Multiple equilibrium overnight rates in a dynamic interbank market game

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Abstract

We analyse a two period model of the interbank market, i.e. the market at which banks trade liquidity. We assume that banks do not take the interbank interest rate as given, but multilaterally negotiate on interest rates and transaction volumes. The solution concept applied is the Shapley value. We show that there is a multiplicity of average equilibrium interest rates of the first period so that the average interest rate in this period does not convey any information on the expected liquidity situation at the interbank market.

Zusammenfassung

Wir analysieren ein Zwei-Perioden-Modell des Interbankenmarktes, d. h. des Marktes an dem Banken untereinander Liquidität handeln. Wir nehmen an, dass die Banken den Zinssatz am Interbankenmarkt nicht als exogen betrachten, sondern Zinssätze und Transaktionsvolumen in multilateralen Verhandlungen festlegen. Als Gleichgewichtskonzept dient der Shapley-Wert. Wir zeigen, dass der durchschnittliche Zinssatz der ersten Periode im Gleichgewicht nicht eindeutig ist und daher keine Informationen über die erwartete Liquiditätssituation am Interbankenmarkt enthält.

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1 Introduction

Central banks normally use the average overnight interbank market rate (the EONIA rate in the European monetary union, the Fed funds rate in the US) as an indicator for the liquidity situation at the interbank market. If the rate is high, the market is assumed to expect a liquidity deficit, if the rate is low, it is assumed to expect a liquidity surplus. This method to assess the liquidity situation at the interbank market can easily be justified theoretically. Ho and Saunders (1985) and Spindt and Hoffmeister (1988) for example discuss models of the Fed funds market, while Välimäki (2001), Quirós and Mendizábal (2001) and Tapking (2002) analyse models of the European interbank market. In all of these models, the equilibrium rate is the higher the more likely a liquidity deficit is. However, most of these models are general equilibrium models and thus based on the assumption that all banks take the interbank rate as given.

At least in Europe, transactions at the interbank market are usually agreed on in direct negotiations between banks, often on the telephone. Sometimes, brokers are involved to help banks to find a transaction partner, but the terms of transactions, i.e. interest rates and transaction volumes, are still a matter of negotiations between the banks.[2] Therefore, one may question whether a general equilibrium model with interest rates taking banks is an appropriate model of the interbank market. Consequently, one may ask whether the overnight interbank market rate is still a good indicator for the liquidity situation at the interbank market, if banks do not take interest rates as given but determine interest rates and transaction volumes in negotiations. The latter question is exactly what we are going to address in this paper.

Why do we have doubts that the overnight rate is a good indicator of the liquidity situation at the interbank market if banks determine the terms

[1] I thank Heinz Herrmann and Joachim Keller for helpful comments. The views expressed in this paper are my own and do not necessarily reflect the view of the Bundesbank or the view of the European Central Bank.

[2] A detailed describtion of the European interbank market can be found in Hartmann, Manna and Manzanares (2001).

of transactions in negotiations? Consider for example two banks that provisionally agree that the first will lend a certain amount of liquidity to the second at a certain interest rate. If the banks now receive new information that indicate that a liquidity deficit is more likely than previously expected, we might expect the banks to agree to adjust the terms of their transaction. But what kind of new agreement will they choose? They maybe agree to leave the transaction volume as it is, but raise the interest rate. However, it also appears to be possible that they agree to leave the interest rate as it is, but reduce the transaction volume. In the latter case, changes in the expectations would not lead to changes in the interest rates.

To put forward this idea in a precise and consistent way, we consider a model of an interbank market with institutional characteristics similar to those of the European interbank market.[3] We look at only one so called maintenance period which lasts two days. With the beginning of the first day of the maintenance period, each bank starts with an initial endowment of liquidity (i.e. deposits on accounts with the central bank). With the beginning of the second day, banks face a random influx or drain of liquidity. The central bank requires each bank to hold on average a certain amount of liquidity at each day of the maintenance period on a so called minimum reserve account with the central bank. On the last day of the maintenance period, banks can lend liquidity to the central banks deposit facility and they can borrow liquidity from the central banks marginal lending facility. The interest rate paid for lending into the deposit facility is called deposit rate, the rate for borrowing from the marginal lending facility is the marginal lending rate. Both facility rates are fixed by the central bank. The marginal lending rate is higher than the deposit rate. On both days of the maintenance period, banks can borrow liquidity from other banks and lend liquidity to other banks. Thus, each banks that has less liquidity at its disposal at the beginning of the second day than remaining reserve requirements at that day must borrow liquidity either from the central bank's marginal lending facility or from other banks at the interbank market. All other banks can lend liquidity to the central bank's deposit facility or to other banks. The objective of each bank is to maximise the (expected) liquidity it has at its disposal after the end of the maintenance period.

The crucial assumption in our model is that no bank takes interest rates

[3]A complete describtion of the ECB's regulatory instruments related to the interbank market is in European Central Bank (2000).

as given. All banks instead negotiate multilaterally on interest rates and transaction volumes on both days of the maintenance period. The negotiations are modelled as a cooperative game and the solution concept applied is the Shapley value. Our model can therefore be described as a dynamic cooperative game. At the last day of the maintenance period, negotiations take place in which the outcome of the negotiations of the first day are taken as given. In the negotiations at the first day, banks take the expected outcome of the negotiations of the second day into consideration.

It is of special importance that the banks do not directly negotiate on interest rates and transaction volumes. Instead, they negotiate on how to share the (maximum expected) joint liquidity of the banking industry after the end of the maintenance period. I.e. the direct outcome of the negotiations is a number for each bank (the Shapley value of the bank), which gives the bank's expected final liquidity. The implicit interest rates and transaction volumes that are compatible with these numbers can then be derived. Finally, one can calculate the implicit average interest rate from these implicit interest rates and transaction volumes.

The main results of our analysis are the following: As soon as the interest rates and transaction volumes of the first day of the maintenance period are given, a unique implicit average interest rate of the second day of the maintenance period can be derived from the outcome of the negotiations of the second day. However, there is a multiplicity of implicit average interest rates of the first day. The average interest rate at that day can be any number between 0 and 1, no matter whether the market is expecting a liquidity deficit or a liquidity surplus. The average overnight interest rate at the first day of the maintenance period is therefore useless as an indicator of the expected liquidity situation at the interbank market.

The reason for the multiplicity of average overnight rates is simple: If for example average rates are fixed at a very low level and the transaction volumes are increasing, borrowing banks are normally getting better off and lending banks are getting worse off. If instead interest rates are rising and the transaction volumes are constant, borrowing banks are normally getting worse off and lending banks are getting better off. If now both the transaction volumes and the interest rates are increasing but the interest rates remain at a relatively low level, the expected profits of all banks may remain unchanged. That implies that there are different combinations of overnight interest rates and transaction volumes that all lead to the same expected final amount of liquidity for all banks. Since both the overnight rates and the transaction volumes are directly determined by the banks and all banks are indifferent between these combinations, they are all compatible with the assumptions of our model.

Note that the Shapley value has often been used as a solution concept for models of exchange economies. The Shapley value has been introduced and justified axiomatically by Shapley (1953). A textbook treatment of the Shapley value can be found for example in Myerson (1991) and Eichberger (1993). Hart and Mas-Colell (1996) give a noncooperative justification of the Shapley value as a bargaining solution concept. First applications of the Shapley value as an equilibrium concept for exchange economies with transferable utility are Shapley (1964) and Aumann and Shapley (1974), especially chapter 6.[4] In the wake of these contributions, many articles have been published which analyse the relation between the general equilibrium of an exchange economy and the core and Shapley value of the related cooperative game, the so called market game.[5] See chapter A and B in Mertens and Sorin (1994) for an overview.

In section 2, we describe our model which is analysed in the sections 3 and4. The last day of the maintenance period is considered in section 3. Going backwards, we deal with the first day of the maintenance period in section 4. In section 5, we try to test empirically a hypothesis that can be motivated by our theoretical findings. We show that the empirical results crucially depend on how expectations are approximated empirically.

2 The model

In this section, we formally describe the main institutional assumptions used in our model. Our assumptions on how prices and transaction volumes are determined in multilateral negotiations among banks will be exposed in later sections. We consider a set $I = \{1, ..., n\}$ of n banks and only one maintenance period lasting for only two days $t = 1$ and $t = 2$. Moreover, we assume that all borrowing and lending of liquidity has a maturity of only one day.

[4]A recent discussion of Aumann and Shapley (1974) can be found in Butnariu and Klement (1996).

[5]This literature has well established that under quite general conditions the utility allocation of the general equilibrium is the unique element of the core of the related market game and coincides with its Shapley value, if no player has significant market power (infinite and non-atomic player sets).

Finally, a day in our model will be thought of as a point of time rather than a period of time.

