NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1297

STATE AND DEVELOPMENT OF FLUTTER CALCULATION

By A. Teichmann

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I. FUNDAMENTALS OF APPROACH TO THE PROBLEM

A. General Treatment

Schwarzmann's expositions showed the flutter problem to be of such outstanding importance that the large amount of work for years expended on it seems more than justified. Nevertheless, the question arises of whether it is not sufficient to eliminate the danger of flutter by simply obeying a number of simple and generally known rules for the design, for instance avoiding rearward positions of the center of gravity, identifying the separate natural frequencies of aerodynamically essential component systems, obtaining rigid constructions free from play, etc.

The strict observance of such rules for design causes, however, in many ways, structural limitations which are by no means necessary. Figure 1 shows, for instance, the critical speed of a wing as a function of the natural frequency of the control surface. According to the rule of design, equality of frequencies between control surface and wing $(\omega_R = \omega_D)$ ought absolutely to be avoided. However, this would be justified only for the model at the right of figure 1, "model in the sense of the two-dimensional problem," whereas for the "actual wing" at the left there is no objection whatsoever to equality of frequencies, since the critical speed there shows only an insignificant reduction compared to the case of vanishing control stiffness which must be taken into account, anyway.

On the other hand, however, there also exists the possibility of flutter occurring in spite of optimum observance of the design rules. Figure 2 shows the critical speed of a power-controlled auxiliary-control-surface arrangement as a function of the position of the control-surface center of gravity. Even for mass-balanced control surface, and all the more for forward positions of the center of gravity, this system flutters at a relatively low critical velocity,

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although the fin-bending frequency is 32 percent above the auxiliary-control-surface frequency; and just for rearward positions of the center of gravity the system exhibits a - insignificant, but still existing - region free from flutter.

Examples of the type mentioned demonstrate the necessity of undertaking an individually prepared investigation of its flutter characteristics for every airplane design.

B. Individual Treatment

In evaluating such individual investigations, it must be noted that for most parameters on which the flutter characteristics of an airplane design depend, reliable numerical values are not available. This applies especially to control stiffnesses and control masses, to torsional stiffnesses of wing and fins which vary according to preloading and corresponding distortion (or wrinkling); above all, however, it applies to the parameters in the air-force law and to the structural-member damping.

Moreover, the separate construction data for an airplane design change repeatedly, particularly at the initial stage where the flutter investigation must start if it is to affect the construction at all. Later on, too, many construction parameters fluctuate considerably due to tolerances or to subsequent modifications as they are required according to flight tests, that is, changes in the tail surfaces, lengthening of the fuselage, displacement of the control-surface center of gravity, variations of aerodynamic control-surface balance, etc.

On the other hand, one must consider how abruptly a construction may pass from a flutter-safe to a flutter-dangerous state when a parameter is changed. (See fig. 3.) Accordingly, it would be fundamentally wrong to "tailor" a flutter investigation only to one particular combination of individual design parameters without considering whether perhaps a parameter combination with distinctly unfavorable flutter characteristics is approached. A meaningful flutter investigation must therefore comprehend, on principle, the entire ranges within which the uncertain design parameters may possibly lie.

Of course, such "variational considerations" are necessary also when it is a question of finding the optimum design arrangement, for instance the optimum position of split tail surfaces, the optimum degree of control-surface mass balance, or the most suitable arrangement of an aerodynamic control-surface balance.

In view of the work expenditure on a conscientious flutter investigation, researchers have probably pondered before whether it would not be advisable to set up detailed curve tables from which to take quickly the flutter characteristics of any construction.

Unfortunately, there is little prospect of realizing an "atlas of graphs" of that kind - even for quite special systems with two to four degrees of freedom the flutter characteristics depend on 10 to 36 freely disposable parameters. Besides, even if such atlases did exist, extensive individual investigations would still be necessary - merely the classification of a certain single construction within such an atlas would require the determination of decisive characteristics of that design, which is an essential part of an individual single investigation.

The uncertainties in the formulation for the air force seem to suggest, for flutter investigations, that all calculations be omitted and all problems be instead clarified by use of wind-tunnel models similar to airplanes. However, one may call such tests superior to calculations only if the model has to a sufficient degree the aerodynamic properties as well as the spring-mass and structural-member damping characteristics of the actual system. This requires, on one hand, performance of the tests at the correct Mach numbers, on the other, an internal model structure which is essentially simulated to the actual system and thus renders the latter's complicated bending, torsional, and camber characteristics - a modern airplane construction is usually not replaceable by a stick with straight elastic axis. Also, the model must nowhere show higher structural-member damping coefficients than those corresponding to the actual system. In view of the technical difficulties in even approximately satisfying such requirements and guaranteeing their fulfillment, the construction of such a model progresses only slowly, according to all experiences. Consequently, the wind-tunnel test with correctly simulated models cannot be used for constructive decisions concerning the flutter problem at the design stage of an airplane, and such decisions can therefore be made in practice only according to calculations.

Tests with correctly simulated models are intrinsically significant in that they make a conclusive checking of the flutter characteristics possible, but here again frequently extensive calculations are required, either because, after all, in the end some model data deviate from actual conditions and thus require an additional calculation for evaluating these deviations, or because unexpected results perhaps make a too high damping of the model or other discrepancies seem probable - aside from the fact that the Mach numbers of such a test most probably will not agree with those for actual conditions. Of course, these objections do not refer to model tests of the fundamental type, such as are necessary for obtaining data for detailed calculations; such basic tests do not depend on the model's actually reproducing one particular airplane.

The considerations above were meant to show from what viewpoints the DVL at present treats the problems of flutter calculations. In closest collaboration with the airplane industry, the attitude of the DVL proved to be right. Only in exceptional cases, single problems encountered by the industry could be clarified immediately from empirical facts, diagrams, etc; usually individual calculations were required for the ultimate decision.

II. NATURE OF THE FLUTTER CALCULATION

A. Mode of Consideration

Because of the addition of the air forces to the inertia, spring, and damping forces of the airplane structure, a flutter calculation must necessarily be more complicated than the otherwise known oscillation calculations. Furthermore, modern airplane structures represent complicated combinations of framework and monocoque structures with cut-outs, hinged control surfaces, and auxiliary control surfaces, locally concentrated single masses, etc. Such structures can no longer be represented so as to be surveyable as easily as, for instance, sticks with straight elastic axis and well-defined bending and torsional stiffness. Accordingly, setting up differential or integral equations of the flutter process generally does not by any means imply a satisfactory answer to the technical problem.

Since it is imperative to introduce as few "unknowns" as possible into technical calculations, it would normally be inexpedient to individually regard as unknown, for instance, all elements of motion (paths, torsions, variations in camber) required in a complete description of the flutter condition. Rather it will be advisable to concentrate only on a few particular combinations of all motion elements (compare fig. 4), that is, certain "elementary forms" $F_k(x,z)$; more will be said below on their selection (x,z = space coordinates). The unknowns then would be the scales a_k at which these elementary forms F_k must be superimposed on one another, in order to describe approximately the actual flutter condition $\delta(x,z;t)$. At first, these scales a_k are, of course, unknown functions $a_k(t)$ of time. However, it should be noted that in a motion according to forms F_k prescribed in this manner a state of dynamic equilibrium can be established only when one takes into consideration the additional forces which would have to attack at the system if it actually should carry out this form of motion. Hence, it is advisable to use formulations stemming from the principle of virtual displacements for in case of displacements in the sense of any forms the restraint forces do not produce work and thus drop out of the mathematical formulation. The equations of motion for calculation of the time-dependent superimposition scales $a_k(t)$ of the separate elementary forms F_k then may be directly set up formally in the form of Lagrange's equations.

