



# Effective collision strengths for allowed transitions among the n

Aggarwal, K. M., Igarashi, A., Keenan, F. P., & Nakazaki, S. (2008). Effective collision strengths for allowed transitions among the n. Astronomy and Astrophysics, 479, 585-588. DOI: 10.1051/0004-6361:20078933

Published in: Astronomy and Astrophysics

**Document Version:** Publisher's PDF, also known as Version of record

**Queen's University Belfast - Research Portal:** Link to publication record in Queen's University Belfast Research Portal

**Publisher rights** © 2008 ESC Aggarwal, K. M.; Igarashi, A.; Keenan, F. P.; Nakazaki, S. In: Astronomy and Astrophysics, Vol. 479, 02.2008, p. 585-588.

#### **General rights**

Copyright for the publications made accessible via the Queen's University Belfast Research Portal is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy The Research Portal is Queen's institutional repository that provides access to Queen's research output. Every effort has been made to ensure that content in the Research Portal does not infringe any person's rights, or applicable UK laws. If you discover content in the Research Portal that you believe breaches copyright or violates any law, please contact openaccess@qub.ac.uk.



# Effective collision strengths for allowed transitions among the $n \le 5$ degenerate levels of AI xill

K. M. Aggarwal<sup>1</sup>, A. Igarashi<sup>2</sup>, F. P. Keenan<sup>1</sup>, and S. Nakazaki<sup>2</sup>

<sup>1</sup> Astrophysics Research Centre, School of Mathematics and Physics, Queen's University Belfast, Belfast BT7 1NN, Northern Ireland, UK

e-mail: K.Aggarwal@qub.ac.uk

<sup>2</sup> Department of Applied Physics, Faculty of Engineering, University of Miyazaki, Miyazaki 889-2192, Japan

Received 26 October 2007 / Accepted 17 December 2007

#### ABSTRACT

*Aims.* In this paper we report on calculations of collision strengths and effective collision strengths for allowed transitions among the  $n \le 5$  degenerate levels of Al XIII.

Methods. The Dirac atomic R-matrix code (DARC) has been adopted for these calculations.

*Results.* Collision strengths are reported over a wide energy range below 300 Ryd, and effective collision strengths are provided for electron temperatures of  $4.4 \le \log T_e \le 6.8$  K.

Key words. atomic data - atomic processes

## 1. Introduction

In a recent paper (Aggarwal et al. 2005) we reported results for energy levels, radiative rates, collision strengths, and excitation rate coefficients for transitions among the  $n \leq 5$  levels of Al XIII. Those calculations were a substantial improvement over our earlier results in the LS coupling scheme for the same ion (Aggarwal et al. 2001). However, even the more recent data for transitions among the fine-structure levels remained limited in scope, particularly for those that are allowed among the degenerate levels of a state, such as  $2s {}^{2}S_{1/2}-2p {}^{2}P_{1/2,3/2}^{\circ}$ . This was because collision strengths were *not* converged within the partial wave range  $(J \le 60)$  considered in that paper. Extending the partial wave range in a collisional code such as DARC is impractical, because these transitions converge very *slowly* as demonstrated by Igarashi et al. (2003). Therefore, a "top-up" based on the formulations of Burgess et al. (1970) has now been fully implemented in the Dirac atomic R-matrix code (DARC) of Ait-Tahar et al. (1996), which enables us to calculate converged values of  $\Omega$  for all allowed transitions, including the "elastic" ones, i.e. those for which  $\Delta E \sim 0$ . However, results for all transitions, forbidden and allowed, remain the same as already reported, except for those which are allowed among the degenerate levels and for which  $\Delta E \sim 0$ . Therefore, in this paper we present values of collision strengths ( $\Omega$ ) as well as effective collision strengths ( $\Upsilon$ ) for all 26 elastic transitions alone over a wide energy/temperature range, so that data for all transitions can be applied with confidence in plasma modelling.