Let $A_{i,t} \geq 0$ be the reserves that bank i holds on its minimum reserve account with the central bank from t to $t+1$ $(t = 1, 2)$. We assume that bank i has to satisfy $A_{i,1} + A_{i,2} \geq \overline{m}_i$, i.e. $\overline{m}_i$ is the bank's (exogenous and aggregated, i.e. not average) minimum reserve requirement for the maintenance period under consideration. The central bank pays no interest on reserves so that bank i holds not more reserves with the central bank than necessary, thus

$$A_{i,1} + A_{i,2} = \overline{m}_i \tag{1}$$

On day $t = 1$, bank i starts with an initial endowment $\overline{L}_i$ of liquidity. We assume that $0 < \sum_{i=1}^{n} \overline{L}_i < \sum_{i=1}^{n} \overline{m}_i$. On both days of the maintenance period, banks can go to the interbank market to lend liquidity to or borrow liquidity from other banks. Let $F_{i,t}^{j}$ be the liquidity bank j borrows from bank i from t to $t+1$ and $r_{i,t}^{j}$ the corresponding interest rate, i.e. by definition we have $F_{i,t}^{j} = -F_{j,t}^{i}$ and $r_{i,t}^{j} = r_{j,t}^{i}$. The terms of trade between two banks i and j at the interbank market, given by $F_{i,t}^{j}$ and $r_{i,t}^{j}$, are a matter of negotiations as will be explained in the following sections. Thus, neither bank i nor bank j takes the interest rate $r_{i,t}^{j}$ as given.

In $t = 2$, bank i realizes a random and exogenous liquidity influx g_i which may be positive or negative. The term g_i is mainly driven by customers of bank i paying in or withdrawing money from their account. Moreover, g_i comprises for example liquidity drains because of dividend payments, payouts of factor income and real investments.

In $t = 2$, the banks can lend a liquidity surplus to and borrow a liquidity deficit from the central bank's standing facilities. The liquidity lent by bank i to the central bank via the deposit facility is D_i, the liquidity borrowed from the central bank via the marginal lending facility is S_i. The deposit rate is r_D and the marginal lending rate is r_S $(r_S > r_D)$. We assume both rates to be exogenous and non-random. The banks have no access to standing facilities in $t = 1$. This assumption is of course not in accordance with the reality at the European interbank market where banks can go to the ECB's standing facilities on every day. But it can be shown that only under quite extreme parameter constellations the banks in our model would use the standing facilities in $t = 1$, if we additionally assumed that they had access to them on every day of the maintenance period. The assumption that banks can go to the standing facilities only on the last day of the maintenance period is thus quite harmless and simplifies our analysis significantly.

From the assumptions described so far, we can derive several identity equations. In $t = 1$, each bank can use its initial endowment either to fulfil reserve requirements or to lend it to other banks at the interbank market. We therefore have

$$\overline{L}_i = A_{i,1} + \sum_{\substack{j=1 \\ j \neq i}}^{n} F_{i,1}^{j} \tag{2}$$

The liquidity bank i can dispose of in $t = 2$ is given by

$$L_{i,2} = A_{i,1} + \sum_{\substack{j=1 \\ j \neq i}}^{n} (1 + r_{i,1}^{j}) F_{i,1}^{j} + g_i \tag{3}$$

or, from the expenditure side, by

$$L_{i,2} = A_{i,2} + D_i - S_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} F_{i,2}^{j} \tag{4}$$

Finally, bank i's liquidity after the end of the maintenance period, i.e. at day $t = 3$, is

$$L_{i,3} = A_{i,2} + \sum_{\substack{j=1 \\ j \neq i}}^{n} (1 + r_{i,2}^{j}) F_{i,2}^{j} + (1 + r_D) D_i - (1 + r_S) S_i \tag{5}$$

Note that $L_{i,3}$ is the final value of bank i's activity at the interbank market in the maintenance period under consideration. This is because we assume that no endogenous liquidity drains occur during the maintenance period, but the whole liquidity surplus is reinvested according to equations 3 and 4. Therefore, we assume that bank i's objective at day t, $t = 1, 2$, is to maximise $E_t[L_{i,3}]$, i.e. its in t expected liquidity after the end of the maintenance period.

In the next two sections, we analyse the banks' behaviour at the interbank market. We consider the interaction between several banks at the interbank market as a cooperative game. Banks are not price taker, but negotiate multilaterally with each other on interest rates and amounts of liquidity lent and borrowed. The main analytical concept used is the Shapley value. To begin with, we consider the last day of the maintenance period.

3 The last day of the maintenance period

On the last day of the maintenance period $t = 2$, each bank i has to satisfy $A_{i,2} = \overline{m}_i - A_{i,1}$, where $A_{i,1}$ is given. Thus, $A_{i,2}$ is given and each bank has to decide only on its recourse to the facilities (D_i and S_i) and its activities at the interbank market. The terms of deals at the interbank market are determined in multilateral negotiations. The payoff some bank i obtains on the basis of these negotiations in $t = 2$ is its Shapley value and will be denoted by $L^*_{i,3}$. To determine the Shapley value for each bank, we first have to go through some hypothetical considerations on coalition behaviour.

Let $\rho = I^2$ denote the set of all subsets of I. Each $k \in \rho$ is called a possible coalitions of banks. Coalitions are to be thought of as follows: Each bank can take part in exactly one coalition per day. If some bank i is in coalition k on day $t = 2$, it can only borrow from and lend to those banks on that day which are also in k. Thus, for some coalition k on day $t = 2$ we have $F^j_{i,2} = 0$ for all $i \in k, j \notin k$. Denote by x_i bank i's liquidity surplus in $t = 2$. It is defined by

$$x_i = L_{i,2} - (\overline{m}_i - A_{i,1}) \tag{6}$$

Now assume that the members of coalition k would act such that they maximise the sum of the payoffs of all banks in k, if coalition k were formed in $t = 2$. They would then choose D_i, S_i, $F^j_{i,2}$ and $r^j_{i,2}$ for all $i \in k$ and $j \in I$ to maximise

$$\begin{aligned} v_{k,2} &\equiv \sum_{i \in k} L_{i,3} \\ &\quad \text{subject to} \\ D_i &\geq 0, S_i \geq 0, D_i - S_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} F^j_{i,2} = x_i \text{ for all } i \in k \\ F^j_{i,2} &= 0 \text{ for all } i \in k, j \notin k \end{aligned}$$

The solution of the above maximization problem leads to a value for $v_{k,2}$ that we denote by $v^*_{k,2}$. The function that assigns the value $v^*_{k,2}$ to each possible coalition $k \in \rho$ is called the characteristic function. It is needed in order to determine the Shapley value for a given bank.

Since recourse to both facilities on the same day is never optimal for a given coalition, the solution to the above problem has to satisfy (1) $\sum_{i \in k} x_i =$

$\sum_{i \in k} D_i$ and $S_i = 0$ for all $i \in k$ if $\sum_{i \in k} x_i \geq 0$ and (2) $\sum_{i \in k} x_i = -\sum_{i \in k} S_{i,T}$ and $D_i = 0$ for all $i \in k$ if $\sum_{i \in k} x_i \leq 0$. With this, we get from equation 5

$$\begin{aligned} v^*_{k,2} &= \sum_{i \in k} (\overline{m}_i - A_{i,1}) + I_k(1 + r_D) \sum_{i \in k} x_i \\ &\quad + (1 - I_k)(1 + r_S) \sum_{i \in k} x_i \end{aligned} \tag{7}$$

where

$$I_k = \begin{cases} 1, \text{ if } & \sum_{i \in k} x_i \geq 0 \\ 0, \text{ if } & \sum_{i \in k} x_i < 0 \end{cases} \tag{8}$$

as the obvious solution of the above problem. Equation 7 gives us the characteristic function of our cooperative game in $t = 2$.

Since we know the characteristic function now, we can apply common equilibrium concepts of cooperative game theory like the core or the Shapley value to our problem. We are going to work with the Shapley value. Let

$$q_k = \frac{(\#k - 1)!(n - \#k)!}{n!} \tag{9}$$

where $\#k$ is the number of banks in k. Bank i's Shapley value is defined as

$$L^*_{i,3} = \sum_{k \in \rho} q_k [v^*_{k,2} - v^*_{k \setminus \{i\},2}] \tag{10}$$

The expression $[v^*_{k,2} - v^*_{k \setminus \{i\},2}]$ can be interpreted as bank i's contribution to coalition k. It is the higher the more the coalition would lose if bank i drops out of the coalition. If $i \notin k$, then we of course have $[v^*_{k,2} - v^*_{k \setminus \{i\},2}] = 0$. The numbers q_k are non-negative weights. Let $\rho_i = \{k \in \rho | i \in k\}$. It is easy to show that $\sum_{k \in \rho_i} q_k = 1$. Bank i's Shapley value is thus a weighted average of i's contributions to the various possible coalitions. We assume that the Shapley value is the payoff bank i obtains from the negotiations with the other banks at the interbank market in $t = 2$. For both axiomatic and non-axiomatic justifications of this assumption, the reader is referred to the publications mentioned in the introduction, the most comprehensive of which is Hart and Mas-Colell (1996).

With equation 7, we get after some rearrangements

$$\begin{aligned} L^*_{i,3} &= x_i[1 + r_S - (r_S - r_D)\sum_{k\in\rho_i} q_k I_k] \\ &\quad -(r_S - r_D)\sum_{\substack{j=1\\ j\neq i}}^{n} x_j[\sum_{k\in\rho_{i,j}} q_k(I_k - I_{k\setminus\{i\}})] \\ &\quad +\overline{m}_i - A_{i,1} \end{aligned} \tag{11}$$

where $\rho_{i,j} = \{k \in \rho | i, j \in k\}$ and ρ_i as defined above.

