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Physical Models of Wind Instruments

A Generalized Excitation Coupled with a Modular Tube Simulation Platform

Abstract

To develop real-time simulations of wind instruments, digital waveguides filters can be used as an efficient representation of the air column. Many aerophones are shaped as horns, which can be approximated using conical sections. Therefore the derivation of *conical waveguide filters* is of special interest. When these filters are used in combination with a generalized reed excitation, several classes of wind instruments can be simulated.

In this paper we present the methods for transforming a continuous description of conical tube segments to a discrete filter representation. The coupling of the reed model with the conical waveguide and a simplified model of the termination at the open end are described in the same way. It turns out that the complete lossless conical waveguide requires only one type of filter.

Furthermore, we developed a digital reed excitation model, which is purely based numerical integration methods, i.e., without the use of a lookup table.

1   Introduction

The last decade has seen a growing interest in the use of physical models of musical instruments for sound synthesis. The increased performance of computer systems (DSP-chips) has made real-time simulation of several classes of instruments possible.

These *virtual instruments* are of interest for several reasons. They are a new way of representing sound; not as a waveform or its spectrum (FFT), but as the mechanism producing it. In principle, they are expected to produce sound with an acoustic high quality (often lacking in other synthesis methods) and to respond to the gestures of a performer in a natural way, allowing for a more intuitive control of the sound. Additionally, the models can be used to study the effects of modifications of historical or contemporary instruments in a simulation; or to test acoustic theories in general, and last but not least, as a method of music synthesis. Furthermore, there is a strong relation of a "natural" sound, produced by an oscillating system liable to physical laws, with the perceptual image of such a sound. Therefore, sounds produced by physical models could be of interest in the field of sound recognition.

Our research is aimed at the development of methods for sound synthesis using these physical models, with specialisation to wind instruments. We have set us some criteria for our models, so they may be used as a basis for the development of a complete sound synthesis environment.

Firstly, the software should run in real-time, so it may be used in a performance. Secondly, the environment should be as general as possible, allowing the design of a wide variety of wind

instruments. One way of obtaining a certain degree of generality is making the environment modular, defining a set of basic elements which can be added together to build a complete instrument. Within these two constraints, the simulation should be as faithful to acoustical laws as possible.

A general scheme for physical models is the division of a musical instrument into a linear and a non-linear part, which usually can be associated with the resonating part of the instrument and the excitation mechanism, respectively. The resonating part of an aerophone is an air column, contained in a tube system. The excitation is done by blowing (more) air in this system, by mouth or mechanically.

With the earlier mentioned criteria in mind the elements for simulating the excitation and resonator of reed-driven wind instruments were developed. This paper presents these modules, together with a method of coupling them.

2 Excitation Mechanisms of Reed-driven Instruments

2.1 How a reed/tube system vibrates

The sound production of reed-driven instruments is based on the interaction between the air column and the reed. The air column (tube) of wind instruments is usually characterized by its linear response. This response can be calculated very exactly by specifying the geometry of the air column, including the effect of toneholes, radiation and wall losses. In testing the *interaction* with the reed, a much more simplified model of the tube is sufficient, namely a lossless digital waveguide simulation of a cylindrical tube. The reed itself can be seen as a clamped bar (see fig. 2.1.)
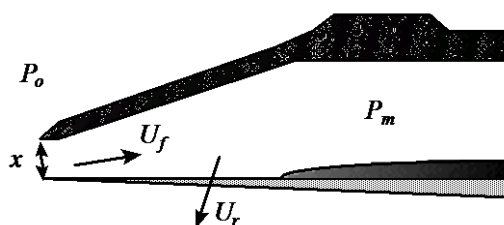

fig. 2.1   *sectional model of the mouthpiece. The reed is represented by the lightgray object and is clamped to the mouthpiece. Only the movement (x) of the reed tip is approximated in our model.*