## Table 1. Energy levels (in Ryd) for Al XIII.

Index	Configuration	Level	Energy
1	1s	${}^{2}S_{1/2}$	0.000000
2	2s	${}^{2}S_{1/2}$	126.988960
3	2p	${}^{2}P_{1/2}^{o}$	126.985252
4	2p	${}^{2}P_{3/2}^{o}$	127.081062
5	3p	${}^{2}P^{0}_{1/2}$	150.534500
6	3s	${}^{2}S_{1/2}$	150.535599
7	3d	${}^{2}D_{3/2}$	150.562851
8	3р	${}^{2}P_{3/2}^{o}$	150.562897
9	3d	$^{2}D_{5/2}$	150.572266
10	4p	${}^{2}P_{1/2}^{o}$	158.772842
11	4s	${}^{2}S_{1/2}$	158.773300
12	4d	$^{2}D_{3/2}$	158.784805
13	4p	${}^{2}P_{3/2}^{o}$	158.784821
14	4d	$^{2}D_{5/2}$	158.788773
15	4f	${}^{2}\mathrm{F}^{o}_{5/2}$	158.788773
16	4f	${}^{2}\mathrm{F}^{\mathrm{o}}_{7/2}$	158.790756
17	5s	${}^{2}S_{1/2}$	162.584579
18	5p	${}^{2}P_{1/2}^{o}$	162.584351
19	5d	${}^{2}D_{3/2}$	162.590469
20	5p	${}^{2}P^{o}_{3/2}$	162.590485
21	5f	${}^{2}\mathrm{F}^{\mathrm{o}}_{5/2}$	162.592499
22	5d	$^{2}D_{5/2}$	162.592499
23	5g	${}^{2}G_{7/2}$	162.593521
24	5f	${}^{2}\mathrm{F}^{0}_{7/2}$	162.593521
25	5g	${}^{2}G_{9/2}$	162.594131

## 2. Collision strengths

The details of our calculations have already been given in our earlier paper (Aggarwal et al. 2005) and hence are not repeated here. Additionally, results for all transitions *except* those

which are allowed among the degenerate levels remain the same. Therefore in the present paper we discuss results for only the elastic transitions, i.e. those which are *allowed* within the  $n \leq 5$  levels of Al XIII, and for which  $\Delta E \sim 0$ .

In Table 1 we list our energy levels, as reported earlier (Aggarwal et al. 2005), which will facilitate the discussion of



**Fig. 1.** Comparison of collision strengths for some "elastic" transitions of Al XIII. Continuous curves: DARC, broken curves: CC+CB, stars: 2–4 ( $2s^{2}S_{1/2}-2p^{2}P_{3/2}^{\circ}$ ), squares: 5–7 ( $3p^{2}P_{1/2}^{\circ}-3d^{2}D_{3/2}$ ), and circles: 8–9 ( $3p^{2}P_{3/2}^{\circ}-3d^{2}D_{5/2}$ ) transitions are from FAC.

our subsequent results. It may be noted here that the energy of the 2s  ${}^{2}S_{1/2}$  level is *higher* than that of 2p  ${}^{2}P_{1/2}^{\circ}$  due to the inclusion of Lamb shift. However, we have retained this ordering (of  $2s {}^{2}S_{1/2}$  level before  $2p {}^{2}P_{1/2}^{\circ}$ ), mainly because DARC provides results in this ordering and more importantly, this is the same ordering as was adopted in our earlier paper (Aggarwal et al. 2005). Table 2 lists our results of  $\Omega$ , at energies above thresholds but below 300 Ryd, for all 26 elastic transitions among the  $n \leq 5$  degenerate levels of Al XIII. To our knowledge, the only other results available in the literature for (qualitative) comparison purposes are those of Zygelman & Dalgarno (1987). Their values of  $\Omega$  cover a wide energy range up to ~250 Ryd, but only for transitions within the n = 2 levels. Similarly, they have performed calculations for several ions with  $2 \le Z \le 18$ , but not for Z = 13. Therefore, we have performed two other independent calculations to verify our results. The first uses the Flexible Atomic Code (FAC) of Gu (2003), available from the website http://kipac-tree.stanford.edu/fac.This is a fully relativistic code (as is DARC) and is based on the well known and widely used distorted-wave (DW) method. FAC provides background values of  $\Omega$  over a wider energy range (up to ~1400 Ryd) but only at six energies. Nevertheless, the data obtained from FAC are helpful in assessing the accuracy of our results. The other calculation is from a combination of the close-coupling (CC) and the Coulomb-Born (CB) programs of Igarashi et al. (2003, 2005), who reported collision strengths for elastic transitions, but again within the n = 2 levels alone. Their CC+CB programs have now been extended to calculate values of  $\Omega$  up to the n =5 levels for a hydrogenic ion of any nuclear charge Z.