We should remain a while at equation 11. Our model predicts that trade of liquidity at the interbank market in $t = 2$ and recourse to the central bank's facilities in $t = 2$ will be such that the amount of liquidity bank i can dispose of at the first day after the maintenance period is $L^*_{i,3}$ as given in equation 11. Note that the liquidity bank i can dispose of at that day originates either from required reserve holdings ($A_{i,2} = \overline{m}_i - A_{i,1}$) or from the usage of x_i. The last line in equation 11 obviously describes the liquidity influx stemming from required reserve holdings. Thus, the first two lines give us the liquidity originating from the usage of x_i. All parts of equation 11 that drop out if $r_S = r_D$ describe the impact of negotiations on $L^*_{i,3}$, since interbank lending is useless if $r_S = r_D$.

Consider the extreme case that $I_k = 1$ for all $k \in \rho$. It is easy to show that in this case we have $\sum_{k\in\rho_i} q_k I_k = 1$. The second line in equation 11 is obviously 0. Thus, the influx of liquidity to bank i in $t = 3$ originating from the usage of $x_{i,T}$ is $x_i(1 + r_D)$. This is because *all* banks have a liquidity surplus, thus no interbank trade can take place and bank i lends its whole surplus x_i to the central bank via the deposit facility. Now fix some bank i and assume that $I_k = 1$ for all $k \in \rho_i$ and $I_k = 0$ for all $k \notin \rho_i$. The first line in equation 11 is still $x_i(1 + r_D)$, while the second line is now

$$-(r_S - r_D)\sum_{\substack{j=1\\ j\neq i}}^{n}[x_j \sum_{k\in\rho_{i,j}} q_k]$$

where $0 < \sum_{k\in\rho_{i,j}} q_k < 1$ and $x_j \leq 0$ for all $j \neq i$. Thus, the influx of liquidity to bank i in $t = 3$ originating from the usage of x_i is higher than $x_i(1+r_D)$. This proves that bank i can now lend parts of its liquidity surplus to other banks at a rate higher than r_D.

Finally assume the other two extreme cases: If $I_k = 0$ for all $k \in \rho$ (i.e. $x_i < 0$ for all i), banks i's influx of liquidity stemming from the usage of x_i is $x_i(1 + r_S)$, since all banks have a liquidity deficit, thus borrowing from other banks is not possible and bank i borrows from the central bank only. If $I_k = 0$ for all $k \in \rho_i$ and $I_k = 1$ for all $k \notin \rho_i$, banks i's influx of liquidity stemming from the usage of x_i is higher than $x_i(1 + r_S)$, since it can now borrow parts of its liquidity deficit from other banks at a rate less than r_S.

Equation 11 only gives us the equilibrium liquidity bank i, $i = 1, ..., n$, can dispose of in $t = 3$. We now want to determine the underlying activities of bank i in $t = 2$ which lead to $L^*_{i,3}$ as described in equation 11. To do so, we need to solve

$$\left.\begin{array}{c} L^*_{i,3} = L_{i,3} \\[2ex] x_i = D_i - S_i + \sum_{\substack{j=1 \\ j\neq i}}^{n} F^j_{i,2} \\[2ex] F^j_{i,2} = -F^i_{j,2},\ r^j_{i,2} = r^i_{j,2},\ D_i \geq 0,\ S_i \geq 0 \\[2ex] i, j = 1, ..., n \end{array}\right\} \tag{12}$$

for D_i, S_i, $F^j_{i,3}$ and $r^j_{i,3}$ for all $i, j \in I$. Here, $L^*_{i,3}$ is given in equation 11, $L_{i,3}$ is given in equation 5 and the second equation in 12 is derived from equations 1, 4 and 6. Note that we do not assume $r^j_{i,2} = r_2$ for all $i, j \in I$ and some number r_2. In a general equilibrium model, two different prices for a homogeneous good are impossible. However, our model is no general equilibrium but a bargaining model and we show now that different prices for the same good are not in contradiction with our assumptions.

First note that 12 has no unique solution. Consider the following case: $n = 3$, $I_{\{1\}} = 1$, $I_k = 0$ for all $k \in \rho\backslash\{1\}$. In this case of only three banks, bank 1 has a liquidity surplus in $t = 2$, while the other two banks have a liquidity deficit. Moreover, the liquidity deficit of both bank 2 and bank 3 is higher than the liquidity surplus of bank 1. It is easy to check that the following is a solution of 12: $r^j_{i,2} = \frac{2}{3}r_S + \frac{1}{3}r_D$ for $i, j = 1, 2, 3$, $F^2_{1,2} = F^3_{1,2} = \frac{1}{2}x_1$, $F^3_{2,2} = 0$, $S_1 = D_i = 0$ for $i = 1, 2, 3$, $S_2 = -x_2 - \frac{1}{2}x_1$ and $S_3 = -x_3 - \frac{1}{2}x_1$. In this solution, all interbank rates are equal. But it is also easy to check that another solution to 12 is: $r^2_{1,2} = \frac{1}{3}r_S + \frac{2}{3}r_D$, $r^3_{1,2} = \frac{7}{9}r_S + \frac{2}{9}r_D$, $F^2_{1,2} = \frac{1}{4}x_1$, $F^3_{1,2} = \frac{3}{4}x_1$, $F^3_{2,2} = 0$, $S_1 = D_i = 0$ for $i = 1, 2, 3$, $S_2 = -x_2 - \frac{1}{4}x_1$ and $S_3 = -x_3 - \frac{3}{4}x_1$. This example shows that (1) there is not necessarily a unique set $\{r^j_{i,2} | i, j \in I, i \neq j\}$ of equilibrium interbank rates and (2) we do not necessarily have $r^j_{i,2} = r_2$ for all $i, j \in I$ and some number r_2 in equilibrium. [6]

To continue, we make the following symmetry assumptions for notational convenience: (1) $sign(x_i) = sign(x_j)$ implies $F^j_{i,2} = 0$ for all $i, j \in I$, i.e. two banks do not trade with each other, if both have a liquidity deficit or if both have a liquidity surplus. (2) $\sum_{i=1}^n x_i \geq 0$ and $x_i \leq 0$ implies $S_i = D_i = 0$. (3) $\sum_{i=1}^n x_i \leq 0$ and $x_i \geq 0$ implies $S_i = D_i = 0$. Now define $I^+ = \{i \in I | x_i > 0\}$, $I^- = I \backslash I^+$ and

$$R_2 = \frac{\sum_{i \in I^+} \sum_{j \in I^-} r^j_{i,2} F^j_{i,2}}{\sum_{i \in I^+} \sum_{j \in I^-} F^j_{i,2}} \tag{13}$$

Note that R_2 is the interbank rate index, i.e. a weighted average of all interbank rates in $t = 2$. It is straightforward to show that 12 implies

$$\begin{aligned} \text{(a) } 0 \ &\leq \ \sum_{i=1}^n x_i \Rightarrow \ R_2 = \frac{\sum_{i \in I^-} [L^*_{i,3} - L_{i,2}]}{\sum_{i \in I^-} x_i} \\ \text{(b) } 0 \ &\geq \ \sum_{i=1}^n x_i \Rightarrow \ R_2 = \frac{\sum_{i \in I^+} [L^*_{i,3} - L_{i,2}]}{\sum_{i \in I^+} x_i} \end{aligned}$$

Note that $L^*_{i,3}$ is uniquely given by equation 11. Thus, R_2 is unique, though $r^j_{i,2}$ for some $i, j \in I$ is not. However, note that there is no trade at all if in case (a) we have $\sum_{i \in I^-} x_i = 0$ or in (b) $\sum_{i \in I^+} x_i = 0$. In both cases, R_2 is not defined.

We now briefly summarize the results of our cooperative game analysis of the interbank market at day $t = 2$. (1) There is not necessarily a unique equilibrium interbank rate for the trade of liquidity among two given banks. (2) But the equilibrium interbank rate index R_2 is unique. (3) The interbank rate in one deal can differ from the rate in another deal, thus $r^j_{i,2} \neq r^l_{k,2}$ for some $i, j, k, l \in I$ $(i \neq j \neq k \neq l)$ is possible.

[6]Note that 12 has often no solution with $r^j_{i,2} = r_2$ for all $i, j \in I$ and some number r_2. If for example $n = 4$, $I_k = 1$ for all $k \in \{\{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,2,3\}, \{1,2,4\}\}$ and $I_k = 0$ otherwise, than there is no solution with an equal interbank rate for all trades.

