasting ≈5 days; a short-lived peak at a 2–25 keV flux of $(1.9$–$2.6) \times 10^{-9}$ erg cm^{-2} s^{-1}; a 10–15 day slow decay in luminosity that lasts until the source reaches $\approx0.7 \times 10^{-9}$ erg cm^{-2} s^{-1}; and finally a rapid fall in luminosity followed by weeks of low-luminosity flares with a ~5 d periodicity, during which the observed flux can vary by a factor of 1000 (Wijnands et al. 2001). This sudden drop has been interpreted as a transition into a "propeller" accretion regime (e.g., Gilfanov et al. 1998) in which accretion is only partially inhibited (Spruit & Taam 1993; Rappaport et al. 2004), although it may also be caused by a change in the viscosity of the accretion disk as it transitions into quiescence (Gilfanov et al. 1998; Gierliński et al. 2002).

Shortly after the discovery of pulsations in SAX J1808.4−3658, Cui et al. (1998) noted the presence of soft phase lags, the tendency of the pulses at lower ("softer") energies to arrive later than the corresponding pulses at higher ("harder") energies. The magnitude of the lags increased approximately linearly between 2–10 keV and saturated at 0.08 rotational cycles (200 μs) above 10 keV. Similar soft lags have been detected in six other AMPs: IGR J00291+5934 (Galloway et al. 2005), XTE J0929−314 (Galloway et al. 2002), XTE J1751−305 (Gierliński & Poutanen 2005), XTE J1807−294 (Kirsch et al. 2004), XTE J1814−338 (Watts & Strohmayer 2006), and HETE J1900.1−2455 (Galloway et al. 2007).

A number of explanations for the AMP soft lags have been proposed. Models that include only Doppler boosting, due to the stellar rotation (Ford 2000;

[1] Kavli Institute for Astrophysics and Space Research, Massachusetts Institute of Technology, Cambridge, MA 02139
[2] National Research Council research associate
[3] New address: Code 7655, Naval Research Laboratory, Washington, DC 20375; Jacob.Hartman@nrl.navy.mil
[4] Astronomical Institute "Anton Pannekoek", University of Amsterdam, Kruislaan 403, 1098 SJ Amsterdam, Netherlands

Weinberg et al. 2001) or the orbiting material in the accretion disk (Sazonov & Sunyaev 2001), have short-comings that are summarized by Gierliński et al. (2002). The two most plausible explanations invoke a two-component emission model with differing angular dependence and spectra (Poutanen & Gierliński 2003; hereafter PG03) or the scattering delay of hot photons from the accretion shock off the cooler disk or stellar surface (Falanga & Titarchuk 2007; hereafter FT07). In this paper we present an energy-resolved analysis of the persistent pulsations for all the outbursts of SAX J1808.4−3658 detected by *RXTE*. We will discuss the implications of our results for these models of the soft lags.

2. DATA ANALYSIS

For our study of the energy dependence of the persistent pulsations, we analyzed the same *RXTE* observations of SAX J1808.4−3658 as in H08 (refer to Table 1 therein). The `E_125US_64M_0_1S` data mode of the *RXTE* Proportional Counter Array (PCA; Jahoda et al. 1996) was used for nearly all the observations. It provides 122 μs time bins and 64 energy channels divided evenly over the detector response, a good combination for discerning the energy dependence of the persistent pulsations. The energy band covered by each channel varied over the 7 yr of observation spanned by this study due to adjustments to the PCA anode voltages (Jahoda et al. 2006); we accounted for this effect throughout our measurements.

For our timing analysis, we shifted the photon arrival times to the solar system barycenter, applied the *RXTE* fine clock correction, and filtered out data during Earth occultations and intervals of unstable pointing. We also removed any data within 5 min of thermonuclear X-ray bursts. To measure the phases and fractional amplitudes of the persistent pulsations for a given energy band E, we folded 512 s intervals of data using the phase timing solution derived in H08. Dividing the resulting pulse profiles into n phase bins with photon counts $x_{E,j}$, $j = 1 \ldots n$, the amplitude ($A_{E,k}$) and pulse phase offset ($\phi_{E,k}$) for the kth harmonic[5] is

$$A_{E,k} e^{2\pi i k \phi_{E,k}} = 2 \sum_{j=1}^{n} x_{E,j} e^{2\pi i j k / n} . \qquad (1)$$

We measure the phase offsets of the energy bands relative to the softest band, E_0: $\Delta\phi_{E,k} = \phi_{E,k} - \phi_{E_0,k}$. Thus negative values of $\Delta\phi_{E,k}$ indicate that pulsations of the kth harmonic within band E arrived earlier than the pulsations within the softest band. The uncertainties of these phase offsets due to Poisson noise are

$$\sigma_{E,k} = \frac{\sqrt{2N_E}}{2\pi k A_{E,k}} , \qquad (2)$$

where N_E is the total number of photons in the energy band (H08). For our analysis, we rejected phase measurements with $\sigma_{E,k} > 0.04$ cycles (i.e., 0.1 ms).