Typical sustained oscillations of aerophones grow because of the regenerative feedback mechanism that is inherent to any reed/tube coupling. The mouth pressure (P_o) forced by the player causes an *airflow* (U_f) through the reed aperture (x). In the case of cane reed, this mouth pressure tends to close the reed, so the aperture decreases while a *pressure wave* (p_i) is "released" into the tube. At the open end of the tube, a part of the wave is reflected and travels back to the mouthpiece (p_r), where it pushes the reed back open again. Then the aperture allows a new pressure wave into the tube and the process repeats itself. Though this is a simplified version of the process, it describes the basic mechanism (see fig 2.2). The character of this interaction process is mainly determined by the physical relation between total airflow (U_b) through the mouthpiece bore and the reed motion (x).
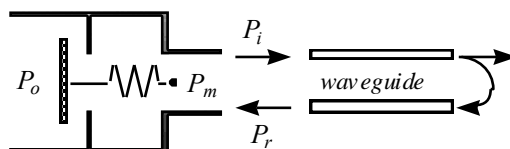

fig. 2.2   *simulation model*

The motion of the reed without the influence of the player and the response of the tube is dominated by the motion of the first mode of vibration of the reed, and can therefore be modelled as a simple harmonic oscillator with a resonance frequency equal to the frequency of the first reed mode. In the model the displacement of the reed is zero when the reed is closed and is positive when opened. The flow or *volume velocity* is taken positive in the direction of the tube end. The reed aperture is not allowed to become negative. This corresponds to the beating of the reed when the instrument is played at higher mouth pressures. The tube response is simulated with a travelling wave or waveguide model.

2.2 The Interaction Model.

The equations that describe the oscillations in a reed/tube system depend on the following constants and variables that relate to physical values and quantities.

U_b	Total volume flow through the bore	S_t	Cross-section of the tube
U_f	Normal Volume flow	S_r	Effective surface of the reed
U_r	Additional volume flow	g_r	Halfpower bandwidth of the reed frequency
P_o	Mouth pressure	w_r	Reed resonance frequency
P_m	Mouthpiece pressure	μ_r	Effective mass per unit area of the reed
dP	Pressure difference	I_r	Inertance of the air above the reed
p_r	Reflected pressure wave from the tube	c	Velocity of sound in air
p_i	Ingoing pressure wave into the tube	ρ	Air Density
x	position of the reed (= reed aperture)	B	airflow constant
H	aperture of the reed in equilibrium		
Z	Characteristic impedance of the tube		

The interaction between the reed motion and the tube response depends on the reed aperture and the *pressure difference* (dP) across the reed, i.e. the difference between mouth and mouthpiece-pressure. Backus (1963) measured this flow through the reed aperture as a function of pressure difference for different rest openings and found the following expression:

$$U_f = B \cdot sign(dP) \cdot |\, dP\, |^{\frac{2}{3}}\, x^{\frac{4}{3}} \qquad\qquad (2.1)$$

The character of this relation is nonlinear due to the exponentials in the equation. The measurements were done with static reed, this means that the effect of the *air inertance* is neglected. If the formula is rewritten including the air inertance one obtains:

$$dP = D \cdot sign(U_f) \cdot |\, U_f\, |^q\, /\, x^2 + I_r \cdot \dot{U}_f \qquad (2.2)$$

where $D = B^{-1.5}$ and $q = 1.5$ for single reed and $q = 2$ for double and lip reed (Keefe 1992). In this equation there is no third root like in equation (2.1) and it is therefore easier to implement on a DSP. Also the inclusion of air inertance makes it a more realistic model.

An additional flow is caused by the motion of the reed; it sweeps an extra amount of air into the tube depending on its velocity and the reed surface:

$$U_r = S_r \cdot \dot{x} \qquad\qquad (2.3)$$

The total flow (U_b) through the bore of the mouthpiece can now be expressed as:

$$U_b = U_f - U_r,$$ (2.4)

To calculate eqs. (2.2) and (2.3) the values of x, $\dot{x}$ and dP have to be known. The pressure difference can be calculated simply as:

$$dP = P_o - P_m$$ (2.5)

The actual pressure in the mouthpiece is equal to the sum of the ingoing and outgoing pressure waves:

$$P_m = p_r + p_i$$ (2.6)

The in- and outgoing waves are related to the *normalized* flow in the mouthpiece:

$$Z \cdot U_b = p_i - p_r$$ (2.7)

with Z being the *characteristic impedance* of the tube:

$$Z = (\rho \cdot c)/S_t$$ (2.8)

Now we can calculate the displacement of the reed which is depending on dP. Several models exist in literature, varying from simple harmonic oscillators to more complicated models of clamped bars (Stewart and Strong, 1980). Adrien (1988) used a harmonic oscillator with variable mass and stiffness coefficients, including the effect of variable reed mass and damping due to lip contact. This way an *embouchure model* of the reed can be developed.