4 The first day of the maintenance period

We now consider day $t = 1$ and start with some introductory considerations to obtain intuition on what is going on at the first day of the maintenance period. With the equations 2, 3 and 6, we easily get

$$x_i = 2\overline{L}_i - \overline{m}_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} (1 - r_{i,1}^j) F_{i,1}^j + g_i \tag{14}$$

If we assume that $0 < r_{i,1}^j < 1$ for all $i, j \in I$, then bank i's liquidity surplus in $t = 2$, i.e. x_i, is decreasing in $F_{i,1}^j$ for all $j \in I$. The reason is the following: Lending more liquidity to other banks in $t = 1$ means (1) holding less reserves in $t = 1$, i.e. more remaining reserve requirements and thus a lower liquidity surplus in $t = 2$, and (2) receiving more interest from other bank and thus a higher liquidity surplus in $t = 2$. Effect (1) is clearly stronger than effect (2). However, what is the effect of $F_{i,1}^j$ on $L_{i,3}^*$? If we replace in equation 11 x_i and x_j for all $j \in I \backslash \{i\}$ by the right hand side of equation 14 and $A_{i,1}$ by means of equation 2, we get

$$\begin{aligned} \frac{\partial L_{i,3}^*}{\partial F_{i,1}^j} &= 1 - (1 - r_{i,1}^j)[1 + r_S \\ &\quad + (r_S - r_D)\Big(\sum_{k \in \rho_{i,j}} q_k (I_k - I_{k \backslash \{i\}}) - \sum_{k \in \rho_i} q_k I_k\Big)] \end{aligned} \tag{15}$$

Lending to some other bank j is of course the more profitable for bank i, the higher $r_{i,1}^j$ is. However, the sign of equation 15 is not clear. It even depends on $F_{i,1}^j$, because the indicator functions in equation 15 change when $F_{i,1}^j$ changes ($L_{i,3}^*$ is piecewise linear in $F_{i,1}^j$). But the right hand side of equation 15 is in principle the higher, the higher the liquidity surplus in $t = 2$ is. For a given $r_{i,1}^j$, equation 15 is the highest if $I_k = 1$ for all $k \in \rho_i$ and the lowest if $I_k = 0$ for all $k \in \rho_i$. There is a simple reason for this: The more some bank i lends to other banks in $t = 1$, the lower is $A_{i,1}$, the higher is its reserve requirement in $t = 2$ and thus the lower is its liquidity surplus in $t = 2$. And having a low liquidity surplus is the better the higher the liquidity surplus of the market is.

We now consider the cooperative game played in $t = 1$. As in day $t = 2$, all banks negotiate multilaterally with each other on interbank rates and on amounts of liquidity borrowed by one bank from another. To determine the outcome of these negotiations, i.e. the Shapley value, we need to determine the value $v^*_{k,1}$ for each possible coalition $k \in \rho$ in $t = 1$, i.e. the characteristic function in $t = 1$. As for day $t = 2$, for any coalition $k \in \rho$ we have $F^j_{i,1} = 0$ for all $i \in k, j \notin k$. Summation of the right hand side of equation 11 for all $i \in k$ after substituting $A_{i,1}$ by means of the equations 1 and 2 and after rearranging with equation 9 gives

$$\begin{aligned}\sum_{i\in k} L^*_{i,3} \quad &= \quad [1 + r_S - (r_S - r_D)I_I] \sum_{i\in k} x_i \\ &+ \sum_{i\in k} [\overline{m}_i - \overline{L}_i + \sum_{\substack{j=1 \\ j\neq i}}^{n} F^j_{i,1}] \\ &+ (r_S - r_D) \sum_{i\in k} x_i [\sum_{j\notin k} \sum_{l\in\rho_{i,j}} q_l (I_l - I_{l\setminus\{j\}})] \\ &- (r_S - r_D) \sum_{i\notin k} x_i [\sum_{j\in k} \sum_{l\in\rho_{i,j}} q_l (I_l - I_{l\setminus\{j\}})]\end{aligned} \tag{16}$$

Note that replacing x_i for all $i \in I$ in equation 16 by the right hand side of equation 14 leads us to a formula for $\sum_{i\in k} L^*_{i,3}$ which contains only variables chosen in $t = 1$ and variables exogenously given in $t = 1$.

Analogously to section 3, assume now that the members of coalition k would act such that they maximise the sum of the (expected) payoffs of all banks in k, if coalition k were formed in $t = 1$. They would then choose $F^j_{i,1}$ and $r^j_{i,1}$ for all $i \in k$ and $j \in I$ to maximise

$$\left.\begin{aligned}&v_{k,1} \equiv E_1[\textstyle\sum_{i\in k} L^*_{i,3}] \\ &\text{subject to} \\ &0 \leq \overline{L}_i - \textstyle\sum_{\substack{j=1 \\ j\neq i}}^{n} F^j_{i,1} \leq \overline{m}_i, \\ &F^j_{i,1} = -F^i_{j,1} \text{ for all } i, j \in I,\ F^j_{i,1} = 0 \text{ for all } i \in k, j \notin k\end{aligned}\right\} \tag{17}$$

Note that $v_{k,1}$ depends on variables that are chosen by banks which are not members of coalition k. Thus, coalition k maximises $v_{k,1}$ given the decisions of other coalitions. But which are the other coalitions? For simplicity we assume that if coalition k were formed in $t = 1$, there would be only one other coalition in $t = 1$, namely $I \backslash k$. Thus, if some coalition k were formed, coalition $I \backslash k$ would also be formed and both coalitions would play a two player Nash game in which coalition k's payoff function is $v_{k,1}$ as defined in problem 17 and its strategy set is described by the constraints presented in problem 17. The value $v^*_{k,1}$ of coalition k is its Nash equilibrium payoff in this game.

It is important to make sure that this game has an equilibrium. Firstly, note that $F^j_{i,1} = -F^i_{j,1}$ for all $i, j \in I$ implies $\sum_{i \in k} \sum_{j=1, j \neq i}^{n} F^j_{i,1} = 0$, i.e. we can simplify equation 16 accordingly. That implies that the only strategy variables for some coalition k are $(1 - r^j_{i,1})F^j_{i,1}$ for all $i, j \in k$. If we assume that the interest rates are restricted such that $a \leq r^j_{i,1} \leq b$ for some numbers a and b and all $i, j \in k$, then the strategy set of k is compact. Secondly, it is very easy to check that $L^*_{i,3}$ as given in equation 11 is a continuous (and piecewise linear) function in x_j for all $j \in I$. Thus, $v_{k,1}$ is also a continuous function in x_j for all $j \in I$. Since x_i is continuous in $(1 - r^j_{i,1})F^j_{i,1}$ for all $j \in I$, we know that coalition k's payoff function $v_{k,1}$ is continuous in the strategy variables. It is well known that a game with a compact strategy set and a continuous payoff function for all players has at least one equilibrium in mixed strategies.[7] We can therefore be sure that the game described above has at least one equilibrium in mixed strategies. However, the payoff function is not always quasi-concave so that we cannot be sure that the game has an equilibrium in pure strategies.

For $k = I$, the last two lines of equation 16 are 0 and the first two lines do not depend on strategy variables. Thus, we always have $v^*_{I,1} = v_{I,1} = E_1[\sum_{i \in k} L^*_{i,3}]$, i.e.

$$v^*_{I,1} = \sum_{i=1}^{n} [\overline{m}_i - \overline{L}_i] + E_1[(1 + r_S - (r_S - r_D)I_I) \sum_{i=1}^{n} x_i] \tag{18}$$

Unfortunately, for any coalition $k \neq I$, it is very hard to determine $v^*_{k,1}$ in general. Therefore we assume from now on that there are only $n = 3$ banks. This is a very simply case indeed, since there is nothing a coalition of only one bank can do so that its opponent, a coalition of two banks, has a very simple problem to solve. Moreover, we assume that there are only two states of the world $s \in \{s_1, s_2\}$ with probabilities p_1 and $p_2 = 1 - p_1$.

[7] See for example Eichberger (1993), page 95.

The random parameters are determined by the states according to $g_1(s_1) = g_2(s_1) = g_3(s_2) = g$ and $g_1(s_2) = g_2(s_2) = g_3(s_1) = -g$ for some $g > 0$. Moreover, we assume $2\overline{L}_i = \overline{m}_i$ for $i = 1, 2, 3$. With these assumptions it is clear that $\sum_{i=1}^{n} x_i(s_1) = g$ and $\sum_{i=1}^{n} x_i(s_2) = -g$. Therefore, there is a liquidity surplus in state s_1 and a liquidity deficit in state s_2. With this example, we immediately get from equation 18

$$v^*_{\{1,2,3\},1} = [p_1(1 + r_D) - p_2(1 + r_S)]g + \sum_{i=1}^{3} \overline{L}_i \tag{19}$$

Now assume that the coalitions $\{1, 2\}$ and $\{3\}$ are formed. The following lemma is proved in the appendix:

Lemma 1 *In $t = 1$, coalition $\{1, 2\}$ would choose $(1 - r^2_{1,1})F^2_{1,1} \geq g$ (or - because of symmetry - $(1 - r^2_{1,1})F^2_{1,1} \leq -g$).*

The economic reason for this result is the following: If $F^j_{i,1} = 0$ for all $i, j \in I$, bank 3 would have a very good bargaining position in $t = 2$, because it would have a deficit if the other two banks have a surplus and a surplus if the other two banks have a deficit, i.e. both bank 1 and 2 would need bank 3 in both states. If instead $(1 - r^2_{1,1})F^2_{1,1} \geq g$ and $F^j_{3,1} = 0$ for $j = 1, 2$, the position of bank 3 in $t = 2$ would be less strong. For in s_1 both bank 1 and 3 would have a deficit and bank 2 a surplus, while in s_2 bank 2 and 3 would have a surplus and bank 1 a deficit.