The measured fractional rms amplitudes are

$$r_{E,k} = \frac{A_{E,k}}{\sqrt{2}\left(N_E - B_E\right)} , \qquad (3)$$

[5] Throughout this paper, we number the harmonics such that the kth harmonic is k times the frequency of the 401 Hz fundamental.
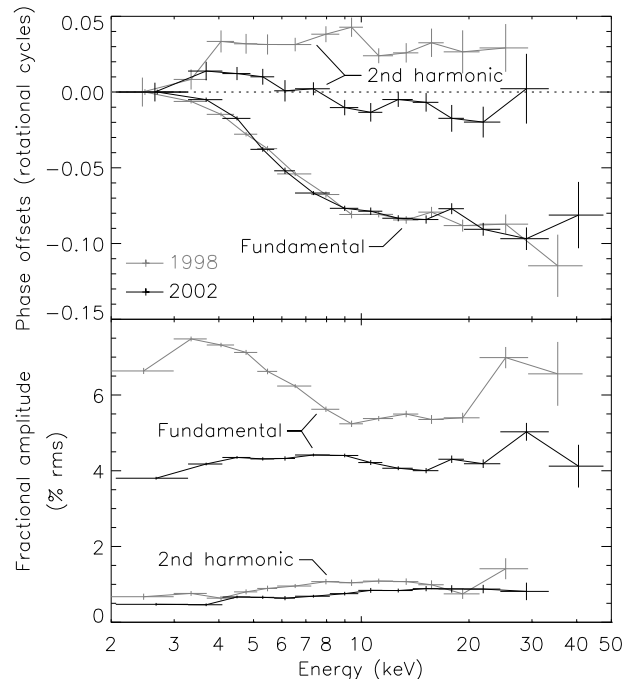

FIG. 1.— Average phase offsets $\Delta\phi_{E,k}$ (relative to the 2.1–3.3 keV band) and fractional amplitudes during the slow-decay stages of the 2002 outburst (black; MJD 52564.1–52575.0) and the 1998 outburst (grey; MJD 50919.0–50932.8, the same pointed observations used by Cui et al. 1998). Negative offsets indicate soft lags.

where B_E is the approximate number of background events within our energy band and time interval, estimated using the FTOOL `pcabackest`.[6] Uncertainties on the fractional amplitudes are computed using the method described by Groth (1975) and Vaughan et al. (1994), which correctly accounts for the addition of noise to the complex amplitude of the signal.

We estimated the 2.5–25 keV fluxes of the persistent emission by fitting the spectra extracted from Standard2 mode data (128 energy channels, 16 s time resolution) with an absorbed blackbody plus power law model, including an iron line if there were significant residuals around 6.4 keV; see Galloway et al. (2008), §2.1, for full details.

3. RESULTS

To assess the energy dependence of the pulse phase and fractional amplitudes, we calculated the average phase offsets and fractional amplitudes during the 11 d slow-decay stage of the 2002 outburst. Figure 1 shows the results, along with measurements from the 1998 outburst for comparison. We chose this time range because it was the longest and most data-rich interval during which the pulse profile of SAX J1808.4−3658 was observed to be reasonably stable. The pulse profiles during the slow-decay stage of the 2005 outburst, although similar to the 2002 profiles, were somewhat less stable (see Fig. 3 of H08).

The soft phase lag of the pulsations is immediately apparent for the fundamental. Between 2 keV and 10 keV, the pulses in harder bands arrive progressively sooner, with a slope of -0.014 ± 0.002 cycles keV^{-1}. Between