The difficulty in this treatment is the selection of appropriate elementary forms. The natural oscillation modes of the system and modes somewhat similar to them are given preference in this respect. As Borkmann has shown recently, a group of n such modes F can be examined as to its suitability by determination of the air-force distributions which would pertain to oscillations according to every single one of these modes F; the question then is whether it is possible, in turn, to approximate these air-force distributions by superimposition of the assumed n modes F.

Unfortunately, quite a number of such groups will usually be found appropriate according to the above procedure so that there is practically no way other than to perform several flutter calculations with different choice of elementary modes until the minimum critical velocity is reliably determined. The lower natural oscillation modes with few node lines are, of course, the most interesting ones, since for them the internal damping is small compared to the absorbable air-force energy.

In starting the design, one is interested first in wings and fins without control surfaces in order to make sure that the "basic structure" is "all right" with regard to flutter. Next, the effects of control surfaces and auxiliary control surfaces, of controls and various fine details, are of interest.

The manipulation of flutter calculations would be made unnecessarily still more complicated if one should, for each separate calculation, rigorously take into consideration that the scale factors $a_k(t)$ are, at first, open functions of time. (Compare fig. 4.) Hence, it is advisable also to interpret the individual scale factors $a_k(t)$ in turn as superimpositions of a series of prescribed functions $\Phi_{kg}(t)$, with $g = 1, 2, ...$ (Compare fig. 4.) The unknowns in the mathematical formulation then are, ultimately, the individual superimposition scales a_{kg} of these time functions $\Phi_{kg}(t)$. The required determining equations are suitably set up formally with the use of Gauss' principle of minimum constraint.

Concerning the question of which time functions are to be considered for a technical flutter calculation, a general indication results, first, from the physically trivial fact that, for sufficiently small relative airspeed, every oscillation originally is necessarily damped, that therefore the transition to an excited oscillation must take place over a purely periodic intermediate state. It can be demonstrated that usually several such intermediate states between damping and excitation or vice versa exist for a system. For the time being, only the lowest of these intermediate states is of technical interest, thus a state which leads from the original damping to the first excitation.[1] In practice, this intermediate state is mentioned as a harmonic state; that is, $\cos \omega t$ and $\sin \omega t$ are selected as prescribed time functions $\Phi_{kg}(t)$ and are suitably interpreted as components of the complex expression $e^{j\omega t}$. It is, however, absolutely to be expected that in the future higher harmonic or any diminishing or increasing time functions, respectively, will also be included in the formulation.

The advantages of a purely harmonic oscillation formulation can of course be utilized only under the presupposition that harmonic forces pertain to harmonic motions. How far this is admissible with respect to the actual air forces and to the structural member damping forces is, at present, for want of systematic tests, an open question. However, it certainly involves considerable arbitrariness in the case of systems containing a spring with initial tension, that is, of power-controlled control-surface mechanisms and of systems with free play.

B. Flutter Equations

After all these restrictions in posing the problem, the flutter equations may be set up formally as linear equation systems for the unknown superimposition scales a_k of the separate elementary modes F_k (compare fig. 5); then the i^{th} equation, for instance, expresses the following: The internal and external forces which would appear in flutter according to a linear combination of all elementary modes F_k all together must not produce work if the system is assumed deformed in the sense of the elementary mode F_i. Correspondingly the separate coefficients B_{ik} of the flutter equations are integral expressions over products of the forces of a state "k" and the motions of a state "i"; they are thus analogous to the coefficients of the so-called elasticity equations, which are familiar to anybody working in statics research; they can be divided into inertia, spring, air, and damping members.

[1]Therein the simplifying assumptions on which the air-force formulation is based may cause certain difficulties; more details on this are contained in a research report by Leiss soon to be published.

Borkmann recently wrote an exhaustive treatise on the nature of these individual component terms (compare fig. 5) and their determination from calculations and tests. He therewith provided the long-needed continuation of various trains of thought, indicated by the author in the Luftfahrtforschung 1939. A. Borkmann calls the separate components "mass, spring, air force, and damping characteristics."

Normally the mass surface distribution will be known. The determination of the mass characteristics then offers no difficulties whatsoever.

Determination of the spring characteristics, however, is less simple; the quantity p_k^F entering there (compare fig. 5) is the loading which would have to attack on the airplane in order to force upon it exactly the deformation F_k. Determination of p_k^F is simplest when the elementary mode F_k is precisely a natural oscillation mode of the system, because in that case p_k^F equals the product of mass surface distribution, amplitude distribution F_k, and the square of the natural circular frequency ω_k pertaining to F_k.

In practice, however - above all in variational considerations - the elementary modes selected often will deviate more or less from the natural oscillation modes. Then it is expedient, first, to determine separately the deformations pertaining to a series of load conditions, and then to superimpose the load conditions in such a manner that their resultant deformation agrees as well as possible with the elementary mode F_k. The former superimposed loading then is practically the required loading p_k^F.

Of course, the spring characteristics thus found are reliable only if in their determination the preloading existing in flutter which is produced by the steady lift of the airplane also is taken into consideration, for the preloading determines, among other things, the wrinkling and may thus cause considerable variations of the elastic properties.

With the mass characteristics known, the spring characteristics may be determined also from the natural-oscillation conditions of the airplane, in practice, for instance, on the basis of ground oscillation tests which have to be performed, if necessary, with a preloading with soft springs. For this purpose, the individual natural-oscillation mode first is split up approximately into components equal to the elementary modes used. The resulting component coefficients of the individual constituents are inserted in the "static oscillation equations" as known amplitudes a, in order to then determine from them the spring characteristics contained in them. Correspondingly, one may of course determine the mass characteristics if the spring characteristics have been determined otherwise.

Borkmann, in his aforementioned treatise, calls the methods indicated "superimposing and splitting-up methods." All details concerning their nature and the manifold possibilities for their use are systematically treated in that report; of special interest are the possibilities discussed therein for using the methods for determination of the air-force characteristics from oscillation tests in the air stream, inasmuch as the mass and elastic characteristics have already been found in another way. Above all, such tests are significant in that they may be used for general statements on the air-force laws of the oscillating wing. Naturally, simple models may be used to which the initially mentioned restrictions regarding the applicability of the model test do not apply.

Flutter calculations of today are characterized by the linearized air-force formulations for the harmonically oscillating wing strip in the - stripwise two-dimensional - flow of an ideal fluid such as were first developed by Küssner and later on extended according to various viewpoints and adapted to practical use by Theodorsen, Ellenberger, Schwarz, Söhngen, and Dietze. The formulas at present ready for use deal, above all, with motions in broken and separated straight lines such as those shown in figure 6. The imaginary models indicated there represent various interpretations regarding the effect of a wing strip with control surface and apply likewise to additional auxiliary control surfaces. Also it is not difficult to insert, if necessary, curvatures into the form of motion. Dietze's expositions[2] contain some ideas on the selection of the suitable imaginary model in the individual case.

In calculations with these air-force formulations executed at the DVL, it proved desirable to be able to obtain all entering functions directly from curve tables. Thus the DVL recently instigated, for maximum time saving, the preparation of suitable curve tables uniform for all functions required in all interpretations of the wing with control surface and auxiliary control surface shown in figure 6.

The few tests which, so far, give information on the validity of the customary air-force formulations show that for systems with control surfaces certain modifications are necessary even for smallest Mach numbers. Thus Voigt recommends, according to his flutter tests, certain reductions of the control-surface chord ratio used.