With lemma 1 and $F^j_{3,1} = 0$ for $j = 1, 2$, we get from 16 and 14:

$$v^*_{\{1,2\},1} = [p_1(1 + r_D) - p_2(1 + r_S)]2g + (r_S - r_D)\frac{1}{2}g + \overline{L}_1 + \overline{L}_2 \tag{20}$$

and

$$v^*_{\{3\},1} = [p_2(1 + r_S) - p_1(1 + r_D)]g - (r_S - r_D)\frac{1}{2}g + \overline{L}_3 \tag{21}$$

The situation occurring if the coalitions $\{1, 3\}$ and $\{2\}$ are formed is described by the following

Lemma 2 *In $t = 1$, coalition $\{1, 3\}$ would choose $(1 - r^3_{1,1})F^3_{1,1} = g$ if $p_1 \geq p_2$ and $(1 - r^3_{1,1})F^3_{1,1} = -g$ if $p_1 \leq p_2$.*

This strategy would imply that $x_1 = x_3 = 0$ in the state with the highest probability, i.e. neither bank 1 nor bank 3 would need to go to the facilities in this state.[8]

With lemma 2 and $F^j_{2,1} = 0$ for $j = 1, 3$ we get

$$v^*_{\{1,3\},1} = -(r_S - r_D)g\frac{1}{6}\min\{p_1, p_2\} + \overline{L}_1 + \overline{L}_3 \tag{22}$$

and

$$\begin{aligned} v^*_{\{2\},1} &= (1 + r_S)g(p_1 - p_2) \\ &\quad -(r_S - r_D)g(p_1 - \frac{1}{6}\min\{p_1, p_2\}) + \overline{L}_2 \end{aligned} \tag{23}$$

Finally, because of symmetry we get

$$v^*_{\{2,3\},1} = -(r_S - r_D)g\frac{1}{6}\min\{p_1, p_2\} + \overline{L}_2 + \overline{L}_3 \tag{24}$$

and

$$\begin{aligned} v^*_{\{1\},1} &= (1 + r_S)g(p_1 - p_2) \\ &\quad -(r_S - r_D)g(p_1 - \frac{1}{6}\min\{p_1, p_2\}) + \overline{L}_1 \end{aligned} \tag{25}$$

Bank i's Shapley value is denoted by $E_1[L^{**}_{i,3}]$ and analogously to equation 10 defined by

$$E_1[L^{**}_{i,3}] = \sum_{k \in \rho} q_k[v^*_{k,1} - v^*_{k \backslash \{i\},1}] \tag{26}$$

where q_k has been defined in equation 9 for all $k \in \rho$. With the characteristic function given by the equations 19 to 25 it is quite easy to determine the Shapley value for the cooperative game played in $t = 1$ as defined in equation 26:

$$\begin{aligned} E_1[L^{**}_{i,3}] &= \overline{L}_i + (1 + r_S)g(p_1 - p_2) \\ &\quad -(r_S - r_D)g(p_1 - \frac{1}{6} - \frac{1}{18}\min\{p_1, p_2\}) \end{aligned} \tag{27}$$

[8]Note that a coalition $\{i, j\}$'s choice of $(1 - r^j_{i,1})F^j_{i,1}$ could lead to a situation with for example $E_1[L^*_{i,3}]$ being lower than it would be if $F^j_{i,1} = 0$. Bank i would agree on such a deal only if i and j simultaneously agree that j pays a transfer to i in $t = 3$ to offset the losses of bank i in a coalition with j. Thus, we have to assume that banks can agree in $t = 1$ on interbank payments in $t = 3$. However, in equilibrium such agreements will not be made.

for $i = 1, 2$, and

$$E_1[L^{**}_{3,3}] = \overline{L}_3 + (1 + r_S)g(p_2 - p_1) - (r_S - r_D)g(\frac{1}{3} - p_1 + \frac{1}{9}\min\{p_1, p_2\}) \qquad (28)$$

Our model implies that the in $t = 1$ expected liquidity bank i, $i = 1, 2, 3$, can dispose of in $t = 3$ is given by the equations 27 and 28. As explained, these expectations are the result of multilateral negotiations between the three banks in $t = 1$ and rational expectations of the outcome of the negotiations in $t = 2$.

The first question we want to addressed is whether there is interbank trade in $t = 1$. Define $v^0_{\{i\},1} = v_{\{i\},1}$ for $F^j_{h,1} = 0$ for all $j, h \in I$. Thus, $v^0_{\{i\},1}$ is the in $t = 1$ expected liquidity of bank i in $t = 3$, if no trade in $t = 1$ takes place. It is very easy to verify that in our example above we have $v^0_{\{i\},1} < E_1[L^{**}_{i,3}]$ for $i = 1$ and $i = 2$ and $v^0_{\{3\},1} > E_1[L^{**}_{3,3}]$, if $0 < p_1 < 1$. Thus, we can tell from the fact that $v^0_{\{i\},1}$ and $E_1[L^{**}_{i,3}]$ are not equal, that trade takes place in $t = 1$, i.e. $F^j_{i,1} \neq 0$ for some $i, j \in I$. The trade increases bank 1's and 2's expected payoff, while it decreases bank 3's expected payoff.

We now want to derive from the equations 27 and 28 the interest rates and the credit volumes the banks have agreed on in the negotiations of day $t = 1$. Thus, we have to solve

$$\left.\begin{array}{l} E_1[L^{**}_{i,3}] = v_{\{i\},1} \\ \\ F^j_{i,1} = -F^i_{j,1},\ r^j_{i,1} = r^i_{j,1}, \\ \\ i, j = 1, ..., n \end{array}\right\} \qquad (29)$$

for $F^j_{i,1}$ and $r^j_{i,1}$ and all $i, j \in I$. It is not hard to verify that 29 has no unique solution. From now on, we concentrate on solutions of 29 that satisfy $F^3_{1,1} = F^3_{2,1}$, $F^2_{1,1} = 0$, $0 < r^3_{1,1} = r^3_{2,1} < 1$. The interbank rate index in $t = 1$ is thus $R_1 = \frac{r^3_{1,1}F^3_{1,1} + r^3_{2,1}F^3_{2,1}}{F^3_{1,1} + F^3_{2,1}} = r^3_{1,1}$. The next proposition is the main result of this section and follows directly from lemma 4, which we will introduced below and prove in the appendix:

Proposition 3 *For every $\omega \in]0, 1[$, there is at least one solution of 29 with $F^3_{1,1} = F^3_{2,1}$, $F^2_{1,1} = 0$ and $r^3_{1,1} = r^3_{2,1} = \omega$, i.e. $R_1 = \omega$.*

This proposition states that there is no unique equilibrium interbank rate index in $t = 1$. The equilibrium rate index can be any number between 1 and 2. There is for instance always an equilibrium with $R_1 = \frac{1}{2}(r_S + r_D)$, i.e. with the average equilibrium rate in $t = 1$ being exactly in the middle between the two facility rates. This is true no matter if the market is expecting a liquidity surplus (p_1 high) or deficit (p_1 low). And the interbank rate index may be both low or high if there is for example an expected liquidity deficit. That implies that in our model, the interbank rate index R_1 does not convey any information about the expected liquidity situation in the market.

The reason for this result is the following: When negotiating with other banks in $t = 1$, each bank is only interested in maximising the expected liquidity it can dispose of in $t = 3$. Each bank is indifferent between two different constellations of interbank rates $r^j_{i,1}$ and amounts of liquidity borrowed and lent $F^j_{i,1}$, $i, j \in I$, if both result in the same expected disposable liquidity in $t = 3$. Proposition 3 simply states that there are many such constellations that all lead to an expected disposable liquidity of $E_1[L^{**}_{i,3}]$ for all three banks $i = 1, 2, 3$.[9]

We finally describe how these constellations that all solve 29 differ from each other. One might think that $r^3_{1,1}$ and $r^3_{2,1}$ have to rise in order to compensate bank 3 if both rates are low and if $F^3_{1,1}$ and $F^3_{2,1}$ are negative and decreasing. And it seems plausible that for similar reasons, $r^3_{1,1}$ and $r^3_{2,1}$ have to fall if they are high and $F^3_{1,1}$ and $F^3_{2,1}$ are positive and increasing. I.e. we might expect that 29 implies $\frac{dF^3_{1,1}}{dr^3_{1,1}} < 0$. However, the following lemma shows that this is not necessary correct:

Lemma 4 *Consider only solutions of 29 and assume* $F^3_{1,1} = F^3_{2,1}$, $F^2_{1,1} = 0$ *and* $0 < r^3_{1,1} = r^3_{2,1} < 1$. *Then there are numbers* α, β, γ *with* $\gamma < \alpha < \beta$ *such that:*

(i) For every and only for $r^3_{1,1} \in]0, \beta]$, *there is a* $F^3_{1,1}$ *such that* $0 > (1 - r^3_{1,1})F^3_{1,1} \geq -g$. *Moreover,* $0 > (1 - r^3_{1,1})F^3_{1,1} \geq -g$ *implies* $\frac{dF^3_{1,1}}{dr^3_{1,1}} < 0$.

(ii) For every and only for $r^3_{1,1} \in [\alpha, \beta]$, *there is a* $F^3_{1,1}$ *such that* $(1 - r^3_{1,1})F^3_{1,1} \leq -g$. *Moreover,* $(1 - r^3_{1,1})F^3_{1,1} \leq -g$ *implies* $\frac{dF^3_{1,1}}{dr^3_{1,1}} > 0$.

[9] Note that it can be shown that R_2 depends on R_1, i.e. the alleged uniqueness of R_2 holds only if R_1 has been fixed.

(iii) For every and only for $r^3_{1,1} \in [\gamma, 1[$, there is a $F^3_{1,1}$ such that $0 < (1 - r^3_{1,1})F^3_{1,1} \leq g$. Moreover, $0 < (1 - r^3_{1,1})F^3_{1,1} \leq g$ implies $\frac{dF^3_{1,1}}{dr^3_{1,1}} < 0$.