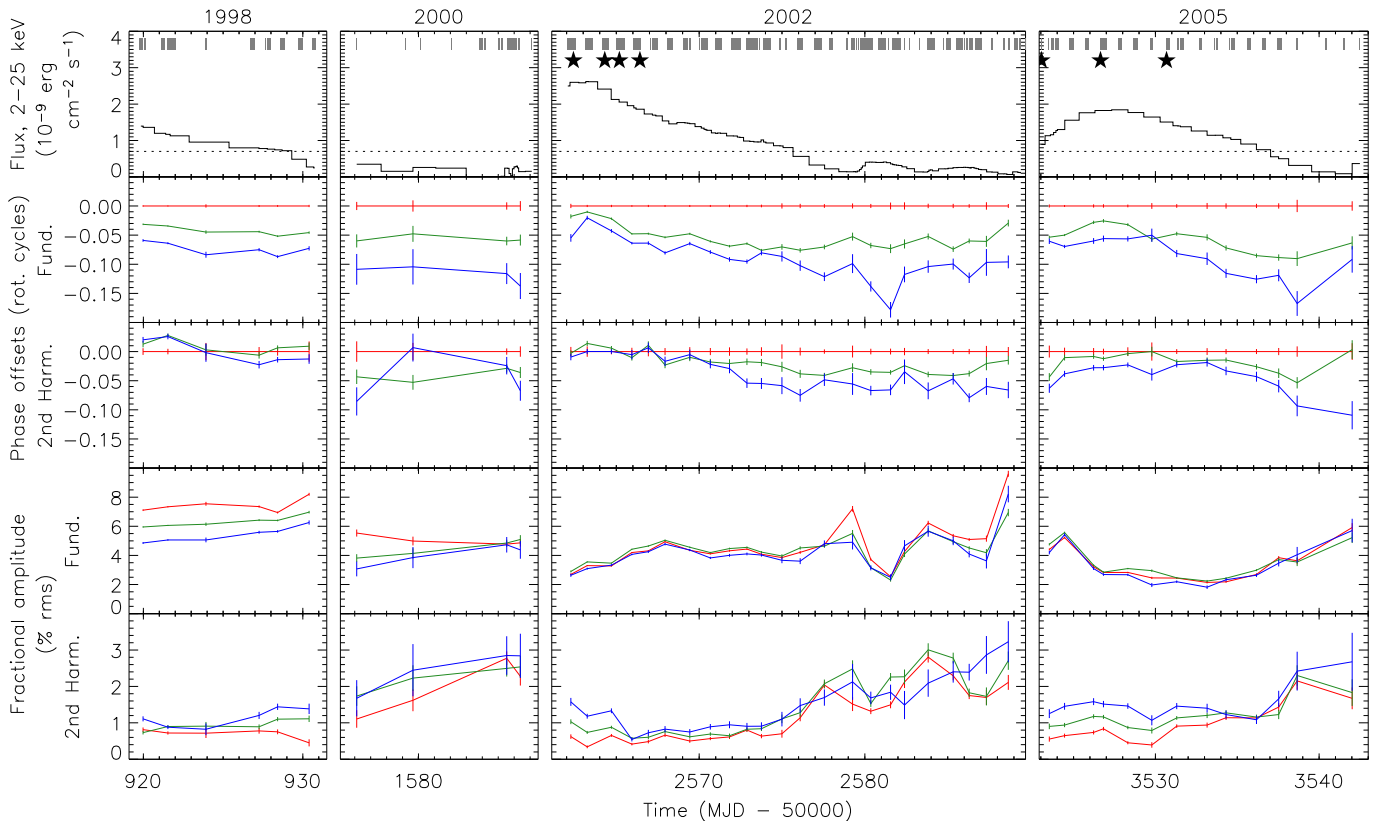[6] `http://heasarc.gsfc.nasa.gov/docs/xte/recipes/pcabackest.html`

Fig. 2.— Phase offsets and fractional amplitudes as a function of energy for the four SAX J1808.4−3658 outbursts observed by *RXTE*. The top panels show the background-subtracted light curves for each outburst. The strips along the tops of the graphs indicate the times of observations; stars indicate the times of the seven observed thermonuclear bursts. The dotted line indicates the critical flux of 0.7×10^{-9} erg cm^{-2} s^{-1}, at which the source transitions into the flaring-tail stage of the outburst. The second and third panels from the top show the phase offsets of the medium energy band (green; roughly 5–10 keV) and hard energy band (blue; $\approx$10–20 keV) pulsations relative to the soft band (red; $\approx$2–5 keV). Negative phase offsets indicate that higher-energy pulses lead the soft pulses. The bottom two panels show the background-subtracted fractional amplitudes for these three energy bands. Each point in the phase and fractional amplitude plots gives a day-long average. (They are not evenly spaced at 1 d intervals due to the uneven spacing of the *RXTE* observations.) Data within 5 min of thermonuclear bursts were excluded from these averages.

10 keV and 20 keV, this trend saturates at a lead of 0.08 cycles (200 μs). There is impressive agreement between the 1998 and 2002 outbursts, which also had nearly identical light curves; the phase lags during the dimmer 2005 outburst are similar in shape but plateau at a smaller magnitude. The phase offsets of the second harmonic are less pronounced, disappearing almost entirely or becoming hard lags when the source is bright, as is the case for the data in Figure 1.