It is quite certain by now that the customary air-force formulations no longer apply in cases of higher Mach numbers; therefore, modern

[2]Dietze, F: Vergleichsrechnungen zum aerodynamischen Ruderinnenausgleich. Lilienthal-Gesellschaft Bericht 135, pp. 70-74.

flutter calculations should take the compressibility of the air into consideration, all the more so since the formulations for Mach numbers ≤ 0.7 set up by Possio can be directly applied to arbitrary imaginary models. It is true that then for any imaginary model that happens to be in use, the numerical evaluation of a complicated integral equation becomes necessary which expresses the adaptation of the flow variation to the dimensions and peculiarities of that imaginary model; this numerical evaluation must be made in succession for all reduced frequencies $\omega_r = \frac{\omega}{v}\frac{l}{2}$ and Mach numbers in question (ω = circular frequency, v = relative airspeed, l = wing chord). Possio has already carried out the evaluation to a larger extent for the wing strip without control surface.

Of course it would be valuable if one could so modify the formulations customary since that time from the assumption of an ideal fluid that they can serve as manageable approximate formulas for compressible fluids; such attempts have been made occasionally, for instance with the use of Prandtl's rule for steady flow. As far as is known, however, these endeavors failed or could not be generalized.

It is also sure at present that the assumption of stripwise two-dimensional flow may fail to work; according to tests and calculations by Cicala, for instance, in case of so-called reduced frequencies $\omega_r = \frac{\omega}{v}\frac{l}{2} < 1.0$. Here also it is, for more accurate calculations, a question of numerically evaluating an integral equation for every individual case and for every required reduced frequency. Here also it will be correct to use first the formulations neglecting as little as possible and to turn to simplifications only after their admissibility has been proved. Accordingly, Küssner's and Possio's formulations are the first to be considered. Here, too, it would, of course, be valuable if one could so modify the expressions for stripwise two-dimensional flow now in use that they would represent manageable approximate formulas for consideration of the induction.

Independent of these analytical possibilities for perfecting the air-force formulations for flutter calculations, it is, of course, imperative to check and develop them by means of tests. This includes, among other requirements, a careful clarification of their linear additivity.

Thus it would be ideal if the interference method developed by Zobel would soon be made disposable for air-force measurements on the oscillating wing. Independent of that, the DVL intends to expedite indirect air-force determination from model oscillation tests, characterized previously in the discussion of the conceptions "superimposing and splitting-up methods." Voigt reports[3] on tests of that type.

Far more uncertain than even the basis for the air-force characteristics are the data so far at disposal for the formulation of the damping characteristics. At the time it is not even known whether it is at all permissible to linearly superimpose on one another the damping components corresponding to different motion components. Doubts concerning this fact arise from the obvious comparison of structural-member dampings with friction forces which surely play a role in it; for friction forces, a linear superimposition certainly is not possible.

Thus, when today it is still regarded frequently as a rule, on principle, to neglect the structural-member damping in flutter calculations, this fact may be justifiable because this neglect will usually amount to a measure on the safe side unless the damping causes, exceptionally, a very far-reaching change in the coupling relationships. However, with the neglect of the structural-member damping, the actual numerical representation of the critical velocity is basically ruled out, and the flutter calculation merely permits separating the available designs into flutter-safe structures and into structures with flutter risk. Especially, recently there arose repeatedly the necessity of making use of structural-member damping, possibly even artificial damping in order to justify sensitive constructions. Taking such cases into consideration, Boelk and Schmidt performed experiments regarding the problem of damping years ago. Systematic investigation of structural-member damping often has been contemplated at the DVL. The first task will be to determine certain minimum values for the damping coefficients which in case of need may unhesitatingly be used in calculation.

III. RATIONALIZATION OF THE CALCULATION

A. Simplified Air-Force Characteristics

In view of the uncertainty of any determination of characteristics, the expenditure of flutter calculations is in an unfavorable proportion to their reliability. In calculation with the air-force formulations customary at present, a considerable part of the work is spent on the numerical evaluation of integral expressions of the form

$$\int D(z)\ T\left[\frac{\bar{\omega}}{v}\ \frac{l(z)}{2}\right]\ dz$$

[3]Voigt, H.: Messung instationärer Luftkräfte. Lilienthal-Gesellschaft Bericht 135, pp. 90-93.

(z = coordinate in direction of width, ω = circular velocity, v = relative airspeed, $l(z)$ = wing chord at point z); for these expressions must be computed for numerous values of the ratio $\frac{\omega}{v}$. (Compare fig. 7.) As Borkmann has shown in the afore-mentioned report, this evaluation may unhesitatingly be shortened, if the transcendent functions appearing in the integrand

$$T\left[\frac{\omega}{v}\frac{l(z)}{2}\right]$$

are approximately replaced by an expression of the form

$$a_0 + a\sqrt{\frac{\omega}{v}\frac{l(z)}{2}}$$

since then only the space function $D(z)$ which is independent of ω/v remains under the integral sign.

Leiss went even further when he replaced – originally only for fundamental considerations – the functions mentioned

$$T\left[\frac{\omega}{v}\frac{l(z)}{2}\right]$$

by constant amounts. It was shown that in this manner, too, practical flutter calculations can still be performed; however, it is then advisable for safety reasons to intersperse occasional spot checks according to more accurate calculations into every series of variations of the parameters of interest – in these spot checks Borkmann's simplifications may of course be used. Figure 8 shows dependence curves calculated according to Leiss' simplification and interspersed with numerous spot checks; the figure gives an idea of the effect of Leiss' approximation. Naturally, only the agreement at the heavily drawn sections of the lower limiting curves of the flutter region is of practical interest.

With Leiss' simplification and a spot check according to Borkmann's, the expenditure of a variational consideration with three degrees of freedom, for instance, may be reduced from 47 to 16 days (under the presupposition that all spring and mass characteristics are prescribed).

B. Method of Calculation

The other reason for the large work expenditure of flutter calculations lies in the search for the critical velocity after the matrix of the flutter equations has been determined, the elements of which are known to contain the critical velocity v_{cr} and the flutter frequency ω_{cr}.

The DVL was repeatedly faced with the necessity of deciding on the fastest method of determining the critical values v, ω without taking into consideration closely related secondary problems, as for instance the increase of the flutter amplitudes after the critical velocity has been exceeded.

On one hand, the critical state may be found from the flutter determinant (it then practically amounts to experimentation to find at which values of v, ω the determinant disappears); on the other hand, however, the critical state may be determined directly from the flutter equations; then it is practically a matter of obtaining by iteration the flutter form and therewith the critical velocity.

The treatment of the determinant may consist in first splitting it up into a real and a purely imaginary component Δ' and Δ'' and then plotting the amounts of the component determinants for different pairs of values of v, ω.

For another treatment of the determinant, one may use the real notation.

Besides these primitive methods, special ones may be used, but only insofar as natural oscillation forms are selected as degrees of freedom, or as suitable transformations are performed first (which is probably impossible at the outset in variational considerations of the type initially described); furthermore, these methods require that the damping be neglected.