In Fig. 1 we compare our results from DARC with those from FAC and CC+CB, but for only three transitions, namely 2–4 (2s  ${}^{2}S_{1/2}$ –2p  ${}^{2}P_{3/2}^{\circ}$ ), 5–7 (3p  ${}^{2}P_{1/2}^{\circ}$ –3d  ${}^{2}D_{3/2}$ ), and 8–9

(3p  ${}^{2}P_{3/2}^{\circ}-3d {}^{2}D_{5/2}$ ), over a wide energy range below 400 Ryd. As is clear from this figure, there is no discrepancy among the different calculations, and the values of  $\Omega$  from any calculation can be safely employed for the determination of excitation rates. Similar comparisons for other transitions have also been made but are not shown here for brevity. However, a comparison of the present results shows that the earlier reported values of  $\Omega$ , with only  $J \leq 60$ , are underestimated for all elastic transitions by over a factor of two, and by up to an order of magnitude for some transitions, such as: 17–18 (5s  ${}^{2}S_{1/2}-5p {}^{2}P_{1/2}^{\circ}$ ), 19–20 (5d  ${}^{2}D_{3/2}-5p {}^{2}P_{3/2}^{\circ}$ ), 21–22 (5f  ${}^{2}F_{5/2}^{\circ}-5d {}^{2}D_{5/2}$ ), and 23–24 (5g  ${}^{2}G_{7/2}-5f {}^{2}F_{7/2}^{\circ}$ ). Finally, based on comparisons of the different calculations for all elastic transitions, we can state with confidence that the results for  $\Omega$  listed in Table 2 are accurate to better than 15%.

## 3. Effective collision strengths

In Table 3 we list our results of  $\Upsilon$  for all 26 elastic transitions among the  $n \le 5$  degenerate levels of Al XIII, over a wide temperature range of 4.4  $\le \log T_e \le 6.8$  K, the same as adopted in our earlier paper (Aggarwal et al. 2005). From these values of  $\Upsilon$ , the corresponding results for excitation and de-excitation rate coefficients can be easily determined from the simple relationships given in Eqs. (2) and (3) of Aggarwal et al. (2001). To our knowledge, there are no other results for comparison purposes, but our earlier reported values of  $\Upsilon$  are underestimated by over a factor of two for almost all elastic transitions, and by up to a factor of eight for some transitions, such as: 12-13 (4d  $^2D_{3/2}-4p$   $^2P_{3/2}^{\circ}$ ), 14-15 (4d  $^2D_{5/2}-4f$   $^2F_{5/2}^{\circ}$ ), 19-20 (5d  $^2D_{3/2}-5p$   $^2P_{3/2}^{\circ}$ ), and 21-22 (5f  $^2F_{5/2}^{\circ}-5d$   $^2D_{5/2}$ ). This is because our corresponding