In contrast, the energy dependence of the fractional amplitudes varies considerably between outbursts. The fractional amplitude of the fundamental during the slow-decay stage of the 1998 outburst (5.2–7.5% rms) was considerably higher than during the 2002 outburst (3.8–5.0% rms), as can be seen in Figure 1; the amplitude of the 2005 outburst (2.0–2.6% rms) was lower still. The morphology of the energy dependence differs as well. The 1998 outburst exhibits a large fractional amplitude peak at 3.5 keV and falling amplitudes up to 20 keV, similar to what is seen in XTE J1814−338 (Watts & Strohmayer 2006). In contrast, during 2002 and 2005 the fractional amplitude vs. energy curves are much flatter, with small but significant peaks at 8 keV. Given the strong similarities between the light curves of the 1998 and 2002 outbursts, it is curious that the energy dependence of their fractional amplitudes is markedly different. All three

outbursts show weak evidence of the fundamental's fractional amplitude increasing again above $\approx$20 keV. The fractional amplitude of the second harmonic generally increases with energy for all the outbursts, indicating less sinusoidal pulse profiles in harder energy bands. XMM observations of the 2008 outburst of this source reveal that the fractional amplitude of both harmonics continues to decrease below 2 keV (Patruno et al. 2009).

Figure 2 illustrates how the energy-dependent phases and fractional amplitudes change over the course of the four observed outbursts. The soft lags in the fundamental are apparent throughout all four outbursts; for the second harmonic, soft lags are apparent everywhere except during the outburst peaks. However, the degree of these lags is not constant: they are much more pronounced during the tails of the outbursts, when the flux is low. Watts & Strohmayer (2006) noted a similar pattern in XTE J1814−338, although that source did not trace as wide a range of fluxes, so the relationship was less clear. The energy dependence of the fractional amplitudes does not exhibit any clear correlation with the source flux or any other measured parameters, apart from the general trend of fractional amplitudes increasing in the outburst tails (see §3.4 of H08).

To further understand the relationship between the soft lags and the flux, we measured the phase vs. en-
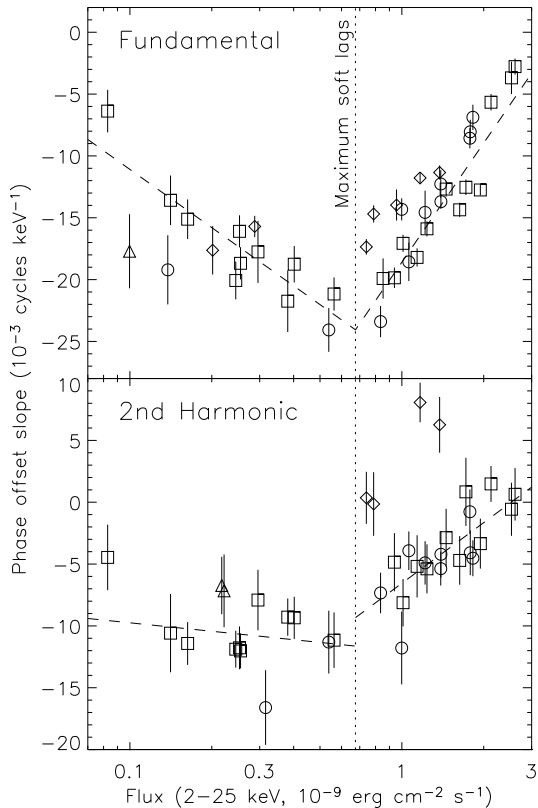
FIG. 3.— Correlation between the slope of the phase lags and the flux in SAX J1808.4−3658. Each slope is determined by a linear fit to the phase offset vs. energy measurements over 2–10 keV, with each point representing a 1 d average. Large negative slopes indicate large soft lags. The data from all four outbursts are shown: 1998 as diamonds, 2000 as triangles, 2002 as squares, and 2005 as circles. The dashed lines indicate the best linear fits above and below the 0.68×10^{-9} erg cm^{-2} s^{-1} turnover. These fits are based on the 2002 and 2005 data only, as the 1998 and 2000 outbursts exhibit greater scatter.

ergy slopes over 2–10 keV. We chose this range because it has the most significant energy dependence for the fundamental and, to a lesser extent, for the second harmonic. Figure 3 plots the resulting slopes as a function of flux. For the fundamental, the magnitude of the soft lags is clearly anticorrelated with the 2–25 keV flux when the source is bright: as the source dims from its peak at 2.7×10^{-9} erg cm^{-2} s^{-1} down to roughly 0.7×10^{-9} erg cm^{-2} s^{-1}, the magnitude of the soft lags grows. This range of fluxes covers the peak and slow-decay stages of the outburst. At lower fluxes, the relation reverses, and the magnitude of the lags becomes correlated with flux. The flux at which this turnover occurs is significant: once it falls to 0.7×10^{-9} erg cm^{-2} s^{-1}, the source dims rapidly and enters the flaring-tail stage of the outbursts.