We will first name a promising method, suggested by Borkmann in his afore-mentioned report, as one of the conceivable possibilities of arriving at the pair of critical values v, ω directly from the flutter equations; it consists in step-by-step improvement of a component system selected from the flutter matrix by adequate use of all flutter equations. Another possibility is to improve an assumed flutter form by means of repeatedly carrying through the energy balance.[4]

In a report, as yet unfinished, Mayer has made estimates regarding the work expenditure for such methods. The points in figure 9 show a preliminary result; one is at liberty to double or halve the indicated work periods. The curves shown there are valid for calculations with formulation of the customary air-force laws and neglect of the structural member damping. One notices first of all the great advantage the iteration methods promise for more than five degrees of freedom. Unfortunately this advantage is counterbalanced by the fact that (due to the entering

[4]Compare Teichmann, Luftfahrtforschung (Aviation Research) 1939.

of the two parameters v and ω) it is quite uncertain in the individual case whether the method will converge toward the group v, ω of minimum velocity, or perhaps toward another group which is of no technical interest, but would then possibly be regarded as the decisive pair of values. For this reason, an entire region for the iteration methods has been plotted in figure 9. Its lower limiting curves are valid for a calculation with a certain initial system by carrying through two iteration processes; its upper limiting curve shows what the expenditure would be if, as a precaution, three different initial systems were tried out.

The question of whether the minimum critical velocity of the system under consideration is actually found in this manner remains open, of course; besides, so far there exists no evidence whether and under what circumstances the iteration methods for the flutter problem converge at all. In view of the present state of knowledge one should forego the use of iteration methods, if there are no reliable comparisons available.

The more involved methods which start directly from the flutter determinant are free from these disadvantages. In case of less than six degrees of freedom, it is there obviously advisable (according to fig. 9) to split the determinant into its real and imaginary part; for more degrees of freedom, however, the use of the real notation seems to be more advantageous with respect to time.

As figure 10 shows, splitting up of the determinant into a real and an imaginary part is advisable also in case of more than six degrees of freedom when Leiss' simplification of the air-force formulation (for instance $T' = T'' = 0$) is used.

Figure 11 shows the work expenditure if the calculator uses instead of the customary air-force formulations air forces determined, for instance, by means of experiment so that various theoretical relations do not enter. It is interesting that, in this case, use of the real determinant is more expedient than use of the determinant split up into real and imaginary parts.

At present, it is still necessary to represent the combined effect of a larger number of degrees of freedom by separately considering numerous partial systems with two or three degrees of freedom each with the expectation that one of them suffice to essentially describe the flutter characteristics of the entire system. In some cases, this might be more or less justifiable by evaluation of the coupling relationship existing just then. In many cases, however, some points remain unclear; therefore, such a procedure should be used only as a last resort. This unfavorable situation can be changed only by considerably restricting the computation times indicated in the figures. In practice, nothing at all is gained by suggesting that these times be reduced by 20 or even 50 percent; rather, it will be necessary to reduce them to 1/10 or 1/20 of the amounts given in the figures.

This is possible only when automatic calculation devices are used such as are already employed for other purposes in America and England.

In flutter problems such devices must accomplish two tasks: On one hand, they must determine the characteristics which requires above all the forming of multiterm product sums; on the other hand, they must solve the flutter determinant or, respectively, the flutter equations for the desired parameters v and ω. Zuse now is developing a device which is to be suitable for both these tasks and many more. His device differs from those known so far by the fact that it expresses all numerical values in yes-no combinations of telephone relays; the course of the desired operations is then controlled by a prepared perforated tape. The DVL has taken over the development of this device.

DISCUSSION TO THE LECTURE OF A. TEICHMANN

Bock.- It seems to me that one of the most important viewpoints is that the lecturer performs his calculations with consideration of different parameter variations and thus finds out which combinations of design parameters are unfavorable. For in the construction of airplane models the individual parameters are not absolutely fixed but vary somewhat from specimen to specimen so that avoidance of such unfavorable parameter combinations is important.

Furthermore, I deem important the efforts toward restricting the calculation expenditure, by mechanization of the calculation, so greatly that one may in the project stage obtain really usable surveys of the critical velocities as speedily as possible.

Quessel.- Teichmann mentioned in his lecture, among other things, the sensitivity of the critical velocity with respect to the design parameters; therefore, I should like to state our opinion on this problem, and illustrate it with a few examples, in particular, just on the example given by Teichmann. We are dealing here not with laws of nature but with conclusions from a calculation experience which, of course, admits exceptions.

A sensitivity of the critical velocity appears ordinarily only in the following cases:

1. When in addition to a stubborn flutter possibility, a harmless flutter possibility exists.

By a stubborn flutter possibility, we understand one that is hard to eliminate, by a harmless flutter possibility, in contrast, one which can easily be eliminated by relatively simple structural measures. The conception "harmless flutter possibility" has no connection with the conception of "benign" flutter. The harmless flutter possibility must be eliminated, as a rule, because it causes too low critical velocities. Then the stubborn flutter possibility with higher values of critical velocity remains which ordinarily is not sensitive with respect to the construction parameters. Thus, a sensitivity no longer exists after the required changes concerning the flutter possibility of the airplane have been carried out.

I should like to take, as an example, the case named by Teichmann for illustration of the sensitivity; we interpret it somewhat differently. Figure 1(a) shows the variation of the critical velocity, calculated with three degrees of freedom bending-torsion-control surface, as a function of the control-surface unbalance with a sensitivity of the critical velocity with respect to the aileron unbalance for rearward position a, as shown in Teichmann's lecture. (I sketched the figure from memory.) Figure 2(a) shows the diagram as it would appear according to our opinion if the dashed part of the curve were left out, and the limitation to minimum values of critical velocity dropped. One recognizes that one deals here with two different types of flutter: first, the harmless bending flutter which, due to the agreement between flutter and bending frequency, shows low values of critical velocity and a limited range of excitation and can easily be eliminated by damping or mass balance of the control surface; second, the stubborn or intractable torsional flutter which remains after elimination of the bending flutter and shows, due to the approximate agreement between flutter and torsional frequency, higher values of critical velocity and no limited range of excitation. This second type of flutter may hardly be wholly eliminated, is insensitive with respect to design parameters, and therefore ordinarily permits only a moderate increase in critical velocity by structural measures.

2. When the critical velocity is very high and, consequently, its dependence on a change in construction parameters is not of interest.

We take as an example the dependence of the critical velocity on the mass coupling for a wing with fixed ailerons (fig. 3(a)) which shows a sensitivity of the critical velocity with respect to the mass coupling in the region a - b. Since flutter of three-dimensional wings with fixed aileron ordinarily sets in - even for the most unfavorable value of mass coupling - only at sufficiently high velocities, this sensitivity is not of interest.

3. When one deals with a physically impossible calculation result which, due to the imperfection of the assumed deformation lines, occurs in a region sensitive for calculation.

We take as an example the dependence of the critical velocity, calculated according to the power criterion, on the bending frequency for a three-dimensional wing with the degrees of freedom bending-torsion-control surface (fig. 4(a)). In the region of curve 2, one could speak of a sensitivity of the critical velocity with respect to the bending frequency. As I explained in detail in my contribution to the discussion of Dimpker's lecture, the critical velocity zero is impossible for a three-dimensional wing if even one single degree of freedom is elastic. In order to eliminate curve 2, one would have to perform an iteration, which is very troublesome. We introduced material damping. It was found that a very small value of material damping was sufficient to eliminate curve 2.

Thus a sensitivity of the critical velocity with respect to the construction parameters - the thing that matters - will hardly occur.

Teichmann expressed the opinion that it is not of importance to calculate a single critical velocity but to evaluate the safety against flutter within a larger scope which is made possible, among other expedients, by variation of different construction parameters. Furthermore, Teichmann pointed out that the number of parameters is very high. We are absolutely of the same opinion; therefore, we always carry out variations of the construction parameters. In selecting the parameters to be varied, we favor those with the following characteristics:

1. The construction parameter has a great influence on the critical velocity.

2. The determination of the parameter is unreliable.

3. Variation of the parameter is convenient with respect to calculation technique.

In this manner, we arrive at a restricted number of variations in construction parameters and thus at a tolerable work expenditure.