According to Thompson (1979) the nonlinearity of the reed dynamics is much smaller than the nonlinearity in the flow control of the reed, and can therefore be thought of as a perturbation in the linear model. This is confirmed by Keefe (1992). and Benade (1988). Backus (1963) already found that the reed acts as a harmonic oscillator driven below its resonance frequency, although the frequency has significant influence on the tone production of wind instruments. We choose the harmonic oscillator model used by Keefe, Thompson and Benade:

$$\ddot{x} + g_r \dot{x} + w_r(x - H) = \frac{\sigma}{\mu_r} dP$$ (2.10)

Where g_r is related to the damping of the reed and μ_r to the reed mass. The sign $\sigma = -1$ is valid for cane reed and $\sigma = 1$ for lip reed, because lip reeds tend to enlarge the reed opening when the mouth pressure is increased (Keefe 1992).

The ten equations together describe the entire system (except for the tube response). To obtain all the variables at each time-step (sample period) in the discrete domain, the differential equations (2.2), (2.3) and (2.10) should be solved *numerically*. All the other equations can simply be integrated into the dynamical system of those three differential equations. This set is mathematically equivalent to the dynamical system described by Keefe (1992), but the elements are kept more separate. This is done to make the model easier to implement and understand.

2.3 Solving the System in Discrete Time

Several numerical integration methods were tried to solve the reed/tube interaction system. Due to the nonlinearities introduced by the flow characteristics and the beating of the reed the *stability* of the numerical integration method is essential. In the mean time the *convergence* of the method can not be neglected, since a physical model should have a quick response to the player's actions, similar to

playing a real instrument. Applying pure *implicit* integration methods (Keefe 1992) fulfils these needs, but this also enforces enormous calculation costs, thereby prohibiting a real-time implementation. On the other hand, even with fourth-order Runge-Kutta *explicit* methods the desired stability could not be ensured. Therefore a special integration scheme was developed, involving both implicit and explicit methods.

The numerical integration results in a single nonlinear equation:

$$D_1 \cdot U_f + D_2 \cdot \text{sgn}(U_f) \cdot \left| U_f \right|^q + D_3 \qquad\qquad (2.11)$$

where D_1, D_2, D_3 are the products of variables in the system, but can be considered as constants in this equation. From this equation U_f is solved using the Newton-Rhapson iteration method.

2.4 Numerical Results

The first results were obtained by connecting a single reed model with a digital waveguide, representing a cylindrical tube, which resulted in clarinet-like oscillations. Fig. 2.3 shows a waveform of the first 1024 samples. The cylindrical shape of the tube with one open end (the side where the reed is connected can be considered as a closed end) forces the sustain of uneven harmonics. This is shown in fig. 2.4.
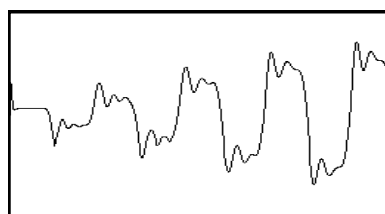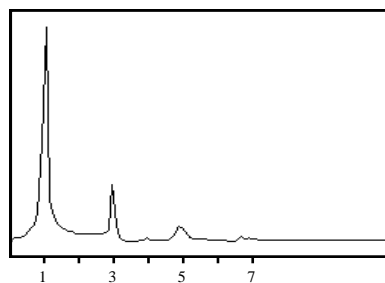

fig. 2.3   *clarinet-like simulation waveform*

fig 2..4   *spectrum of 2048 samples. On the horizontal the harmonic number is shown.*

3   Digital waveguide Modelling of Acoustical Tubes

3.1 Introduction

Modelling with digital waveguide filters (hereafter refered to as waveguides) is a particularly efficient method for synthesizing the sound of musical instruments (Smith, 1993). The method is based on the solution of the one-dimensional wave equation in *travelling waves*. The travelling waves are represented by two parallel delay-lines (rails) propagating in opposite direction. Waveguides are closely related to classical lattice filters and transmission line descriptions of acoustical systems.

