

ORBIT DISTORTION AND ITS CORRECTION IN A NON-SCALING FFAG

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Abstract

The large variation in betatron tune over the energy range of the EMMA non-scaling FFAG, and the rapidity of the acceleration, result in novel features in the properties of orbit distortion. The crossing of many integer tune resonances is achieved through fast crossing. It is clear that standard harmonic correction is not applicable since the phase advance between lattice elements varies with momentum. Two correction methods that reduce orbit distortion due to transverse magnet misalignments are presented - local correction of the error sources and optimisation of the injection coordinates and corrector magnet strengths.

INTRODUCTION

Bending and focusing in the EMMA FFAG is achieved using quadrupole doublets in which the beam pipe is offset from the magnet centres. There are 42 cells and in every second cell a 1.3 GHz rf cavity sits in the middle of the long drift (apart from at injection and extraction). For the purposes of this report the 19 cavities are each set to 120kV allowing acceleration from 10-20 MeV to be completed in 6 turns. A novel 'serpentine' mode of acceleration is followed in which the rf frequency remains fixed. The details of the EMMA lattice is discussed in [1].

The quadrupoles are mounted on mechanical sliders which enable correction of misalignments in the horizontal direction. There is also the possibility to install vertical corrector magnets at various points in the lattice. The dipole field of these magnets will remain constant during the short acceleration time.

In this study the beam optics code MAD-X and the tracking code PTC (Polymorphic tracking code) [2] are employed to calculate the orbit distortion. PTC is a single particle symplectic integrator that takes fully into account the three dimensional structure of a lattice. In this way, complex ring geometries that cannot be properly treated by MAD-X, can be handled by PTC. Unlike MAD-X, the code allows modelling of offset quadrupoles and includes an analytic model of the quadrupole fringe field [3].

ORBIT DISTORTION

A feature of a non-scaling FFAG with rapid acceleration is that the orbit distortion does not see the integer tune resonances that, in a slow cycling machine, are excited by magnet misalignments. As the tune in EMMA varies by approximately one integer for every turn, the concept of resonance is not applicable (Fig. 1). As is clear from the

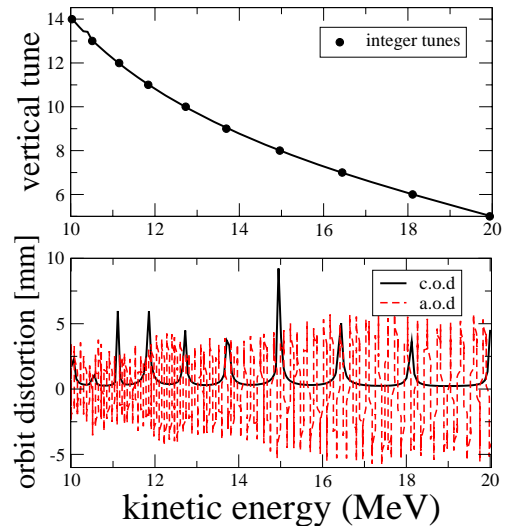


Figure 1: Vertical tune as a function of kinetic energy (top) and a comparison of the closed and accelerated orbit distortion in the vertical plane due to magnet misalignments (bottom). The peaks in the closed orbit distortion coincide with integer vertical tune (circles, top). It is clear that no such structure exists in the case with acceleration.

figure, the peaks in the closed orbit distortion that correspond to integer values of total tune do not appear when acceleration is included, i.e. a non-scaling FFAG with rapid acceleration does not experience these resonances. Instead the orbit distortion is excited by random dipole kicks due to the magnet misalignments [4]. This orbit distortion we may refer to as the 'accelerated orbit distortion' to make a distinction with the closed orbit distortion that is calculated at fixed momentum.

In order to find the level of magnet misalignments that can be tolerated in EMMA, the amplification factor, defined as the ratio of the maximum orbit distortion to the standard deviation of the input misalignments, is calculated. The beam is tracked through 300 lattices, each with different random misalignments. The standard deviation of the magnet misalignments is increased from 1-150 μm and the maximum orbit distortion noted. A linear fit through this set of data yields the amplification factor (Fig. 2). This is found to be 89 in the horizontal plane and 72 in the vertical when operating at 120kV per cavity.