The intention of the DVL to follow up the problem of material damping is very commendable. For the time being, one is dependent on the tests compiled by Küssner. These tests result in a minimum value of the material damping $\alpha = 0.03$. The number of airplanes investigated is not sufficiently large, most of them are not all-metal airplanes, and all of them are old models. Although the material damping usually does not amount to much, knowledge of a reliable minimum value of material damping is very desirable for the following reasons:

1. If large masses participate in the motion, the influence of internal damping may become decisive.

2. In certain frequency ranges the material damping may increase the critical velocity very considerably.

3. Sometimes the calculation arrives at physically impossible results which disappear with introduction of material damping, so that involved investigations become unnecessary.

4. In exceptional cases, sensitivities occur when none of the presuppositions treated above are met. These sensitivities produce difficulties. As a rule, such sensitivities cease with introduction of a moderate material damping.

Boelk.- Teichmann mentioned a method of Borkmann according to which an iterative solution of the determinant is suggested, in such a manner that a partial system of the determinant is solved with respect to v,ω and this solution is then improved by bringing in the rest of the determinant. This iteration will, no doubt, turn out to be correct if one has started out by assuming the possibilities of motion as the fundamental oscillation mode. However, if one has started out with a higher-harmonic oscillation, it will reduce to a suborder harmonic. How can that be prevented?

Borkmann.- As already discussed in the research report FB 1338, no proof of convergence for the iteration method mentioned exists so far. Just as for any flutter calculation of an airplane, no more than three or four are singled out as decisive from the large number of degrees of freedom; here also a partial determinant is singled out from the entire determinant in the expectation that the flutter mode of the total system will not greatly differ from that of this partial system. As far as this expectation comes true, the iteration method offers a good chance of improvement. Otherwise, of course, the application of this iteration method may encounter difficulties.

Leiss.- Teichmann's remarks on the sensitivity of a system to fluctuations in construction parameters purport that it is not sufficient to investigate a system for one particular combination of construction parameters only but that, rather, it is necessary to include a certain variation range of the separate construction parameters in order to find possibly existing sensitivities. Obviously, such sensitivities are to a large extent avoidable, but to that end they must first be determined.

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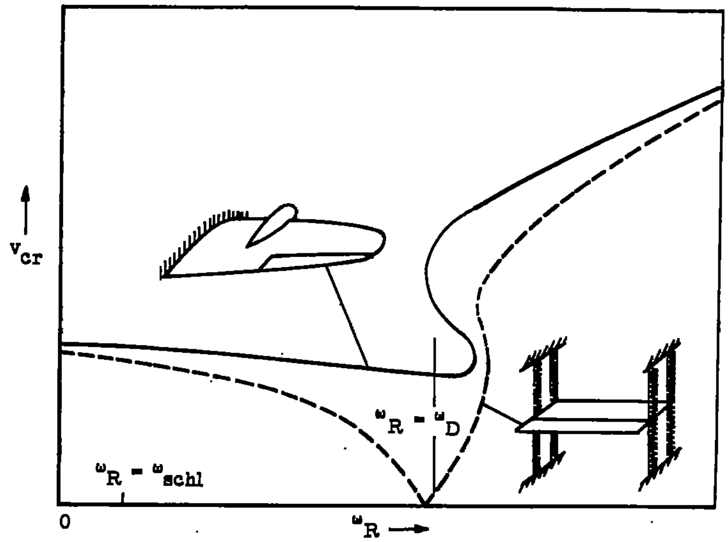
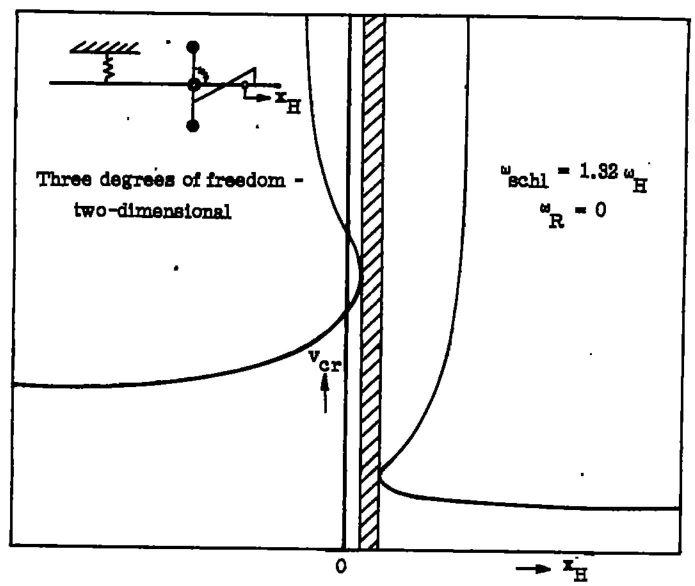


v_{cr} = critical velocity

ω_R = natural circular frequency of the control-surface torsion
ω_D = natural circular frequency of the wing torsion
ω_{schl} = natural circular frequency of the wing bending
} with the remaining degrees of freedom eliminated

Figure 1.- Harmful and harmless frequency region (systems with three degrees of freedom: wing bending, wing torsion, control-surface torsion).

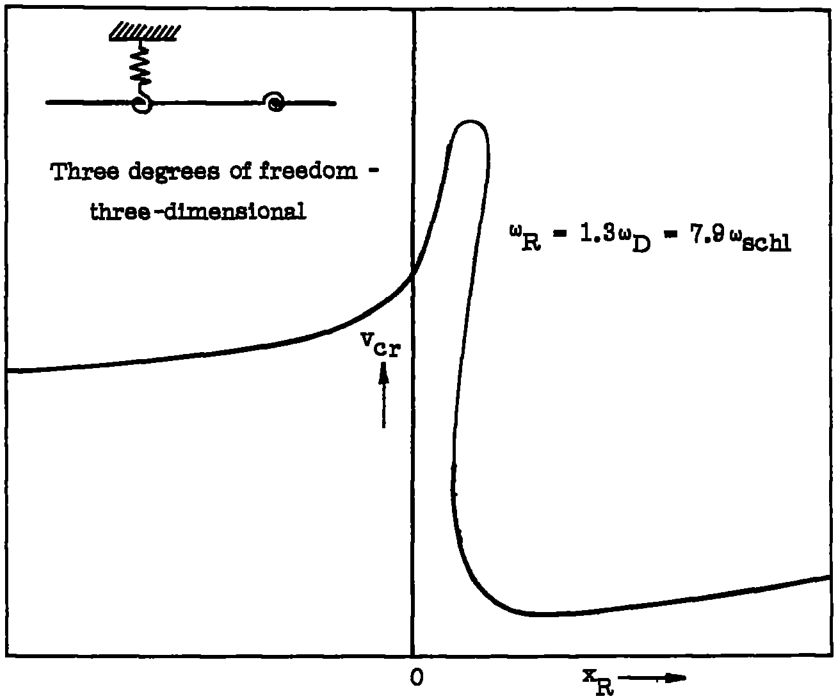


v_{cr} = critical velocity

x_H = position of the auxiliary control-surface center of gravity. Compare sketch of system

ω_{schl} = natural circular frequency of wing bending
ω_R = natural circular frequency of control-surface torsion
ω_H = natural circular frequency of auxiliary control-surface torsion
} with the remaining degrees of freedom eliminated