The trend for the second harmonic is somewhat less clear. Measuring a slope through the 2–10 keV fractional amplitude points does not work as well since the energy dependence is less linear (top panel of Fig. 1), so there is more scatter. Even so, the data strongly suggest a similar relationship at high fluxes, particularly when the relatively scattered 1998 data are excluded.

The slopes of these trends differ significantly for the two harmonics. In the low-flux regime, the slopes are

-16 ± 2 for the fundamental and -2 ± 2 for the second harmonic; at high fluxes, they are 32 ± 4 for the fundamental and 17 ± 3 for the harmonic. (The units in all cases are 10^{-3} cycles keV^{-1} [log flux]$^{-1}$.) These slopes were fit using the 2002 and 2005 data only, as the two earlier outbursts had appreciably more scatter and less data. Note that these units reflect *rotational* cycles of the star, so a mechanism that induces the same phase lag in all photons of a given energy would result in the slopes being equal for the two harmonics. For the high-flux slopes, we can conclude with 8 σ significance that this is not the case.

We also looked for correlations between the lags and the spectral parameters from our blackbody plus power law fits. The only parameter for which a correlation is apparent is the normalization of the hard power law component. This relation is not surprising, because the hard normalization and the total X-ray flux are linearly proportional: during the 1998–2005 outbursts, $\log(N_{\mathrm{hard}}) = (1.02 \pm 0.03) \log(f_x)$, with a reduced χ^2 near unity. The uncertainties of N_{hard} are much greater than the flux uncertainties, particularly when the source is dim, and the large scatter obscures whether a turnover is present in the lag vs. N_{hard} relation.

4. DISCUSSION

The energy dependence of the pulsations from SAX J1808.4−3658 exhibit strong variability both between and during the four outbursts observed by *RXTE*. The overall fractional amplitudes and the shape of the amplitude vs. energy curves differ substantially between outbursts, with no obvious correlation with flux; the only clear trend is that the pulsations generally become less sinusoidal with energy in the observed 2–30 keV range. In contrast, the soft lags of the pulses have a consistent energy dependence, increasing in magnitude in the 2–10 keV band and saturating at 10–30 keV, with a strong dependence on flux. At the highest fluxes the lags are near zero, but they increase steadily as the flux falls. Below a 2–25 keV flux of $\approx 0.7 \times 10^{-9}$ erg cm^{-2} s^{-1}, this trend reverses and lags get smaller again. This particular flux also marks the point in the outburst where the luminosity decays rapidly and the source enters the flaring tail.

Soft lags are a common feature of AMP pulsations, and two plausible models have emerged to explain their existence. PG03 suggested a two-component model involving soft emission from a surface hot spot and a hard Comptonized component from the shocked layer in the accretion funnel. Each component has a different beaming pattern, which is affected in a different way by the Doppler effect. This results in the soft pulse lagging the hard pulse. While this model works well for SAX J1808.4−3658 and XTE J1751−305 (Gierliński & Poutanen 2005), it does not predict the reduction in soft lags at very high energies seen in IGR J00291+5934 (Falanga et al. 2005). As a result FT07 proposed an alternative model in which the disk plays a prominent role. In this model, the main pulsed element is the hard component from the accretion column: this radiation is then downscattered by cold plasma in the inner disk (or on the neutron star surface) to gener-

ate a soft lagging component.[7] These soft lags get larger as the density of the scattering plasma falls.

Let us now explore the factors that might cause the flux dependence of the soft lags, assuming that flux directly traces accretion rate. We then need to ask how the properties of the system are expected to change as the accretion rate falls. For a star with a magnetic field that is strong enough to channel the accretion flow, the reduction in accretion rate should cause an increase in inner disk radius (e.g., Ghosh et al. 1977). This will have two relevant effects. First, the density of scattering plasma near the star will fall: within the model of FT07 this would cause the soft lags to increase. In addition the relative angular velocity of the inner disk and star will reduce. This will affect the angle at which the accretion funnel impacts the surface (Romanova et al. 2004), changing the column density above any surface hot spot. This type of change might work within the model of PG03 to affect the soft lags.

If these changes continue steadily, however, it is hard to see what could cause the turnover seen in Figure 3. One intriguing possibility that would give a very natural explanation for such a turnover is that the system reaches the point where the magnetospheric radius r_m exceeds the corotation radius r_c. This is often referred to as the "propeller" regime, since it was originally thought that matter would then be ejected from the system, with the star going from spin-up to spin-down (Illarionov & Sunyaev 1975). In fact, the transition is probably not sharp, and r_m needs to be substantially larger than r_c for ejection to occur (Spruit & Taam 1993). At intermediate accretion rates, continued accretion is possible even while the neutron star starts to spin down (Spruit & Taam 1993; Rappaport et al. 2004). Simulations by Long et al. (2005) indicate that the transition between spin up and spin down should occur when $r_m/r_c \approx 0.7$ for AMPs. The observed turnover happens at an accretion rate of $\dot{M} = 1.8 \times 10^{-10} \, M_\odot \, \mathrm{yr}^{-1}$, giving $r_m/r_c = 0.6$ for the magnetic field $B = 1.5 \times 10^8$ G from H08.[8] This close agreement between the simulations and the $\dot{M}$ at which the flux rapidly drops and the soft lags turnover strongly suggests that SAX J1808.4−3658 passes through this transition as the accretion rate falls. In this eventuality there are two effects that could cause