Since the orbit distortion is due to the cumulative effect of random dipole kicks at the misaligned quadrupoles, the amplification factor should scale with the square root of the number of quadrupoles, i.e. the number of turns taken to

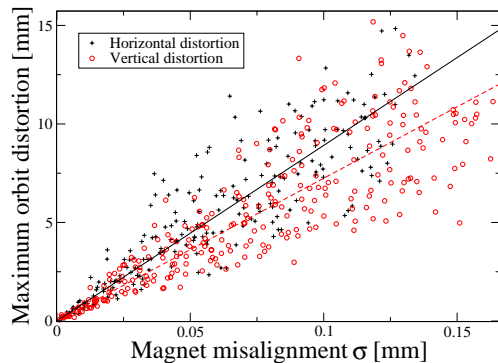


Figure 2: Dependence of maximum horizontal (pluses) and vertical (circles) accelerated orbit distortion on the standard deviation of random magnet misalignments. The lines show the linear fit to the data in the horizontal (solid) and vertical (dash) cases.

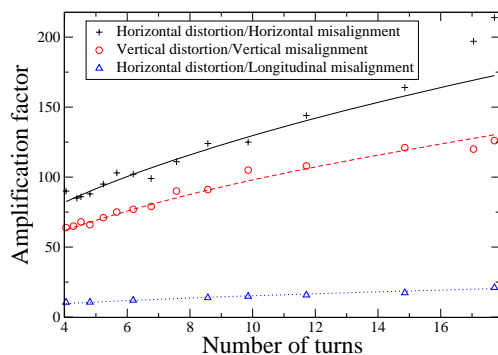


Figure 3: Dependence of the amplification factor on the number of turns required to complete acceleration corresponding to horizontal (pluses), vertical (circles) and longitudinal (triangles) misalignments. The amplification factor is calculated based on 100 random magnet misalignments with standard deviation in the range $1 - 60 \mu\text{m}$. The lines show the predicted amplification factor based on the square root of the number of turns in the horizontal (solid), vertical (dash) and longitudinal (dot) cases.

complete acceleration. Therefore a prediction of the amplification factor can be made at different acceleration rates, scaling from the already calculated value at 120kV per cavity. This scaling is applied to predict the amplification factor in the range 55kV – 180kV per cavity, corresponding to a range of 4 – 18 in the number of turns required to reach 20MeV (Fig.3). It is clear that the prediction is consistent with the tracking results. With amplification factor of order 100, a horizontal magnet misalignment of $50 \mu\text{m}$ will lead to a maximum orbit distortion of 5mm. This level of orbit distortion is significantly above tolerance $\approx 1\text{mm}$.

ORBIT CORRECTION

Conventional orbit correction relies on a constant phase advance between the corrector magnet and beam position monitors (BPM). In EMMA the phase advance varies strongly with momentum and so alternative correction schemes must be considered.

Local Correction

Local Correction

The inclusion of horizontal sliders under each quadrupole enables correction of misalignments in that plane. Determining the magnet misalignments is the subject of this section. Since a quadrupole misalignment is equivalent to a dipole kick, the BPM measurements m_i may be written

$$m_i = m_{ideal} + \sum_{j=1}^{n_{quad}} \Delta x_j R_{ij} + \sum_{j=1}^{n_{corr}} \theta_j T_{ij} \quad (1)$$

where Δx_j is the misalignment of quadrupole j , m_{ideal} is BPM measurement that would be made with perfectly aligned magnets and R_{ij} is the response coefficient that gives the closed orbit distortion at the measurement points due to the magnet misalignments, T_{ij} is the response matrix relating closed orbit distortion to corrector magnet kick angle $\theta_j = \frac{B_j l_j}{B\rho}$. The beam optics code MAD-X includes the MICADO correction algorithm that will find the set of corrector kick angles θ_j that minimises the closed orbit distortion at the BPMs. The magnet misalignments can be calculated from the corrector kick angles by placing 'virtual' corrector magnets in the centre of each quadrupole and noting $\Delta x_j = \theta_j / (k_j l_j)$, where k_j is the normal quadrupole coefficient and l_j its length.

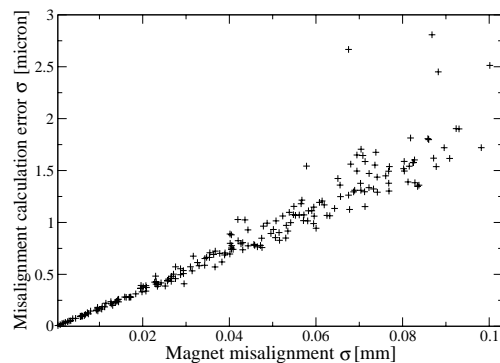


Figure 4: Dependence of standard deviation in magnet misalignment calculation on the standard deviation of the input magnet misalignments. Each data point uses a different misalignment error pattern. The BPM measurements are assumed to be perfect.