**Table 2.** Collision strengths for the "elastic" transitions of Al XIII.  $(a \pm b \equiv a \times 10^{\pm b})$ .

Trans	sition				H	Energy (Ryd	l)			
i	j	170.0	180.0	190.0	200.0	220.0	240.0	260.0	280.0	300.0
2	3	2.728+0	2.753+0	2.774+0	2.791+0	2.818+0	2.839+0	2.856+0	2.871+0	$2.883 \pm 0$
2	4	3.090+0	$3.189 \pm 0$	$3.268 \pm 0$	3.336+0	$3.443 \pm 0$	3.526+0	3.594 + 0	3.651+0	3.700+0
5	6	1.617 + 1	1.647 + 1	1.666 + 1	1.681 + 1	1.702 + 1	1.716 + 1	1.728 + 1	1.737 + 1	1.745 + 1
5	7	1.180 + 1	1.252 + 1	1.301 + 1	1.337 + 1	1.389 + 1	1.426 + 1	1.455 + 1	1.478 + 1	1.497 + 1
6	8	1.867 + 1	1.982 + 1	2.058+1	2.116+1	2.198 + 1	2.257 + 1	2.302 + 1	2.339 + 1	2.370+1
7	8	4.255+0	4.328 + 0	4.378 + 0	4.415 + 0	$4.468 \pm 0$	4.506+0	4.535+0	$4.559 \pm 0$	4.579 + 0
8	9	2.550+1	2.683 + 1	2.773 + 1	2.839 + 1	2.934 + 1	3.000+1	3.053 + 1	3.095 + 1	3.130 + 1
10	11	6.119+1	6.266 + 1	6.348 + 1	6.405 + 1	6.478 + 1	6.529 + 1	6.570 + 1	6.599 + 1	6.626 + 1
10	12	5.035 + 1	5.497 + 1	5.757 + 1	5.935 + 1	6.175 + 1	6.338 + 1	6.461 + 1	6.560 + 1	6.640 + 1
11	13	6.254 + 1	6.829 + 1	7.151 + 1	7.372 + 1	7.670 + 1	7.873 + 1	8.026 + 1	8.148 + 1	8.249 + 1
12	13	2.038+1	2.086+1	2.113 + 1	2.132+1	2.156+1	2.173 + 1	2.185 + 1	2.195 + 1	2.203 + 1
12	15	6.410 + 1	6.910 + 1	7.188 + 1	7.377 + 1	7.633 + 1	7.807 + 1	7.937 + 1	8.041 + 1	8.127 + 1
13	14	1.088+2	1.173 + 2	1.221 + 2	1.253+2	1.296+2	1.326+2	1.349+2	1.366+2	1.381 + 2
14	15	$8.467 \pm 0$	8.642 + 0	8.739 + 0	8.808 + 0	8.901 + 0	$8.962 \pm 0$	9.009+0	9.050+0	9.079 + 0
14	16	1.013 + 2	1.084+2	1.124 + 2	1.151+2	1.188+2	1.213 + 2	1.231+2	1.247 + 2	1.258+2
17	18	1.534+2	1.582 + 2	1.605 + 2	1.620+2	1.638+2	1.651+2	1.660+2	1.668 + 2	1.674 + 2
17	20	1.576+2	1.763 + 2	1.853 + 2	1.911+2	1.986+2	2.037+2	2.075+2	2.105+2	2.129+2
18	19	1.385 + 2	1.549 + 2	1.628 + 2	1.679 + 2	1.745 + 2	1.789 + 2	1.822 + 2	1.849 + 2	1.871 + 2
19	20	6.264 + 1	6.431+1	6.513 + 1	6.563 + 1	6.634 + 1	6.678 + 1	6.709 + 1	6.738 + 1	6.754 + 1
19	21	2.293 + 2	2.523 + 2	2.633 + 2	2.703 + 2	2.795 + 2	2.856+2	2.901 + 2	2.938+2	2.967 + 2
20	22	2.990+2	3.291 + 2	3.435 + 2	3.527 + 2	3.648 + 2	3.727 + 2	3.787 + 2	3.835 + 2	3.874 + 2
21	22	2.733 + 1	2.816+1	2.855 + 1	2.881 + 1	2.914 + 1	2.936+1	2.952+1	2.965 + 1	2.975 + 1
21	23	2.056+2	2.243 + 2	2.331+2	2.388+2	2.462 + 2	2.510+2	2.547 + 2	2.577+2	2.601 + 2
22	24	3.621 + 2	3.952 + 2	4.109 + 2	4.210+2	4.341 + 2	4.428 + 2	4.494 + 2	4.545 + 2	4.587 + 2
23	24	1.166 + 1	1.199 + 1	1.215 + 1	1.226+1	1.239 + 1	1.248 + 1	1.255 + 1	1.261 + 1	1.265 + 1
24	25	2.853+2	3.095+2	3.210+2	3.284+2	3.380+2	3.443+2	3.491+2	3.528 + 2	3.560+2

**Table 3.** Effective collision strengths for the "elastic" transitions of Al XIII.  $(a \pm b \equiv a \times 10^{\pm b})$ .