---

[7] Ibragimov & Poutanen (2009) recently argued that photoelectric absorption in the accretion disk would prevent the large number of scattering events required by the FT07 model. However, standard disk models give a temperature of ∼300 eV in the inner disk (Shakura & Syunyaev 1973); the actual temperature will be higher due to the irradiation. At these temperatures and the expected electron densities ($n_e \sim 3 \times 10^{19}$ cm^{-3}; see FT07 and Fig. 4 here), all elements through oxygen will easily be fully ionized by the hard tail of the thermal distribution. The only significant source of photoelectric absorption is Fe, but for this n_e only Fe XXV and higher remain at $T \geq 300$ eV, with the transition to full ionization occuring at 400–600 eV (Rubiano et al. 2004). Its He-like and H-like forms have K edges at 9 keV, so they will not absorb the softer photons responsible for the lags. Since the FT07 model is not ruled out *a priori*, this discussion attempts to address both models without preference.

[8] Accretion rates were estimated in the usual manner, by equating the bolometric and infall luminosities: $4\pi d^2 f_x c_{\mathrm{bol}} = GM\dot{M}/R$, using $d = 3.5$ kpc and the bolometric correction $c_{\mathrm{bol}} = 2.12$ from Galloway & Cumming (2006). The Alfvén radius was estimated as $r_m = k_A (2GM)^{-1/7} \dot{M}^{-2/7} (BR^3)^{4/7}$, with $k_A = 0.5$ as per Long et al. (2005). $M = 1.4 \, M_\odot$ and $R = 10$ km were used in all cases.
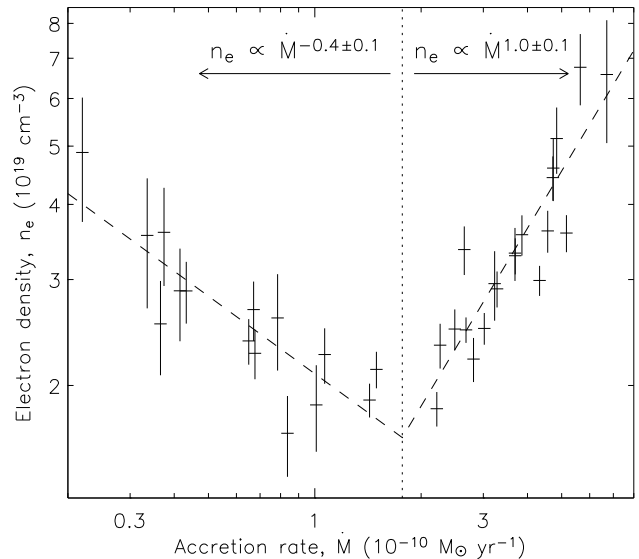

FIG. 4.— Electron density n_e of the inner accretion disk, as fit using the model of FT07, versus the mass accretion rate for the 2002 and 2005 outbursts of SAX J1808.4−3658. The vertical dotted line marks the $\dot{M}$ at which the magnitude of the soft lags is greatest and below which the outburst is in its flaring-tail stage; a transition from normal accretion to the "propeller" regime can explain this change in the observed behavior. The dashed lines give the best fits of $n_e \propto \dot{M}^\gamma$ in each region.

the turnover in soft lags.

The first is due to the fact that the relative angular velocity of the inner disk and the star changes sign. The accretion funnel will go from being dragged forwards to being dragged backwards (assuming that it settles rapidly; Romanova et al. 2004). The column depth above any surface hot spot should therefore go through a maximum near the zero torque point, causing a turnover in the scattering properties of the accretion column. If the accretion column plays a major role as discussed in PG03, this could cause the observed turnover. Another explanation within the PG03 model is the suppression of the lagged soft component at high fluxes due to the increased optical depth of the accretion shock when $\dot{M}$ is large. However, this mechanism would require the normalization of the soft component to decrease during the outburst peaks, contrary to what is seen (cf. Fig. 2 of Gierliński et al. 2002).