Figure 2.- Rules for design fail to work (system in the sense of the two-dimensional problem with three degrees of freedom: wing bending, control-surface torsion, auxiliary control-surface torsion).



v_{cr} = critical velocity
x_R = position of control-surface center of gravity
ω_R = natural circular frequency of the control-surface torsion
ω_D = natural circular frequency of the wing torsion
ω_{schl} = natural circular frequency of the wing bending
(with the remaining degrees of freedom eliminated)

Figure 3.- Variation of a construction parameter (system with three degrees of freedom: wing bending, wing torsion, control-surface torsion).

$$
\begin{aligned}
\delta(x,z;t) &= \sum_k a_k(t) \cdot F_k(x,z) \\
&= \sum_k \left[\sum_g a_{kg} \cdot \Phi_{kg}(t)\right] \cdot F_k(x,z) \\
&= \sum_k \left[a_k' \cdot \cos\omega t - a_k'' \cdot \sin\omega t\right] \cdot F_k(x,z) \\
&\underset{R}{=} \sum_k \underline{a}_k \cdot e^{j\omega t} \cdot F_k(x,z) \\
&\left(= \sum_k a_k \cdot \Phi_k(x,z;t)\right)
\end{aligned}
$$

Figure 4.- Formula for the flutter process.

Motion formula: $\delta_{cr}(x,z;t) = \sum_{i=1}^{n} a_i \cdot F_i(x,z) \cdot e^{j\omega t}$

Flutter equations:

$$a_1 \cdot B_{11}(v,\omega) + a_2 \cdot B_{12}(v,\omega) + \ldots + a_n \cdot B_{1n}(v,\omega) = 0,$$

$$a_1 \cdot B_{21}(v,\omega) + a_2 \cdot B_{22}(v,\omega) + \ldots + a_n \cdot B_{2n}(v,\omega) = 0,$$

$$\vdots$$

$$a_1 \cdot B_{n1}(v,\omega) + a_2 \cdot B_{n2}(v,\omega) + \ldots + a_n \cdot B_{nn}(v,\omega) = 0$$

Unknowns: a_i = amplitudes of $F_i(x,z)$

Coefficients:

$$B_{ik}(v,\omega) = \int p_k(v,\omega;\, c\ldots) \cdot F_i \cdot dV$$

$$= \underbrace{\int p_k^{F}(c\ldots) \cdot F_i \cdot dV}_{-\varphi_{ik}} + \underbrace{\int p_k^{T}(\omega;c\ldots) \cdot F_i \cdot dV}_{+\omega^2 \mu_{ik}} + j\underbrace{\int p_k^{D}(\omega;c\ldots) \cdot F_i \cdot dV}_{+\vartheta_{ik}(\omega)}$$

$$+ \underbrace{\int p_k^{BR}(\omega;c\ldots) \cdot F_i \cdot dO}_{+\omega^2 \rho \cdot \lambda_{ik}^{(0)}} + \underbrace{\int p_k^{BS'}(v,\omega;c\ldots) \cdot F_i \cdot dO}_{-v^2 \rho \cdot \bar{\lambda}_{ik}\left(\frac{\omega}{v}\right)} + j\underbrace{\int p_k^{BS''}(v,\omega;c\ldots) \cdot F_i \cdot dO}_{-v\omega\rho \cdot \bar{\bar{\lambda}}_{ik}\left(\frac{\omega}{v}\right)}$$

Figure 5.- Flutter equations.

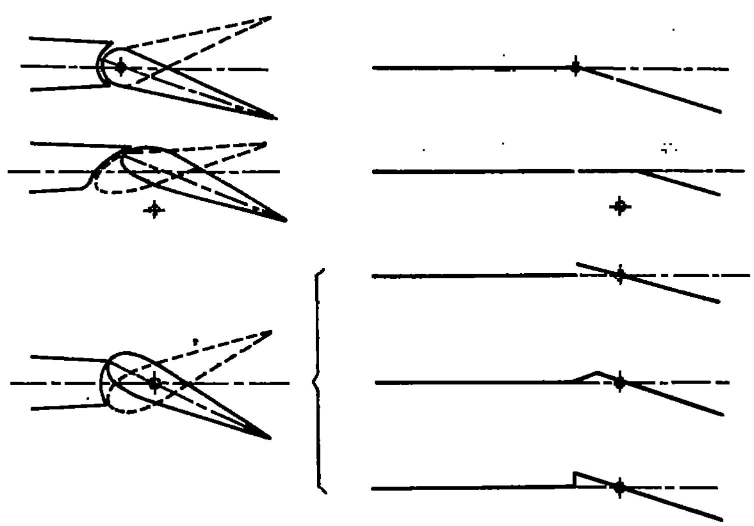


Figure 6.- Aerodynamic control-surface balance (interpretations).

$$\lambda_{ik}\left(\frac{\omega}{v}\right) = \ldots . + \int D_{ik}(z) \cdot T'\left[\omega_r(z)\right] dz + \ldots .$$

$$\omega_r(z) = \frac{\omega}{v} \cdot \frac{l(z)}{2}$$

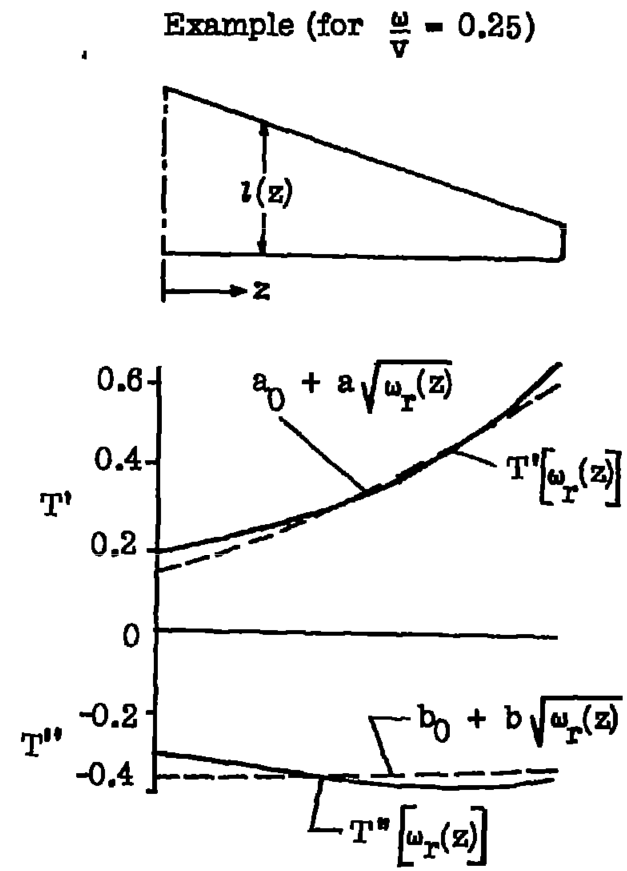
Simplification $\left[\text{for } D_{ik}(z) > 0\right]$:

$$\int D_{ik}(z) \cdot T'\left[\omega_r(z)\right] dz \approx T'(\omega_r^*) \cdot \int D_{ik}(z) dz$$

$$\omega_r^* = \frac{\omega}{v} \cdot \frac{l^*}{2} \text{ with } \sqrt{l^*} = \frac{\int D_{ik}(z) \cdot \sqrt{l(z)}\, dz}{\int D_{ik}(z) dz}$$