The wave propagation in cylindrical and conical tubes can be simulated with a single waveguide system. Many air columns are not perfectly cylindrical or conical, but are shaped as horns. The horn shape can be approximated with conical sections. *Impedance changes* occur at the cone junctions and at the open end (termination) of the air column. These impedance changes are simulated by placing filters in the delay-line system. (see fig. 3.1).
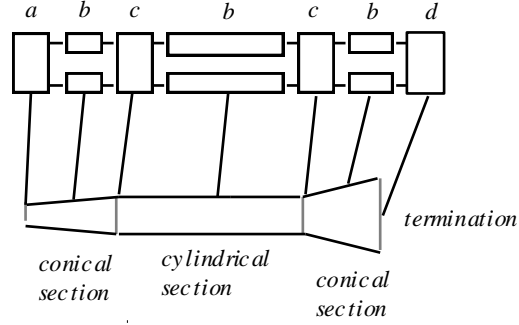
fig. 3.1   *waveguide representation of an air column. The letters represent the following elements: (a) reed   coupling   filter. (b) waveguide. (c) scattering junction. (d) termination junction.*

The waveguide junctions are called *scattering junctions*. For our model a special termination junction was developed, as well as a waveguide filter for the coupling of a reed excitation model with a conical tube. Thermo-viscous losses can be introduced in the system by including special filters (Agulló et al., 1993 , Amir et al., 1994). This is not yet included in our model.

3.2 Conical Waveguides

The one dimensional wave equation for spherical waves in a conical tube is:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial P}{\partial r}\right) = \frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} \qquad (3.1)$$

Where P stands for pressure as a function of time (t) and place (r), with $r = x + x_0$, the distance from the cone apex (see fig. 3.2).
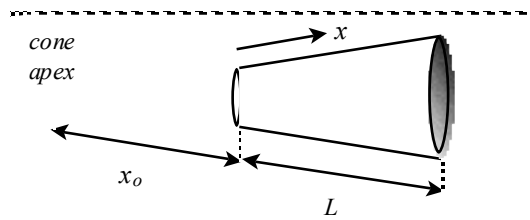

fig. 3.2   *A conical air column.*

Solution of the wave equation in travelling waves reveals the waveguide representation:

$$P(r,t) = \frac{p^+}{r} + \frac{p^-}{r} \qquad (3.2)$$

with
$$\begin{cases} p^+ = Ae^{-jkr+j\omega t} \\ p^- = Be^{jkr+j\omega t} \end{cases} \qquad (3.3)$$

where p^+, p^- represent the pressure wave components in positive (upper waveguide rail) and negative (lower waveguide rail) direction. The solution in volume velocity waves in the cone yields:

$$U(r,t) = \frac{u^+}{r} - \frac{u^-}{r} \qquad (3.4)$$

with
$$\begin{cases} u^+ = Z_x^{-1}\left(1 - \frac{j}{kr}\right)Ae^{-jkr+j\omega t} \\[2mm] u^- = -Z_x^{-1}\left(1 + \frac{j}{kr}\right)Be^{jkr+j\omega t} \end{cases} \qquad (3.5),$$

Where wave propagation $k = \omega / c$, and with the standard characteristic impedance $Z_x = \rho \cdot c / S_x$ at point x of the air column.

From these equations we can derive the local *characteristic impedance*. Impedance (Z), or the reciprocal value of impedance, admittance (Y), is a frequency dependent value that indicates the resistance to motion at a certain place in the system. In acoustical systems the definitions are:

$$\begin{aligned} Z &= P/U \\ Y &= U/P \end{aligned} \qquad (3.6)$$

In cylindrical tubes the characteristic impedance is a constant value depending on the tube diameter. In conical tubes the impedance also is a function of frequency and location. Kinsler & Frey (1950) give the admittances for upper and lower rail. They can be calculated from eqs. (3.2) , (3.4) and (3.6):

$$\begin{aligned} Y^+ &= Z_x^{-1}(1 - j/kr) \\ Y^- &= Z_x^{-1}(1 + j/kr) \end{aligned} \qquad (3.7)$$

Note that these expressions are also valid for the cylindrical case, because then $r \to \infty$.

From the characteristic impedances we can calculate the *coordinate transformation* from transmission line coordinates to waveguide coordinates. The transformation translates a representation in pressure (P) and flow (U) values to upper (p^+) and lower (p^-) travelling waves. Combining (3.3) and (3.7) gives:

$$\begin{aligned} p^+ &= \frac{(1 + j/kr)P + Z_x U}{2r} \\[2mm] p^- &= \frac{(1 - j/kr)P - Z_x U}{2r} \end{aligned} \qquad (3.8)$$

The transformation is quite similar to the one used in *wave digital filters* (Lawson and Mirzai, 1990), using the equivalents of pressure and flow in electrical circuits, voltage and current. The main difference is that within the wave digital filter transformation no frequency dependent resistance can be defined. The frequency dependency is assimilated in adaptors, which represents an element in a electrical circuit model of the acoustical system. These models are widely available in literature (Benade, 1988), (Stewart and Strong, 1980), from which wave digital filter simulations could be made. We choose to use waveguide filter representations, because they are more directly related to acoustical systems.