We assume that the BPMs have no errors associated with them. We also make the assumption that the BPM measurements m_{ideal} in the perfect lattice are known. The accuracy of the magnet misalignment calculation is shown in Fig. 4. A linear fit to the data implies that the standard deviation in the misalignment calculation error is about 2% of the standard deviation of the misalignments themselves. However, if BPM errors are included in the calculation then the

accuracy of the magnet misalignment calculation is significantly reduced and work is under way to address this.

Overall Correction

A method that, on average, reduces the accelerated orbit distortion calculated over the entire energy range is proposed. The parameters to be adjusted in this optimisation are the corrector magnet strengths and the injection phase space variables. A set of Taylor coefficients A_{ij} corresponding to the linear dependence of each BPM measurement y_i on each corrector magnet j that preceded it can be created using differential algebra in PTC, i.e. $A_{ij} = \frac{\delta y_i}{\delta \theta_j}$. Note that the number of measurements is the product of the number of BPMs and the number of turns. We may write

$$A \cdot \theta = -y_{bpm} \quad (2)$$

where θ is the set of corrector strengths and y_{bpm} the measured distortion. The least squares problem can be solved for the corrector strengths θ (or similarly for the injection phase space variables) via QR Decomposition. Fig. 5 show how the vertical orbit distortion is, on average, reduced by optimising the initial phase space variables.

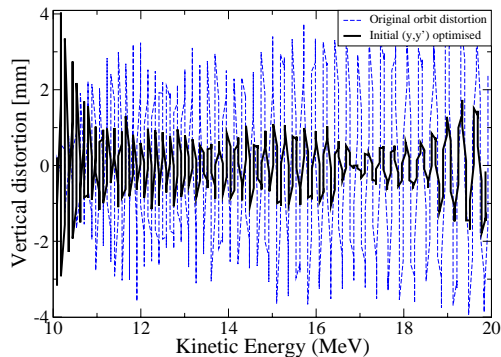


Figure 5: Vertical accelerated orbit distortion measured at BPMs against kinetic energy with initial (y, y') from the closed orbit at the injection energy (dash) or set to values optimised by overall correction (solid). The magnets were misaligned with a standard deviation of $50\mu\text{m}$.

Further improvements in the orbit distortion can be made by optimising the corrector magnet strengths. It is of interest to determine the dependence of the orbit distortion reduction on the number of corrector magnets added. The number of corrector magnets is increased from 1 to 16 and in each case the best corrector locations are found for each of 100 misalignment error patterns. The mean improvement in orbit distortion with number of correctors is shown in Fig. 6. It is apparent that each corrector magnet added produces a smaller improvement in orbit distortion than the last and that the bulk of the improvement is achieved by the injection optimisation.

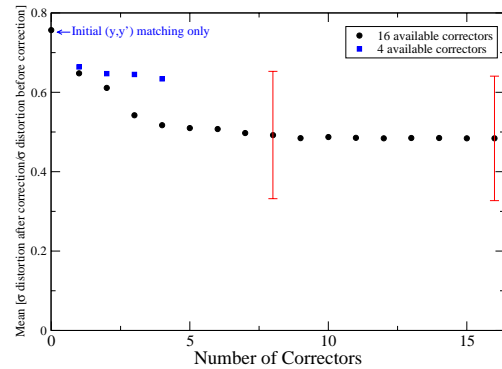


Figure 6: Mean ratio of orbit distortion σ with and without corrector magnet optimisation over 100 misalignment error patterns against number of correctors included. In each case the best set of corrector magnets from 16 locations (black) and 4 locations (blue) was used. The first point shows the mean improvement with injection optimisation only. The error bars show 1σ over the 100 cases and are almost identical for each data point. The magnets were misaligned with a standard deviation of $50\mu\text{m}$.

SUMMARY

It has been shown that in EMMA the amplification factor that relates the maximum orbit distortion to the level of transverse magnet misalignments is about 90 in the horizontal plane and 70 in the vertical plane at the maximum rate of acceleration (120kV per cavity). placing stringent requirements on orbit correction accuracy.

It was demonstrated that although traditional harmonic correction of orbit distortion will not work in a non-scaling FFAG, other approaches can be adopted. The misalignments themselves are found and corrected by finding the closed orbit distortion at fixed energy and adapting the standard MAD-X correction algorithm. The orbit distortion can also be reduced using the so called overall correction method. Optimisation of the injection phase space parameters results in a substantial reduction in orbit distortion ($\sim 25\%$). The vertical orbit distortion can be further improved, though we less effect, by including correctors whose strengths are determined by the overall correction.

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