A second possibility stems from the fact that the properties of the disk must also change at this point. For accretion to continue after zero-torque point, the inner edge stops retreating, and the density at the inner edge must rise (Spruit & Taam 1993). In the scenario of FT07, this rise in inner disk density would cause the magnitude of the soft lags to decrease again. In the absence of Compton upscattering, which would introduce hard lags not observed in SAX J1808.4−3658, their model predicts that the scattering from the disk will modify the arrival times as $\Delta t(E) \approx (m_e c/\sigma_T n_e)(E_0^{-1} - E^{-1})$, where σ_T is the Thompson cross-section and n_e is the electron density of the scattering region. This model provides reasonable fits to the observed phase lags; Figure 4 shows the resulting n_e.

The fitted electron densities probe the relationship between the inner disk and the accretion rate. $n_e \sim$

$10^{19.5}$ cm^{-3} gives a mean free path of $\ell = (\sigma_T n_e)^{-1} \sim$ 0.5 km and time $\ell/c \sim 2$ μs, requiring $\sim$100 scattering events to produce the observed soft lags. This ℓ rules out the thin neutron star atmosphere as a scattering medium. On the other hand, simulations by Romanova et al. (2004) suggest an inner disk thickness of order the neutron star radius, and $(R/\ell)^2 \sim 400$ is more than adequate to accomodate the number of needed scattering events. From the fits in Figure 4, we find that $n_e \propto \dot{M}^{1.0\pm0.1}$ during the main outburst; during the presumed propeller stage, $n_e \propto \dot{M}^{-0.4\pm0.1}$. An interesting consequence of the former relationship is that at higher accretion rates before the turnover, when $r_m/r_c \lesssim 0.7$, the flow rate (the volume of material per unit time crossing the inner edge of the disk) must be constant: since $\dot{M} = \Omega m_p n_e r_m^2 v_r$, where Ω is the solid angle of the inner edge and v_r is the radial velocity of material crossing it, the flow rate $\Omega r_m^2 v_r$ is fixed.

If the phase lags in SAX J1808.4$-$3658 are entirely due to Compton downscattering, as suggested by FT07, then this model must explain why the magnitudes of the soft lags for the second harmonic are roughly half the magnitudes for the fundamental over a wide range of $\dot{M}$, as can be seen in Figure 3. Following the formalism of Sunyaev & Titarchuk (1980), we model the timing effects of Comptonization with a distribution $P(\Delta t)$ of the photon escape times from the scattering medium. For the kth harmonic of the spin frequency ν_s, we convolve the harmonic component $x_k(t) = A_k \exp(2\pi i k\nu_s t)$ with $P(\Delta t)$ to find the new amplitude A'_k and the time lag Δt of the scattered pulses:

$$A'_k e^{2\pi i k\nu_s(t-\Delta t)} = \int_{-\infty}^{t} A_k e^{2\pi i k\nu_s t'} P(t'-t) dt' \quad (4)$$

In general, the choice of $P(\Delta t)$ accounts for the geometry of the emitting and scattering regions. However, for the large number ($\sim$100) of scatterings previously noted,

the geometry becomes unimportant and a photon diffusion distribution of $P(\Delta t) = \tau^{-1} \exp(\Delta t/\tau)$ is appropriate (Sunyaev & Titarchuk 1980). Applying this form to equation (4), the time lag of the kth harmonic is

$$\Delta t_k = (2\pi k\nu_s \tau)^{-1} \tan^{-1} 2\pi k\nu_s \tau . \quad (5)$$

The diffusion timescale cannot be longer than the lags themselves: $\tau \lesssim \Delta t_{\max} = 200$ μs. This sets a lower limit of $\Delta t_2/\Delta t_1 \gtrsim 0.85$ due to scattering alone, contrary to the observation that $\Delta t_2/\Delta t_1 \approx 0.5$. Thus Compton scattering cannot by itself fully explain the observed lags: a change in the energy dependence of the initial pulses is still necessary.

Clearly, detailed calculations are required to check whether the two models really would predict the outcomes that we suggest, but they are at least in principle plausible. One would also need to test whether the models would predict any reversal in other pulsation properties at propeller onset, given that no such changes are observed.

The interesting behavior of the soft lags in this source suggests that the lags of the other accretion-powered millisecond pulsars should be revisited to check for flux dependence. The only source for which this has been done is XTE J1814$-$338: an increase in soft lags was observed in the tail of the outburst (Watts & Strohmayer 2006), but the rapid drop off and shorter tail of this source made it very difficult to study the pulsations at the lowest fluxes. The other AMPs may be more promising as candidates for further study.