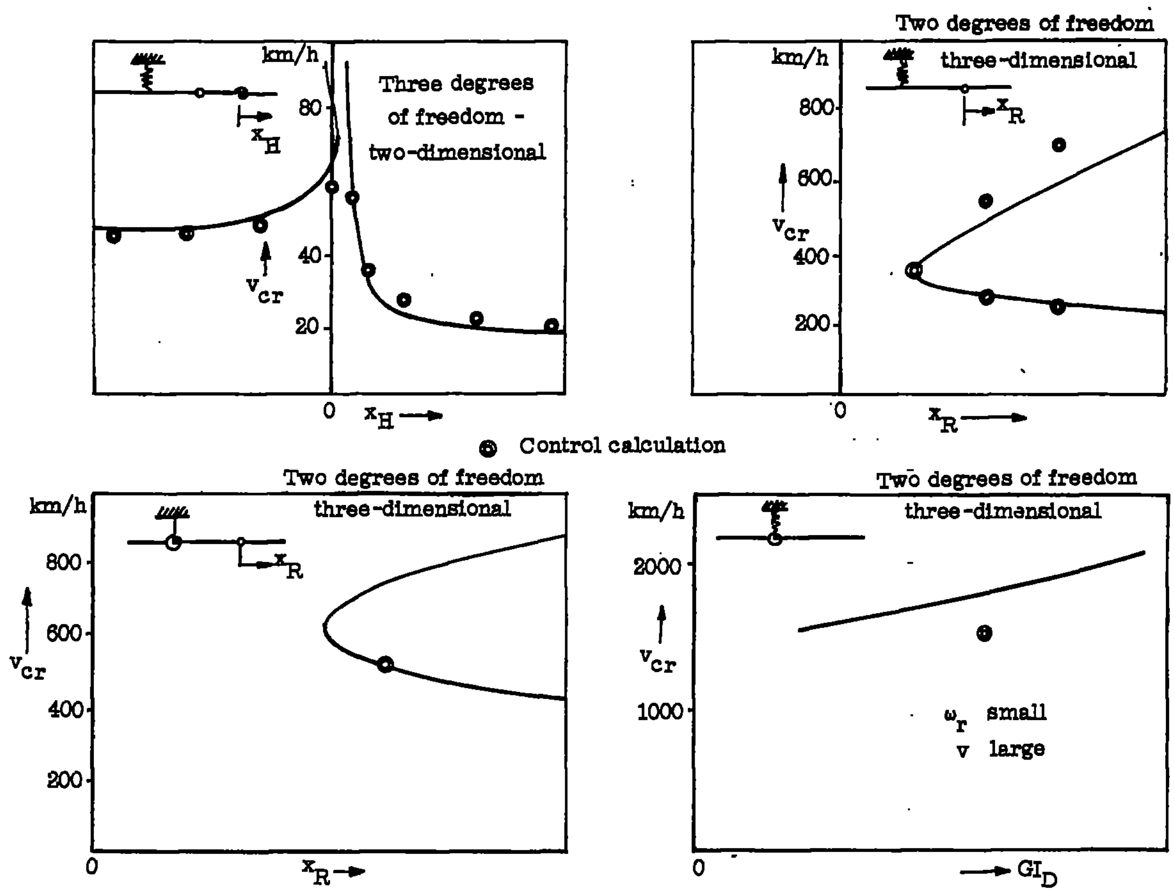
Assumption used as basis:

$$T'\left[\omega_r(z)\right] dz \approx a_0 + a\sqrt{\omega_r(z)}$$



z = coordinate in direction of span width
$l(z)$ = wing chord at point z

Figure 7.- Simplified determination of the air-force characteristics.



v_{cr} = critical velocity

x_R, x_H = position of control-surface and auxiliary control-surface center of gravity

GI_D = torsional stiffners

Figure 8.- Air forces simplified.

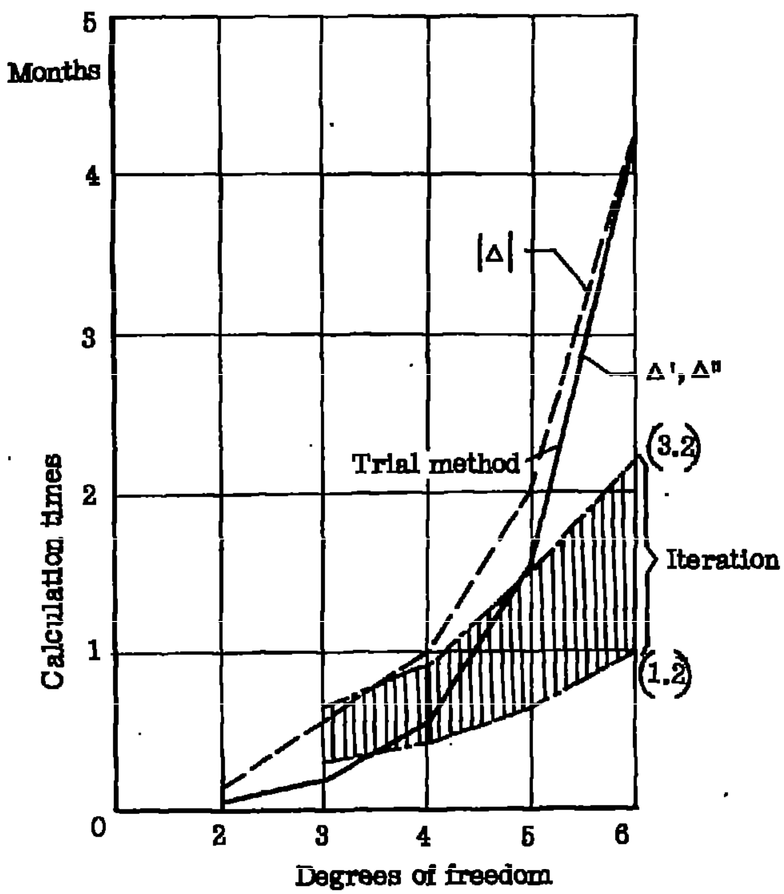


Figure 9.- Calculation times required for solution of the flutter equations with use of the complete air-force formulations (calculation times of one single calculator in months).

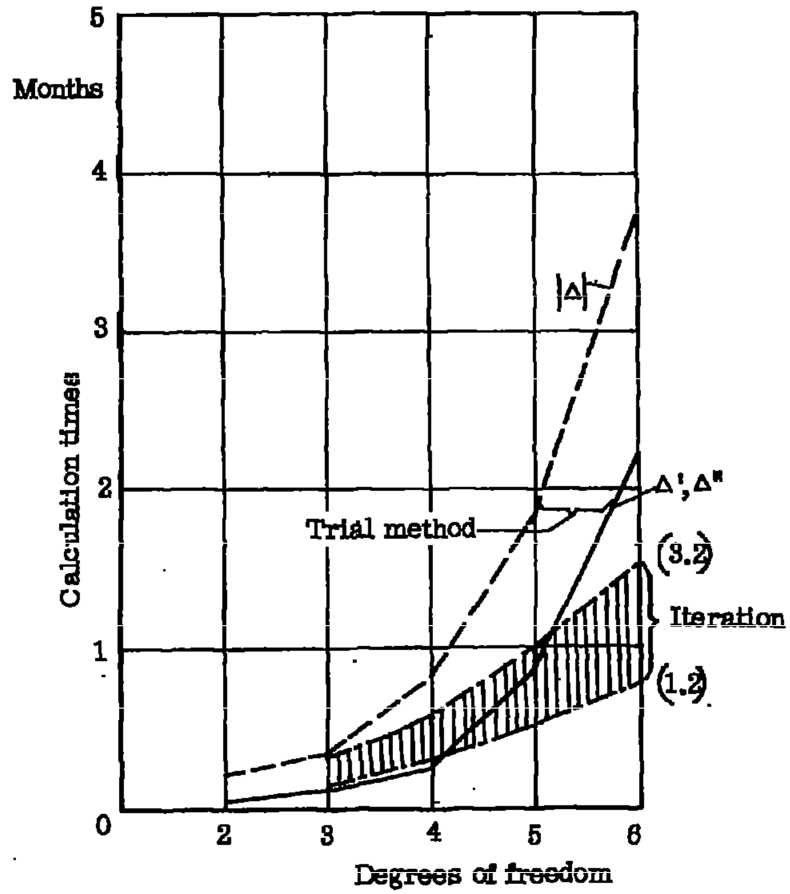


Figure 10.- Calculation times required for solution of the flutter equations with use of simplified formulations for the air forces (calculation times of one single calculator in months).