(iv) For every and only for $r^3_{1,1} \in [\gamma, \alpha]$, there is a $F^3_{1,1}$ such that $(1 - r^3_{1,1})F^3_{1,1} \geq g$. Moreover, $(1 - r^3_{1,1})F^3_{1,1} \geq g$ implies $\frac{dF^3_{1,1}}{dr^3_{1,1}} > 0$.

The parts (i) and (iii) of the lemma are in line with our suspicion. If for example $0 > (1 - r^3_{1,1})F^3_{1,1} \geq -g$, then we have of course $F^3_{1,1} < 0$ (the banks 1 and 2 are borrowing from bank 3), $r^3_{1,1} \in]0, \beta]$ and $r^3_{1,1}$ is decreasing in $F^3_{1,1}$. Thus, as long as $r^3_{1,1}$ is relatively low (i.e. smaller than β) and $0 > (1 - r^3_{1,1})F^3_{1,1} \geq -g$, increasing the amount the banks 1 and 2 borrow from bank 3 and keeping the interest rate constant reduces bank 3's expected profit and increases the expected profits of the banks 1 and 2. To compensate bank 3, $r^3_{1,1}$ has to rise when bank 3 is lending more.

However, for any $r^3_{1,1} \in]\alpha, \beta]$ there are two equilibria with $F^3_{1,1} < 0$, namely one with $0 > (1 - r^3_{1,1})F^3_{1,1} \geq -g$ and one with $(1 - r^3_{1,1})F^3_{1,1} \leq -g$. As just described, $r^3_{1,1}$ has to rise when bank 3 is lending more and $0 > (1 - r^3_{1,1})F^3_{1,1} \geq -g$. But if $(1 - r^3_{1,1})F^3_{1,1} \leq -g$, $r^3_{1,1}$ has to fall when bank 3 is lending more. Thus, if $(1 - r^3_{1,1})F^3_{1,1} \leq -g$, increasing the amount the banks 1 and 2 borrow from bank 3 and keeping the interest rate constant increases bank 3's expected profit and reduces the expected profits of the banks 1 and 2. To compensate the banks 1 and 2, $r^3_{1,1}$ has to fall when bank 3 is lending more. Thus, whether an increase in the amount borrowed by 1 and 2 has to be accompanied by a rise or by a fall in the related interest rate does not only depend on how high the interest rate is, but also on $F^3_{1,1}$. Note that the reason for this result has been given in our discussion of equation 15. There, we have seen that the impact of changes of $F^j_{i,1}$ on $L^*_{i,3}$ does not only depend on $r^j_{i,1}$, but also on the values of the indicator functions I_k, $k \in \rho_i$. Because changing $F^j_{i,1}$ alters the position the banks i and j have in the negotiations in $t = 2$.

Finally note that $F^3_{1,1}$ is negative if $r^3_{1,1}$ is low and $F^3_{1,1}$is positive if $r^3_{1,1}$ is high. This is plausible given that we already know that the trade in $t = 1$ makes the banks 1 and 2 better off and bank 3 worse off.

5 Empirical results

The interbank market with the institutional characteristics described in this paper can easily be analysed in a general equilibrium model with all banks taking (expected) interest rates as given.[10] In such a model, the interest rate follows a martingale-like process $R_1 \simeq E_1[R_2]$. There is empirical evidence that there are small deviations from the martingale hypothesis. Angelini (2002) and Bindseil, Weller and Wuertz (2002) for example find that the EONIA rate is on average relatively high at the end of the month. However, all in all the martingale hypothesis seems to work quite well.

Nevertheless, in this section we try to define an alternative hypothesis that may appear to work even better. This hypothesis is motivated by the model of this paper. To begin with, two crucial assumptions of our model should be noted. The first assumption is that banks do not negotiate pairwise and successively, but multilaterally and simultaneously with each other. The second is that those banks that negotiate with each other all have the same information. There are hundreds of banks at for example the euro interbank market. It is of course hard to imagine that they all negotiate simultaneously and multilaterally and that they all have the same information. However, our model may still appear realistic if one can show that the interbank market is highly segmented so that there are many small groups of banks that maintain relations almost exclusively with members of their own group. Cocco, Gomes and Martins (2001) have shown empirically that this may indeed be the case. Focusing on the Portuguese interbank market, they find that many banks trade with only a few other banks over a long period.[11] In the context of these findings, our model may be interpreted as the model of one of these segments instead of a model of the whole interbank market.

Now look at one such segment of the interbank market. If our model is a good description of this segment, the average interest rate in this segment at the day before the last day of the maintenance period (in our model R_1) can in principle be any number between 0 and 1. However, banks might in reality be reluctant to agree on a zero or a one hundred percent interest rate. It appears to be more likely that they agree on an interest rate that is somehow "plausible" in the eyes of observers. The most plausible interest rates on the day before the last day of a maintenance period may be the

[10] See for example Tapking (2002).

[11] Another example is Germany where a large number of small savings and of small co-operative banks tend to trade liquidity only with their respective "head" institutions.

expected (average) interest rate on the last day of the maintenance period (in our model $E_1[R_2]$) and the mid point between the facility rates (in our model $\frac{1}{2}(r_D + r_S)$).

Accordingly, we now assume that there are m segments at the interbank market. Banks in a segment negotiate and trade only with banks of the same segment. Let R_t^s be the average interest rate at day t in segment s. In the first k segments, there is a habit to set bilateral interest rates in $t = 1$ (the day before the last day of the maintenance period) equal to $\frac{1}{2}(r_D + r_S)$, i.e. $R_1^s = \frac{1}{2}(r_D + r_S)$ for $s = 1, ..., k$. In the other segments, there is a tradition to set interest rates according to the martingale hypothesis, i.e. $R_1^s = E_1[R_2^s]$ for $s = k+1, ..., m$. Let a_s be the relative volume of segment s in the whole interbank market so that $\sum_{s=1}^m a_s = 1$. The EONIA rate in t is defined as $R_t = \sum_{s=1}^m a_s R_t^s$. Thus

$$R_1 = \sum_{s=1}^{k} a_s \frac{1}{2}(r_D + r_S) + \sum_{s=k+1}^{m} a_s E_1[R_2^s]$$

Assuming that the expected liquidity situation is similar over all segments (otherwise a new formation of segments takes place in the medium term), i.e. $E_1[R_2^s] = c$ for all $s = 1, ..., m$ and some parameter c, we get

$$E_1[R_2] = E_1[\sum_{s=1}^{m} a_s R_2^s] = c \sum_{s=1}^{m} a_s = c$$

i.e.

$$R_1 = a\frac{1}{2}(r_D + r_S) + (1-a)E_1[R_2] \tag{30}$$

with $a \equiv \sum_{s=1}^k a_s > 0$. Equation 30 is the theoretical model we want to test empirically against the martingale hypothesis $a = 0$.

The first problem we face is how to handle $E_1[R_2]$ empirically. Two approaches have been prevailing in the literature. The first simply replaces $E_1[R_2]$ by R_2 so that the econometric model is

$$R_{1,p} = a\frac{1}{2}(r_{D,p} + r_{S,p}) + (1-a)R_{2,p} + \varepsilon_p \tag{31}$$

where p stands for the respective maintenance period. The other approach replaces $E_1[R_2]$ by the so called tomorrow-next futures rate F_1. If two banks agree in $t = 1$ to trade tomorrow-next at F_1, they agree that the one bank borrows from the other banks in $t = 2$ overnight at the rate F_1. That leads to an alternative econometric model

$$R_{1,p} = a\frac{1}{2}(r_{D,p} + r_{S,p}) + (1 - a)F_{1,p} + \varepsilon_p \tag{32}$$

To test our theoretic model by means of the two alternative econometric models, we use data from the European interbank market. There were 47 maintenance periods between January 1999 and December 2002. $R_{1,p}$ is the EONIA rate on the day before the last day of maintenance period p, $R_{2,p}$ is the EONIA rate on the last day of maintenance period p, $r_{D,p}$ and $r_{S,p}$ are the deposit rate and the marginal lending rate on the day before the last day of the maintenance period p and $F_{1,p}$ is the tomorrow-next EONIA futures rate on the day before the last day of the maintenance period p. The tomorrow-next rate is calculated as the average rate of one-day-in-advance EONIA futures traded on the Italian e-MID platform.