3.3 Reed Coupling

The above transformation is necessary for the coupling of the reed model with a conical waveguide, since the reed model is calculated in transmission line parameters; it gives flow as output, which has to

be translated into waveguide coordinates. An equation can be derived that expresses the coupling between the reed and the tube involving one filter (see fig. 3.3 for signal flow):

$$p^+ = Z_o U + p^- - R_c(\omega)\left[Z_o U + 2p^-\right] \qquad (3.9)$$

where $R_c(\omega)$ is a first order lowpass filter:

$$R_c(\omega) = \frac{\alpha_c}{j\omega + \alpha_c} \qquad (3.10)$$

with $\quad \alpha_c = c/x_o$

So the coupling filter transfer function only depends on the distance to the cone apex (x_o). In the cylindrical case there is no filtering, because then $x_o \to \infty$ and $R_c(\omega) \to \infty$, which corresponds to equation (2.7).
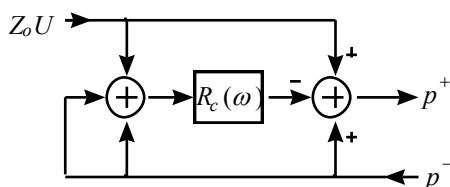

fig 3.3   *reed coupling signal flow.*

3.4 Cone Junctions

The filters structures that represent the coupling of two tube sections are called *scattering junctions*. An example junction of the cones *a* and *b* is depicted in fig. 3.4. The conical scattering equations can be derived by following Kirchoff's law, which expresses the condition of conservation of volumetric flow and pressure. The equations are expressed in *reflection coefficients* ($R^{\pm}(\omega)$) and *transmission coefficients* ($T^{\pm}(\omega)$). The reflection coefficient indicates how much of a travelling wave is reflected, whereas the transmission coefficient represents the part of the signal that is guided through the junction.
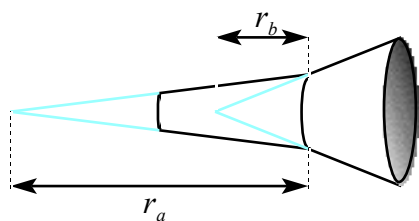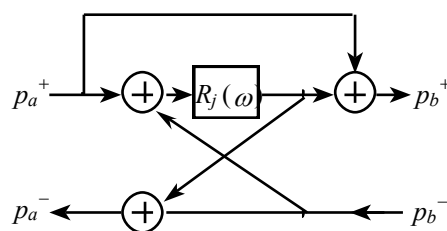

fig. 3.4   *Junction of two cones*

fig 3.5   *conical scattering junction without taper discontinuity*

Välimäki (1994a) and Agulló (1993) have derived the general conical scattering equations, which involve two filters. We are mainly interested in junctions with equivalent cone diameters, since most instrument air columns do not have taper discontinuities. Fortunately in this case there is only one filter necessary:

$$p_a^{\ -} = p_b^{\ -} + R_j(\omega)(p_a^{\ +} + p_b^{\ -})$$
$$p_b^{\ +} = p_a^{\ +} + R_j(\omega)(p_a^{\ +} + p_b^{\ -})$$
(3.10)

This filter is the same first order lowpass type as $R_c(\omega)$:

$$R_j(\omega) = \frac{-\alpha_j}{j\omega + \alpha_j}$$
(3.11)

with $\alpha_j = \dfrac{c(r_a - r_b)}{2r_a r_b}$

One (or both) of the tubes may be cylindrical, for example when $r_a \to \infty$, then $\alpha_j = c/2r_b$. Furthermore, the scattering junction is valid for both tapering $(x_o < 0)$ and flaring $(x_o > 0)$ cones (Välimäki 1994b).