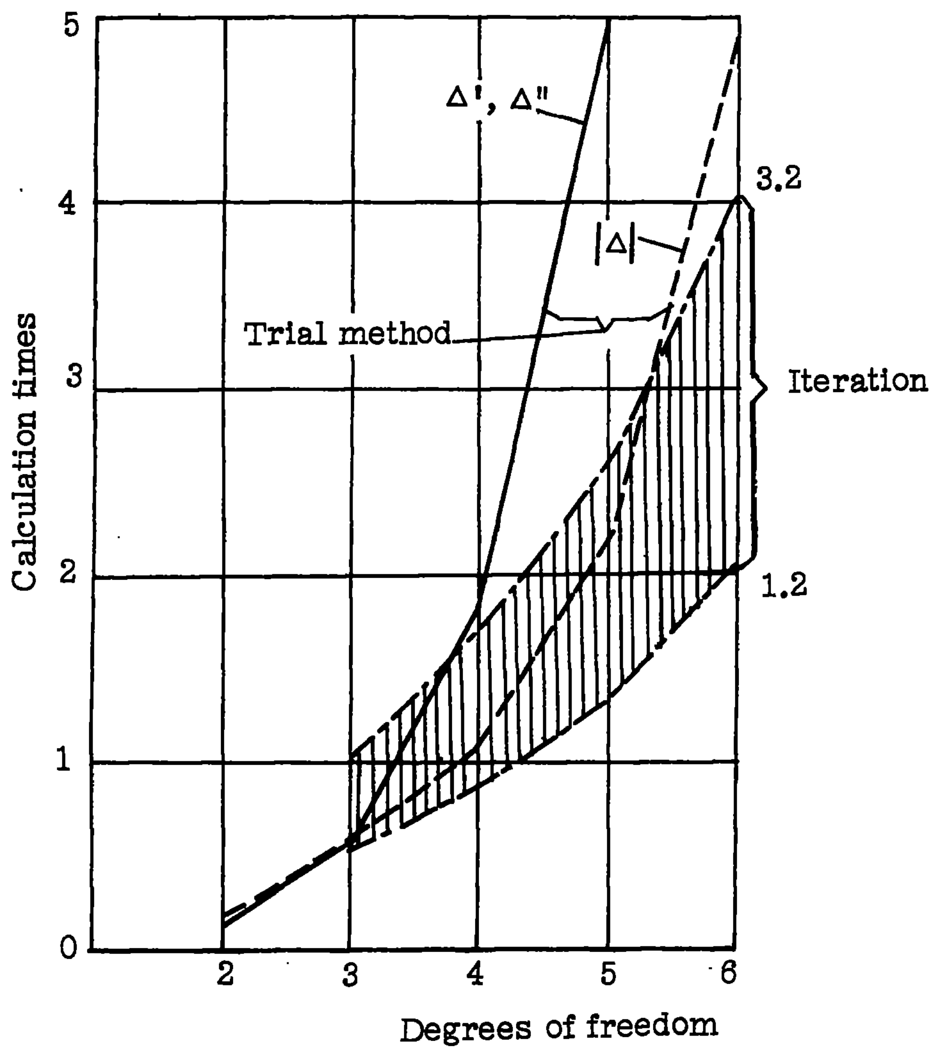


Figure 11.- Calculation times required for solution of the flutter equations without limit rule for the air forces (calculation times of one single calculator in months).

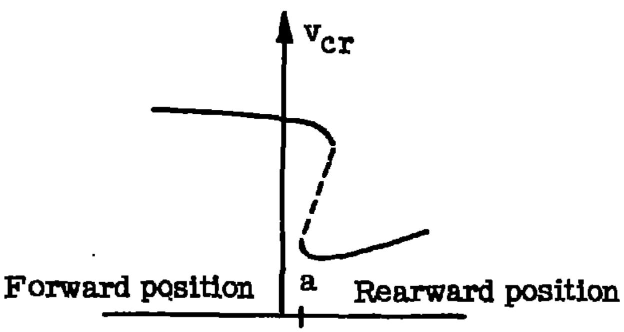


Figure 1(a).

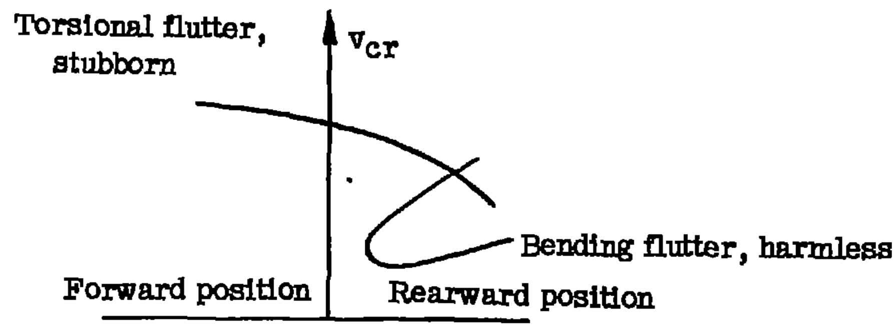


Figure 2(a).

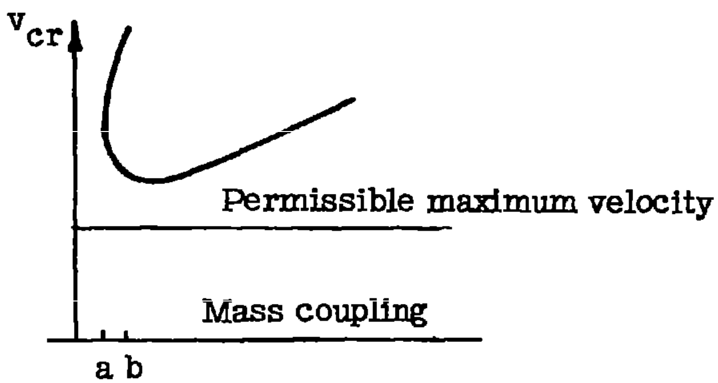


Figure 3(a).

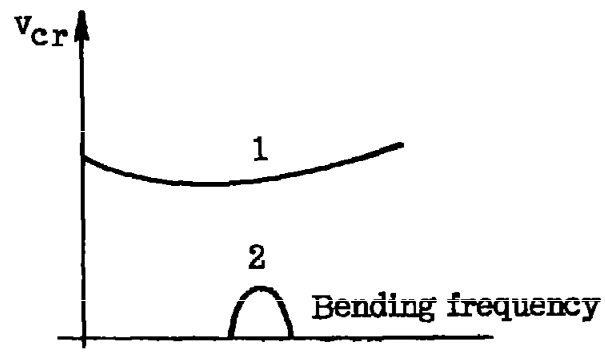


Figure 4(a).

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