REFERENCES

Chakrabarty, D. & Morgan, E. H. 1998, Nature, 394, 346

Cui, W., Morgan, E. H., & Titarchuk, L. G. 1998, ApJ, 504, L27

Di Salvo, T., Burderi, L., Riggio, A., Papitto, A., & Menna, M. T. 2008, MNRAS, 389, 1851

Falanga, M., Kuiper, L., Poutanen, J., Bonning, E. W., Hermsen, W., Di Salvo, T., Goldoni, P., Goldwurm, A., Shaw, S. E., & Stella, L. 2005, A&A, 444, 15

Falanga, M. & Titarchuk, L. 2007, ApJ, 661, 1084 (FT07)

Ford, E. C. 2000, ApJ, 535, L119

Galloway, D. K., Chakrabarty, D., Morgan, E. H., & Remillard, R. A. 2002, ApJ, 576, L137

Galloway, D. K. & Cumming, A. 2006, ApJ, 652, 559

Galloway, D. K., Markwardt, C. B., Morgan, E. H., Chakrabarty, D., & Strohmayer, T. E. 2005, ApJ, 622, L45

Galloway, D. K., Morgan, E. H., Krauss, M. I., Kaaret, P., & Chakrabarty, D. 2007, ApJ, 654, L73

Galloway, D. K., Muno, M. P., Hartman, J. M., Savov, P., Psaltis, D., & Chakrabarty, D. 2008, ApJS, 179, 360

Ghosh, P., Pethick, C. J., & Lamb, F. K. 1977, ApJ, 217, 578

Gierliński, M., Done, C., & Barret, D. 2002, MNRAS, 331, 141

Gierliński, M. & Poutanen, J. 2005, MNRAS, 359, 1261

Gilfanov, M., Revnivtsev, M., Sunyaev, R., & Churazov, E. 1998, A&A, 338, L83

Groth, E. J. 1975, ApJS, 29, 285

Hartman, J. M., Patruno, A., Chakrabarty, D., Kaplan, D. L., Markwardt, C. B., Morgan, E. H., Ray, P. S., van der Klis, M., & Wijnands, R. 2008, ApJ, 675, 1468 (H08)

Ibragimov, A. & Poutanen, J. 2009, MNRAS, submitted, arXiv: 0901.0073

Illarionov, A. F. & Sunyaev, R. A. 1975, A&A, 39, 185

Jahoda, K., Markwardt, C. B., Radeva, Y., Rots, A. H., Stark, M. J., Swank, J. H., Strohmayer, T. E., & Zhang, W. 2006, ApJS, 163, 401

Jahoda, K., Swank, J. H., Giles, A. B., Stark, M. J., Strohmayer, T., Zhang, W., & Morgan, E. H. 1996, in Proc. SPIE Vol. 2808, p. 59-70, EUV, X-Ray, and Gamma-Ray Instrumentation for Astronomy VII, Oswald H. Siegmund; Mark A. Gummin; Eds., ed. O. H. Siegmund & M. A. Gummin, 59–70

Kirsch, M. G. F., Mukerjee, K., Breitfellner, M. G., Djavidnia, S., Freyberg, M. J., Kendziorra, E., & Smith, M. J. S. 2004, A&A, 423, L9

Long, M., Romanova, M. M., & Lovelace, R. V. E. 2005, ApJ, 634, 1214

Patruno, A., Rea, N., Altamirano, D., Linares, M., Wijnands, R., & van der Klis, M. 2009, MNRAS, accepted, arXiv: 0903.3210

Poutanen, J. & Gierliński, M. 2003, MNRAS, 343, 1301 (PG03)

Psaltis, D. & Chakrabarty, D. 1999, ApJ, 521, 332

Rappaport, S. A., Fregeau, J. M., & Spruit, H. 2004, ApJ, 606, 436

Romanova, M. M., Ustyugova, G. V., Koldoba, A. V., & Lovelace, R. V. E. 2004, ApJ, 610, 920

Rubiano, J. G., Florido, R., Rodriguez, R., Gil, J. M., Martel, P., & Minguez, E. 2004, Journal of Quantitative Spectroscopy and Radiative Transfer, 83, 159

Sazonov, S. Y. & Sunyaev, R. A. 2001, A&A, 373, 241

Shakura, N. I. & Syunyaev, R. A. 1973, A&A, 24, 337

Spruit, H. C. & Taam, R. E. 1993, ApJ, 402, 593

Sunyaev, R. A. & Titarchuk, L. G. 1980, A&A, 86, 121

Vaughan, B. A., van der Klis, M., Wood, K. S., Norris, J. P., Hertz, P., Michelson, P. F., van Paradijs, J., Lewin, W. H. G., Mitsuda, K., & Penninx, W. 1994, ApJ, 435, 362
Watts, A. L. & Strohmayer, T. E. 2006, MNRAS, 373, 769
Weinberg, N., Miller, M. C., & Lamb, D. Q. 2001, ApJ, 546, 1098

Wijnands, R., Méndez, M., Markwardt, C., van der Klis, M., Chakrabarty, D., & Morgan, E. 2001, ApJ, 560, 892
Wijnands, R. & van der Klis, M. 1998, Nature, 394, 344