3.5. Termination

The radiation impedance of the spherical wave can be used as an approximation of the real termination impedance at the open end. Comparison with the radiation formulas given by Levine and Schwinger (1948) shows that this impedance is not correct in the lower frequencies. Nevertheless it is a useful test model that represents the basic acoustic mechanism. Fletcher and Rossing (1991) give the impedance of a spherical wave with end radius a_l:

$$Z_l = Z_e \frac{jka_l + (ka_l)^2}{(ka_l)^2 + 1}$$
(3.12)

From this equation we can calculate the termination admittance:

$$Y_l = Z_e^{-1}\left[1 + \frac{1}{jka_l}\right]$$
(3.13)

This admittance is equal to the admittance of an infinite cone with an apex distance r_l equal to the radius a_l. This means that, within our model, we can consider the impedance transition from a cone to the open air as a junction of that cone with an infinite cone (see fig 3.6). There is no reflection at the end of an infinite cone, so the radiation scattering junction only reflects $p_a^{\ +}$:
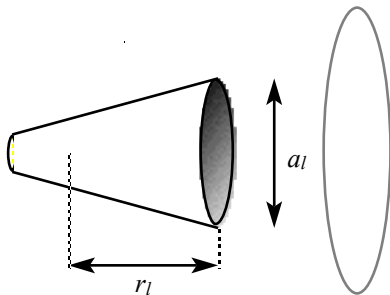

fig. 3.6   Termination model: junction of a cone
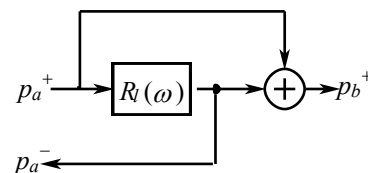with an infinite cone junction.

fig. 3.7   Termination scattering   junction.

$$p_a{}^- = R_l(\omega)p_a{}^+$$
$$p_b{}^+ = p_a{}^+ + R_l(\omega)p_a{}^+ \qquad\qquad (3.14)$$

$R_l(\omega)$ is the same as $R_j(\omega)$, $\alpha_l = \dfrac{c(r_a - a_l)}{2ar_a}$:

$$R_l(\omega) = \frac{-\alpha_l}{j\omega + \alpha_l} \qquad\qquad (3.15)$$

3. 6 Discretisation

The complete model requires only one type of analog filter :

$$H(s) = \frac{-\alpha}{s + \alpha} \qquad\qquad (3.16)$$

(with $s = j\omega$)

This filter has to be mapped into the discrete domain. Since we are simulating a physical system, the transformation should map both the magnitude and the phase properties as correct as possible to the Z-domain. We made a comparison between the impulse-invariance method and the bilinear transform of this filter type. The magnitude response of the bilinear transform underestimates that of the analog filter, while the impulse-variation method causes an overestimation. Both methods have quite accurate (error of less than 3 dB) magnitude responses up until 10 kHz.

The impulse-invariant method causes a linear phase component of half a sample period in the phase response. This means that the waveguide on which a filter is placed becomes half a sample longer. If more filters are used the total extra delay must be compensated within the waveguide. With the use of the bilinear transform this problem does not arise. This allows a more intuitive waveguide filter implementation.

3.7 Fractional Delay Waveguides

To be able to connect tubes of arbitrary length the number of delays required in the waveguide simulation is not fitting to the integer delay-steps provided by the waveguide implementation. Therefore interpolation and deinterpolation techniques must be used to obtain *fractional delay* values (Välimäki, 1994b). In our current model we use a first order allpass filter for this interpolation.

3.8 Conclusions

A real-time simulation model of reed-driven instruments is developed by coupling a reed model to a waveguide simulation of acoustical tubes. The shape of these tubes is approximated with conical sections. The effects of impedance transitions are simulated with filters. In our current model the complete waveguide can be modelled with only one type of filter.

A general digital reed model has been developed without using a lookup table as is usually done in implementations, that yet includes the effects of dynamical reed, reed inertia and reed beating. The model does not only show a good steady state behaviour and frequency domain characteristics that match with existing instruments, it also is capable of simulating authentically the fast transients, which occur at every parameter change caused by a player. It is the opinion of the authors that these transients reveal a great deal of the characteristic 'acoustic' nature of instruments.

What is excluded from this model is the complex effect of turbulent airflow, which is, for example, an

important effect in clarinets according to Hirschberg (1994). Another simplification is the representation of the reed: its motion is reduced to the influence of the first mode. Furthermore, frequency dependent losses can be included in a future model.

The major advantage of the model lies in the generality, because of the modular structure of the waveguide representation and the flexibility of the reed simulation: within the same DSP-model single reed, double reed and lip reed can be simulated.

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