Transit	ion						Temp	erature (le	og, K)					
i	j	4.400-0	4.600-0	4.800-0	5.000-0	5.200-0	5.400-0	5.600-0	5.800-0	6.000-0	6.200-0	6.400-0	6.600-0	6.800-0
2	3	8.827-1	9.764-1	1.071+0	1.190+0	1.345+0	1.488+0	1.576+0	1.650+0	1.747+0	1.866+0	1.995+0	2.126+0	2.256+0
2	4	1.332+0	1.357+0	1.392 + 0	1.443+0	1.514+0	1.608+0	1.727+0	$1.867 \pm 0$	2.026+0	2.201+0	2.392+0	2.596+0	2.809+0
5	6	6.325 + 0	7.014+0	7.707+0	8.554+0	9.619+0	1.056+1	1.109 + 1	1.149 + 1	1.201 + 1	1.265+1	1.335+1	1.406+1	1.476 + 1
5	7	5.355+0	5.514 + 0	5.742 + 0	6.057+0	$6.469 \pm 0$	$6.972 \pm 0$	7.555+0	8.200+0	$8.885 \pm 0$	9.599+0	1.034+1	1.111+1	1.189 + 1
6	8	$8.512 \pm 0$	$8.769 \pm 0$	9.137+0	$9.645 \pm 0$	1.031+1	1.112+1	1.205+1	1.308 + 1	1.418 + 1	1.532 + 1	1.650+1	1.772 + 1	1.896 + 1
7	8	2.665+0	2.944+0	3.197 + 0	3.483+0	3.831+0	4.120+0	4.248 + 0	4.317+0	4.430+0	$4.589 \pm 0$	4.768+0	4.951+0	5.132 + 0
8	9	1.050+1	1.099+1	1.168 + 1	1.260+1	1.373 + 1	1.501 + 1	1.638 + 1	1.781 + 1	1.930+1	2.081 + 1	2.233+1	2.387 + 1	2.540+1
10	11	2.377+1	2.643 + 1	2.901 + 1	3.207 + 1	3.583 + 1	3.907 + 1	4.074 + 1	4.185 + 1	4.338 + 1	4.534+1	4.747 + 1	4.963 + 1	5.176 + 1
10	12	2.456+1	2.544 + 1	2.668 + 1	2.838 + 1	3.052+1	3.302 + 1	3.578 + 1	3.871 + 1	4.174 + 1	4.483 + 1	4.795 + 1	5.112 + 1	5.429 + 1
11	13	2.930+1	3.053 + 1	3.227 + 1	3.460+1	3.751+1	4.082 + 1	4.437 + 1	4.810 + 1	5.196 + 1	5.588 + 1	5.984 + 1	6.385 + 1	6.785 + 1
12	13	1.213+1	1.334+1	1.441 + 1	1.562 + 1	1.708 + 1	1.828 + 1	1.876 + 1	1.898 + 1	1.939+1	1.999+1	2.067+1	2.138+1	2.208 + 1
12	15	2.746+1	3.084 + 1	3.406+1	3.789 + 1	4.272 + 1	4.704 + 1	4.958 + 1	5.159 + 1	5.422 + 1	5.744 + 1	6.090+1	6.443 + 1	6.794 + 1
13	14	4.662 + 1	5.238 + 1	5.787 + 1	6.441 + 1	7.263 + 1	8.000+1	8.433 + 1	8.774 + 1	9.220+1	9.767 + 1	1.036+2	1.096+2	1.155+2
14	15	5.304 + 0	5.794 + 0	6.214 + 0	$6.685 \pm 0$	7.259+0	7.722+0	7.888+0	7.945+0	8.080+0	8.300+0	8.557+0	$8.823 \pm 0$	9.087 + 0
14	16	4.662 + 1	5.124 + 1	5.594 + 1	6.180+1	6.931 + 1	7.602 + 1	7.981 + 1	8.267 + 1	$8.645 \pm 1$	9.114 + 1	9.620+1	1.013+2	1.064+2
17	18	6.551+1	7.284 + 1	7.975 + 1	8.771 + 1	9.736+1	1.055+2	1.094+2	1.117 + 2	1.151+2	1.196+2	1.245+2	1.295+2	1.345 + 2
17	20	7.673 + 1	8.112 + 1	8.714 + 1	9.485 + 1	1.036+2	1.124+2	1.208+2	1.292+2	1.383+2	1.480+2	1.579 + 2	1.679+2	1.778+2
18	19	6.706+1	7.086+1	7.607 + 1	$8.273 \pm 1$	9.035+1	9.799+1	1.053+2	1.127 + 2	1.207+2	1.291+2	1.378+2	1.465+2	1.552+2
19	20	3.806+1	4.157 + 1	4.456+1	4.788 + 1	5.192 + 1	5.516+1	5.626 + 1	5.657 + 1	5.742 + 1	5.886 + 1	6.057 + 1	6.235 + 1	6.412 + 1
19	21	1.129+2	1.240+2	1.352+2	1.492 + 2	1.670+2	1.828+2	1.916+2	1.980+2	2.066+2	2.174+2	2.290+2	2.408+2	2.526+2
20	22	1.475+2	1.621 + 2	1.767 + 2	1.950+2	2.183+2	2.389+2	2.504+2	2.588+2	2.700+2	2.841 + 2	2.992+2	3.147 + 2	3.300+2
21	22	2.497 + 1	2.691 + 1	2.846+1	3.018 + 1	3.232+1	3.399+1	3.438 + 1	3.432 + 1	3.459 + 1	3.524 + 1	3.606+1	3.694 + 1	3.780 + 1
21	23	1.018+2	1.126+2	1.233+2	1.361+2	1.522+2	1.663+2	1.737+2	1.789 + 2	1.859+2	1.947 + 2	2.042+2	2.139+2	2.236+2
22	24	1.794+2	1.984+2	2.173+2	2.401+2	2.685 + 2	2.933+2	3.065+2	3.156+2	3.279+2	3.434+2	3.602 + 2	3.773+2	3.943 + 2
23	24	1.047 + 1	1.127 + 1	1.192 + 1	1.264 + 1	1.354+1	1.423 + 1	1.440 + 1	1.437 + 1	1.448 + 1	1.476+1	1.511+1	1.547 + 1	1.584 + 1
24	25	1.434+2	1.588+2	1.739+2	1.919+2	2.141+2	2.333+2	2.431 + 2	2.496+2	2.585+2	2.699+2	2.824+2	2.951+2	3.076+2