Instead of estimating the restricted models of the equations 31 and 32, we estimate the unrestricted models

$$R_{1,p} = c_1\frac{1}{2}(r_{D,p} + r_{S,p}) + c_2R_{2,p} + \varepsilon_p \tag{33}$$

and

$$R_{1,p} = c_1\frac{1}{2}(r_{D,p} + r_{S,p}) + c_2F_{1,p} + \varepsilon_p \tag{34}$$

and test the hypotheses $c_1 + c_2 = 1$ and $c_1 > 0$. Moreover, we estimate the martingale models

$$R_{1,p} = c_2R_{2,p} + \varepsilon_p \tag{35}$$

and

$$R_{1,p} = c_2F_{1,p} + \varepsilon_p \tag{36}$$

and compare the goodness of fit of these two with the models in the equations 33 and 34. The results of the OLS estimations of the four models are given in the following table:

	equation 33	equation 34	equation 35	equation 36
$\widehat{c}_1$	0.349	-0.163	–	–
t for $H_0 : c_1 = 0$	4.065	-3.085	-	-
$\widehat{c}_2$	0.653	1.156	0.999	0.996
t for $H_0 : c_2 = 0$	7.634	22.129	86.008	208.42
t for $H_0 : c_2 = 1$	-	-	-0.104	-0.909
t for $H_0 : c_1 + c_2 = 1$	0.198	-1.401	-	-
R^2	0.927	0.986	0.900	0.983
adjusted R^2	0.926	0.986	0.900	0.983
Akaike info criterion	0.124	-1.521	0.394	-1.371
Schwarz criterion	0.202	-1.442	0.433	-1.332

For all hypothesis tested, the critical value on the 5% level is at about 2.016. To begin with, we see that in the models of the equations 35 and 36 the hypothesis $c_2 = 1$, i.e. the martingale hypothesis cannot be rejected. However, we learn from the adjusted R^2, the Akaike info and the Schwarz criterion that the model of equation 33 yields a better fit than the model of equation 35 and the model of equation 34 yields a better fit than the model of equation 36. We therefore concentrate on the first two models given in the equations 33 and 34.[12]

The results for equation 33 clearly support our theoretical model of equation 30. Both hypothesis $c_1 = 0$ and $c_2 = 0$ can be rejected, we have $\widehat{c}_1 > 0$ and $\widehat{c}_2 > 0$ and we cannot reject the hypothesis $c_1 + c_2 = 1$. However, the results for equation 34 are less compelling. We still cannot reject the hypothesis $c_1 + c_2 = 1$. However, we have $\widehat{c}_1 < 0$ and the hypothesis $c_1 = 0$ must be rejected.

Which of the two econometric models, equation 33 or 34, is more appropriate depends on whether R_2 or F_1 is a better proxy for $E_1[R_2]$. Theoretically, the rate F_1 appears to be a good proxy for $E_1[R_2]$ if all banks take the futures rate as given at the futures market. However, in the light of the model of this paper, it may be questioned that $F_1 \simeq E_1[R_2]$ if the terms of trades at the EONIA futures market are fixed in multilateral negotiations. In this case, R_2 might be the better proxy for $E_1[R_2]$, i.e. equation 33 is more appropriate for

[12]The squared correlation between $\frac{1}{2}(r_S + r_D)$ and R_2 is 0.777. It is 0.874 between $\frac{1}{2}(r_S + r_D)$ and F_1. Thus, there may be a multicollinearity problem in the regression of the equations 33 and 34.

the model in equation 30. Empirically, however, we have no reason to believe that F_1 is a bad proxy for $E_1[R_2]$: Estimating the mean μ of the expectation error $R_2 - F_1$ leads to $\widehat{\mu} = -0.0332$ with a t-statistic of $t = -0.765$ so that the hypothesis $\mu = 0$ cannot be rejected.

6 Concluding remarks

We have discussed a model of the interbank market that is based on the assumption that banks do not take the interest rate as given, but multilaterally negotiate at each day of a two day maintenance period on interest rates and on the amount of liquidity one bank borrows from another bank. The theoretic concept we have used to model these negotiations is the Shapley value. We have shown that the implicit average interbank rate on the first day of the maintenance period is not unique. It is any number between 0 and 1 and therefore conveys no information on the expected liquidity situation in the market.

One conclusion is that the market may expect a liquidity deficit even if the average interbank rate is exactly in the middle between the two facility rates. It could expect a liquidity deficit even if the average interbank rate is low and it could expect a liquidity surplus even if the average rate is high. Central bankers who believe that interbank interest rates are a matter of multilateral negotiations would therefore need to take into consideration additional information, for example information on transaction volumes, if they want to assess the liquidity situation at the interbank market.

However, whether our model is a good description of the reality is hard to say. Our empirical results crucially depend on how expectations are approximated. Estimations of an econometric model that can be motivated by our theoretical model lead to very good results if the expected interest rate on some day t is approximated by the true interest rate of that day. However, if we instead use forward rates to approximate expected rates, the empirical results do not support our econometric model.

7 Appendix

Proof of the lemmas 1 and 2:

Consider two coalitions of the form $\{i, h\}$ and $\{j\}$. With $F^h_{j,1} = F^i_{j,1} = 0$, we get

$$\begin{aligned} & \frac{\partial v_{\{h,i\},1}}{\partial(1-r^h_{i,1})F^h_{i,1}} \qquad (37)\\ = \quad & (r_S - r_D)\frac{1}{6}[p_1(I_{\{i\}}(s_1) - I_{\{i,j\}}(s_1) - I_{\{h\}}(s_1) + I_{\{h,j\}}(s_1)) \\ & +p_2(I_{\{i\}}(s_2) - I_{\{i,j\}}(s_2) - I_{\{h\}}(s_2) + I_{\{h,j\}}(s_2))] \end{aligned}$$

Note that $v_{\{h,i\},1}$ is piecewise linear, i.e.37 is only defined if $\sum_{i\in k} x_i(s_z) \neq 0$ for $k \in \{h\}, \{h,j\}, \{i\}, \{i,j\}$ and all $z \in \{1,2\}$. But it is a continuous function.

Lemma 1: Let $i = 1$, $h = 2$. For $0 < (1-r^2_{1,1})F^2_{1,1} < g$, we have $I_{\{1\}}(s_1) = I_{\{2\}}(s_1) = I_{\{2,3\}}(s_1) = I_{\{2,3\}}(s_2) = 1$ and $I_{\{1\}}(s_2) = I_{\{2\}}(s_2) = I_{\{1,3\}}(s_1) = I_{\{1,3\}}(s_2) = 0$, i.e. the right hand side of equation 37 is $(r_S - r_D)\frac{1}{6} > 0$ and $v_{\{1,2\},1}$ is not maximised. For $(1-r^2_{1,1})F^2_{1,1} > g$, we have $I_{\{2\}}(s_1) = I_{\{2\}}(s_2) = I_{\{2,3\}}(s_1) = I_{\{2,3\}}(s_2) = 1$ and $I_{\{1\}}(s_1) = I_{\{1\}}(s_2) = I_{\{1,3\}}(s_1) = I_{\{1,3\}}(s_2) = 0$, i.e. the right hand side of equation 37 is 0. A further increase of $(1-r^2_{1,1})F^2_{1,1}$ would not change $v_{\{1,2\},1}$ anymore, thus any $(1-r^2_{1,1})F^2_{1,1} \geq g$ or - because of symmetry - $(1-r^2_{1,1})F^2_{1,1} \leq -g$ maximises $v_{\{1,2\},1}$.

Lemma 2: Let $h = 1$ and $i = 3$. (1) For $0 < (1-r^3_{1,1})F^3_{1,1} < g$, we have $I_{\{1\}}(s_1) = I_{\{3\}}(s_2) = I_{\{1,2\}}(s_1) = I_{\{2,3\}}(s_1) = I_{\{2,3\}}(s_2) = 1$ and $I_{\{1\}}(s_2) = I_{\{3\}}(s_1) = I_{\{1,2\}}(s_2) = 0$, i.e. the right hand side of equation 37 is $(r_S - r_D)p_1\frac{1}{6} > 0$ and $v_{\{1,3\},1}$ is not maximised. (2) For $g < (1-r^3_{1,1})F^3_{1,1} < 2g$, we have $I_{\{3\}}(s_1) = I_{\{3\}}(s_2) = I_{\{2,3\}}(s_1) = I_{\{2,3\}}(s_2) = I_{\{1,2\}}(s_1) = 1$ and $I_{\{1\}}(s_1) = I_{\{1\}}(s_2) = I_{\{1,2\}}(s_2) = 0$, i.e. the right hand side of equation 37 is $-(r_S - r_D)p_1\frac{1}{6} < 0$. Thus, $(1-r^3_{1,1})F^3_{1,1} = g$ is a local maximum. (3) For $(1-r^3_{1,1})F^3_{1,1} > 2g$, we have $I_{\{3\}}(s_1) = I_{\{3\}}(s_2) = I_{\{2,3\}}(s_1) = I_{\{2,3\}}(s_2) = 1$ and $I_{\{1\}}(s_1) = I_{\{1\}}(s_2) = I_{\{1,2\}}(s_2) = I_{\{1,2\}}(s_1) = 0$, i.e. the right hand side of equation 37 is 0. A further increase of $(1-r^3_{1,1})F^3_{1,1}$ would not change $v_{\{1,3\},1}$ anymore. (4) For $-g < (1-r^3_{1,1})F^3_{1,1} < 0$, we have $I_{\{1\}}(s_1) = I_{\{3\}}(s_2) = I_{\{1,2\}}(s_1) = 1$ and $I_{\{1\}}(s_2) = I_{\{3\}}(s_1) = I_{\{1,2\}}(s_2) = I_{\{2,3\}}(s_1) = I_{\{2,3\}}(s_2) = 0$, i.e. the right hand side of equation 37 is $-(r_S - r_D)p_2\frac{1}{6} > 0$ and $v_{\{1,3\},1}$ is not maximised. (5) For $-2g < (1-r^3_{1,1})F^3_{1,1} < -g$, we have $I_{\{1\}}(s_1) = I_{\{1\}}(s_2) = I_{\{1,2\}}(s_1) = 1$ and $I_{\{3\}}(s_1) = I_{\{3\}}(s_2) = I_{\{1,2\}}(s_2) = I_{\{2,3\}}(s_2) = I_{\{1,2\}}(s_1) = 0$, i.e. the right hand side of equation 37 is$-(r_S - r_D)p_2\frac{1}{6} > 0$. Thus, $(1-r^3_{1,1})F^3_{1,1} = -g$ is a local maximum. (6) For $(1-r^3_{1,1})F^3_{1,1} < -2g$, we have $I_{\{1\}}(s_1) = I_{\{1\}}(s_2) = I_{\{1,2\}}(s_1) = I_{\{1,2\}}(s_2) = 1$ and $I_{\{3\}}(s_1) = I_{\{3\}}(s_2) = I_{\{2,3\}}(s_2) = I_{\{2,3\}}(s_1) = 0$, i.e. the right hand side of equation 37 is 0. A further decrease of $(1-r^3_{1,1})F^3_{1,1}$ would not change $v_{\{1,3\},1}$ anymore. It is clear that the local maximum (2) is higher (lower) than the local maximum (5) if and only if $p_1 > p_2$ ($p_1 < p_2$).