values of  $\Omega$  were also underestimated, as stated earlier. However, based on the comparisons made in our earlier paper for the forbidden (and other allowed) transitions, and the comparisons

of  $\Omega$  values discussed above in Sect. 2, we expect our present results for  $\Upsilon$  to be accurate to better than 15% at all temperatures. Furthermore, the results presented here, along with those already

reported for radiative rates and effective collision strengths in our earlier work (Aggarwal et al. 2005), form a complete set of atomic data for all transitions in Al XIII, and will hopefully be useful for plasma modelling.

*Acknowledgements.* The work reported in this paper has been financed by the EPSRC and STFC of the UK, and F.P.K. is grateful to AWE Aldermaston for the award of a William Penney Fellowship. K.M.A. would like to thank the hospitality of the University of Miyazaki during a visit in March 2006 when a part of this work was initiated. We thank Mr. Keita Hamada for his help in running the CC+CB program.

#### References

Aggarwal, K. M., Keenan, F. P., & Rose, S. J. 2001, Phys. Scr., 63, 95

Aggarwal, K. M., Keenan, F. P., & Rose, S. J. 2005, A&A, 432, 1151

Ait-Tahar, S., Grant, I. P., & Norrington, P. H. 1996, Phys. Rev. A, 54, 3984

Burgess, A., Hummer, D. G., & Tully, J. A. 1970, Phil. Trans. Roy. Soc. A, 266, 225

Gu, M. F. 2003, ApJ, 582, 1241

- Igarashi, A., Horiguchi, Y., Ohsaki, A., & Nakazaki, S. 2003, J. Phys. Soc. Japan, 72, 307
- Igarashi, A., Ohsaki, A., & Nakazaki, S. 2005, J. Phys. Soc. Japan, 74, 321 Zygelman, B., & Dalgarno, A. 1987, Phys. Rev. A, 35, 4085