■

Proof of proposition 3 and lemma 4:

To prove lemma 4, we firstly consider cases with $0 \geq (1-r^3_{1,1})F^3_{1,1} \geq -\frac{1}{2}g$, i.e. $I_{\{1\}}(s_1) = I_{\{2\}}(s_1) = I_{\{1,2\}}(s_1) = I_I(s_1) = I_{\{3\}}(s_2) = 1$ and $I_k(s_z) = 0$ for all other k and z. It is easy to show that in this case $v_{\{i\},1} = E_1[L^{**}_{i,3}]$ is equivalent to

$$F^3_{1,1} = \frac{(r_S - r_D)\frac{1}{18}g\min\{p_1,p_2\}}{1-(1-r^3_{1,1})[1+r_S-(r_S-r_D)(\frac{1}{6}p_1+\frac{1}{3})]}$$

for all $i \in \{1,2,3\}$. We obviously have $\frac{dF^3_{1,1}}{dr^3_{1,1}} < 0$. To be consistent, we have to ensure

$$0 \geq \frac{(r_S - r_D)\frac{1}{18}g\min\{p_1,p_2\}(1-r^3_{1,1})}{1-(1-r^3_{1,1})[1+r_S-(r_S-r_D)(\frac{1}{6}p_1+\frac{1}{3})]} \geq -\frac{1}{2}g$$

It is easy to check that this is equivalent to

$$r^3_{1,1} \leq \frac{r_S-(r_S-r_D)(\frac{1}{6}p_1+\frac{1}{3}+\frac{1}{9}\min\{p_1,p_2\})}{1+r_S-(r_S-r_D)(\frac{1}{6}p_1+\frac{1}{3}+\frac{1}{9}\min\{p_1,p_2\})} \equiv \beta'$$

Similar considerations lead us to the following:

For $-\frac{1}{2}g \geq (1-r^3_{1,1})F^3_{1,1} \geq -g$, we get

$$F^3_{1,1} = \frac{(r_S-r_D)g(\frac{1}{6}-\frac{1}{6}p_1+\frac{1}{18}\min\{p_1,p_2\})}{1-(1-r^3_{1,1})[1+r_S-(r_S-r_D)\frac{1}{2}p_1]}$$

$\frac{dF^3_{1,1}}{dr^3_{1,1}} < 0$ and $\beta' \leq r^3_{1,1} \leq \beta$ with

$$\beta \equiv \frac{r_S-(r_S-r_D)(\frac{1}{3}p_1+\frac{1}{6}+\frac{1}{18}\min\{p_1,p_2\})}{1+r_S-(r_S-r_D)(\frac{1}{3}p_1+\frac{1}{6}+\frac{1}{18}\min\{p_1,p_2\})}$$

For $(1-r^3_{1,1})F^3_{1,1} \leq -g$, we get

$$F^3_{1,1} = \frac{(r_S-r_D)g(\frac{1}{3}p_1-\frac{1}{3}+\frac{1}{18}\min\{p_1,p_2\})}{1-(1-r^3_{1,1})[1+r_S-(r_S-r_D)\frac{1}{2}]}$$

$\frac{dF^3_{1,1}}{dr^3_{1,1}} > 0$ and $\alpha < r^3_{1,1} \leq \beta$ with

$$\alpha \equiv \frac{r_S - (r_S - r_D)\frac{1}{2}}{1 + r_S - (r_S - r_D)\frac{1}{2}}$$

For $0 \leq (1 - r^3_{1,1})F^3_{1,1} \leq \frac{1}{2}g$, we get

$$F^3_{1,1} = \frac{(r_S - r_D)g\frac{1}{18}\min\{p_1, p_2\}}{1 - (1 - r^3_{1,1})[1 + r_S - (r_S - r_D)(\frac{1}{2} + \frac{1}{6}p_1)]}$$

$\frac{dF^3_{1,1}}{dr^3_{1,1}} < 0$ and $r^3_{1,1} \geq \gamma'$ with

$$\gamma' \equiv \frac{r_S - (r_S - r_D)(\frac{1}{6}p_1 + \frac{1}{2} - \frac{1}{9}\min\{p_1, p_2\})}{1 + r_S - (r_S - r_D)(\frac{1}{6}p_1 + \frac{1}{2} - \frac{1}{9}\min\{p_1, p_2\})}$$

For $\frac{1}{2}g \leq (1 - r^3_{1,1})F^3_{1,1} \leq g$, we get

$$F^3_{1,1} = \frac{(r_S - r_D)g(\frac{1}{6}p_1 + \frac{1}{18}\min\{p_1, p_2\})}{1 - (1 - r^3_{1,1})[1 + r_S - (r_S - r_D)(\frac{1}{2} + \frac{1}{2}p_1)]}$$

$\frac{dF^3_{1,1}}{dr^3_{1,1}} < 0$ and $\gamma' \geq r^3_{1,1} \geq \gamma$ with

$$\gamma \equiv \frac{r_S - (r_S - r_D)(\frac{1}{3}p_1 + \frac{1}{2} - \frac{1}{18}\min\{p_1, p_2\})}{1 + r_S - (r_S - r_D)(\frac{1}{3}p_1 + \frac{1}{2} - \frac{1}{18}\min\{p_1, p_2\})}$$

For $(1 - r^3_{1,1})F^3_{1,1} \geq g$, we get

$$F^3_{1,1} = \frac{(r_S - r_D)g(\frac{1}{3}p_1 - \frac{1}{18}\min\{p_1, p_2\})}{1 - (1 - r^3_{1,1})[1 + r_S - (r_S - r_D)\frac{1}{2}]}$$

$\frac{dF^3_{1,1}}{dr^3_{1,1}} > 0$ and $\alpha > r^3_{1,1} \geq \gamma$.

It is easy to show that $0 < \gamma < \gamma' < \alpha < \beta' < \beta$.

Note that proposition 3 follows from $\gamma < \beta$ and part (i) and (iii) of lemma 4.

■

References

ANGELINI, P. (2002), "Liquidity and announcement effects in the euro area", Banca d'Italia Discussion Paper No. 451.

AUMANN, R. J. and L. S. SHAPLEY (1974), "Values of non-atomic games", Princeton University Press, Princeton.

BINDSEIL, U., B. WELLER, F. WUERTZ (2002), "Central bank and commercial banks' liquidity management - what is the relationship?", Economic Notes, forthcoming.

BUTNARIU, D. and E. P. KLEMENT (1996), "Core, value and equilibria for market games: On a problem of Aumann and Shapley", International Journal Of Game Theory 25, 149-160.

COCCO, J. F., F. J. GOMES and N. C. MARTINS (2001), "A friend in need! The role of relationships in the interbank market", mimeo.

EICHBERGER, J. (1993), "Game theory for economists", Academic Press, San Diego.

EUROPEAN CENTRAL BANK (2000), "The single monetary policy in stage three: General documentation of Eurosystem monetary policy instruments and procedures", Frankfurt.

HART, S. and A. MAS-COLELL (1996), "Bargaining and value", Econometrica 64, 357-380.

HARTMANN, P., M. MANNA and A. MANZANARES (2001), "The microstructure of the European money market", European Central Bank, Working Paper No. 80.

HO, T. S. Y. and SAUNDERS, A. (1985), "A micro model of the federal funds market", The Journal Of Finance XL, 977-990.

MENDIZABAL, H. R. and G. P. QUIROS (2001), " The daily market for funds in Europe: Has something changed with the EMU?", European Central Bank, Working Paper No. 67.

MERTENS, J.-F. and SORIN, S. (1994), "Game-theoretic methods in general equilibrium analysis", Kluwer Academic Publisher, Dordrecht, Boston, London.

MYERSON, R. B. (1991), "Game Theory: analysis of conflict", Harvard University Press, Cambridge (MA), London.

SHAPLEY, L. S. (1953), "A value for n-person games", in: Contributions To The Theory Of Games II (Annals Of Mathematics Studies 28), ed. by H. W. Kuhn and A. W. Tucker, Princeton University Press, Princton, 307-317.

SHAPLEY, L. S. (1964), "Values of large games VII: A general exchange economy with money", The Rand Corporation, RM 4248.

SPINDT, P. A. and HOFFMEISTER, J. R. (1988), "The micromechanics of the federal funds market: Implications for day-of-the-week effects of funds rate volatility", Journal Of Financial And Quantitative Analysis 23, 401-416.

TAPKING, J. (2002), "The Eurosystem's standing facilities in a general equilibrium model of the European interbank market", Deutsche Bundesbank Discussion Paper 20/02.

VÄLIMÄKI, T. (2001), "Fixed rate tenders and the overnight money market equilibrium", Bank of Finland, Discussion Paper 8/2001.

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